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MULTIPLE OBJECTIVE DECISION MAKING: GENERATING TECHNIQUES OR GOAL PROGRAMMING?

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Abstract. Two approaches to formal multiple objective decision analysis are described, illustrated and compared. These two approaches, generating techniques and goal programming, are illustrated by the solution of a simple numerical example. This solution combined with some lessons from the field of psychology provide a basis for some judgments on the relative merits of the approaches.

INTRODUCTION

Dozens of techniques have been advanced during the past decade or so for assisting with public decision making based on multiple conflicting objectives. The field of mathematical programming of relevance for this purpose is called vector optimization. Some of the techniques in this field are confined to the generation of the set of noninferior solutions and do not require the prior articulation of preferences by the ultimate decision maker (or investigator). The remaining approaches do require a prior or a sequential articulation of preferences and they result in a single *optimum* solution rather than a set of noninferior solutions.

Unfortunately, applications of these techniques, particularly the generating techniques, have lagged. In Castle's (p. 3) words, "Despite considerable recent work on multiple objective planning, we have not progressed very far unless we can reduce objectives to some common denominator and maximize net social product." And later he suggests the existence of an "alarming gap" (p. 16) between what the academic suggests for public policy and what is done in practice. Bromley's comments are similar. What kind of multiple objective analysis is it when we impose a utility function (perhaps with economic efficiency given a weight of one and all other objectives zero), solve for the *optimal* solution, and then tell decision makers what decision to make?

This paper describes and contrasts vector optimization techniques of the type that impose a value system and then solve for the optimal solution and the type which provides alternative "efficient" solutions and requires the decision maker to make choices in the face of conflicting objectives. Specifically, we compare goal programming as an example of the former type of approach with techniques designed to generate the efficient solution set. The two approaches are briefly described;¹ the solution procedures are illustrated with a numerical example; and the lessons from psychology are brought to bear on the issue of relative merit of the alternative approaches.

VECTOR OPTIMIZATION AND NONINFERIOR SOLUTION SPACE

The multiple objective problem can be described in its most general form as:

- (1) $\max z(x) = G[z_1(x), z_2(x), \dots, z_k(x)]$,
subject to
(2) $x \in X$
(3) $x \geq 0$,

where $z(x)$ is the k -dimensional objective function (there are k objectives); x is an n -dimensional vector of decision variables; (2) is the set of m constraints operating on the problem; and (3) is the set of nonnegativity conditions. The region defined by (2) and (3) in n -dimensional Euclidean space is referred to as the feasible region in decision space. Each feasible solution to this problem implies a value for each objective ($z_i(x)$, $i = 1, \dots, k$) and the k -dimensional objective function (1) maps the feasible region in decision space (X) onto the feasible region in objective space $z(X)$, which is defined in k -dimensional Euclidean space.

Of course, a vector like (1) cannot be optimized, at least not without information about preferences (a specification of the operator G) with which to combine objectives. Without such information the objectives are *incommensurable* and some solutions cannot be compared. Thus, it is only by introducing value judgments into the solution approach that a complete ordering, and an optimal solution, can be obtained for a vector optimization problem. This introduction of value judgments underlies the multiple objective decision-making approaches other than the generating techniques.

Even in the absence of value judgments, however, a portion of the feasible solutions can be dismissed on the basis of the incomplete ordering associated with the vector objective function. With the assumption that more of each objective is preferred to less, only the noninferior solutions are relevant. A noninferior solution is a feasible solution to the problem, $x \in X$, such that no other feasible solution, $\bar{x} \in X$, exists for which

$$z_p(\bar{x}) > z_p(x)$$

for some $p = 1, 2, \dots, k$, and

$$z_i(\bar{x}) \geq z_i(x)$$

for all $i \neq p$. Noninferior solutions can be defined in decision space (X^*) as well as in objective space ($z(X^*)$), since each noninferior solution $x \in X^*$ implies a value for each of the k objectives $z(x)$.

