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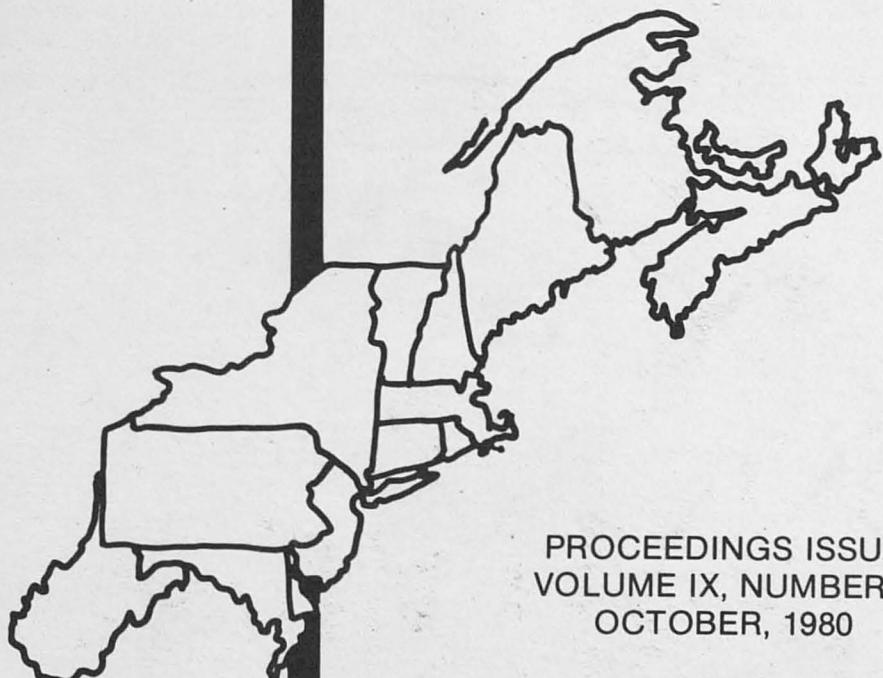
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THE USE OF OPTIMAL CONTROL TECHNIQUES FOR MANAGING THE INTERNATIONAL RADIO SPECTRUM

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INTRODUCTION

Outer space would appear to be a natural resource whose use is characterized by open access and a minimum of congestion externalities. However, over the past two decades, there has been an increase in demand for the limited number of "orbital slots" for communications satellites, and over-crowding has become a problem in space. Recent studies commissioned by the National Aeronautics and Space Administration (Gamble *et al.*, 1979 and Gabriszeski *et al.*, 1979) concluded that demand for communications satellite circuits will increase many-fold between 1982 and the end of this century. This projected increase in demand is the result of the growing number of non-military uses for satellite activities such as weather and climate data collection and remote-sensing of the earth's resources. Another is simply the growing number of nations with interests and capabilities in advanced space technology. To illustrate, membership in the International Telecommunications Satellite Organization, (Intelsat), the organization responsible for coordinating global telephone and television relays, has grown from 14 nations in 1964 to 101 in 1980.

Moreover, the radio spectrum, i.e. that portion of the electromagnetic spectrum used to transmit information, is becoming congested and is experiencing growing problems with interference. The basis of all electronic communication is electromagnetic waves passing through space or along wires. Electromagnetic waves vary in the number that can be sent per second. Their frequency is measured in cycles per second; the higher the frequency, the shorter the wavelength. The entire electromagnetic spectrum extends from zero cycles per second to X-rays which measure more than one quintillion cycles per second. When use of the electromagnetic spectrum began in the early 1900's, the usable portion of the spectrum lay between 20,000 and 1,500,000 cycles per second. Over the years, use of the spectrum has expanded to higher frequencies by means of advancements in technology. At present, the radio spectrum extends from zero cycles per second up to 100 billion cycles per second (Brown *et al.*, 1977).

The radio spectrum may be viewed as a common property resource in that ownership of or property rights to the resource are not clearly defined. Thus, the resource can be exploited by any member of the international community possessing the required technology. Use of the radio spectrum, however, is unlike the use of other common property resources because the spectrum itself cannot be diminished by use. When a particular signal ceases, the frequency is restored to its original capacity.

