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NON-PARAMETRIC MEASURES OF TECHNICAL EFFICIENCY IN MILK PRODUCTION

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ABSTRACT

This paper presents estimates of technical efficiency in milk production based on Farrell's non-parametric frontier production function methodology. Data from a sample of Maine and Vermont dairy farms, included in the ELFAC business analysis for 1979, were used to derive Efficient Unit Isoquants for capital-labor and concentrate feed-roughage combinations. The analysis indicates that milk production in Maine and Vermont farms was characterized by significant technical inefficiencies during 1979.

I. INTRODUCTION

The degree of technical efficiency with which resources are used at the aggregate and micro levels is a key indicator of economic performance. Technical inefficiency, which occurs whenever there is a divergence between the existing use of resources and any theoretical or empirical optimum, hinders overall economic growth and thus is costly to society and to individual economic agents (Heady). In order to minimize such costs, empirical estimates of actual performance and of efficiency gains that could be achieved by individual firms or entire industries are required.

The purpose of this study is to establish an empirically optimum use of resources and then measure the gap that exists between technically efficient and inefficient firms in a sample of northern New England dairy farms. A major consideration will be to derive efficiency measures that are meaningful and useful not only to researchers, but also to students, extension personnel and milk producers.

The organization of the paper is as follows. Section two contains a summary of Farrell's approach for estimating efficiency followed by a description of the data and definition of variables in section three. The analysis of technical efficiency in milk production is presented in section four and the final section contains the usual concluding remarks.

II. FARRELL'S APPROACH TO MEASURING EFFICIENCY

Empirical work measuring technical efficiency has relied extensively on the use of average factor productivity, efficiency indexes, and economic-engineering methodology. Although the average factor productivity criterion has been widely used it has the major shortcoming of considering only one factor at a time, thus ignoring the presence of other inputs "which affect average (and marginal) productivity" (Lau and Yotopoulos, p. 94). Attempts to surmount this shortcoming led to the development of elaborate efficiency indexes in which a weighted average of inputs is compared with output. Even though these weighted averages have been refined over time, they are plagued with the usual index number problems (Domar).

The economic-engineering approach has been used extensively to study the relationship between cost (efficiency) and farm size under specific assumptions (eg., Buxton and Jensen; Madden; Miller et al.). Estimates obtained using this technique do not reflect actual farm situations and are realistic only to the extent that specific assumptions are met (Hall and LeVeen).

In a seminal paper written almost three decades ago, Farrell introduced a measure of efficiency which reflects actual firm performance, can include all relevant factors of production, and is not subject to index number problems. The method, as originally presented, yields a deterministic non-parametric frontier production function (FPF) defined as "an empirical function based on the best results observed in practice" (Farrell, p. 255). The frontier function has the added advantage of being consistent with the conventional definition of a theoretical production function, and thus has a direct economic interpretation. It should be noted that most empirical work is based on 'average' production functions[1] even though their economic meaning is much less clear-cut than that of frontier functions (Timmer 1970).

In order to measure efficiency Farrell started by assuming that a group of firms in an industry produce a single homogeneous output (Y) using two inputs (X_1 and X_2) under conditions of constant returns to scale. Dividing each input by the level of output the three dimensional Y, X_1, X_2 surface becomes a two dimensional plane in X_1/Y and X_2/Y which can be represented in a simple isoquant map. One special feature of this representation is that a particular isoquant, the Efficient Unit Isoquant (EUI), can be drawn reflecting those firms that use the least amount of inputs per unit of output. Curve SS' in Figure 1, which is a reproduction of Farrell's Figure 1, is a graphical illustration of an EUI and represents an estimate of a non-parametric frontier production function.

Firms using input combinations that coincide with the EUI, points Q and Q' on Figure 1, are said to be 100 percent technically efficient.

