J. OF THE NORTHEASTERN AGR. ECON. COUNCIL VOL. XII, NO. 1, SPRING, 1983

MODELLING THE RESIDENTIAL DEMAND FOR ELECTRICITY IN NEW ENGLAND

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ABSTRACT

Accurate forecasts of energy demand are required for public policy formation, but estimation of the residential demand for electricity presents a number of conceptual and statistical problems. This paper focuses on two interrelated issues in electricity demand analysis: model specification with respect to the price variable and the level of data aggregation. From an empirical study of demand in New England, our principal conclusions are: (a) price elasticities, estimated using state level data, differ from those at the utility level; (b) at the state level of aggregation, alternative model specifications of demand give markedly different results; (c) there appears to be significant differences between the New England states in the demand for electricity; and (d) it was not possible to discern whether consumers respond to average price or marginal price.

INTRODUCTION

Significant lead times and risks are associated with all forms of electricity generation and the need for accurate demand forecasts is obvious. Unfortunately, electricity demand contains a number of features which are difficult to model. In this paper we focus on two interrelated issues in electricity demand analysis; model specification with respect to the price variable and level of aggregation.

Background and Previous Research: Taylor [1975] notes that most econometric electricity demand studies use ex post average price (calculated by dividing total expenditure by quantity consumed) as the only price variable. However, because of the nature of electricity rates, simultaneity exists between ex post average price and quantity and an upward bias in the estimate of the responsivness of demand to price may result.

The correct specification requires that both average and marginal prices be used in the demand function. Taylor suggests that the average price variable could be calculated as the average per kilowatt-hour of electricity consumed up to, but not including, the final block. Alternatively, the total payment for blocks other than the final one could be used as an explanatory variable in a demand function. More recently, Nordin [1976] demonstrated that it is better to use marginal price and a difference variable which is the actual electricity bill minus what the bill would have been if all electricity were sold at the marginal price. The empirical studies by Smith and Cicchetti [1975], Smith [1980], Halvorsen [1978], Taylor [1977], and Houthakker [1962] have, however failed to demonstrate bias in single equation ex post average price models. Although these results appear somewhat comforting, most studies on the choice of the appropriate price variable(s) for empirical analysis utilized aggregate data. As noted by Mount and Chapman [1973], "No unique marginal or average price exists for a city or state, consequently the distinction between the two pricing systems may have been obscured" (p. 6).

The problem with aggregate data in the case of declining block pricing can be demonstrated by assuming two consumers reacting to the hypothetical rate structure below:

Π_1 = 150¢ sevice charge,

Π_2 = 6¢ for first 50 kwh,

Π_3 = 2¢ for next 500 kwh.

Assume that one individual consumes 45 kwh while the other consumes 53 kwh. Next, assume an increase in Π_3 (marginal price) to 3¢ causing the latter individual to reduce consumption to 51 kwh. In this case, the average or aggregate individual is observed to consume 49 kwh before the rate change and 48 kwh after, but the marginal price for the average or aggregate consumer remains constant. Moreover, the percentage change in the aggregate ex post average price differs from that associated with each individual consumer.

Choice of the appropriate demand model specification, therefore, remains an unsettled issue which is obscured by the nature of electricity rates and by level of aggregation. The purpose of this paper is to provide additional empirical evidence regarding the importance of these interrelated issues.

EMPIRICAL EVALUATION

The demand models to be examined here are: (1) Opaluch [1982] decomposed average price, ex post average price, marginal price, and Nordin [1976] difference models estimated using state level data for the six New England states; and (2) decomposed average price, ex post average price, marginal price, and Nordin difference models using utility level data for most of the electrical utilities in New England.

Data were obtained for a representative household (defined by kwh consumed) for most of the electrical utilities in New England for the 1967-1975 period (N = 405 observations). The use of a single representative household per electrical utility allowed the marginal price, ex post average price and Nordin's [1976] difference variable to be obtained from actual rate schedules. The basic data set comprises: electricity consumption for the average household per year (KWH); the ex post average electricity price in cents (PA); marginal price of electricity in cents (PM); the electricity bill (in cents) minus what the bill would have been if all electricity were sold at the marginal price (DIFF); income in

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dollars (Y); the fuel adjustment change per kwh in cents (FADJ): an oil price index (OILI); and an appliance price index (API). All price and income variables were deflated by the consumer price index (1967 = 100). Finally, a set of five state dummy variables is appended to the data set (R_i; i = 2,..., 6).[1]

In order to investigate the impact of estimating the demand models with a more aggregate set of data, the individual utility level data were averaged over utilities in each state to produce a state level data set for the years 1967-1975 (N = 54).[2] That is to say, for each year, KWH, PA, PM, Y and FADJ were averaged over utilities in each of the six states in turn. OILI and API do not vary across utilities and DIFF was recomputed using the aggregate data set, i.e., using the mean levels of total expenditure and PM.

