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DYNAMIC vs. STATIC MODELS FOR EROSION CONTROL POLICY RESEARCH

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ABSTRACT

In recent studies of erosion control in agricultural production it has been standard practice to use timeless or static models of agricultural production processes. In this paper implications of a dynamic model of agricultural production for erosion control decision making are compared to those of a static model. On the basis of this comparison it is suggested that future research on the economics of erosion control and erosion control policy may benefit from the use of dynamic modeling methods.

INTRODUCTION

The economic consequences of public policy measures to reduce cropland erosion as mandated by Federal water pollution control policy developments during the 1970's have been the subject of a number of studies (e.g., Alt and Heady; Miranowski, et al.; Taylor and Frohberg; Wade and Heady; Walker and Timmons; and White and Partenheimer). Among the issues considered have been least-cost management practices for securing specified erosion levels, the responses of profit maximizing farmers to alternative policy strategies, and the costs of alternative policy strategies. In addressing these and other issues the principal approach has involved the analysis of static linear models of agricultural production. Levels of aggregation have ranged from individual representative farms to the nation as a whole. The issue considered in this paper pertains to the typical treatment of erosion control decision making in these models.

While the details of model construction differ from study to study, erosion is typically incorporated as an outcome of production decisions which does not in turn influence the decision process. In these models the soil conservation benefits associated with erosion control have no bearing upon the solution because they are not allowed for in model construction. The models emphasize the costs of erosion control in the form of foregone profits due either to changes in farming practices requiring no outlays for implements or erosion control structures, or to other methods which do require investment expenditures as well as process modifications. Further, these costs are typically considered for a range of alternative erosion rates without modification of model parameters to reflect the changes in soil productivity and erosivity which result from changes in the erosion rate. But erosion control is not without benefits. Accelerated erosion rates deplete topsoil, alter soil structure, and have other impacts which typically reduce soil productivity while increasing soil erosivity. In considering the responses of farmers to policy initiatives and the costs of those initiatives, the incentives arising from conservation benefits may have an important bearing on the results.

The presence of the intertemporal interdependence between the costs and returns to agricultural production at different points in time implies that in investigating the economics of erosion control the appropriate analytical approach calls for dynamic modeling, at least to the extent that the changes occuring over time are significant. This view is emphasized by early works on soil conservatuon such as those of Bunce, Ciriacy-Wantrup, and Heady. This has not, however, been the approach taken in research on erosion control except in a very few cases (e.g., Burt; Frohberg and Swanson; and Shortle). Instead, the fundamental underlying soil conditions are essentially treated as being invariant with respect to the rate of erosion. Representative farm firms, considered individually or in aggregation, are in essence modeled as wealth maximizers with wealth maximized by independent periodic profit maximizing behavior. Stated differently, static analysis has been used to analyze a problem inherently dynamic in character.

The purpose of this paper is to note weaknesses in static treatments of erosion control which limit their usefulness to policy makers. A simple abstract dynamic model of erosion control is first presented. The analysis then proceeds to consider the implications for erosion control decision making of a related static model. Comparisons between the two models are then drawn and implications for policy research noted.

DYNAMIC EROSION CONTROL

Soils, as noted by Ciriacy-Wantrup, are a composite of many interrelated stock and flow resources. Consequently, when modeling soil management a full treatment requires as a foundation an inventory of the resources which together form the composite, a system of equations of motion which describe the behavior of the inventory in response to internal and external forces, natural or otherwise, and a relationahip between the inventory at a point in time, production inputs, and the flow of outputs. The inventory may be referred to as a state vector, as it describes the state of the resources, and the relationship between product and input flows given the soil resource inventory or state vector is, of course, a production function.

The complexities and complications inherent in a full treatment are not, however, required to address the issues which are the subject of this analysis. Thus, to focus on the issues at hand, several simplifying assumptions are made. The first is that the productivity and erosivity of a particular soil are dependent upon the depth of the topsoil. This assumption captures the crucial concern: Erosion, by depleting the topsoil,

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reduces the productivity of the soil while increasing the erosivity of the soil. Further, this assumption is not without some basis, for in agronomic discussions of the issue, reference is often made to rules of thumb regarding yield losses per inch of topsoil loss. The time rate of change in soil depth is written:

$$\dot{S}(t) = R(t) - E(t) \tag{1}$$

where $S(t)$ is soil depth at time t, $E(t)$ is the depletion of soil depth by erosion at time t, and $R(t)$ is the augmentation of soil depth by soil building processes at time t. It is assumed here that the rate of soil genesis, $R(t)$, is determined by natural processes independent of soil depth and management. This assumption simplifies the analysis without detracting from the results.