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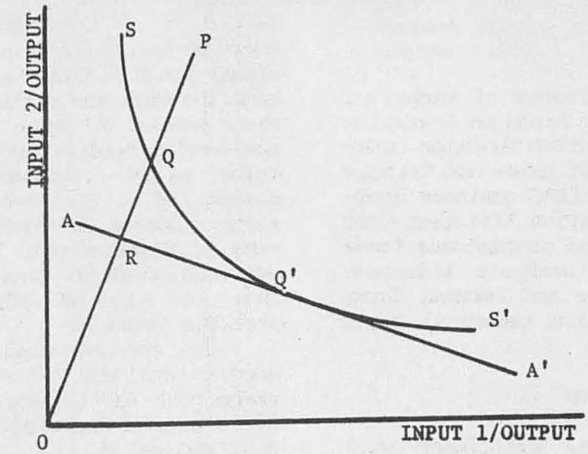
[1] 'Average' production functions are "...those estimated by a statistical technique such as least squares that minimizes errors on both sides of the estimated function..." (Timmer 1970, p. 117).

Figure 1. The Efficient Unit Isoquant (Farrell, p. 254)



SS' : Efficient Unit Isoquant

AA' : Relative input price ratio

Q,Q': Technically efficient firms

P : Technically inefficient firm

Q' : Technically and price efficient (i.e., economic efficient) firm

$\frac{OQ}{OP}$ = **Measure of technical efficiency of firm P**

$\frac{OR}{OQ}$ = Measure of price efficiency of firm Q

$\frac{OR}{OP} = \frac{OQ}{OP} \times \frac{OR}{OQ}$ = Economic efficiency of firm P

Any firm observed operating off the EUI, such as point P on Figure 1, is considered technically inefficient.[2] A measure of the technical inefficiency of firm P is given by the ratio OQ/OP which represents the minimum level of inputs actually used by firm P.

The EUI framework can also be used to measure price or allocative efficiency provided that input prices are known. Assuming that the slope of line AA' in Figure 1 represents the prevailing relative input price ratio, then a 100 percent technically and price efficient firm is one that operates where the price line is tangent to the EUI. It follows that firm Q' in Figure 1 is both technically and price efficient, while firm Q is technically efficient but price inefficient. Price inefficiency for firm Q is measured by the ratio OR/OQ. Finally, a measure of (overall) economic efficiency can be derived by multiplying technical efficiency times price efficiency. Thus, economic efficiency for firm P in Figure 1 is equal to OQ/OP x OR/OP.

The EUI was generalized by Farrell to several inputs and outputs and made more operational by Boles and Carlson through the use of linear programming techniques. The greater generality achieved by including more than two inputs is offset by the difficulty of visualizing the non-parametric production function unless it is constrained to a specific algebraic form. "Then the constraint of a functional form must be balanced against the ease of visualizing the production surface" (Timmer 1970, p. 110).

A further generalization of Farrell's technique, which has been extended by Farrell and Fieldhouse, Boles, and Carlson, relates to the assumption of constant returns to scale required to develop the EUI. Farrell relaxed this assumption by using the 'grouping method' which simply consists in segmenting the observations based on output level and then estimating the EUI for each output group separately. A comparison of the resulting EUIs provides an estimate of scale economies.

2 It should be clear that points lying on the EUI and to the northeast of it are attainable while all points to the southwest of the EUI are infeasible.

Farrell's method was also the basis for the parametric deterministic methodology developed to measure technical efficiency by Aigner and Chu, and extended by Timmer (1970 and 1971). More recent work dealing with deterministic statistical and stochastic frontiers can also be traced to Farrell's initial paper (e.g., Aigner et al.; Bagi; Lesser and Greene; Schmidt).

The preceding discussion indicates that several procedures for estimating frontier functions have evolved from Farrell's initial contribution. In spite of this methodological progress, King has argued that the original EUI representation of a frontier production function can be a very useful tool in teaching, research and extension activities and that much can be gained by more general use of this approach. Hence in this paper the EUI concept is applied to measure technical efficiency in milk production. Specifically, EUIs are used to investigate the substitutability between pairs of inputs, identify the presence of economies of size, and determine whether technical efficiency in milk production differs between the states of Maine and Vermont.