As indicated above, our purpose is to compare alternative formulations of electricity demand for different levels of data aggregation. Throughout the empirical analysis, double-log specifications have been utilized. Thus, the estimated coefficients on the price and income variables may be interpreted as elasticities. Care should be taken, however, if an interpretation in percentage terms is sought for the estimated coefficients on the state dummy variables (Halvorsen and Palmquist). Ordinary least squares and ridge regression are used to generate estimates of the unknown parameters. For convenience, the presentation begins with the empirical results using state level data. The utility level demand equations follow and we conclude with a brief examination of the impacts of using regionally aggregated data.

1. State Level Results: Equations 1.1 and 1.4 of Table 1 conform to specifications of the ex post average price and Nordin difference models respectively, which are commonly estimated using aggregate data and single equation models. These equations, with own price, income and oil and appliance price indices as the regressors, appear to be quite satisfactory in that they explain a fairly high proportion of the variation in household consumption of electricity. In addition, the estimated price and income coefficients have, in general, the expected signs, they are statistically significant, and the own price elasticities appear to conform with estimates from previous research. One drawback of both specifications, however, is that they preclude the possibility of consumption varying systematically across states, reflecting differing regional conditions, tastes and preferences. In the absence of data on socio-economic variables which may account for differing preference structures, a set of five state dummy variables have been introduced (equations 1.2 and 1.5). The resultant increase in the explanatory power of each estimated equation, as judged by the value of the adjusted R^2, is quite striking and the statistical significance of the set of dummy variables is verified by the usual F tests.[3]

While the inclusion of the state dummy variables makes a marked contribution to the overall fit of each model, in several instances the estimated coefficients of the dummy variables, taken individually, are not statistically significant and the standard errors of the estimated coefficients of the other explanatory variables have increased. This may suggest the presence of multicollinearity in the data set and indeed this is perhaps to be expected since the oil and appliance price indices do not vary across states and both prices and income follow much the same time path.

Ridge regression was employed to deal with the estimation problems that result from multicollinearity. That is to say, since the moments matrix $(X'X)$ approaches singularity, each diagonal element of the matrix is multiplied by $(1 + k)$ where $0 < k < 1$. While the ridge estimates are biased, a suitable choice of k will yield parameter estimates with lower mean square error than OLS estimates (Hoerl and Kennard [1970]). The optimal value of k, it is suggested, can be determined by starting with small values of k and increasing k until the coefficients stabilize. The results of applying ridge regression, with k = 0.25, are presented in equations 1.3 and 1.6 of Table 1. The map of the principal estimated coefficients of interest as a function of k, which is called a ridge trace, is illustrated in Figure 1. It should be noted that in ridge regression the ratio of an estimated coefficient to its standard error, given in parenthesis below each coefficient, is not distributed exactly as Student's t under the null hypothesis that the population coefficient is zero. These ratios are approximate, however, and serve as good relative indicators of departures from equality. It is, therefore, encouraging to note that, in general, the ratios of coefficient estimates to standard

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[1] Massachusetts (R_1) was chosen as the base state. Connecticut, Rhode Island, New Hampshire, Maine and Vermont are represented by R_2, R_3, R_4, R_5, R_6, respectively. By construction $R_i = 1$ if the observation is in state i; = 0 otherwise.

[2] Strictly, if we believe that the log-linear specification is the true model at each level of aggregation, the geometric means of the individual utility data should be used (Grunfeld and Griliches, p. 3). However, the researcher rarely has control over the method of aggregation and it is our purpose here to construct a "typical" set of aggregated data with which the researcher might have to work.