There is a wide range of factors influencing erosion on cropland, including the erosivity of the soil, slope length and steepness, and farming practices. On any parcel of cropland, the exercise of erosion control can be accomplished in a variety of manners. These range from changes in crop management practices requiring little or no direct erosion control expenditures to investments in major erosion control structures, such as terraces, requiring considerable direct control expenditures. Because the range of alternatives is broad, and because control can be secured without direct expenditure of resources, it is useful to simplify the analysis by proceeding in the discussion with the term "erosion control effort." The idea "effort" is meant to convey is simply that, relative to a baseline situation, additional erosion control requires positive action, a fact well established by the research. Effort may be thought of as an index of the extent of control measures. For example, an index of sorts of erosion control effort is provided by the "C" or "crop management" factor of the Universal Soil Loss Equation of Wischmeier and Smith. While a variety of control measures can be adopted to secure additional control, control measures can basically be subdivided into two types: structural measures, such as installation of terraces, and nonstructural measures, such as changes in crop mix or tillage practices. Thus, erosion control effort is further subdivided into "structural effort" and "nonstructural effort."

With these comments assume that erosion at any point in time is given by

$$E(t) = g(X(t), K(t), S(t), t) \tag{2}$$

where $X(t)$ is nonstructural erosion control effort and $K(t)$ is structural erosion control effort at time t. Factors other than erosion control effort and the state of the soil resource, as described by soil depth, influencing erosion are taken to be known functions of time. As soil erosivity generally increases as erosion reduces soil depth it is assumed the $g_s > 0$. The first-order partial derivatives of structural and nonstructural control effort are assumed negative ($g_x < 0$ and $g_k < 0$) implying increased erosion control effort leads to reduced erosion.

At any point in time, the maximum profit obtainable from agricultural production is assumed to be given by

$$\pi(X(t), K(t), S(t), t). \tag{3}$$

This profit function is notionally obtained by solving the instantaneous profit maximization problem for time t treating $X(t)$, $K(t)$, and $S(t)$ parametrically. Other production decisions influencing profits are solved in specifying the profit function and thus the function gives the profit for time t given specified values of $X(t)$, $K(t)$, and $S(t)$. This specification permits a focus on erosion control decision making for it allows the analysis to abstract from other production decisions. Assuming that the profit obtainable is positively related to soil productivity which is assumed to be positively related to soil depth, it follows that $\pi_s > 0$. Erosion control costs at a point in time can take two forms. The first is outlays for structures and implements used in conservation management practices. But in addition, both structural and nonstructural practices involve process modifications and existing research generally shows these to be costly. The second form of control costs is incorporated in the profit function and accordingly it is assumed that $\pi_x < 0$ and $\pi_k < 0$.

Investment outlays are assumed only to be associated with structural control effort to simplify the analysis. This assumption does not detract from the principal results. To introduce these outlays, assume that structural effort is proportional at a rate of one-to-one with a stock or erosion control capital. The capital stock and structural effort at time t are both denoted by $K(t)$. This assumption is in keeping with the standard treatment of capital in production functions. What appropriately enters production functions is not the capital stock but the flow of capital services. But assuming the flow to be proportional to the stock, the latter may replace the former. The time rate of change of the stock is assumed to be given by

$$\dot{K}(t) = I(t) - \delta K(t) \tag{4}$$

where $I(t)$ is investment in erosion control capital at time t and δ is a depreciation factor. The term $\sigma(t)I(t)$ then gives investment outlays where $\sigma(t)$ is the constant marginal investment cost. Adding $\sigma(t)I(t)$ to (3), a complete specification of the profit at time t is obtained.