Coordination of radio spectrum use is necessary because interference occurs when two or more signals are sent over the same frequency at the same time in the same geographical area. At the present time, coordination is controlled by the International Telecommunications Union, an agency of the United Nations. The major function of the ITU is performed by its International Frequency Registration Board (IFRB) and involves allocation, registration, and coordination of activities requiring use of the radio spectrum in order to avoid interference between radio stations of various countries (Brown *et al.*, 1977).

Although the ITU has met with moderate success in coordinating radio spectrum use, its current policies are hindering efficient and equitable allocation. Firstly, the ITU is structured in such a way that it is dominated by telecommunications interests, with the U.S. industry playing a major role. Secondly, users of orbital spaces pay no fees. Historically, slots have been allocated on a first-come, first-served basis, and there exists no mechanism for reallocating slots once they are assigned, even if they are not fully utilized by their registered "owners." Thirdly, new technology has been developed which allows more efficient use of the radio spectrum. However, under the current allocation scheme, users have no incentive to upgrade their equipment. The result of these policies is that the radio spectrum is being used inefficiently and that new users, particularly third-world nations are finding it increasingly difficult to avail themselves of it. Clearly, under the current system, the radio spectrum is being allocated inefficiently and, with increasing international demands for its use, mechanisms must be established to permit allocation on the basis of economic efficiency criteria, rather than on a first-come, first-served basis.

In 1954, H. Scott Gordon, using the fishery resource as an example, developed an economic theory of common property resource use. Gordon maintained that optimum use of the fishery would occur only if conditions of private property prevailed and the fishery was managed by a single owner intent on maximizing resource rent. Such a model might well be applied to management of the radio spectrum.

Gordon's work is essentially a static analysis, but optimal resource management must be viewed as a problem in intertemporal resource allocation. Thus, its proper solution may require the use of dynamic optimization techniques. One such technique is optimal control which has been used by some authors, notably Quirk and Smith (1969), Cummings (1969), Plourde (1970, 1971), Brown (1974), Smith (1974, 1977), Altobello (1975), and Pindyck (1978), in analyzing aspects of intertemporal resource management.

Viewed within the context of optimal control, management of the radio spectrum becomes a problem of choosing values for decision or control variables, i.e. number of orbital slots allocated to each nation, in such a way that a specified objective function is maximized over a given time horizon. The values of these decision variables, along with a given initial inventory of slots and an equation of motion describing the change in the number of slots, determine the values of the state variable or total availability of slots.

The purpose of this paper is to outline the development of a general model which may be used by a central decision maker, such as the International Telecommunications Union, in managing use of the radio spectrum by the world community. The development of such a model involves specifying relevant economic relationships and constraint functions in the most general sense, combining these

with the principles of optimal control theory, and deriving solutions which lead to maximization of resource rent. Utilizing the continuous time examples cited above as a basis, a discrete time model for determining optimal allocation of the radio spectrum is developed and solutions are presented.

EQUATION OF MOTION, ECONOMIC RELATIONSHIPS AND STATEMENT OF PROBLEM

An important aspect of the model is the equation of motion. The one used here is essentially a discrete-time equivalent of that used in previous continuous-time, intertemporal resource allocation problems:

$$S_{t+1} = f(S_t) - \sum_{i=1}^{n} V_{i,t} \qquad t = 0, \dots, T-1 \quad (1)$$

where S_t and S_{t+1} represent the number of orbital slots available in successive time periods and $V_{i,t}$ represents total slots assigned to country i in time period t. Since more than one nation is utilizing the radio spectrum, $\sum_{i=1}^{n} V_{i,t}$ represents the sum of orbital slots occupied by all nations in time period t.

Important economic relationships are the demand or price relationship, and the total revenue and total cost functions. It is assumed that the total number of available orbital slots is sold in a single market, and that price per slot, P_t, may be expressed as a function of total slots occupied in time t:

$$P_t = \phi \left(\sum_{i=1}^{n} V_{i,t} \right) \qquad t = 0, \dots, T-1 \quad (2)$$

Thus, total revenue from the sale of orbital slots in time t, or price per slot times the total number in use, can be written as follows:

$$R_t = \left[\phi \left(\sum_{i=1}^{n} V_{i,t} \right) \right] \left[\sum_{i=1}^{n} V_{i,t} \right] \qquad t = 0, \dots, T-1 \quad (3)$$

Finally, in order to express the varying cost of utilizing the radio spectrum by each nation, we let the total cost function, C_t, be the sum of the individual cost functions of each nation:

$$C_t = \sum_{i=1}^{n} c_i \left(V_{i,t} \right) \qquad t = 0, \dots, T-1 \quad (4)$$

