A Nonparametric Approach to Multi-product and Product-specific Scale Economies, Economies of Scope, and Cost Efficiency for Kansas Farms

1. Introduction

Reducing costs through understanding economies of scale and economies of scope is fundamental in producer theory. Estimation of cost functions has allowed the measurement of economies of scale and scope using duality theory developed by Samuelson (1938) and Shephard (1953), and later work from Baumol et. al. (1982). The full employment of land, labor, and capital make this particularly important in agriculture where resource expansion is costly. Knowing where potential cost savings exist, and for which farms, provides economists and producers with valuable information as they make investment decisions. However, typically scale and scope measures are not calculated from cost frontiers.

In instances where frontiers have been estimated parametrically (Atkinson and Halversen 1984), a common method used is the stochastic frontier developed by Aigner et al (1977). This method has since been modified to estimate cost frontiers (Coelli and Battese 1994). Using the stochastic frontier cost method, Mafoua and Hossian (2001) examined economies of scale and economies of scope cost savings with a multi-product analysis of corn and soybeans using a panel data set.

An alternative method for frontier estimation uses a series of linear segments to envelope the data (Farrell 1957). From Farrell's original model, the nonparametric method evolved until Färe, Grassokopf and Lovell (1985) formally established a linear program for cost frontier estimation. Chavas and Aliber (1993) used the non-parametric cost frontier method to estimate economies of scope by measuring cost savings from multiple outputs instead of producing each output individually using the data envelope analysis (DEA).

The DEA approach to cost frontier estimation has some attractive advantages over parametric methods. Particularly advantageous is that there is not a need to specify a potentially technologically restrictive functional form. The nonparametric approach is consistent with economic theory by ensuring curvature of the cost function is not violated during the estimation process. Lusk et al. examined the relative variability needed in the estimation of dual cost functions to recover the underlying technology. They found that the relative variability necessary to accurately estimate a dual cost function parametrically requires more than 20 years of data based on observed data. Thus, some estimates of scope and scale may be fragile due to the inability to trace out that underlining production process.

Economies of scale estimations from nonparametric methods have been limited to measuring scale efficiency by estimating the model assuming constant returns to scale, and comparing with it with variable returns to scale (Cooper et. al. 2007). The results of the estimates are compared using the ratio of the two cost estimations yielding a measure known as scale efficiency. Paul et. al. (2004) noted however that scale efficiency is not the same as the multi-product scale economies explained by Baumol et. al. and cannot be interpreted as such. Also, there are no measures for product-specific economies of scale reported from nonparametric frontier estimations. Thus, those using nonparametric methods to estimate cost frontiers have been forced to parametrically estimate traditional scale measures (Paul et. al. 2004, Kumar, Sunil, and Gulati 2008).

Recently Parman et al. have shown that the DEA approach can estimate cost efficiency, economies of scale, and economies of scope relatively close to the "true" values of these economic measures relative to other parametric methods. Their approach reduces the need to conduct estimations using multiple methods, and provides scale measures consistent with Baumol et. al.

The primary objective of this research is to estimate economies of scale using the nonparametric approach through estimations of multi-product and product-specific scale economies and cost efficiency for Kansas farms. Product-specific scale economies are evaluated to determine if farm size is related to cost savings for individual products. From the cost frontier, it is possible to determine what type and size of farms make up the frontier and how far other farms are from the most efficient producers (cost efficiency). Using the nonparametric methods of Parman et al. for estimating scale allows the trade-off between cost efficiency and multi-product economies of scale to be examined to determine those farms that will reduce costs more by increasing output versus becoming cost efficient.

The second objective of this study evaluates estimating a panel of Kansas farm's economic measures by year. Specifically, this objective addresses if scale and scope remain consistent across years as the cost frontier shifts due to technology improvement and/or weather variability. This has important implications for understanding how the cost function behaves over time.

2. Methods

Following Parman et. al., to estimate the frontier, economies of scope, and scale economies, the minimum cost (C_i) of producing the farm output mix is determined using DEA.

Costs are minimized for a given set of input prices (w_i) and outputs (y_i) with the choice being the optimal input bundle (x_i^*).

$$\begin{aligned}
&\min C_i = w_i' x_i^* \\
&s.t \\
&Xz \le x_i^* \\
&y' z \ge y_i \\
&z_1 + z_2 + \ldots + z_n = 1 \\
&z_i \in \mathbb{R}^+
\end{aligned} \tag{1}$$

where there are "*n*" farms. The vector *Z* represents the weight of a particular farm with the sum of Z_i*'s* equal to 1 for variable returns to scale. The output quantities (y_i) constrain the cost minimizing input bundle to be at or below that observed in the data. Total cost from the model (C_i) is the solution to the cost minimization problem including the production of all outputs for the i^{th} farm. The cost of producing all outputs except one ($C_{i,all-p}$) where *p* is the dropped output is determined by dropping the p^{th} output constraint. The marginal costs ($MC_{i,p}$) are obtained from the shadow prices on the output constraint (equation 1). Using the cost and output measures obtained from the previous program, economies of scope, multi-product economies of scale, cost efficiency and product-specific economies can be calculated.

Cost efficiency (*CE*) identifies a farm's proximity to the cost frontier for a given input/output bundle. It is the quotient of the estimated frontier cost (equation 1) and the actual total cost (*ATC*) the farm incurred while producing their output bundle. This measure must be greater than 0 but less than or equal to 1.

$$CE_i = \left[\frac{C_i}{ATC_i} \right] \tag{2}$$

The calculation of multi-product economies of scale (MPSE) uses the total cost of producing all outputs ($C_{i,all}$), the marginal costs defined above, and the output levels produced. The MPSE is the change in total cost for a proportional change in the production of all outputs. For each output constraint (equation 1), the $MC_{i,p}$ is determined by the shadow price on the pth constraint.

$$MPSE_i = \left[\frac{C_{i,all}}{\sum_p MC_{i,p} Y_{i,p}} \right] \tag{3}$$

Product specific economies of scale (PSE) require the calculation of the incremental costs ($IC_{i,p}$) that are the cost of producing all outputs minus the sum of the costs of all individual outputs except output (p).

$$IC_{i,p} = C_i - \sum_j C_{i,j \neq p} \forall j \tag{4}$$

Average incremental costs ($AIC_{i,p}$) are determined by dividing incremental costs by individual output:

$$AIC_{i,p} = \frac{IC_{i,p}}{y_{i,p}} \tag{5}$$

Using the average incremental cost and the marginal cost calculation above, PSEs are calculated by:

$$PSE_{i,p} = \frac{AIC_{i,p}}{MC_{i,p}} \tag{6}$$

The calculation of scope economies (SC_i) identifies the potential for cost savings through product diversification.

$$SC_i = \left[\frac{(\sum_p C_{i,p}) - C_{i,all}}{C_i} \right] \quad (7)$$

where $C_{i,\mathrm{p}}$ is the cost of producing output p for farm i, and $C_{i,all}$ is the cost of joint production of all outputs for farm i.