Assume now a wealth maximizing farmer or soil resource manager with a planning horizon $0 \leq t \leq T$. Let $V[K(T), S(T), T]$ denote the terminal market value of the resource at time T given a soil depth of $S(T)$ and an erosion control capital stock of $K(T)$ in place.[1] The management problem

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[1] A finite planning horizon is assumed as a matter of convenience for interpreting optimality conditions and also as a realistic consideration. However, it is inappropriate to assume the soil resource and the erosion control capital stock to be without value beyond the end of the planning horizon. Thus, the terminal value function is introduced. Assuming the resource manager to be a wealth maximizer, the market value of these assets at the terminal time is seemingly the appropriate terminal value. If the market performs efficiently the

is to choose trajectories for the two control variables, nonstructural erosion control and investment in erosion control capital, to maximize the present value of the resource

$$\int_0^T \{\pi(X(t),\, K(t),\, S(t),\, t) - \sigma(t)I(t)\}e^{-rt} + V(K(T),\, S(T),\, T)e^{-rT} \tag{5}$$

subject to the equations of motion

$$\dot{S}(t) = R(t) - g(X(t),\, K(t),\, S(t),\, t) \tag{6}$$

$$\dot{K}(t) = I(t) - \delta K(t) \tag{7}$$

and subject to the initial conditions $K(0) = K_o$ and $S(0) = S_o$. A solution to this problem, if one exists, may be characterized by the first-order necessary conditions of the "maximum principal technique" developed by Pontryagin. To obtain these let the Hamiltonian be

$$\begin{aligned} H = \{&\pi(X(t),\, K(t),\, S(t),\, t) - \sigma(t)I(t) \\ &+ \lambda(t)[R(t) - g(X(t),\, K(t),\, S(t),\, t)] \\ &+ \rho(t)[I(t) - \delta K(t)]\}e^{-rt}. \end{aligned} \tag{8}$$

The conditions of interest for an interior solution are:

$$\frac{\partial H}{\partial X(t)} = \pi_x - \lambda(t)g_x = 0; \tag{9}$$

$$\frac{\partial H}{\partial I(t)} = \rho(t) - \sigma(t) = 0; \tag{10}$$

$$-\frac{\partial H}{\partial S(t)} = -(\pi_s - \lambda(t)g_s) = \dot{\lambda}(t) - r\lambda(t); \tag{11}$$

$$\begin{aligned} -\frac{\partial H}{\partial K(t)} &= -(\pi_k - \lambda(t)g_k - \rho(t)\delta) \\ &= \dot{\rho}(t) - r\rho(t); \end{aligned} \tag{12}$$

$$\text{and } \lambda(T) = \partial V/\partial S \text{ and } \rho(T) = \partial V/\partial K. \tag{13}$$

To interpret these conditions it is useful to assume g_x, g_k, and g_s to be constants to simplify notation. Using (11), (12) and (13), it may be established that in an optimal program

$$\lambda(t) = \int_t^T \pi_s e^{-(r+g_s)(s-t)} ds + \frac{\partial V}{\partial S} e^{-(r+g_s)(T-t)} \tag{14}$$

and

$$\rho(t) = \int_t^T \{\pi_k - \lambda(t)g_k\}e^{-(r+\delta)(s-t)} ds + \frac{\partial V}{\partial K} e^{-(r+\delta)(T-t)} \tag{15}$$

In an optimal program the multipliers $\lambda(t)$ and $\rho(t)$ are interpreted as the marginal values of soil depth and erosion control capital respectively at time t. From (14) it is evident that the former is equivalent to the discounted marginal terminal value of the stock of soil depth plus the stream of discounted marginal profits generated by a marginal change in the stock. As such it may be defined as the marginal user cost of the stock, following Scott, and as the marginal cost of erosion or the marginal benefit of erosion control. Note that the effective discount factor is adjusted downward to reflect the reduction in future erosivity and thus the increase in future soil productivity resulting from increases in the stock. From (15) it is evident that the marginal value of erosion control capital or structural erosion control at time t is equivalent to the present marginal terminal value of the stock plus the stream of marginal erosion control benefits generated (as indicated by the presence of the product of the marginal benefit of erosion control and the marginal product of structural erosion control $-\lambda(s)g_k$) less the stream of discounted marginal profits foregone by structural control. The effective discount factor in this case is adjusted upward to reflect the decay in the stock of capital over time.