III. DATA AND DEFINITION OF VARIABLES

This section contains EUIs for milk production based on a sample of Maine and Vermont dairy farms included in the ELFAC business analysis for 1979. After eliminating incomplete records, 67 farms located in Maine and 96 in Vermont remained in the data set and were used in the analysis. In order to investigate the impact of farm size on technical efficiency, the sample was divided into small and large farms. Farms with a herd size equal to or below the sample median of 60 cows were defined as small, and farms with more than 60 cows were classified as large.

Five variables were included in the analysis reflecting milk output and four inputs used by each farm in the sample. The specific variables selected are:

- Output - Annual milk production per farm measured in pounds adjusted to a 3.5 percent butterfat basis divided by 100,000.
- Labor - Measured in annual full-time man equivalents per farm divided by output
- Capital - A measure of the flow of annual machinery and equipment services per farm divided by output. The flow measure includes an opportunity cost on the machinery and equipment capital stock (.06 x value of the machinery and equipment stocks), plus annual expenses on gas, oil, repairs, and maintenance.
- Concentrate Feed - Pounds of purchased dairy concentrates used annually per farm divided by output.
- Roughage - Tons of hay equivalent produced per farm divided by output. (Using hay equivalent production instead of consumption could lead to misleading results if these figures vary considerably from each other. However, only the production data were available.)

Table 1 contains a summary of descriptive statistics of the variables used in the analysis.

IV. EFFICIENT UNIT ISOQUANTS FOR MILK PRODUCTION

This section presents a series of EUIs for concentrate feed-roughage and capital-labor combinations. These EUIs are the basis for comparing technical efficiency in milk production in dairy farms in the states of Maine and Vermont. It should be stressed that the measures of technical efficiency presented below are based on the use of only two inputs at a time and thus do not provide an overall measure of technical efficiency for the farms in question.

By construction, the position of an EUI is determined solely by a subset of the data which makes it very sensitive to extreme observations and measurement errors (Førsund et al.). In constructing the EUIs presented in Figures 2 through 4 it was arbitrarily determined that at least four observations should lie on the isoquant. In order to accomplish the latter and still draw convex isoquants compatible with conventional neoclassical production theory, it was necessary to delete seven extreme observations from the original data set[3].

Figures 2A-2C represent concentrate feed and roughage combinations used to produce one unit of milk in Vermont and Maine dairy farms. The wide scatter in the observations shown in Figures 2A and 2B reveals considerable technical inefficiency in the use of these two inputs in both states, but particularly in Vermont. A comparison of the EUIs in the top two figures, as shown in Figure 2C, reveals that Vermont farms achieve higher efficiency when concentrate feed is higher than 31,000 (lbs./output) and roughage is lower than 35 (ton/output). By contrast, when concentrate feed is below 31,000 (lbs./output) and roughage exceeds 35 (ton/output) Maine farms exhibit a higher level of technical efficiency.

Figures 3A-3C present EUIs for concentrate feed and roughage when the sample is divided into small and large farms. A comparison of figures 3A and 3B suggests that, overall, large farms tend to be more efficient than small ones. Figure 3C, however, shows that the EUI for small farms is closer to the origin when concentrate feed is higher than 32,000 (lbs./output) and roughage is lower than 29 (ton/output).

The technical efficiency of Maine and Vermont farms regarding capital and labor combinations is shown in figures 4A-4C. The scatter of points in Figure 4A and 4B reveals a wide range of efficiency in the use of these two inputs in both states. When the EUIs for each state are compared in one diagram, as in Figure 4C, the data suggest that the level of efficiency attained in Vermont is higher than in Maine.

A final set of comparisons, shown in Figures 5A-5C, correspond to capital-labor combinations when the sample is classified into small and large farms. The relative scatter of individual farm observations in Figures 5A and 5B shows that

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[3] A more elaborate approach for handling extreme observations is given by Timmer (1970) pp. 115-116.

Table 1. Descriptive Statistics of Variables Used to Derive Efficient Unit Isoquants Based on a Sample of Northern New England Dairy Farms, 1979.