Estimating the frontier nonparametrically using a data set with no single output farms reveals difficulty estimating the incremental costs by forcing one of the output constraints to zero (equation 1). Thus, the only alternative is to drop one of the constraints. However, when an output constraint is dropped, the program may allow some of the output for the dropped constraint to be produced resulting in an overstatement in the cost of that one output ($C_{i,p}$) that will cause an over statement in economies of scope (equation 7) and an understatement in product specific scale economies (equation 6).

The additional product-specific production costs from an output being produced when it should be zero must be removed. The cost of producing y_1 only ($C_{i,1}$) assumes that only (y_1^1) is being produced. However, DEA allows some $y_{i,2}^1$ to be produced in this situation overstating the cost of producing y_1 only ($C_{i,1}$). To remove the additional cost, the percentage of $y_{i,1}^{1}$ is multiplied by the cost of producing y_1 only, yielding an adjusted cost ($C^a_{i,1}$). This new adjusted cost is then used in the calculation of incremental costs and associated economic measures (equation 8).

$$C_{i,1}^a = C_{i,1}\left(\frac{y_{i,1}^1}{y_{i,1}^1 + y_{i,2}^2}\right) \quad (8)$$

3. Data

The data for this study contains 241 Kansas Farm Management Association farms (KFMA) for the years 2002-2011. Input quantities are aggregated into categories including seed, fertilizer, chemicals, feed, fuel, labor, land, and machinery. Associated prices for each input are indexed by year using the NASS[1] website or information from *Agricultural Outlook*. The land price is the Kansas cash rental rate from *Kansas Farm Facts*.

Outputs are aggregated into two categories including crops and livestock using output prices from NASS. Accrual revenue is divided by corresponding prices to obtain output quantities. Table 1 reports descriptive statistics for production quantity indices while Table 2 shows the price indices for both inputs and outputs for all ten years. The DEA model is estimated for the 241 farms for each year individually.

Estimating the cost frontier each year may cause some farms that operate on or close to the frontier in some years to be off the frontier in others due to the randomness of weather, rate of technology adoption, or other unforeseen phenomenon as the frontier shifts from year to year. This is important if the model includes data from an area where a drought occurs in isolated regions, and does not affect all farms universally.

Using the traditional USDA sales classes, of the 2,410 total observations, 92 fell into the category of gross revenues less than $100k, approximately 4% of the total while, the $100k-$250k categories includes 481 observations or nearly 20% of farms. The largest category is the $250k-$500k in annual gross revenues group that accounts for nearly 35% of farms or 837 observations. The $500k-$1m category is the next largest with 705 observations or 29% .Farms with gross revenues greater than 1 million had 295 observations or 12% of the total.

[1] http://www.nass.usda.gov/Statistics_by_Subject/index.php

4. Results

Two types of analysis were completed; One estimating all years together, 2,410 individual observations, and yielded 2,363 marginal cost estimations for crops. Therefore crop-specific scale economy calculations are reported for 2,363 observations. Though all observations produced crops, crop marginal cost estimates that were non-unique for farms on the frontier were also dropped[2]. Livestock-specific scale economies are reported for 1,749 observations which is significantly less than crops because many of the observations do not produce livestock. The calculations for multi-product scale economies include 1,671 observations, the number of observations that yielded marginal cost estimates for both crops and livestock. Economies of scope were calculated for 1,694 total observations. Cost efficiency is calculated for all 2,410 observations.

From the analysis that estimated each year individually, there were 2,271 observations yielding unique marginal costs for crops and 1,714 for livestock with 1,630 observations having marginal cost estimations for both. Thus, there are 1,630 estimates of each individual year's multi-product scale economies. Economies of scope were calculated from 1,684 total observations. The disparity in the number of observations of each economic measure between the single frontier and annual estimations arises because there are ten frontiers in the 2nd analysis and one in the 1st analysis affecting the number of non-unique marginal cost and incremental cost estimations.

After dropping the observations with non-unique marginal costs or zero output observations for livestock, the number of observations in the calculation of multi-product scale economies (and economies of scope approximately) for the combined years estimation are: 31

[2] Also, some farms had a marginal cost calculation equal to zero for crops and livestock if they are small and highly inefficient not fully utilizing current resource allocation

with gross revenues less than \$100k (2%), 380 with gross revenues between \$100k and \$250k (24%), 543 had gross revenues between \$250k and \$500k (36%), 491 with gross revenues between \$500k and \$1m (31%), and 226 with gross revenues above \$1m (14%). Observations estimating each year individually are as follows: 28 with gross revenues less than \$100k (1.7%), 298 with gross revenues between \$100k and \$250k (18%), 613 with gross revenues between \$250k and \$500k (38%), 489 with gross revenues between \$500k and \$1m (30%), and 226 with gross revenues above \$1m (%12). Table 3 presents the summary statistics for the economic cost measures for both the annual and combined estimates. Table 4 shows the summary statistics for each year for the annual analysis.

F-tests were conducted for each economic measure to determine if the economic measures estimated annually were statistically different from the measures estimated with a single frontier. This was done by creating dummy variables for each year and regressing them on each economic measure. For all the economic measures including MPSE, cost efficiency, economies of scope, and the PSEs, the tests revealed that at a significance level of 5% these measures were statistically different (Table 5).

4.1 Cost Efficiency

The cost efficiency calculation for each farm represents its current distance from the frontier. A cost efficiency of 1 is on the frontier while those further from 1 are less cost efficient. From the single frontier analysis, average cost efficiency levels were highest for farms greater than \$1m (0.55) and for farms less than \$100k (0.55). Farms with gross revenues between \$500k and \$1m had an average cost efficiency of 0.48 while the categories \$100k to \$250k and \$250k to \$500k had averages of 0.43 and 0.42 respectively (Table 6). The standard deviation is relatively high for farms less than $100k (0.20) compared to the other for revenue categories which had standard deviations of less than 0.15 (Table 6).

Estimation of each year yielded a higher overall average cost efficiency (Table 3) and higher average cost efficiencies for each gross revenue category (Table 6). This implies that farms are closer to each year's frontier on average than an overall frontier which is to be expected if the frontier is shifting. Each gross revenue category however retained its respective rank for overall average cost efficiency, i.e. farms with greater than $1m in gross revenues had the highest average cost efficiency while farms in the $100k to $250k range had the lowest (Table 6). Examination of the annual cost efficiency averages (Table 4) reveals that average cost efficiencies have been lower in recent years than between the years 2003 to 2008.

Figure 1 shows the cumulative density or the amount of observations below a given cost efficiency level for the size categories. The slope of each curve indicates the variation observed for each group where a steeper slope represents less variability. Figure 1 shows an obvious flatter cumulative density for farms with gross revenues less than $100k indicating a large disparity for cost efficiency levels in this revenue group which is true for both the annual estimations and the single frontier. However, in the single frontier estimation, the cumulative density for cost efficiency of farms less than $100k in gross revenues crosses the curve for farms greater than $1m in gross revenues at a cumulative density of 0.7. For the annual estimations this does not occur indicating that the largest farms are strictly closer to the frontier than any smaller revenue category. This implies that in the year with the lowest total cost, there were relatively many small farms close to the frontier however, in each year on average, the largest farms are closer to the frontier. The results for cost efficiency remain similar for the annual estimation and the single frontier for the rest of the revenue categories.