Equation (11) is the marginal condition for the optimal stock of soil depth at time t. This condition may be rewritten as

$$\pi_s - \lambda(t)g_s + \dot{\lambda}(t) = r\lambda(t). \tag{16}$$

The quantity π_s is marginal profit generated by the stock at time t. The quantity $-\lambda(t)g_s$ is the marginal value of the stock in reducing soil erosion at time t and thus the marginal value of the stock in preserving future soil productivity. The term $\dot{\lambda}(t)$ is the time rate of change in the marginal value of the stock and thus the rate of capital gain. Rearranging (16) to obtain

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = r + g_s - \frac{1}{\lambda(t)}\pi_s \tag{17}$$

it is evident that the percentage growth rate in the marginal value of the stock of soil depth will be less than the rate of discount. Thus, in equilibrium under the assumptions made here the own rate of return on soil depth is less than the discount rate reflecting the presence of flow benefits from holding soil as an asset. The three terms on the left-hand side of (16) compose the marginal benefit of holding the stock of soil at time t. The right-hand term $r\lambda(t)$ is the marginal opportunity cost of holding the stock. Thus, at any time $0 \leq t \leq T$ the optimal stock of soil depth must be such that the marginal benefit from holding the stock is equivalent to the marginal opportunity cost.

Using (10) and (12) the marginal condition for the optimal erosion control captial stock at time t may be written

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market value will be the willingness-to-pay for the assets. But in equilibrium this will be the maximum present, value as of the specified date, of the flow of net returns generated by the assets. This present value is contingent upon the remaining stocks at the terminal time.

$$\pi_k - \lambda(t)g_k = \sigma(t)(r+\delta) - \dot{\sigma}(t). \tag{18}$$

The right-hand side of (18) is the usual way of representing the marginal cost of holding reproducible capital at a point in time. The left-hand side term π_k is the marginal process modification cost or foregone profit at the margin from holding erosion control capital at time t and $-\lambda(t)g_k$ is the marginal value of the stock in reducing erosion and thus in preserving future soil productivity. Thus, at any time $0 \leq t \leq T$ it is necessary in an optimal program that the marginal benefit of the stock or erosion control capital net of the marginal process modification cost equal the marginal opportunity cost of holding the stock.

Equation (9) is the marginal condition for nonstructural erosion control at time t. The quantity π_x is the marginal profit foregone by undertaking nonstructural control or the marginal process modification cost. The quantity $-\lambda(t)g_x$ is the marginal value at time t of nonstructural erosion control. Thus, at any time $0 \leq t \leq T$ in an optimal program the marginal process modification cost must be equivalent to the marginal value of nonstructural erosion control. Equation (10) is the marginal condition for erosion control investment and is simply the equality of the marginal cost of investment and the marginal value of the erosion control capital stock.

For the purpose of this analysis the important implication of the foregoing results is that when soil erosion depletes the soil resource, incentives arise for erosion control effort. In the marginal conditions for both structural and nonstructural effort the presence of these incentives is indicated by the presence of the marginal user cost of the soil stock. This marginal user cost reflects both the direct gains to soil conservation in the form of productivity benefits and the indirect gains in the form of diminished soil erosivity. It is to be noted, too, that the marginal user cost is an endogenous quantity obtained by solving the dynamic soil management problem.

STATIC EROSION CONTROL

It was noted previously that the conservation benefits of erosion control have largely been ignored in previous research. The essence of the implications for erosion control decision making of not accounting for these benefits can be illustrated in an oversimplified fashion as follows: Assume the profit and erosion functions to be invariant with respect to time and $\sigma(t)$ to be constant for all time. Viewing the production set to be uninfluenced by erosion then implies that the present value maximizing erosion control plan can be identified by solving the initial period profit maximization problem:

$$\max \pi(X(0), K(0), S(0), 0) - \sigma(0)K(0). \tag{19}$$

Allowing now for the nonnegativity of erosion control effort and for boundary solutions, the Kuhn-Tucker conditions characterizing the solution are:

$$\pi_x \leq 0, \ (\pi_x)X(0) = 0; \tag{20}$$

$$\pi_k - \sigma(0) \leq 0, \ (\pi_k - \sigma(0))K(0) = 0. \tag{21}$$

It is evident that since $\pi_x < 0$, $\pi_k < 0$, and $\sigma(0) > 0$ the solution to this problem will entail $X(0) = K(0) = 0$. Thus, this static model indicates no erosion control effort to be profit maximizing erosion control effort.