[3] The null hypothesis (NH) is that there are no state level differences in the value of the intercept. The F statistic for the average price model is computed as 39.06; for the Nordin model, F = 56.13. Both exceed the critical value of F, $F_{5,40} = 3.51$ at the 99% confidence level. The NH is rejected. A similar test is performed on the ridge regression results. The test statistic for each model, which is distributed with a non-central F distribution (Wallace and Toro-Vizcarrondo), greatly exceeds the critical value at the 95% confidence level and again the NH is rejected.

Table 1: Regression Coefficients: State Level*

	Average Price			Nordin		
	1.1	1.2	1.3	1.4	1.5	1.6
Const.	7.891 (10.93)	-28.328 (1.42)	8.236 (47.15)	9.164 (8.68)	-2.712 (0.11)	8.125 (36.4)
Ln PA	-1.008 (9.33)	-.309 (3.49)	-.468 (12.29)			
Ln PM				-.858 (5.36)	-.276 (2.79)	-.322 (9.81)
Ln DIFF				-.219 (3.05)	-.037 (1.30)	-.006 (0.45)
Ln Y	.167 (2.08)	4.159 (1.86)	.074 (3.80)	.178 (1.75)	1.302 (0.46)	.070 (3.11)
Ln OILI	.397 (2.93)	-.049 (0.67)	.151 (4.72)	.106 (0.56)	-.171 (2.46)	-.026 (0.64)
Ln API	-.547 (2.07)	-1.466 (9.91)	-.870 (14.01)	-.856 (2.33)	-1.446 (8.13)	-.817 (10.51)
Ln FADJ				.016 (1.57)	-.002 (0.57)	-.0005 (0.13)
R_2		-1.006 (1.82)	-.008 (.58)		-.292 (0.42)	-.003 (0.17)
R_3		.267 (1.36)	-.070 (4.57)		.002 (.07)	-.093 (5.39)
R_4		.547 (2.13)	.625 (4.18)		.238 (0.74)	.077 (4.51)
R_5		1.019 (1.59)	-.125 (9.79)		.198 (0.25)	-.136 (9.15)
R_6		.743 (2.55)	.150 (10.23)		.404 (1.09)	.186 (11.09)
	OLS	OLS	Ridge K=0.25	OLS	OLS	Ridge K=0.25
$\bar{R}^2$	.78	.96	.94	.66	.96	.92

* The ratio of the estimated coefficient to its standard error is given in parentheses.

errors in 1.3 and 1.6 are rather large and, with the exception of the elasticity with respect to the oil price in the Nordin model, the coefficients have the correct signs. Both equations suggest a similar pattern of distribution of consumption by state, i.e., average household consumption is highest in Vermont and New Hampshire and lowest in Maine, *ceteris paribus*.

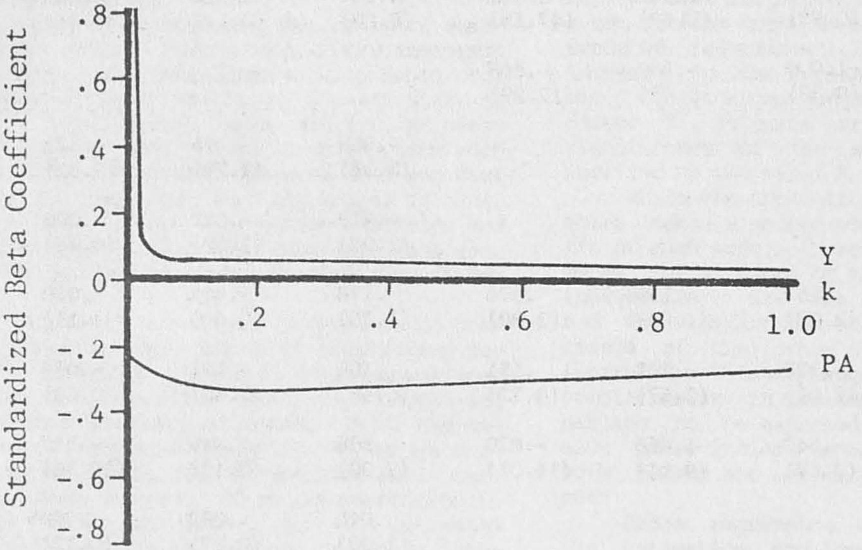
However, the specifications presented in Table 1 provide relatively little information concerning the measure of price to which consumers actually respond. Following Opaluch [1982], model 1.3 was reestimated using a decomposed measure of average price. The demand function estimated was:

$$\ln Q = B_0 + B_1 \ln \chi + B_2 \ln PM + B_3 \ln (PA - PM) + B_4 \ln (Y - DIFF).$$