4.2 Multi-product Economies of Scale

Multi-product scale economies represent potential cost saving by reducing average per unit cost through spreading it over larger quantities. Because MPSE is calculated as total cost divided by the sum of the products of marginal costs and their associated output levels, an MPSE greater than 1 implies that increasing production uniformly across outputs will reduce average costs resulting in economies of scale. For MPSEs to be greater than one, the existence of economies of scope, and/or product-specific economies of scale (Fernandez-Cornejo et al 1992) are required (Baumol et al.). If the MPSE equals 1, then the farm is at constant returns to scale. However, if the MPSE is less than 1 for a given farm, then that farm can reduce average cost by proportionately reducing outputs since that farm lies in the diseconomies of scale region.

Single frontier estimation revealed that MPSE for each gross revenue category is highest for the smallest farm revenue category and gets progressively smaller for larger farms (Table 7). MPSE averages ranged from 2.7 (farms less than $100k) to 0.9 (revenues $500k-$1m and farms greater than $1m). Farms with sales of $100k to $250k had an average MPSE at 1.7 and farms between $250k and $500k were closer to unity at 1.1.

The overall average estimated annually was similar to the single frontier at 1.171 compared to 1.142 respectively (Table 3). The MPSEs are also smaller for each gross revenue category overall estimated yearly relative to the single frontier estimates while retaining the same relative rank of each category (Table 7). Yearly average MPSE estimates show farms, on average, remaining close to constant returns to scale each year (Table 4).

Figure 2 Panels A and B present the distribution for the single frontier analysis and the multiple frontier analysis respectively. The results are similar except that MPSE is lower when estimated annually as reflected by MPSE density curves closer to one. Standard deviation is relatively low farms in the $500k to $1m and farms greater than $1m as illustrated by the nearly vertical cumulative density curves. Within these two groups, MPSE is relatively constant among large farms. For the smallest two categories, the density curves are relatively flatter, especially for the smallest farm category showing a disparity.

4.3 Economies of Scope

Economies of scope represent cost savings through the production of crops and livestock. This savings may be due to the use of resources required for the production of both products such as equipment or storage resources. An economy of scope calculation greater than 0 implies cost savings are realized though multi-product operations. Results show greater difference for economies of scope than for cost efficiency between the farm revenue categories (Table 8). From the data estimated for the single frontier, the highest average level of cost savings from economies of scope is for farms between $100k and $250k with average economies of scope of 30% (Table 8). Large farms including farms with revenues over $1m and farms between $500k and $1m had relatively low economies of scope figures of 13% and 12% respectively. Economies of scope for the smallest category were also high (26%). Using annual frontiers, the measurement of economies of scope is less than those estimated from a single frontier. Annual averages for the scope measures range from 0.06 in 2004 to 0.17 in 2002 (Table 4).

Standard deviations for the economies of scope calculations were below 0.10 for the single frontier but higher for the two smallest gross revenue categories for the annual estimations (Table 8). Figure 3 Panel A shows that the cumulative density for farms in the $100k to $250k category is relatively flatter indicating more overall disparity among economies of scope calculations for this revenue group. The largest gross farm revenue category (greater than $1m) had a relatively low average and relatively high standard deviation at 0.08.

One key difference between the two estimations of scope (annual frontiers versus a single frontier) was that annual estimations yielded negative economies of scope for some of the observations in the larger gross revenue categories. While nearly all of the observations of cost savings from scope estimated annually were lower than the simultaneously estimated data set, none of the simultaneous estimates yielded negative scope economies (diseconomies of scope).

4.4 Product-specific Economies of Scale

If a product-specific economies of scale (PSE) measure is greater than 1, it implies that there exists potential cost savings from increasing that output, and a PSE less than 1 implies cost savings by reducing that output. The overall average product-specific economies of scale measure for livestock (LSE) is higher than the product-specific economies of scale measure for crops (CSE) at 0.83 and 0.77 respectively (Table 3) using the single frontier. However the reverse is true for the PSE estimations from annual analysis, though the difference is relatively small (0.01). All farms operate either at constant returns to scale for CSE and LSE or in the region of diseconomies of scale for crops and livestock.

For CSE under a single frontier, the smallest farm revenue group (less than $100k) was the closest to constant returns to scale on average at 0.85 where the furthest group was the $500k to $1m with an average CSE of 0.74 (Table 9). There was not much difference in average CSE within the four groups with gross revenues greater than $100k. From annual frontiers, the revenue group of less than $100k was also highest but closer to constant returns to scale than in the previous estimation at 0.97. The greater than $1m sales group had a PSE of 0.83. Yearly overall averages for CSE (Table 4) are between 0.8 and 0.9 during the ten year sample showing relatively little variation in overall crop-specific economies of scale from year to year.

The cumulative density curves (Figure 4 Panel A) for CSE for the four largest gross revenue categories estimated simultaneously overlap with small differences in slope indicating the relative variation within groups is also small. However, the CSE density curve for the group containing farms with revenues less than $100k is relatively flat for 50% of the farms and steep for the other 50%. This indicates that many farms in the less than $100k category are operating at a low CSE while others are at or close to constant returns to scale for crops with a single frontier. In Figure 4 Panel B the CSE for the smallest gross revenue categories is not as flat illustrating a tighter distribution with an annual frontier.

Single frontier estimates reveal that the averages between groups for LSE were highest among smaller revenue grossing farms with the three smallest categories all having an average LSE higher than 0.84 (Table 10). The two largest revenue grossing categories had nearly identical LSE averages at approximately 0.80. Annual frontier analysis shows that the smallest revenue group (less than $100k) is close to constant returns to scale on average with the other revenue groups averaging between 0.84 and 0.87. Annual averages (Table 4) for LSE yield results similar to CSE in that the lowest LSE estimate occurs in 2002 and the rest are approximately between 0.8 and 0.9 indicating relative stability for livestock-specific scale economy estimates from year to year.

For the single frontier data set, the standard deviations for LSE were higher than for CSE indicating more variability in the product-specific scale economies for livestock than crops (Table 10). The highest standard deviation was for the gross revenue category less than $100k

(0.23) and lowest was for the greater than $1m category (0.18). However, the density curves for all five categories are similar, without the obvious differences between groups that the other economic measures show (Figure 5 Panel A). The annual frontier analysis shows similar results for standard deviations among revenue categories with the exception of farms with revenues less than $100k.

5. Implications

5.1 Differences between Annual Frontier and Single Frontier Analysis

The statistical test used to determine if the means from the model estimating single frontier was different from those estimating the frontier annually indicated statistical differences at the 5% level. However, the results show that the means are not that economically different. Overall average MPSE was around constant returns and did not vary much from year to year (Table 5). Crop-specific and livestock-specific scale differences from both estimations were similar in mean and relative rank with relatively little variation from year to year.