Variable	Unit 1/ (per year)	Mean	Standard Deviation	Minimum Value	Maximum Value
A. Maine Farms (N=67)					
Milk Production	lbs/100,000	10.66	6.48	2.47	37.37
Labor	Man-Equivalent	0.32	0.10	0.18	0.68
Capital	Dollars	1,437.60	461.13	577.10	2,745.37
Concentrate Feed	Pounds	43,094.08	10,906.47	20,565.90	65,385.20
Roughage	Ton	53.17	18.53	8.90	101.00
Cows	Head	77.16	51.71	25.00	350.00
B. Vermont Farms (N=96)					
Milk Production	lbs/100,000	9.27	5.28	3.61	28.41
Labor	Man-Equivalent	0.30	0.10	0.14	0.56
Capital	Dollars	1,219.37	481.10	492.59	2,593.30
Concentrate Feed	Pounds	41,978.80	8,139.16	26,413.00	60,942.26
Roughage	Ton	59.12	27.97	16.90	169.80
Cows	Head	65.98	33.68	25.00	202.00
C. Small Farms (N=89)					
Milk Production	lbs/100,000	6.10	1.54	2.47	10.44
Labor	Man-Equivalent	0.35	0.10	0.18	0.68
Capital	Dollars	1,287.02	439.86	655.70	2,745.37
Concentrate Feed	Pounds	42,571.25	9,111.34	24,525.60	63,250.20
Roughage	Ton	56.87	28.72	8.90	169.80
Cows	Head	45.25	8.55	25.00	60.00
D. Large Farms (N=74)					
Milk Production	lbs/100,000	14.33	5.91	4.53	37.37
Labor	Man-Equivalent	0.26	0.08	0.14	0.53
Capital	Dollars	1,335.59	459.30	492.59	2,505.55
Concentrate Feed	Pounds	42,276.04	9,711.06	20,565.90	65,385.20
Roughage	Ton	56.43	18.81	20.90	108.80
Cows	Head	101.03	46.38	61.00	350.00

1/ Labor, Capital, Concentrate Feed, and Roughage are divided by Milk Production.

Figure 2. Efficient Unit Isoquants for Concentrate Feed/Output (CFEED) and Roughage/Output (ROUGHAGE) Combinations for Maine and Vermont Farms (1979).

2A. Maine Farms

2B. Vermont Farms

2C. Maine & Vermont Farms

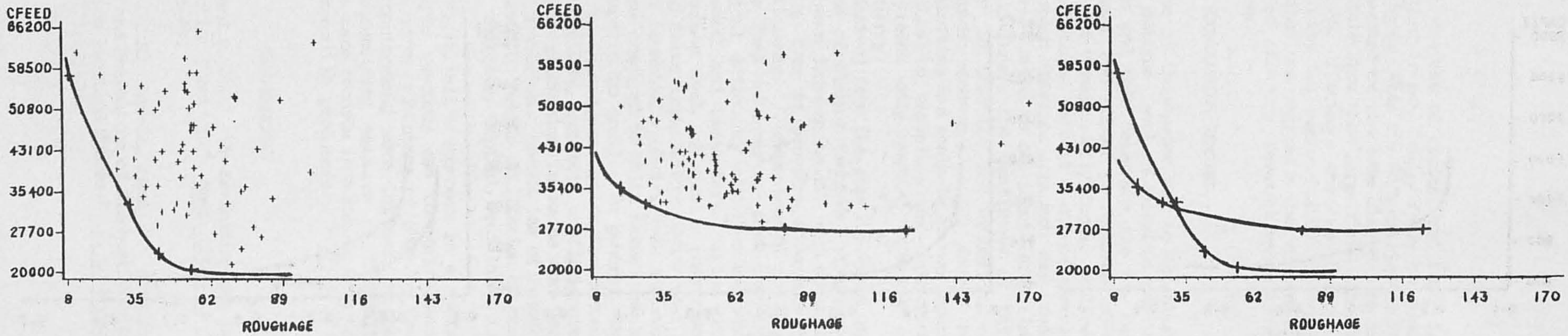


Figure 3. Efficient Unit Isoquants for Concentrate Feed/Output (CFEED) and Roughage/Output (ROUGHAGE) Combinations for Small and Large Farms (1979).