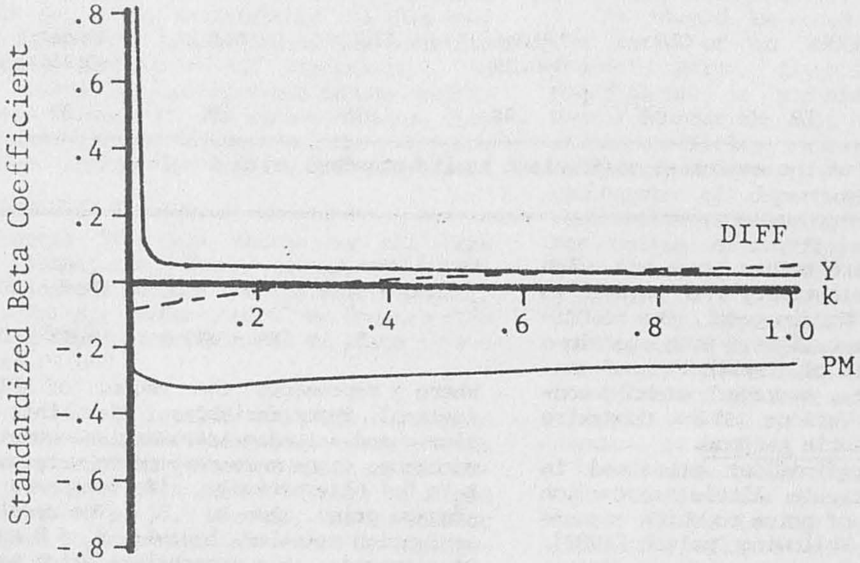
Where χ represents the vector of API, OILI and regional dummy variables. Given this specification, and assuming intramarginal quantities held constant, consumers respond to marginal price if $B_3 = 0$. Alternatively, if consumers respond to average price, then $B_2 = B_3$. The results of this estimation revealed, however, $B_3 \neq 0$ and $B_3 \neq B_2$. Consequently, the appropriate price variable was found to be indeterminate.

Figure 1

Ridge Traces



(i) Ex Post Average Price Model



(ii) Nordin Difference Model

Further information concerning the appropriate specification was obtained by estimating two types of simultaneous equation models. The first simultaneous equation model consisted of the demand equation 1.2 and an ex post average price equation with ex post average price a function of KWH, time, and the regional dummy variables. Results, using 3SLS, revealed an average price coefficient which was not statistically significant at the 90 percent level. The second simultaneous model consisted of the demand equation 1.5 and a marginal price equation with marginal price a function of KWH, time and the regional dummy variables. The results in the case yielded a marginal price coefficient estimate of -.80.

2. Utility Level Results: In order to examine the effect of aggregation on the estimated demand models, the alternative specifications discussed above were re-estimated using utility level data. The results of this exercise are presented in Table 2.

Again, a substantial increase in the value

Table 2: Regression Coefficients: Utility Level*

	Average Price			Nordin		
	2.1	2.2	2.3	2.4	2.5	2.6
Const.	9.538 (18.11)	11.598 (14.31)	9.894 (23.85)	8.483 (14.12)	9.809 (11.14)	9.077 (19.70)
Ln PA	-.766 (12.36)	-.684 (10.40)	-.539 (12.57)			
Ln PM				-.264 (4.98)	-.205 (4.05)	-.190 (5.92)
Ln DIFF				-.021 (2.41)	-.017 (2.13)	-.011 (1.88)
Ln Y	-.049 (0.85)	-.290 (3.25)	-.110 (2.37)	.026 (0.39)	-.134 (1.37)	-.050 (0.96)
Ln OILI	.239 (2.40)	.184 (2.00)	.181 (4.02)	-.230 (2.25)	-.247 (2.67)	-.026 (0.50)
Ln API	-.993 (5.41)	-1.088 (6.20)	-.899 (10.23)	-1.485 (7.03)	-1.572 (8.13)	-.994 (9.77)
Ln FADJ				-.005 (0.57)	-.008 (1.01)	-.008 (1.22)
R_2		.076 (2.29)	.036 (1.67)		.083 (2.25)	.046 (1.94)
R_3		-.082 (2.16)	-.064 (2.15)		-.117 (2.78)	-.095 (2.86)
R_4		.066 (2.10)	.070 (2.94)		.112 (3.17)	.092 (3.47)
R_5		-.229 (5.50)	-.145 (5.44)		-.179 (3.91)	-.132 (4.48)
R_6		.107 (2.46)	.134 (4.44)		.260 (5.78)	.217 (6.52)
	OLS	OLS	Ridge K=0.25	OLS	OLS	Ridge K=0.25
$\bar{R}^2$	.46	.53	.51	.29	.42	.40