The largest difference between economic measure estimates occurred with respect to cost efficiency and economies of scope. In the case of cost efficiency, the difference in overall average from estimating a single frontier versus annual frontiers occurs due to the cost frontier shifting from year to year. Estimating a single frontier assumes the frontier does not shift and thus movement of farms closer to, and further from the frontier is due to efficiency. Estimating the frontier annually allows the frontier to shift and average cost efficiency to remain constant assuming farms are not changing their relative efficiency.

Allowing the frontier to shift from year to year will also affect calculations of economies of scope. Economies of scope are based on estimations of the intercept and when estimated for a single frontier will not change. Thus, increase in the cost of producing each output individually will appear from a single frontier as higher cost savings from joint production rather than changes in the intercepts as the frontier shifts.

It appears that annual frontiers suggest that CSEs and LSEs are closer to one and the scope is closer to zero than the single frontier results. While there is some variation in the annual economic measures, they are relatively stable from year to year.

5.2 Implications for KFMA Farms

Despite the differences between the estimation of cost efficiency and scope between the annual and single frontier estimations, the implications are the same in that larger farms, and the smallest category, are typically closer to the frontier and economies of scope diminish as farms grow larger. Further, economies of scale exist for small farms and tend to be exhausted for farms with sales greater than $250k.

For the smallest farm category (less than $100k), the estimates for cost efficiency and MPSE suggest that these farms have a greater incentive to increase in size rather than move closer to the frontier. Estimated annually, the cost efficiency for this group is 0.66 and the MPSE is 1.99. This shows costs can be reduced by on-average 50% by increasing in size and 34% by becoming more efficient. Economies of size are clearly important for these farms.

For the $100k to $250k group, the implications are also similar in that the benefits are nearly equal in becoming more efficient versus increasing output. From the annual frontier estimates, the overall average cost efficiency is 0.56 indicating that they can save 44% becoming more efficient. Potential cost savings from scale are around 41% indicating a closeness between the two.

For largest three gross revenue categories, the results show cost savings from reaching the frontier is more important than adjusting farm size. All three categories are near constant returns to scale, or slightly in the diseconomies of scale region. The average cost efficiencies range from 0.57 to 0.75 indicating that there is room for cost savings by becoming more efficient.

Economies of scope are more important. Multi-product smaller farms realize greater cost savings through joint production than larger farms. At some point, the advantage of joint production is exhausted. Farms with less than $250k in gross revenues tend to experience greater cost savings with joint livestock and crop production. As farm sales increase however, the incentive to grow larger due to additional cost savings from scale, and savings from joint production diminish.

Product-specific scale economies from annual frontiers are between 0.75 and 0.95 for crop-specific economies of scale and livestock-specific economies of scale. These measures do not vary as much based on farm size as the other measures. Perhaps the conclusion is that the individual enterprises are more size neutral. When arranged in a multi-product farm, multi-product scale measures differ due to level of scope economies. Multi-product farms reap the benefits of joint production (scope) and are not as far from constant returns to scale for livestock or crops specifically. However, the large potential for cost savings illustrated by small farms typically having high MPSE suggests the importance of economies of scope for these operations.

6. Conclusions

The objectives of this research were to determine the level of cost savings from cost efficiency, economies of scale and economies of scope based on farm size for Kansas farms.

This research also evaluated the difference between estimating the frontier yearly versus a single frontier.

The results suggest that there exists a larger incentive for small farms to expand and exploit cost savings through multi-product scale economies. Scale economies are larger than potential saving from becoming more efficient farms with sales less than $100k. As farms move past the $100k in sales range, the potential cost savings from efficiency is about the same as from adjusting size. After sales reach $250k, most of the economies of size are exhausted and cost differences occur due to inefficiency (not being on or close to the frontier).

Estimating the measures as a single multi-year frontier yielded results that were statistically different than from estimating annual frontiers. The measures of economies of scope were lower when estimated annually and the PSEs are higher. Interestingly, while those measures were statistically different, the variability in measures from year to year was not large and the measures of multi-product scale economies were nearly the same. For example, multi-product scale economies for Kansas Farms were between 0.97 and 1.17 for the ten year period and cost efficiency measures were between 0.55 and 0.67 for the 2002 to 2011 time period. This indicates that while the cost frontier may shift from year to year, its shape remains relatively consistent.

References

Aigner, D.J., C.A.K. Lovell, and P. Schmidt. "Formulation and estimation of Stochastic Frontier Production Models." *Journal of Econometrics*, 6(1977): 21-37.

Atkinson, Scott E. and Robert Halvorsen. "Parametric Efficiency Tests, Economies of Scale, and Input Demand in U.S. Electric Power Generation" *International Economic Review*. 25(4)(October 1984):647-662.

Baumol, W. J., Panzar, J. C. and Willig, R. D. (1982) Contestable Markets and the Theory of Industry Structure. Harcourt Brace Janaovich, Inc. New York

Chavas, Jean-Paul and T.L. Cox. "A Nonparaetric Analysis of Agricultural Technology." *American Journal of Agricultural Economics*, 70(1988): 303-310.

Chavas, Jean-Paul and M. Aliber. "An Analysis of Economic Efficiency in Agriculture: A Non-parametric Approach." *Journal of Agricultural and Resource Economics*, 18(1993): 1-16.

Cooper, Wiliam W., Lawrence M. Seiford, and Kaoru Tone. 2007. Data Envelopment Analysis. Springer Science+Media LLC. New York, New York.

Färe, R., S. Groskopf, and C.A.K Lovell. The Measurement of Efficiency of Production, Boston: Kluwer-Nijhoff, 1985.

Farrell, M. J. "The Measurement of Productive Efficiency." *Journal of the Royal Statistical Society*,. 120, 3(1957): 253-290.

Fernandez-Cornejo, J., C.M. Gempesaw II, J.G. Elterich, and S.E. Stefanou. "Dynamic Measures of Scope and Scale Economies: An Application to German Agriculture" *American Journal of Agricultural Economics*, 74(1992): 329-342.

Kumar, Sunil, and Richita Gulati. "An Examination of Technical, Pure Technical, and Scale Efficiencies in Indian Public Sector Banks using Data Envelopment Analysis" *Eurasian Journal of Business and Economics*, 1,(2)(2008):33-69.

Lusk, Jayson L., Allen M. Featherstone, Thomas L. Mash and Abdullahi O. Abdulkadri. " The Empirical Properties of Duality Theory." *The Australian Journal of Agricultural and Resource Economics*, 46(2002): 45-68.

Mafoua, Edouard K. "Scope and Scale Economies for Multi-Product Farms: Firm-level Panel Data Analysis" Selected Paper AAEA August 2001.