The principal distinction between the optimality conditions for erosion control effort between the static model and the dynamic model is the absence of the incentives for erosion control in the former. The reason for this absence is that the gains from erosion control are realized in the future but the future is unaccounted for in this static model. This deficiency of the static approach could be remedied by imputing a value of erosion control effort and incorporating the imputation in the static model. But the correct imputation is the marginal user cost and as this is an endogenous quantity obtained by solving the dynamic problem, it remains that the dynamic approach is in principal appropriate when the damages of soil erosion are significant.

In many respects, the existing research on erosion control has been directed to estimating functions analogous to (19). Specifically, an aspect of most recent studies has been to identify management practices which can achieve specified erosion levels at least cost, given a specified set of soil conditions. However, (19) alone does not provide enough information to formulate conclusions about management responses to policy initiatives and the costs of alternative control strategies if there are significant benefits from soil erosion control.

IMPLICATIONS

In formulating effective and efficient erosion control policy measures it is important that policy makers have an understanding of both the responses of farmers to policy initiatives and the costs of those responses. The static model presented above, while a considerable oversimplification, is in keeping with the fundamental economic characteristics of the economic optimization models constructed to examine erosion control policy issues as they relate to erosion control decision making. While investigations based on static optimization models have generated valuable information regarding both responses to and costs of alternative policy approaches, the foregoing analysis suggests the presence of qualifying weaknesses in the approach when the conservation benefits of erosion control are significant.

The most obvious weakness stemming from the absence of erosion control incentives associated with conservation benefits is that the accuracy of predictions of the responses to alternative policy strategies, and the costs of those responses may be adversely affected because the decision process is mischaracterized. For example, for a given set of specified conditions at a given point in time there may be a bias towards overstating erosion because the incentives for erosion control are understated. This would suggest in turn that the degree of control required to achieve target levels of erosion may be overstated. As another illustration, consider the implications for an analysis of the subsidies necessary to secure the adoption of specified conservation practices. If in modeling the decision process the existing incentives for adoption are understated because the model is not constructed to generate conservation benefits of erosion control, it is likely that the subsidies necessary to yield adoption will be overstated. And by implication, the economic costs of adoption will be overstated.

As noted above, biases arising in static analyses due to the absence of endogenously generated soil conservation benefits from erosion control can be adjusted for by introducing imputed values. However, it was also noted that the correct imputation is endogenously determined in the solution to the dynamic problem. Further, there are potential advantages to examining erosion control issues in a dynamic framework which are beyond potential improvements in the accuracy of estimated outcomes of erosion control policy. One important feature is that the solution to a dynamic optimization problem is composed of a set of time paths for control and state variables. This type of solution may be of considerable value to policy makers for it will be inherently indicative of future trends under specified circumstances. The static optimization models typically used to examine the issues are limited in this respect.

Based on these kinds of considerations it is suggested that when investigating issues in the economics of erosion control and erosion control policy, the preferred approach is one in which the soil conservation benefits from erosion control are incorporated into the analysis. And as suggested above, the preferred manner for incorporating these is by constructing dynamic models of erosion control which explicitly account for the impacts of erosion on the soil resource and for the impacts of changes in the soil resource on the costs and returns in agricultural production. In general this will require in practice the construction of dynamic programming models which are generally unwieldy and costly to solve for problems of any complexity. This is doubly true of stochastic dynamic programming, but as there are stochastic features inherent in erosion control problems, such as the occurrence of rainfall and other natural events which drive erosion processes, stochastic dynamic programming may be appropriate for the investigation of some problems. Of course, in choosing the appropriate types of models, informational gains from more complex techniques must be weighed against the costs. For many soils the damages associated with high erosion rates may be so small as to have no important economic bearing. The difficult decisions in modeling arise when the damages are significant and a priori it would, as a matter of general principal, be preferable to adopt dynamic models in future research in these situations.

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