3A. Small Farms

3B. Large Farms

3C. Small & Large Farms

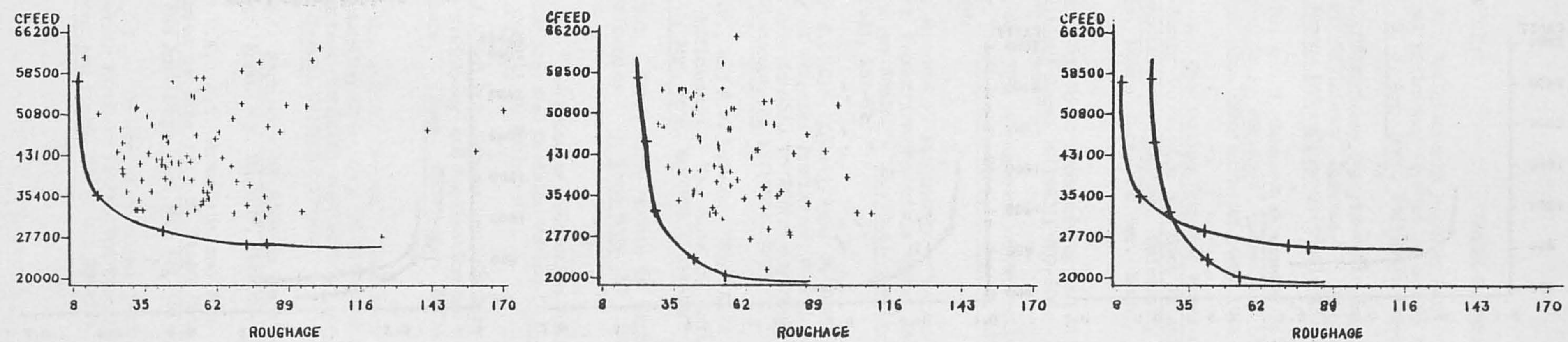


Figure 4. Efficient Unit Isoquants for Capital/Output (CAPIT) and Labor/Output (LABOR) Combinations for Maine and Vermont Farms (1979).