Parman, Bryon J., Allen M. Featherstone, Brian K. Coffey. "A Nonparametric Approach to Multi-product and Product-specific Economies of Scale" 2013. Forthcoming

Parman, Bryon J., Allen M. Featherstone, Vincent Amanor-Boadu. "A Comparison of Parametric and Nonparametric Estimation Methods of Production Frontiers and Multi-product Scale, Product-Specific Scale, and Scope Economies." 2013. Forthcoming

Paul, Catherine Morrison, Richard Nehring, David Banker, and Agapi Somwaru. “Scale Economies and Efficiency in U.S. Agriculture: Are Traditional Farms History?” *Journal of Productivity Analysis*, 22(2004): 185-2005.

Samuelson, P., 1938. Foundations of Economic Analysis, Harvard University Press, Cambridge

Shephard, R.W. 1953. Cost and Production Functions, Princeton University Press, Volume 1, Issue 2, Princeton NJ.

Tables

Table 1
Summary statistics for Kansas Farm Management Farms of input and output quantity indices, 2002 to 2011.

	Mean	Standard Deviation	Minimum	Maximum
		----------*Inputs*----------		
Seed	188	199	1	2010
Fertilizer	298	286	0	3328
Chemicals	229	222	0	2457
Machinery	530	437	24	4163
Feed	401	1497	0	30454
Fuel	162	160	4	1906
Labor	192	306	0	3753
Land	2514	1628	127	11797
		----------*Outputs*----------		
Crops	2458	2182	30	50140
Livestock	1514	3386	0	26277

N=2410

Table 2
Price indices for farm inputs and outputs for each year 2002-2011.

Year / Product	Seed	Fertilizer	Chemicals	Machinery	Feed	Fuel	Labor	Rent	Crops	Livestock
2002	154	124	121	151	114	140	157	123	109	103
2003	158	140	121	162	121	165	160	126	120	116
2004	168	164	123	173	117	216	165	129	111	118
2005	182	176	128	182	124	239	171	141	134	116
2006	204	216	129	191	149	264	177	147	186	118
2007	259	392	139	209	194	344	183	165	259	117
2008	299	275	149	222	186	229	188	184	186	106
2009	310	252	144	230	180	284	189	190	177	123
2010	332	328	145	244	226	362	192	205	239	151
2011	359	333	153	257	260	360	199	212	246	160

Source: http://www.nass.usda.gov/Statistics_by_Subject/index.php

Table 3
Overall summary statistics for estimated cost measures for Kansas Farm Management Farms estimated from a single frontier and annually.

	N	Average	Standard Deviation	Minimum	Maximum
		----- *Single Frontier*------			
Cost Efficiency	2410	0.462	0.136	0.138	1.000
Multi-product Economies of Scale	1571	1.142	0.407	0.588	4.210
Economies of Scope	1571	0.175	0.093	0.003	0.553
Crop-specific Economies of Scale	2363	0.768	0.167	0.023	1.000
Livestock-specific Economies of Scale	1649	0.830	0.190	0.010	1.000
		------ *Annual Frontiers*------			
Cost Efficiency	2410	0.608	0.168	0.138	1.000
Multi-product Economies of Scale	1630	1.171	1.328	0.072	3.079
Economies of Scope	1630	0.110	0.101	-0.220	0.639
Crop-specific Economies of Scale	2271	0.862	0.182	0.105	1.000
Livestock-specific Economies of Scale	1714	0.854	0.183	0.016	1.000

Table 4
Annual averages for cost efficiency, MPSE, PSEs, and economies of scope for Kansas Farm Management Farms

Year	Cost Efficiency	Multi-product scale economies	Economies of scope	PSE Crops	PSE Livestock
2002	0.546	1.061	0.170	0.752	0.796
2003	0.639	1.066	0.085	0.940	0.906
2004	0.635	0.999	0.063	0.937	0.906
2005	0.668	0.992	0.074	0.914	0.869
2006	0.610	1.068	0.124	0.848	0.852
2007	0.606	1.060	0.112	0.916	0.810
2008	0.653	1.022	0.096	0.926	0.803
2009	0.596	0.967	0.098	0.832	0.810
2010	0.546	1.155	0.157	0.795	0.892
2011	0.586	1.170	0.108	0.866	0.898

Table 5
F-Test results evaluating statistical differences in cost frontiers.

Measure	F-Statistic	P-Value
Cost Efficiency	10.28	0.000
Multi Product Economies of Scale	1.98	0.046
Economies of Scope	36.20	0.000
Crop-specific Economies of Scale	11.97	0.000
Livestock-specific Economies of Scale	7.43	0.000

Table 6
Summary statistics for cost efficiency for Kansas Farm Management Farms estimated from a single frontier and annually.

Gross Revenues	N	Average	Standard Deviation	Minimum	Maximum
			----- *Single Frontier*------		
Less than $100k	92	0.549	0.204	0.198	1.000
$100k-$250k	481	0.432	0.125	0.151	1.000
$250k-$500k	837	0.421	0.110	0.138	1.000
$500k-$1m	705	0.483	0.130	0.218	1.000
Greater than $1m	295	0.552	0.146	0.280	1.000
			------ *Annual Frontiers*------		
Less than $100k	92	0.660	0.206	0.261	1.000
$100k-$250k	481	0.559	0.155	0.225	1.000
$250k-$500k	837	0.567	0.145	0.198	1.000
$500k-$1m	705	0.627	0.155	0.219	1.000
Greater than $1m	295	0.749	0.179	0.357	1.000

Table 7
Summary statistics for multi-product economies of scale for Kansas Farm Management Farms estimated from a single frontier and annually.

Gross Revenues	N	Average	Standard Deviation	Minimum	Maximum
		------ *Single Frontier*------			
Less than $100k	31	2.691	0.542	1.938	3.732
$100k-$250k	380	1.619	0.380	0.965	4.210
$250k-$500k	543	1.106	0.224	0.711	1.708
$500k-$1m	491	0.916	0.121	0.658	1.359
Greater than $1m	226	0.918	0.070	0.588	1.010
		------*Annual Frontiers*------			
Less than $100k	28	1.991	0.586	1.266	3.079
$100k-$250k	298	1.406	0.892	0.729	3.053
$250k-$500k	613	1.048	0.188	0.576	1.670
$500k-$1m	489	0.941	0.140	0.575	1.250
Greater than $1m	202	0.850	0.128	0.072	1.075

Table 8
Summary statistics for economies of scope from Kansas Farm Management Farms estimated from a single frontier and annually.