* The ratio of the estimated coefficient to its standard error is given in parentheses.

of adjusted R^2 accompanies the inclusion of the set of state dummy variables and, as a group, they prove to be highly significant.[4] The pattern of consumption by state which is suggested by equations 2.2 and 2.5 is similar, though not identical, to that of the state level models.

Since there is more cross-sectional variation in the utility level data than in the previous data set, it is anticipated that multicollinearity will be less problematic. Indeed, equations 2.2 and 2.5 seem quite satisfactory in terms of overall fit and the statistical significance of individual coefficients. Moreover, the ridge regressions (equations 2.3 and 2.6) do not appear to improve the statistical fit of either model.

While the equations estimated using disaggregated data seem to perform less well than their counterparts at the state level, this is partly[5] because there is more variation to explain. Some of the individual differences reflected in the utility level data tend to cancel when aggregated to the state level. It is perhaps more worrisome to note the presence of incorrect signs on estimated coefficients in Table 2, namely those on the oil price index in the Nordin model and on the income coefficients in both models.

Model 1.2 was re-estimated utilizing the Opaluch specification (outlined above), with an indeterminate result obtained concerning whether people respond to average or marginal price. Simultaneous equation marginal price and average price models were also estimated. The marginal price coefficient was found to be -.67 while the average price model yielded a coefficient of -.301.

IMPACTS OF USING REGIONALLY AGGREGATED DATA

The analysis of the impacts of using aggregated data will focus on two questions: (a) Do the estimated demand elasticities vary systematically with the level of aggregation? and (b) Do the average price, Nordin, and simultaneous equation models provide consistent results at each level of aggregation?

Given the construction of the income data variable, detailed in the Appendix, little importance can be attached to the observed differences in the estimated income elasticities at each level of aggregation. The results with respect to the own price and cross price elasticities can be viewed with more confidence and a relationship between these estimates of price response and the level of aggregation is discernible. Namely, at the utility level, the estimated ex post average price elasticities tend to be larger in absolute value than their counterparts at the state level. To the extent that there is relatively more time series variation in the state level data set, the elasticities in Table 1 may be given a "short run" interpretation, as is common practice in time series analysis. On the other hand, the utility level elasticities, having been computed from a data set in which cross section variation is a more prominent feature, may be viewed as "long run" measures and as such would be expected to be larger in magnitude.

The comparison of the results of the Nordin difference models, the ex post average price models, and the simultaneous equation models is complicated by the simultaneity (in single equation models) between quantity consumed and price(s). Comparisons are straightforward only under rather restrictive conditions. For example, the consumer response to a uniform proportional change in the entire rate structure may be obtained, as suggested by Billings and Agthe [1980], by adding the marginal price and difference elasticities only if intra-marginal quantities remain constant. A more general comparison of Nordin models with the ex post average price models can be made as follows. Let the estimated average price model be written as:

$$\text{Ln } Q = \hat{a} + \hat{b} \text{ Ln } PA + \hat{c} \text{ Ln } Z,$$

and Nordin's model as:

$$\text{Ln } Q = \hat{\alpha} + \hat{\beta} \text{ Ln } PM + \hat{\gamma} \text{ Ln } DIFF + \hat{\delta} \text{ Ln } Z,$$

where Z denotes exogenous variables other than electricity price(s). A one percent change in all rates, with intra-marginal quantities held constant, would induce a $(\beta + \gamma)$ percentage change in quantity consumed by the Nordin model (.22 percent at the utility level, model 2.5; .33 percent at the state level, model 1.6). An indication of whether both models yield consistent results at each level of aggregation was obtained by fitting the following regressions:

Utility level:

$$\text{Ln } PA = .189 + \underset{(13.3)}{.403 \text{ Ln } PM} + \underset{(10.5)}{.057 \text{ Ln } DIFF} \qquad R^2 = .33$$

State level:

$$\text{Ln } PA = -2.377 + \underset{(16.33)}{1.048 \text{ Ln } PM} + \underset{(13.41)}{.332 \text{ Ln } DIFF} \qquad R^2 = .84$$

The utility level regression result implies that a one percent change in all rates induces a .46 percent change in PA and, using 2.2, a .31 percent change in consumption. Consequently, the Nordin model predicts a .22 percent change in quantity while the ex post average price model predicts a .31 percent change. The simultaneous equation ex post average price model predicted a -.301 percent change, while the equivalent simultaneous equation marginal price model predicted a -.308 percent change. Although such comparisons must be viewed with caution, the elasticity values computed at the utility level appear to be much closer than would appear from an examination of Table 2 alone. However, the consistency between the average price, Nordin and simultaneous equation models is much less apparent at the

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[4] For the average price model, F = 12.8 and for the Nordin Model, F = 18.6. Both exceed the critical value $F_{5,\infty} = 3.02$ at the 99% confidence level. The NH of no state level differences in intercepts is rejected.

[5] It can be shown that a natural concomitant of grouping is an increase in R^2. See Cramer [1964].

state level. That is to say, the state level regression above implies that a one percent change in all rates leads to a 1.4 percent change in PA and, from equation 1.3, a .65 percent change in consumption. On the other hand, the state level Nordin model (1.6) would suggest only a .33 percent change in quantity. It may be concluded that while the average price, Nordin and simultaneous equation models yield similar results using a disaggregated data set, divergent results may be obtained in the process of aggregation.

CONCLUSIONS

There has been some controversy on the gains and losses produced by aggregation. Orcutt [1968] has argued that disaggregation always results in more information and a loss in information must accompany aggregation. However, as Grunfeld and Griliches [1960] argue, if the microdata are subject to large errors, compared with macrodata, and if the microrelations are likely to be poorly specified, there could be a gain from using aggregate data rather than the disaggregated data. This paper does not address these broader issues but rather seeks to present some empirical evidence on the impact of aggregation in modelling residential electricity demand. The results presented here suggest that, in general, the use of state level data yields lower ex post average price estimates of own price and cross price elasticities than would be generated at the utility level. In addition, while the average price, Nordin, and simultaneous equation demand models appear to produce quite consistent results at the utility level, the estimated models diverge markedly at the higher level of aggregation. Finally, there appears to be significant state level differences in household consumption behavior and, to the extent that these differences tend to be ignored in regional and national models, the latter may not be adequate for the formulation and evaluation of public policy with respect to the provision of residential electricity.

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APPENDIX

KWH -- The number of kilowatt-hours consumed per year by the average customer, i.e., total residential KWH sold divided by the number of customers. Source: U.S. Federal Power Commission, Statistics of Privately Owned Electric Utilities in the United States and Statistics of Publically Owned Electric Utilities in the United States.

PM -- The marginal price variable in cents. The customer is assumed to consume an equal amount of electicity each month. The price attached to the final block associated with this amount is then the marginal price. Source: United States Federal Power Commission, National Electric Rate Book for each state.

PA -- The ex post average price in cents per KWH, i.e., total residential revenue divided by total residential sales. Source: Statistics of Privately (Publically) Owned Electric Utilities in the United States, 1976 and earlier.

Y -- Income in dollars was determined for three digit zip code areas for 1969 from the U.S. Internal Revenue Service's Statistics of Income: Three Digit Zip Code Data. It is assumed that all towns within the three digit zip code range had the same average income. For those utilities which serve towns across a wide area (such as Massachusetts Electric which serves towns in every county in the state) the state average income is used. It is further assumed that incomes change at a rate equal to the change in cost of living. An additional limitation of this data source is that the figures determined represent average income per income tax return filed separately. Thus the move to two income households in recent years is not taken into account.

CPI API OILI -- The consumer price index, electric appliance price index, and number 2 fuel oil index are all taken from the March 1978 edition of the U.S. Department of Labor, Bureau of Labor Statistic's CPI Detailed Report 1977. These indices are U.S. city averages on a scale where 1967 = 100.

FADJ -- The fuel adjustment change in cents was taken from the National Electric Rate Book as its average through the year.