A numerical example may be useful in illustrating these definitions. Suppose we are faced with the following two-objective problem:

$$(4) \max z(x) = [z_1(x), z_2(x)],$$

where

$$z_1(x) = 2x_1 - x_2$$

$$z_2(x) = -x_1 + 3x_2,$$

subject to

$$(5) x_1 + x_2 \leq 8$$

$$(6) x_1 - 2x_2 \leq 4$$

$$(7) x_1 \leq 5$$

$$(8) x_2 \leq 6$$

$$(9) x_1, x_2 \geq 0$$

Since decision space and objective space are both two-dimensional, it is a simple matter to depict these feasible regions graphically. Figure 1 shows the feasible region (X) and the set of noninferior solutions (X^*) in decision space and Figure 2 features the feasible region ($z(X)$) and the noninferior set ($z(X^*)$) in objective space.

For this simple example, the feasible region in objective space ($z(X)$) was found by enumerating all extreme points and

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¹Willis and Perlack provide a more detailed description. To facilitate translation, the same notation is adopted here.

calculating the values of each objective at these solutions. The noninferior set ($z(X^*)$) was then obtained by application of the definition of noninferiority. Obviously, this procedure is seldom feasible for realistic problems, and some other technique must be used.

Generating Techniques

The weighting and constraint methods are the most common of the generating techniques. The weighting method involves solving for noninferior solutions by converting the original problem (1)-(3) to a scalar optimization problem in which the objective function is simply a weighted sum of the arguments of (1). That is:

$$(10) \max \sum_{i=1}^k w_i z_i(x)$$

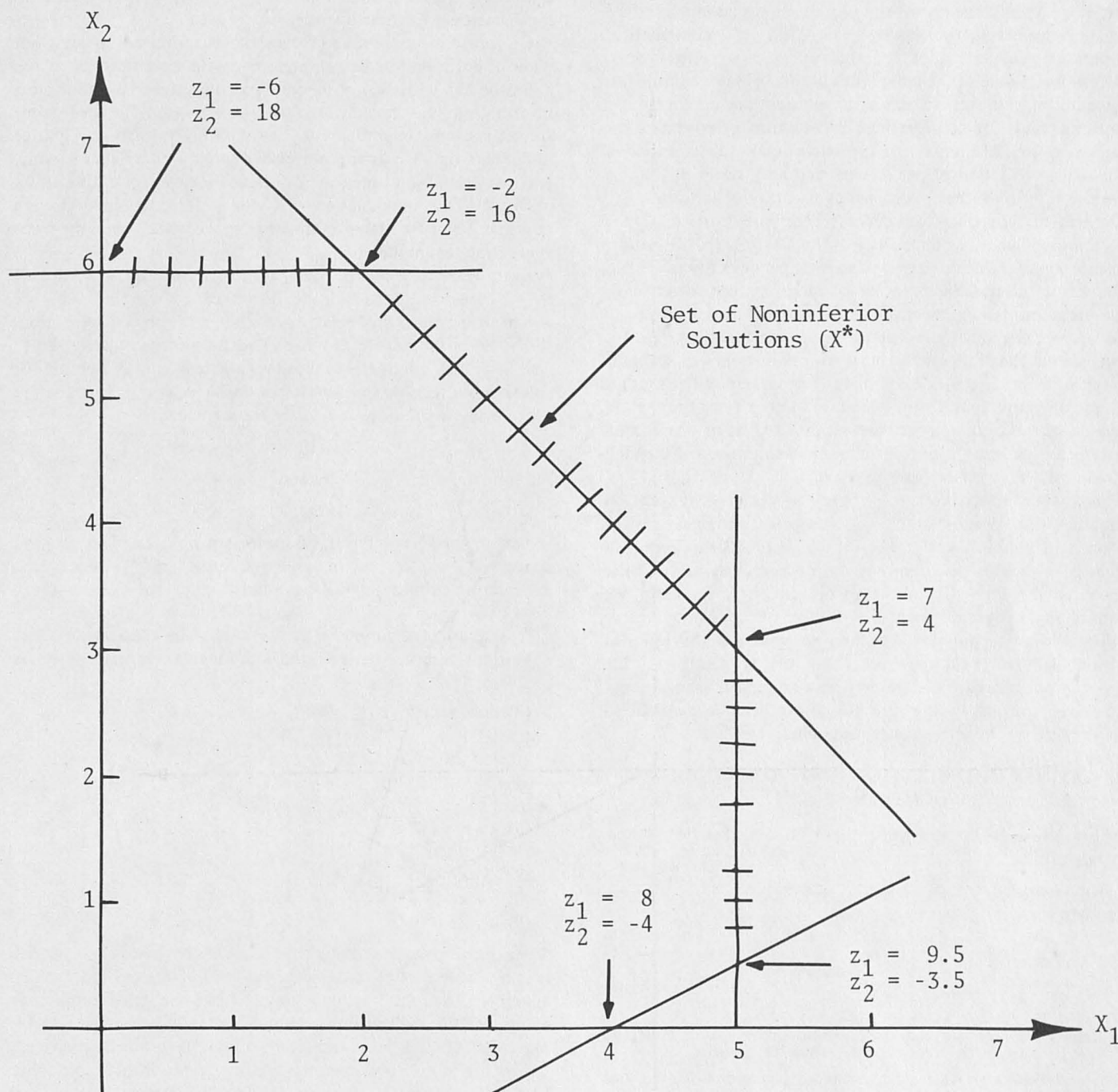
subject to

$$(11) x \in X$$

$$(12) x \geq 0,$$

where $w_i \geq 0$ for all i and strictly positive for at least one. Thus X^* and $z(X^*)$ can be generated by parametric variation of w_i with existing mathematical programming software packages.

Figure 1.
Decision Space Feasible Region X
and Noninferior Solution Set X^*



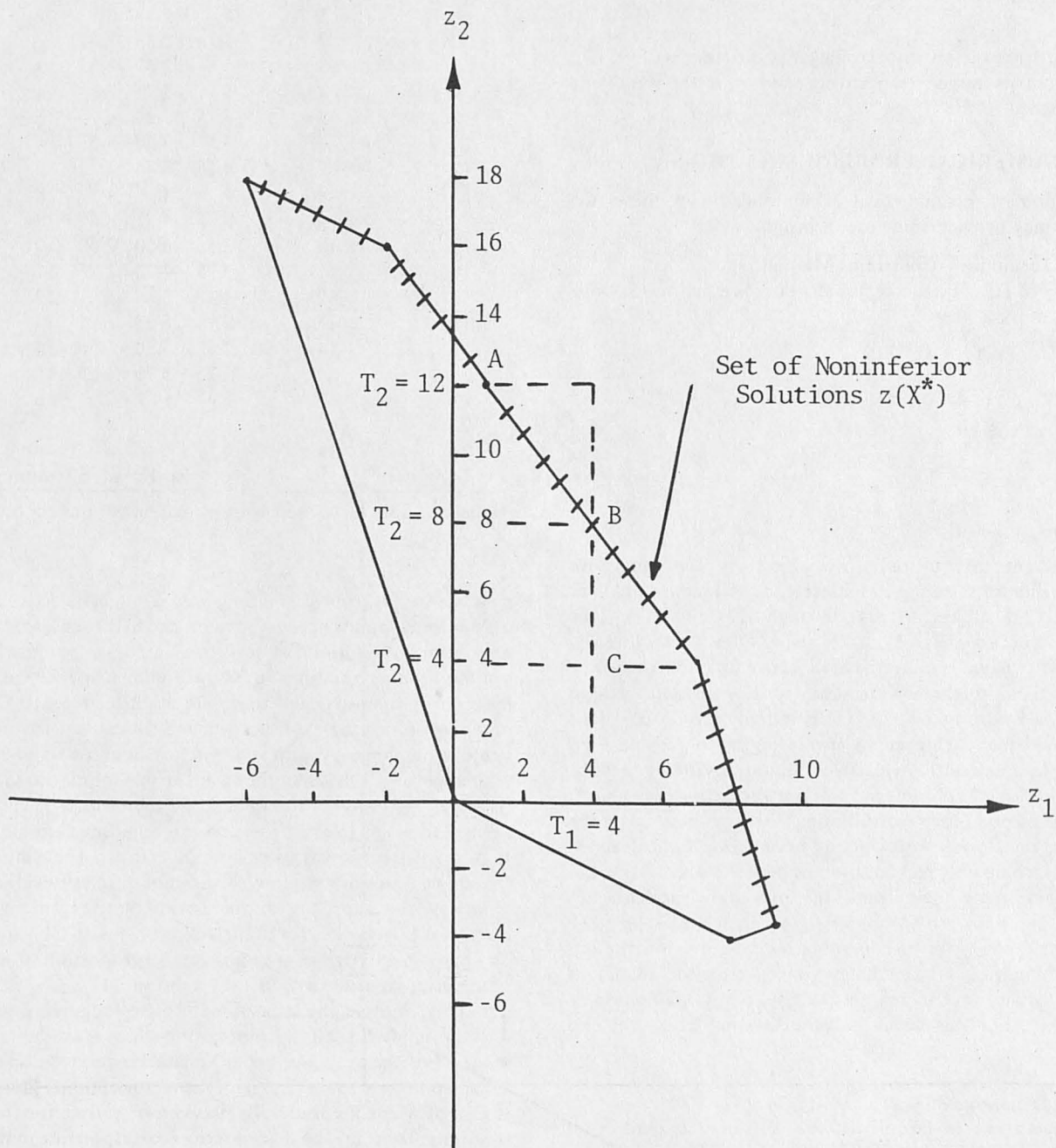
The constraint method is essentially the dual of the weighting approach. It also involves finding noninferior solutions by solving a scalarized version of the original multiple objective problem. Since it is relative values of weights that are important, the approach involves selecting the j^{th} objective to maximize subject to the problem constraints plus lower bound constraints on the remaining $k-1$ objectives. Thus, the original problem becomes:

- (13) $\max z_j(x)$
 subject to
 (14) $x \in X$
 (15) $z_i(x) \geq b_i \quad i \neq j,$
 (16) $x \geq 0,$

where b_i are the lower bounds on the $k-1$ objectives. The initial lower bounds can be set by solving the k problems of maximizing (13) subject to (14) and (16), where each j ($j = 1, \dots, k$) is used in (13) in the k separate solutions, substituting the values of x for each of the k optimal solutions into $z_i(x)$, and selecting for each $z_i(x)$ the lowest of the k values as b_i . The noninferior solution set is then found by solving (13)-(16) with parametric variation of b_i and substitution of each $z_i(x)$, $i \neq j$, into (13).

The constraint method is generally preferred to the weighting method. While they appear quite similar, the weighting method will not span the noninferior solution space if the objective space is

Figure 2.
 Objective Space Feasible Region $z(X)$
 and Noninferior Solution Set $z(X^*)$



not strictly convex. This limitation does not apply to the constraint method.²

Goal Programming

Goal programming is one of the first and most popular of the techniques which require an articulation of preferences prior to solution. Developed by Charnes and Cooper in 1961, it is based upon minimization of the sum of weighted absolute deviations of objectives ($z_i(x)$) from targets (T_i). Thus, the goal programming formulation of the initial multiple objective problem is:

$$(17) \min \sum_{i=1}^k w_i(d_i + e_i)$$

subject to

$$(18) x \in X$$

$$(19) x \geq 0$$

$$(20) z_i(x) - d_i + e_i = T_i, \quad i = 1, \dots, k$$

$$(21) d_i, e_i \geq 0,$$

where d_i and e_i are the positive and negative differences of the i^{th} objective from its target, respectively, and w_i is the weight or priority attached to the i^{th} goal.

NUMERICAL PROBLEM SOLUTIONS

The solution of the numerical example (4)-(9) by these two approaches may provide some useful insights.

Generating Technique—Constraint Method

If we set j of (13) equal to (1) and i of (15) equal to (2),³ our problem becomes:

$$(23) \max z_1 = 2x_1 - x_2$$

subject to

$$(24) x_1 + x_2 \leq 8$$

$$(25) x_1 - 2x_2 \leq 4$$

$$(26) x_1 \leq 5$$

$$(27) x_2 \leq 6$$

$$(28) -x_1 + 3x_2 \geq b_2$$

$$(29) x_1, x_2 \geq 0.$$

For the present case of only two objectives, the procedure described earlier for selecting an initial bound (b_2) merely involves maximizing (23) subject to (24) through (27) and (29). This maximization occurs at $x_1 = 5$, $x_2 = .5$, and $z_1 = 9.5$. Substitution of these values for x_1 and x_2 into the z_2 objective function yields $z_2 = -3.5$. This, then, is the lowest value of z_2 consistent with a noninferior solution, and b_2 of (15) is initially set at -3.5 . This constraint was then tightened by increasing b_2 to -3 , and then successively by a unit until no feasible solution was obtained (at $b_2 = 19$). The resulting 23 noninferior solutions are shown in Table 1.