Gross Revenues	N	Average	Standard Deviation	Minimum	Maximum
		----- *Single Frontier*------			
Less than $100k	31	0.255	0.075	0.063	0.443
$100k-$250k	380	0.301	0.091	0.108	0.529
$250k-$500k	543	0.169	0.060	0.057	0.553
$500k-$1m	491	0.123	0.050	0.020	0.307
Greater than $1m	226	0.134	0.083	0.003	0.332
		------*Annual Frontiers*------			
Less than $100k	28	0.201	0.161	0.000	0.481
$100k-$250k	298	0.196	0.133	-0.011	0.639
$250k-$500k	613	0.116	0.072	-0.128	0.323
$500k-$1m	489	0.075	0.059	-0.122	0.218
Greater than $1m	202	0.037	0.092	-0.220	0.558

Table 9
Summary statistics for crop-specific economies of scale categorized by gross revenues estimated simultaneously and individually by year

Gross Revenues	N	Average	Standard Deviation	Minimum	Maximum
			----- *Single Frontier*------		
Less than $100k	77	0.854	0.233	0.076	1.000
$100k-$250k	476	0.780	0.155	0.023	1.000
$250k-$500k	834	0.775	0.175	0.028	1.000
$500k-$1m	703	0.743	0.162	0.120	1.000
Greater than $1m	273	0.764	0.141	0.282	1.000
			------*Annual Frontiers*------		
Less than $100k	53	0.974	0.095	0.387	1.000
$100k-$250k	465	0.873	0.191	0.192	1.000
$250k-$500k	821	0.902	0.145	0.149	1.000
$500k-$1m	676	0.847	0.145	0.282	1.000
Greater than $1m	256	.0826	0.147	0.105	1.000

Table 10
Summary statistics for livestock-specific economies of scale categorized by gross revenues estimated simultaneously and individually by year

Gross Revenues	N	Average	Standard Deviation	Minimum	Maximum
			----- *Single Frontier*------		
Less than $100k	31	0.850	0.232	0.082	1.000
$100k-$250k	380	0.877	0.192	0.046	1.000
$250k-$500k	543	0.842	0.182	0.010	1.000
$500k-$1m	491	0.799	0.190	0.031	1.000
Greater than $1m	226	0.807	0.179	0.029	1.000
			------*Annual Frontiers*------		
Less than $100k	45	0.969	0.140	0.094	1.000
$100k-$250k	310	0.846	0.230	0.020	1.000
$250k-$500k	625	0.836	0.181	0.016	1.000
$500k-$1m	512	0.862	0.167	0.048	1.000
Greater than $1m	222	0.873	0.142	0.095	1.000

Figures

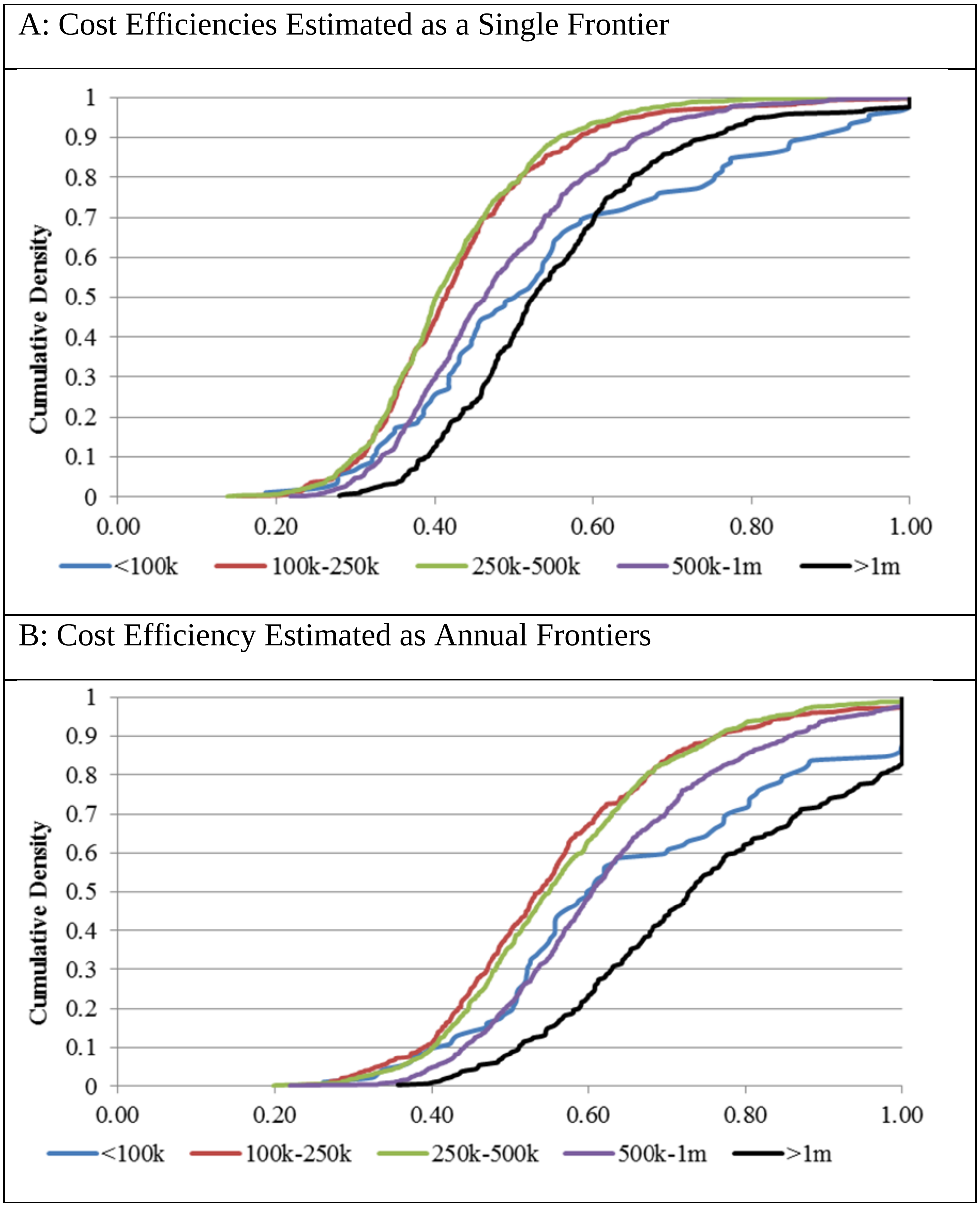
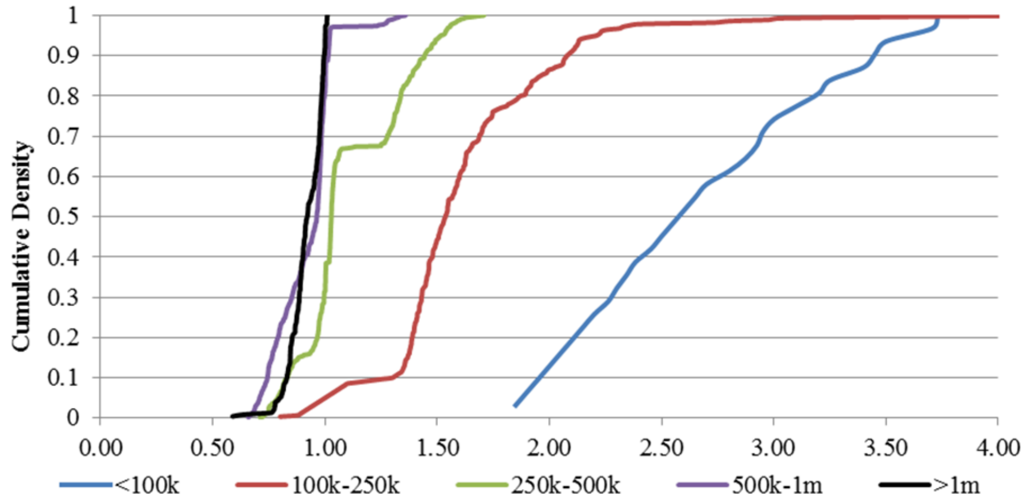