The objective of the central decision-maker is to maximize the sum of discounted net revenues over T time periods plus some function of availability of slots in time T, $F[S_T]$, subject to constraint imposed by the equation of motion. Utilizing (1), (3) and (4) and given $\rho = \dfrac{1}{1+r}$

where r is a specified rate of discount, the problem is formalized below:

$$\text{MAX NR} = \sum_{t=0}^{T-1} \rho^t \{R_t - C_t\} + F[S_T]$$

subject to:

$$S_{t+1} = f(S_t) - \sum_{i=1}^{n} V_{i,t} \qquad t = 0, \dots, T-1$$

or, substituting for R_t and C_t:

$$\text{MAX NR}_{V_{1,t} \cdots V_{n,t};\, S_T} = \sum_{t=0}^{T-1} \rho^t \left\{ \left[\phi \left(\sum_{i=1}^{n} V_{i,t} \right) \right] \left[\sum_{i=1}^{n} V_{i,t} \right] \right. \\ \left. - \sum_{i=1}^{n} c_i \left(V_{i,t} \right) \right\} + F[S_T] \quad (5)$$

subject to:

$$S_{t+1} - f(S_t) + \sum_{i=1}^{n} V_{i,t} = 0 \qquad t = 0, \dots, T-1$$

$$V_{i,t} \geq 0 \ \forall \ i = 1, \dots, n; \quad t = 0, \dots, T-1$$

$$S_t \geq 0 \ \forall \ t = 0, \dots, T$$

DERIVATION OF THE OPTIMAL SOLUTION

The above problem can be viewed as a discrete-time optimal control problem with S_t representing the state variable, thereby permitting the constraint function to be described as follows:

$$S_{t+1} = h_t \ (V_{1,t}, \dots, V_{n,t}; S_t) \qquad t = 0, \dots, T-1 \quad (6)$$

where $V_{1,t}, \dots, Vb_{n,t}$ are the control variables which, along with S_t, determine the value of S_{t+1}.

The objective functional in (5) can be rewritten in the following general form:

$$\pi = F(S_T) + \sum_{t=0}^{T-1} \rho^t \{L_t \ (V_{1,t}, \dots, V_{n,t}; S_t)\} \quad (7)$$

The problem here, given an initial value for S_0, is to find the values of the control variables $V^*_{1,t}, \dots, V^*_{n,t}$, t=0, …, T-1, and the implied values of the states, S^*_t, t=1, …, T, which maximize the sum of discounted net revenues, π. In order to accomplish this, (6) must be affixed to (7) with multiplier sequence λ_t, in the following manner:

$$\overline{\pi} = F(S_T) + \sum_{t=0}^{T-1} \rho^t \{L_t \ (V_{1,t}, \dots, V_{n,t}; S_t) + \rho\lambda_{t+1} \\ \left[h_t \ (V_{1,t}, \dots, V_{n,t}; S_t) - S_{t+1} \right]\} \quad (8)$$

Then the current value Hamiltonian may be defined as:

$$H_t = \rho^t \{L_{1,t}, \dots, V_{n,t}; S_t) + \rho\lambda_{t+1} \\ \left[h_t \ (V_{1,t}, \dots, V_{n,t}; S_t) \right] \} \quad (9)$$

so that (8) may be rewritten as follows:

$$\overline{\pi} = F(S_T) + \sum_{t=0}^{T-1} \left[H_t - \rho^{t+1} \lambda_{t+1} S_{t+1} \right] \quad (10)$$

Totally differentiating $\overline{\pi}$ yields:

$$d\overline{\pi} = \left[\frac{\partial F}{\partial S_T} - \rho^T \lambda_T \right] dS_T + \sum_{t=0}^{T-1} \left\{ \left[\frac{\partial H_t}{\partial S_t} \rho^t \lambda_t \ dS_t \right] \right. \quad (11)$$
$$\left. + \frac{\partial H_t}{\partial V_{1,t}} dV_{1,t} + \dots + \frac{\partial H_t}{\partial V_{n,t}} dV_{n,t} \right\}$$