4A. Maine Farms

4B. Vermont Farms

4C. Maine & Vermont Farms

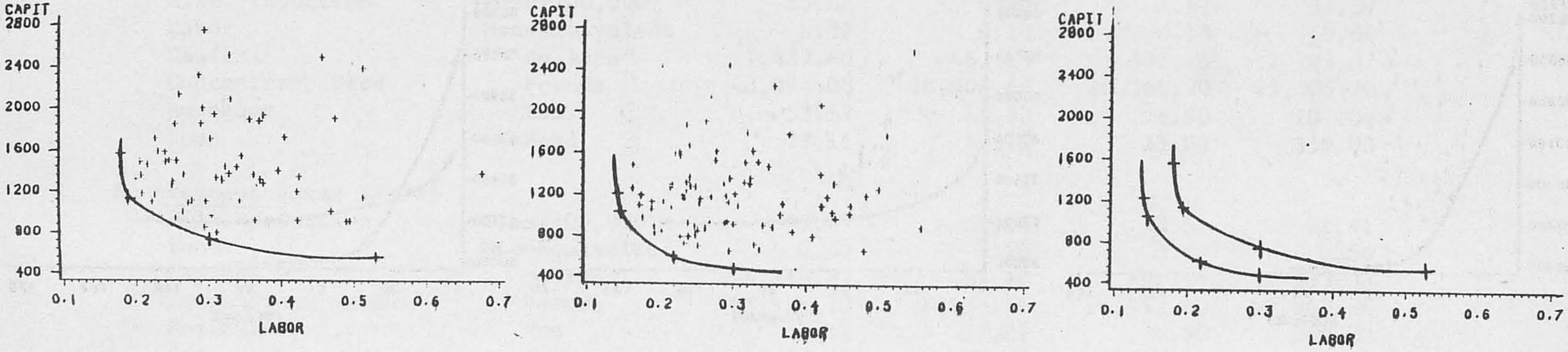
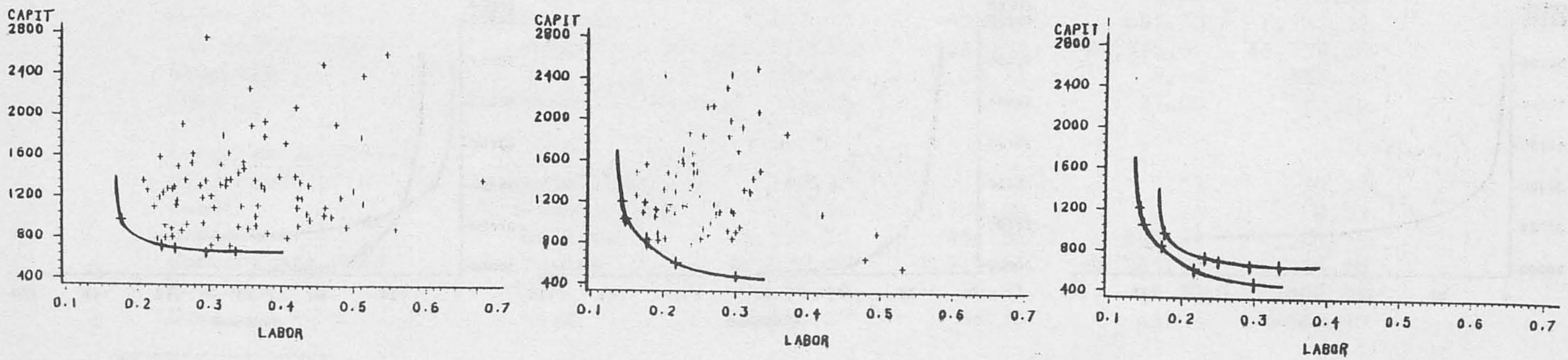


Figure 5. Efficient Unit Isoquants for Capital/Output (CAPIT) and Labor/Output (LABOR) Combinations for Small and Large Dairy Farms (1979).

5A. Small Farms

5B. Large Farms

5C. Small & Large Farms



inefficiency in the use of capital and labor is considerably higher in small dairy farms compared to large operations. When the EUIs of Figures 5A and 5B are presented on the same diagram, as in Figure 5C, the data show that large farms achieve a higher level of efficiency than small farms. In other words, the EUI representation of capital-labor combinations supports the contention that economies of size are prevalent among efficient dairy farms.

V. CONCLUDING REMARKS

The purpose of this paper was to establish an empirically optimum level of resource use in milk production and then determine the gap between the optimal and actual performance of a group of Maine and Vermont dairy farms. Following King's suggestion, Farrell's non-parametric frontier production function approach was used. The analysis was limited to two inputs at a time in order to illustrate the usefulness of Farrell's approach in teaching and extension, as well as research endeavors. It should be stressed that limiting the analysis to two inputs makes it possible to visualize the production surface, but yields only partial measures of technical efficiency.

The EUIs derived from the data represent empirical measures of maximum technical efficiency while the distance between an EUI and observations lying away from it provides a measure of technical inefficiency. A comparison of the EUIs for concentrate feed and roughage show that the relative technical efficiency of Maine and Vermont, and of small and large farms varies with the level of inputs used. A similar comparison for capital and labor combinations shows that a higher level of technical efficiency is achieved by Vermont farms compared to Maine farms, and by large farms compared to small. In general, the EUIs presented in this paper support the contention that considerable technical inefficiencies characterize milk production in Maine and Vermont, and that significant quantities of resources could be saved by better utilization of concentrate feed, roughage, capital, and labor inputs.

In summary, Farrell's approach is a useful method to analyze resource use in agricultural production. A major advantage of EUIs is that they provide considerably more information than the average productivity measures traditionally used to analyze farm records in a manner that can be easily understood by producers.

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