A number of points are worth noting. First, without using the information we already knew about noninferior basic feasible solutions (which we only had because the problem was so trivial as to allow complete enumeration), the procedure mechanically found these solutions—viz., solutions 1, 9, 21, and 23. The other solutions in Table 1 are also noninferior solutions on the X^* boundary of Figure 1. If the increments on b_2 were made smaller, of course, one could move to any desired level of approximation of X^* and $z(X^*)$. For this linear, two-dimensional case, reducing

these increments would be without substantial value. For the nonlinear situation of higher dimensionality, the size of the increments (degree of approximation) becomes an important question.

Table 1
Noninferior Solutions

Solution Number	Constrained Level of Objective 2 b_2	Decision Space		Objective Space		Shadow Price*
		x_1	x_2	z_1	z_2	
1	-3.5	5.	.50	9.50	-3.5	0
2	-3	5.	.67	9.33	-3	.33
3	-2	5.	1.	9.	-2	.33
4	-1	5.	1.33	8.67	-1	.33
5	0	5.	1.67	8.33	0	.33
6	1	5.	2.	8.	1	.33
7	2	5.	2.33	7.67	2	.33
8	3	5.	2.67	7.33	3	.33
9	4	5.	3.	7.	4	.75
10	5	4.75	3.25	6.25	5	.75
11	6	4.5	3.50	5.5	6	.75
12	7	4.25	3.75	4.75	7	.75
13	8	4.	4.	4.	8	.75
14	9	3.75	4.25	3.25	9	.75
15	10	3.5	4.50	2.50	10	.75
16	11	3.25	4.75	1.75	11	.75
17	12	3.	5.	1.	12	.75
18	13	2.75	5.25	.25	13	.75
19	14	2.5	5.50	-.50	14	.75
20	15	2.25	5.75	-1.25	15	.75
21	16	2.	6.	-2.	16	.75
22	17	1.	6.	-4.	17	2.
23	18	0	6.	-6.	18	2.
24	19	No Feasible Solution				

*Dual variable associated with primal constraint on objective 2.

While in linear programming, with a single objective function, only extreme points need to be examined in the sense that if there is an optimum solution, at least one extreme point will be an optimum; other solutions in X^* of Figure 1 or $z(X^*)$ of Figure 2 may be of interest when there are additional goals. Clearly, a mapping of the usual convex social indifference curves on Figure 2 could show tangency with $z(X^*)$ at any point on its boundary. A related observation is that the basic feasible solution at $x_1 = 4$, $x_2 = 0$ ($z_1 = 8$, $z_2 = -4$) was not found in the 23 solutions of Table 2. This is because it is an inferior solution (an adjacent extreme point of Figure 1 has better values of both objectives). The elimination of this single (inferior) extreme point is not dramatic evidence of the ability of the approach to dismiss alternatives on the basis of dominance. However, for nontrivial problems (which are difficult or impossible to depict graphically) the proportion of inferior to noninferior solutions is generally substantial.

Finally, without the imposition of value judgments, generating techniques provide all the information which is available from the model. For the example, Table 1 characterizes decision space (x_1 , x_2) and objective space (z_1 , z_2) for the vector optimization problem. It also provides the trade-offs between objectives for the various solutions. These are the shadow prices corresponding to the second objective treated as an inequality constraint. The machine has

²This is demonstrated by Cohon and Marks.