Thus, first order necessary conditions for maximization of π require that:

$$\frac{\partial H_t}{\partial S_t} = \rho^t \lambda_t \qquad t = 0, \ldots, T-1 \quad (12)$$

$$\frac{\partial F}{\partial S_T} = \rho^T \lambda_T \quad (13)$$

and

$$\frac{\partial H_t}{\partial V_{i,t}} = 0 \qquad i = 1, \ldots, n \quad (14)$$

From (9) note that:

$$\frac{\partial H_t}{\partial S_t} = \rho^t \frac{\partial H_t}{\partial S_t} + \rho^{t+1} \lambda_{t+1} \frac{\partial H_t}{\partial S_t} \quad (15)$$

and

$$\frac{\partial H_t}{\partial V_{i,t}} = \frac{\partial L_t}{\partial V_{i,t}} + \rho \lambda_{t+1} \frac{\partial h_t}{\partial V_{i,t}} \quad (16)$$

Equating the right-hand side of (14) and the right-hand side of (16) yields an expression for the set of first-order necessary conditions:

$$\frac{\partial L_t}{\partial V_{i,t}} + \rho \lambda_{t+1} \frac{\partial h_t}{\partial V_{i,t}} = 0 \quad (17)$$

Equating the right-hand sides of (12) and (15) gives the following result:

$$\lambda_t = \frac{\partial L}{\partial S_t} + \rho \lambda_{t+1} \frac{\partial h_t}{\partial S_t} \quad (18)$$

The interpretation of (18) is that the value of an incremental addition to the number of orbital slots in time t, (λ_t), will equal its marginal contribution to the objective function in time t, $\left(\frac{\partial L_t}{\partial S_t} \right)$

plus the discounted value of its incremental contribution to the number of orbital slots in period t+1, $\left(\rho \lambda_{t+1} \frac{\partial h_t}{\partial S_t} \right)$

To find the solution of this problem, that is, the optimal control vector sequences, $V^*_{1,t}, \ldots, V^*_{n,t}, t = 0, \ldots, T-1$, and the values of the states implied by the optimal controls, $S^*_t, t = 1, \ldots, T$, it is necessary to solve the two-point boundary problem described by the following system of $T(n+2)-1$ equations, from (6), (16), and (18):

I. $\quad S_{t+1} = h_t (V_{1,t}, \ldots, V_{n,t}; S_t) \qquad t = 0, \ldots, T-1$

II. $\quad \lambda_t = \frac{\partial L}{\partial S_t} + \rho \lambda_{t+1} \frac{\partial h_t}{\partial S_t} \qquad t = 1, \ldots, T-1$

III. $\quad \frac{\partial H_t}{\partial V_{i,t}} = \frac{\partial L_t}{\partial V_{i,t}} + \rho \lambda_{t+1} \frac{\partial h_t}{\partial V_{i,t}} = 0 \ \forall \quad i = 1, \ldots, n$
$$t = 0, \ldots, T-1$$

with boundary conditions S_0 (given) and $\rho^T \lambda_T = \frac{\partial F}{\partial S_T}$

CONCLUSIONS AND IMPLICATIONS FOR FUTURE RESEARCH

It has been demonstrated that the radio spectrum is an example of a common property resource characterized by inefficient utilization. Demand for the limited number of orbital slots for communications satellites is expected to increase substantially over the next two decades, especially as more and more third-world nations acquire the advanced technology necessary for communications via satellite. The model outlined above represents an attempt to develop an allocation scheme for the radio spectrum which is based on economic efficiency criteria, and as such, represents a general framework for use in qualitative analyses. It provides a basis for conducting the quantitative analyses necessary for actual implementation of a management program for the radio spectrum.

There are, of course, some problems associated with quantifying the model presented here. These problems are related to data availability, the effects of technological progress, model construction and solution, and equity considerations. Firstly, there is a lack of data, particularly with respect to the cost functions faced by various nations in using the radio spectrum. Another data-related problem would involve comparing and reconciling measures of costs compiled by various countries. Secondly, the model does not explicitly examine the effects of technological progress on both fixed and operating costs and on the availability of orbital slots. It is generally conceded that technological progress will increase the number of available slots, but the associated effects on costs is unclear. Thirdly, the mathematical properties of specific formulations of the objective and constraint functions might pose problems in deriving actual numerical solutions. Fourthly, the allocation mechanism as it is derived here does not necessarily guarantee an equitable distribution of available slots among user-nations. This is something that would have to be worked out by international agreement.

Within the ITU exists the institutional framework for developing and implementing programs designed to bring about efficient and equitable use of the radio spectrum. The problems mentioned here suggest areas where future research efforts might be intensified so that effective international management of the radio spectrum can be achieved.

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