Figure 1. Cumulative Density of Cost Efficiency Estimates for Kansas Farms Categorized by Farm Gross Revenue

A: Multi-product Scale Economies Estimated as a Single Frontier



B: Multi-product Scale Economies Estimated as Annual Frontiers

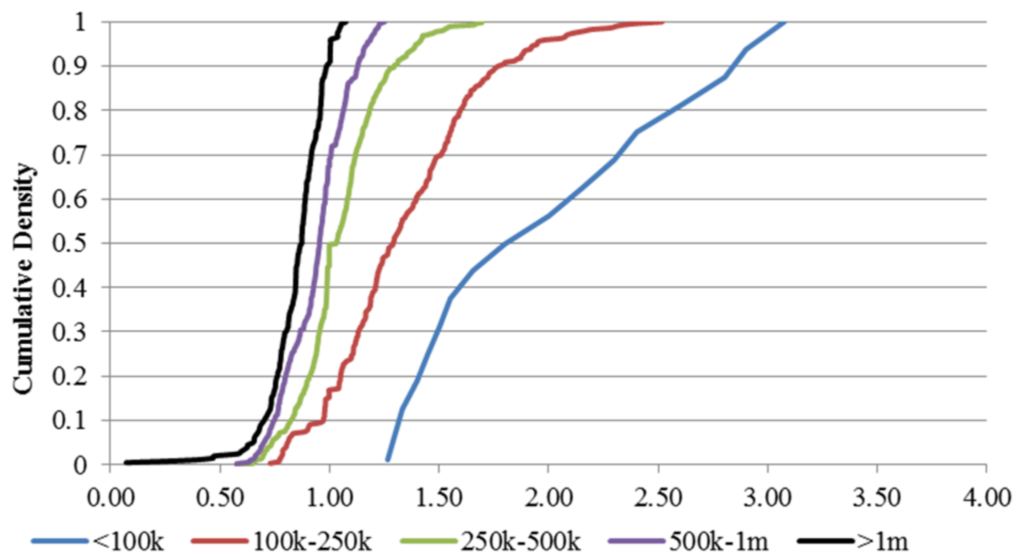
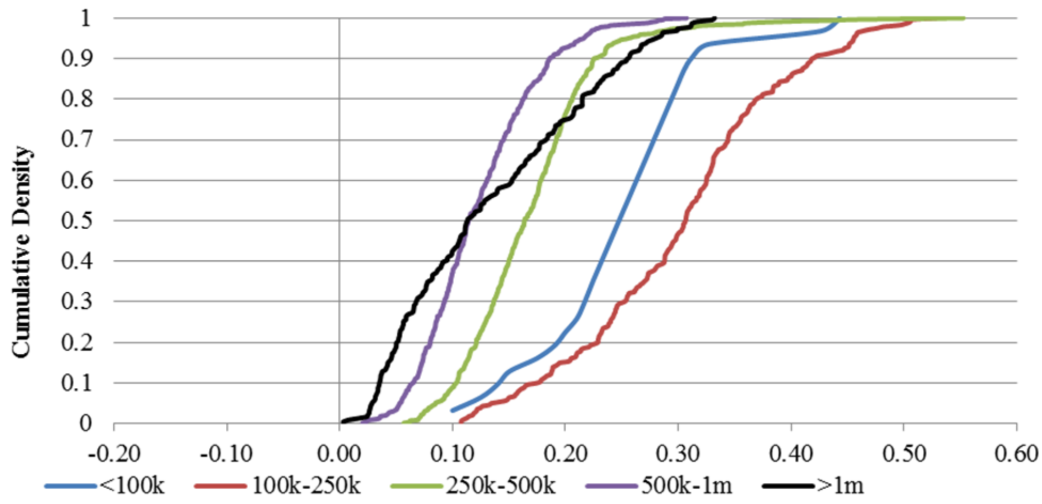


Figure 2. Cumulative Density of Multi-product Scale Economies Estimates for Kansas Farms Categorized by Farm Gross Revenue

A: Economies of Scope Estimated as a Single Frontier



B: Economies of Scope Estimated as Annual Frontiers

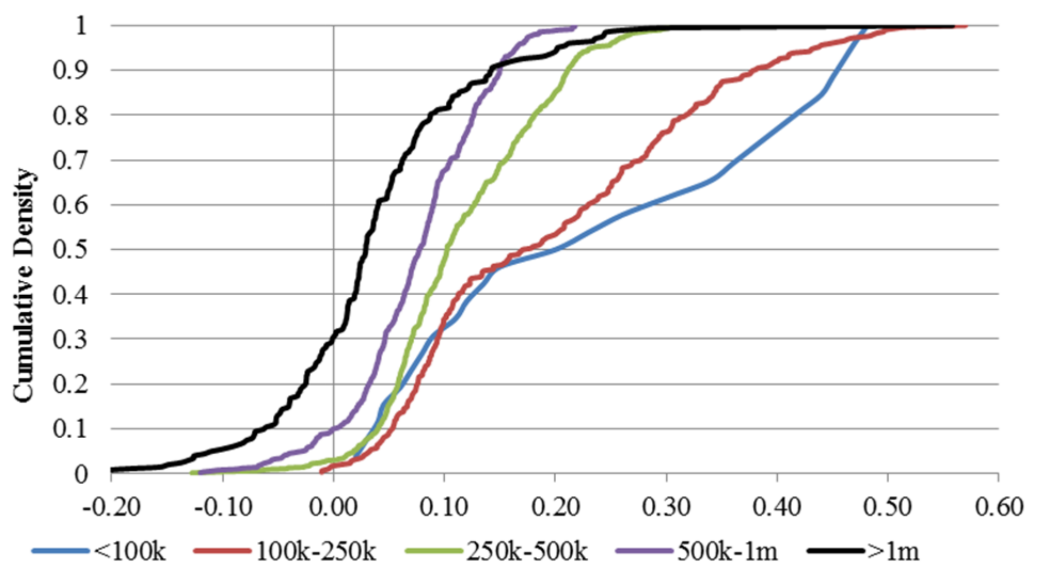
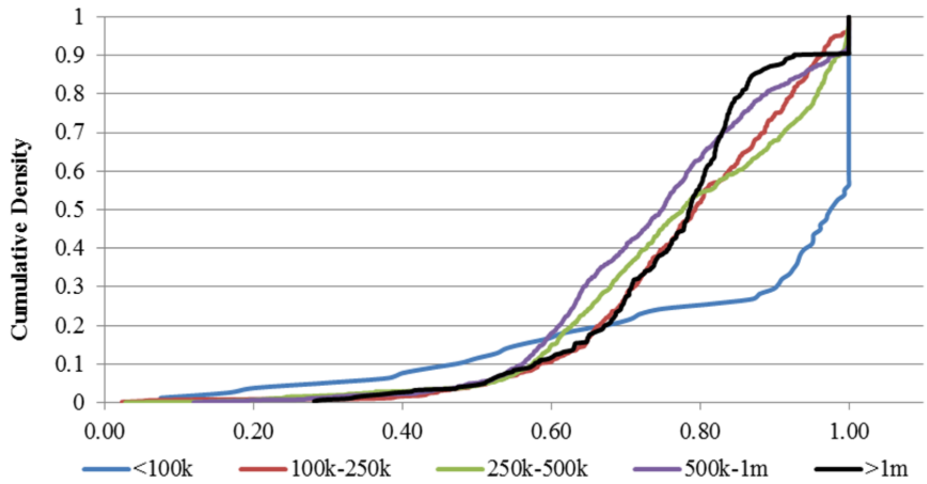