³The selection of z_1 or z_2 to enter the objective (23) is arbitrary and does not affect the generation of noninferior solutions. See Haimes, *et al.* for proof that this approach provides noninferior solutions for the bicriterion case.

produced the sort of information most natural to decision makers. The decision makers might look, for example, at solution 7. They would observe the decision and the implied levels of objectives (7-2/3 units of the first and 2 of the second). Further, the shadow price of .33 suggests that a small unit relaxation of the constraint on goal two would bring about a one-third unit improvement in the first goal. If that seems a good trade, then solution 6 is better, and vice versa. The same evaluation can be made between other pairs of solutions in winnowing the available alternatives.

Goal Programming

Recall that the GP method involves transforming a vector optimization problem (1)-(3) into a scalar optimization problem (17)-(21), in which the weighted absolute difference between the levels of the objectives and their preassigned targets is minimized. For the same problem, this appears as:

$$(30) \min \sum_{i=1}^2 w_i(d_i + e_i)$$

subject to

$$(31) x_1 + x_2 \leq 8$$

$$(32) x_1 - 2x_2 \leq 4$$

$$(33) x_1 \leq 5$$

$$(34) x_2 \leq 6$$

$$(35) 2x_1 - x_2 - d_1 + e_1 = T_1$$

$$(36) -x_1 + 3x_2 - d_2 - e_2 = T_2$$

$$(37) x_1$$

where w_i is the penalty weight attached to exceeding or falling short of the target on the i^{th} objective, d_i is the quantity by which the target on objective i is exceeded and e_i is the quantity by which objective i falls short of its target.

While the generating technique mechanically traces out X^* and $z(X^*)$, with no information required of the decision maker or knowledge of the location of the feasible region, the goal programming procedure is at an impasse until the decision maker supplies w_i and T_i . Now, if he or she is very clear about values of these targets and weights, and does not wish to explore the noninferior space, obtaining the solution is straightforward. The GP approach excels in these situations. In the more usual case in which they are uncertain about targets and weights, especially since they are ignorant of the range of feasible alternatives (including X^* and $z(X^*)$), sensitivity analysis on w_i and T_i is generally prescribed. But this is no simple (and mechanical) task. Referring back to Figure 2, if knowledge of the set of feasible solutions shown there were *not* available, how would one go about setting values of w_i , T_i which would reveal $z(X^*)$? There would be ways of generating this information in most cases by GP, but this is not what GP was developed for and it would be relatively inefficient at finding the noninferior space.

To illustrate the technique, three different target pairs are selected. These correspond to the pairs highlighted in Figure 2: the inferior pair ($T_1 = 4$, $T_2 = 4$), the noninferior solution ($T_1 = 4$, $T_2 = 8$), and the infeasible solution ($T_1 = 4$, $T_2 = 12$). Since the first two target pairs will be insensitive to weights, they are given equal weightings ($w_1 = w_2 = .5$). The third case is solved with equal weights as well as with a situation each in which the deviation of one objective from its target is penalized three times as heavily as the other. The results of these five solutions are provided in Table 2.

Table 2
Goal Programming Solutions

Situation	Targets		Weights		Decisions		Objectives		Deviations			
	T_1	T_2	w_1	w_2	x_1	x_2	z_1	z_2	d_1	e_1	d_2	e_2
1	4	4	.50	.50	3.2	2.4	4	4	0	0	0	0
2	4	8	.50	.50	4	4	4	8	0	0	0	0
3	4	12	.50	.50	3	5	1	12	0	3	0	0
4	4	12	.25	.75	3	5	1	12	0	3	0	0
5	4	12	.75	.25	4	4	4	8	0	0	0	4

The first situation is the solution labeled C in Figure 2. Since both targets are met exactly, the solution is insensitive to w_i values. Without further solutions with different targets, the decision maker would be unaware that this solution is inferior. The second solution is at B of Figure 2. It dominates situation 1 and, unknown to the decision maker at this point, it is a noninferior solution. Situation 3 is the solution labeled A in Figure 1. Target 2 is met and objective 1 falls three units short of target 1. When a heavier penalty is imposed on missing the target on objective 2 (situation 4), the solution remains unchanged. However, when deviations from the target on objective 1 are penalized threefold those on objective 2, as in situation 5, the GP solution moves to point B of Figure 2. The target on objective 1 is met, while the level of the second objective falls four units short of its target. Indeed, for this linear two-objective example, either A or B will be the optimum solution under all possible pairs of weights. Since the slope of the relevant section of $z(X^*)$ is $-4/3$, any combination of w_i with w_1/w_2 less than $4/3$ will produce a solution at A and for w_1/w_2 greater than $4/3$ the solution will be at B. If w_1/w_2 equals $4/3$, solutions A, B, and any intermediate solutions in $z(X^*)$ have identical objective function values.

The information provided by the sorts of solutions in Table 2 is qualitatively weaker than from Table 1. Not only was the noninferior space not mapped out, and it would be relatively difficult to do this with goal programming, the kind of trade-off information previously supplied is unavailable here. Moreover, extensive sensitivity analysis may be required to find that situations like the first are inferior.

The comparison of these approaches suggests that goal programming is computationally superior if T_i and w_i are known to the decision maker. However, there is evidence from the field of psychology to suggest that decision makers may be more capable of processing the information contained in the set of noninferior solutions than in articulating targets and weights to attach to goals without knowledge of the noninferior solution space.⁴

Information Overload Problem

Goal programming implicitly presumes a decision maker who is in possession of a multiattribute utility function which is separable, additive, and stable over decision iterations. Not only is this a restrictive specification of the utility function, it is unlikely that this preference structure is completely known. In combination with the results of a great deal of research in psychology indicating a rather limited information processing capacity of the human brain, this suggests that the decision maker may have great difficulty in articulating weights (w_i) associated with the various objectives.

⁴See Morse (1977) for a discussion of this point.

Morse (1976) provides a useful summary of the human choice theory literature, featuring such main works as Thurstone, Luce, Coombs, Tversky, and Keeney and Raiffa. For present purposes, a number of studies on the information overload problem are referenced. Perhaps the most well-known paper on the subject is by Miller, who reviewed a host of experiments and concluded that human short-term memory is capable of holding active only about seven symbols. This number is corroborated in his judgment by its frequent appearance in the literature and in many cultures (seven day weeks, seven digit telephone numbers). To be sure, Miller admits the possibility of recoding (aggregating) symbols, but even here weights must be established. Thus, to assess weights or trade-offs, vast numbers of symbols must be evaluated simultaneously, and Miller's conclusions are pessimistic regarding our ability to do so. The instability of weights in many experiments bears this out.

Quite a number of other studies have some bearing on our information processing abilities as well. For example, Wilkie and Weinreich demonstrated that expanding the number of attributes to be considered can be harmful to decision-making and that decision-making is also affected by the kind of attributes and the order of consideration of attributes. Wright's work showed how time pressure results in decision-makers weighting negative evidence more heavily and, unsurprisingly, causes individuals to use less of the data which is available to them. Troutman and Shanteau found that without time pressure favorable information was given heavier weight than neutral information. Two other recent articles take different approaches in showing how decisions are hampered by an information overload. Scott and Wright used regression analysis to compare buyers' decisions with their self-reported weights in evaluating products. Generally, increasing the number of attributes to six led to instability in the weighting process. In a separate study, Gehrlein and Fishburn developed an algorithm to show that in many cases, better decisions will result when some information is withheld.

In brief, the literature suggests that choosing is difficult, that large amounts of data aggregate poorly and are associated with trade-offs (weights) that are unstable over time, and that they are not representable by separable additive utility functions. As Zeleny argues, these weights are not *a priori* in the possession of the decision maker. Rather, they are, or should be, learned through the decision process and are not independent of the set of feasible alternatives.

While the decision-makers in a goal programming setting are asked to perform the most difficult task of articulating their preference structures in advance of knowing the set of feasible alternatives, the decision-makers in a generating technique context are aided in their search for the preference structure by being supplied pairs of noninferior feasible alternatives, the objective values, and trade-offs and being asked to compare these solution pairs. The generating technique, in contrast to the goal programming structure, offers an intuitive approach to the problem. It leaves to the computer what it does best (generating alternatives) and to the decision-makers what they do best (evaluating pairs). This preference information searching aspect of generating techniques can be of use not only to decision-makers in their quest for making better decisions, but also to those who are concerned with knowing *ex post* how decision-makers traded off the objectives in question.

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