Figure 3. Cumulative Density of Economies of Scope Estimates for Kansas Farms Categorized by Farm Gross Revenue

A: Crop-Specific Economies of Scale Estimated as a Single Frontier



B: Crop-specific Economies of Scale Estimated as Annual Frontiers

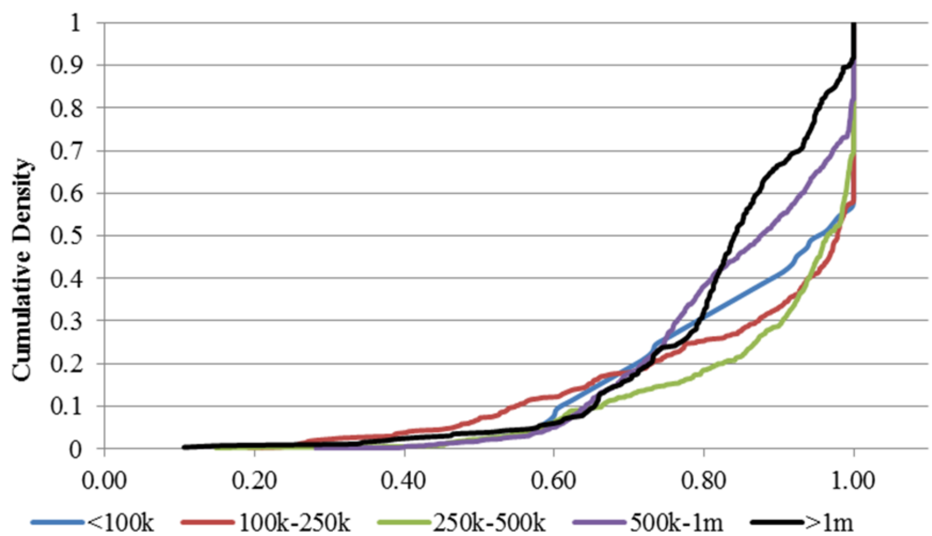
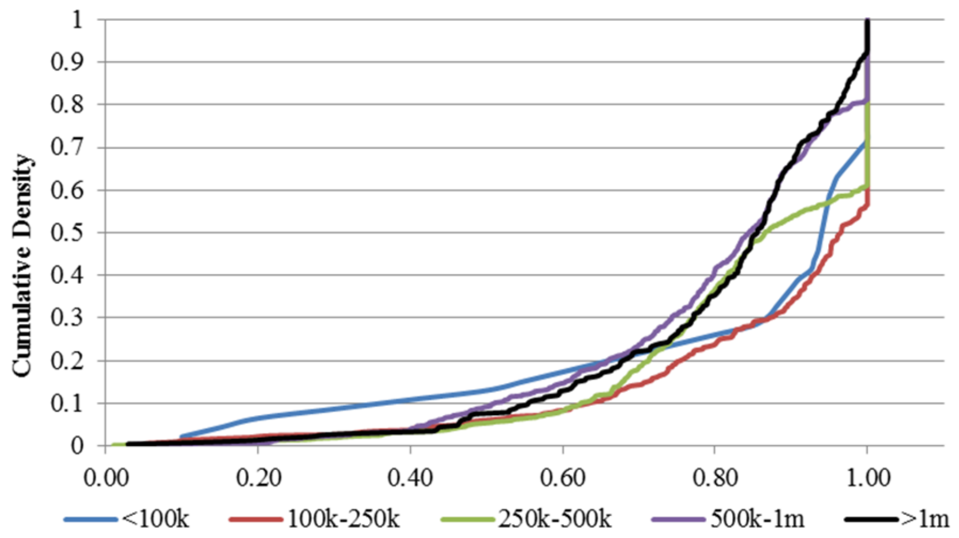


Figure 4**.** Cumulative Density of Crop-specific Scale Economy Estimates for Kansas Farms Categorized by Farm Gross Revenue

A: Livestock-Specific Economies of Scale Estimated as a Single Frontier



B: Livestock-specific Economies of Scale Estimated as Annual Frontiers

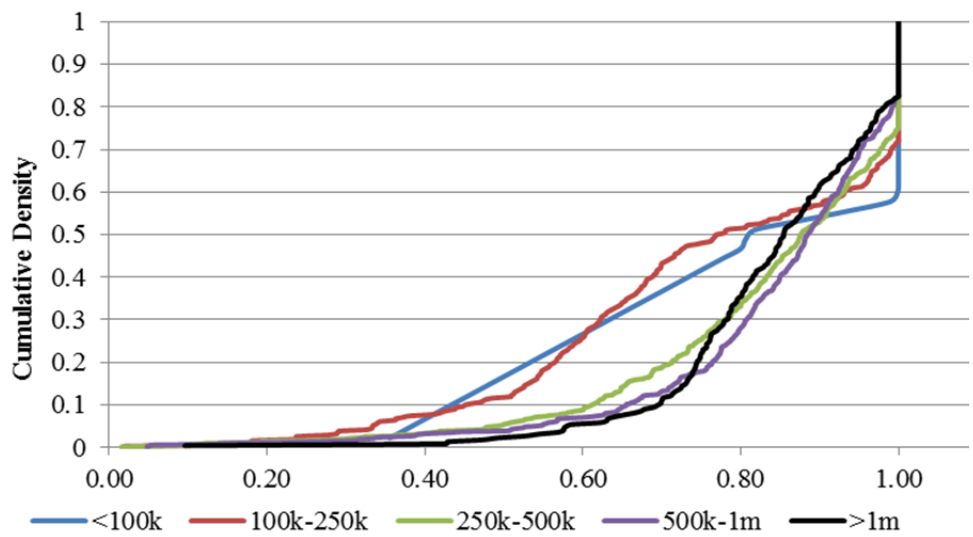


Figure 5. Cumulative Density of Livestock-specific Scale Economy Estimates for Kansas Farms Categorized by Farm Gross Revenue