AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

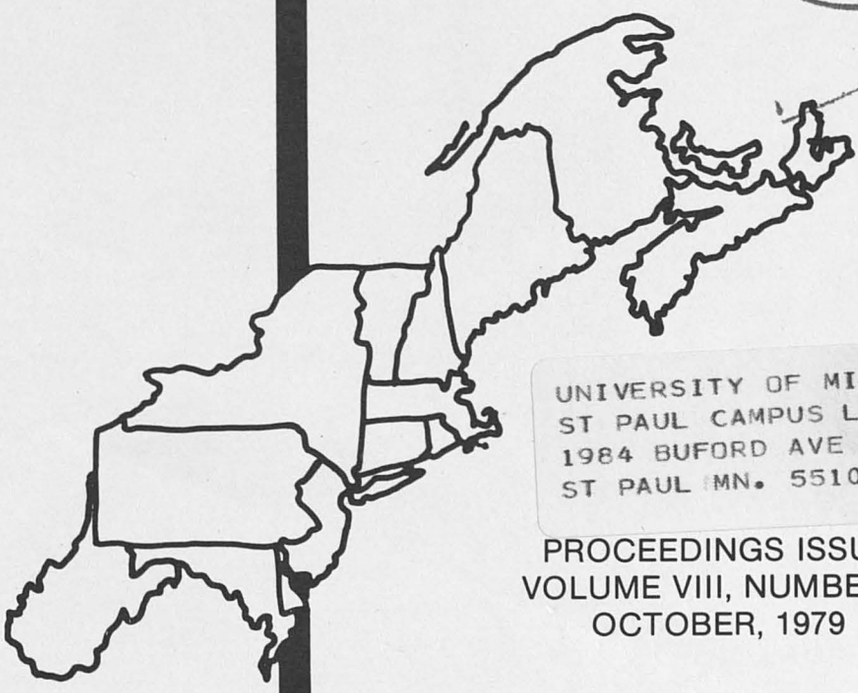
AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

# JOURNAL OF THE

# Northeastern Agricultural Economics Council

UNIVERSITY OF MINNESOTA
REC'D
AUG 21 1981
ST. PAUL CAMPUS LIBRARIES



UNIVERSITY OF MINNESOTA
ST PAUL CAMPUS LIBRARY
1984 BUFORD AVE
ST PAUL MN. 55108

PROCEEDINGS ISSUE
VOLUME VIII, NUMBER 2
OCTOBER, 1979

# AN EXAMINATION OF DEMAND FUNCTIONS FOR BEEF, PORK, AND BROILERS

John F. Yanagida
and
Roger K. Conway

Empirical research on single equation, competitive demand models has employed either price dependent or quantity dependent forms. The price dependent models assume that total output is predetermined in the short-run due to the role of past prices and to production cycles as the case of the livestock industry (Heien). On the other hand, quantity dependency can be described as a supply response system.

Phlips and Intrilligator have shown that there is a dual relationship between the price and quantity dependent equations. The quantity dependent demand equation is derived from a direct utility function whereas the price dependent equation is deduced from an indirect utility function.

This study concentrates on analyzing the predictive ability of alternate models of the livestock sector when demand equations are price dependent or quantity dependent. The methodology uses the same set of independent variables, except that the price and quantity of the commodity in question are switched according to the dependency form. Theoretically, one would expect similar results since these two demand relationships are derived from the same functional form. Whether or not this is true statistically remains to be tested.

## Modeling Methodology

The livestock model used in this study is the current specification utilized by the Forecast Support Group/USDA. The estimated model is recursively solved for an equilibrium solution by the Gauss-Seidel iterative technique (see Heien, Matthews, and Womack for a technical discussion of this procedure). The demand equations in the livestock model are in price dependent form. These price dependent equations are derived from respective quantity dependent forms. Generally, these procedures can be summarized as follows:

---

John F. Yanagida and Roger K. Conway are Economist and Economic Assistant, respectively, with the Forecast Support Group, Commodity Economics Division, U.S. Department of Agriculture. The authors wish to give special thanks to John Baritelle for suggestions and support throughout this research process and acknowledge suggestions from Harry Baumes Jr., Lloyd Teigen, and an anonymous referee.

$$(1) \quad Y_1 = \alpha - \beta_2 P_1 + \beta_3 P_3 + \beta_4 I + \beta_5 \overline{P}$$

where:

$Y_1$ = per capita consumption of good $Y_1$,

$P_1$ = price of good $Y_1$,

$P_2$ = price of substitute good $Y_2$,

$P_3$ = price of substitute good $Y_3$,

$I$ = per capital personal expenditure on nondurable goods and services, and

$\overline{P}$ = consumer price index for nondurables less food.

Equation (1) is then transformed into a homogeneous equation of degree zero by deflating by I (Friedman and Houthakker).

$$(2) \quad Y_1 = \delta - \beta_1 \frac{P_1}{I} + \beta_2 \frac{P_2}{I} + \beta_3 \frac{P_3}{I} + \beta_5 \frac{\overline{P}}{I} \quad \text{where } \delta = \alpha + \beta_4.$$

Then solving for the "own" price of good $Y_1$,

$$(3) \quad \frac{P_1}{I} = \zeta_1 - \zeta_2 Y_1 + \zeta_3 \frac{P_2}{I} + \zeta_4 \frac{P_3}{I} + \zeta_5 \frac{\overline{P}}{I}$$

where:

$$\zeta_1 = \frac{\delta}{\beta_1}; \ \zeta_2 = \frac{1}{\beta_1}; \ \zeta_3 = \frac{\beta_2}{\beta_1}; \ \zeta_4 = \frac{\beta_3}{\beta_1}; \text{ and } \zeta_5 = \frac{\beta_5}{\beta_1}$$

In the livestock model, demand equations for pork and chicken are estimated in this form. However, beef utilizes a "mixed" demand equation (Samuelson) where per capital consumption of some substitute products are included on the right-hand-side with prices of other substitutes.

To incorporate quantity dependent demand equations in the livestock model, equation (2) was estimated for each livestock commodity. Then a corresponding price dependent form was solved algebraically by using the coefficients from this equation. This produced livestock demand equations similar to equation (3). These new demand equations were then substituted for the existing price dependent equations in the livestock model.

## Comparison of Statistical Results

This study analyzes demand equations for beef, pork, and broilers. The

sample periods are 1955-76 for beef and pork and 1960-76 for broilers. The beef retail demand equation in price dependent form is written as (see Appendix 1 for variable descriptions):

$$(4)\quad \frac{BEEIR}{CAPY} = \underset{(4.78)}{.824004} - \underset{(-4.86)}{.00438240} * BEECP - \underset{(-1.86)}{.000821084} * PORKCON$$

$$- \underset{(-4.54)}{.0191155} * VEALCON + \underset{(1.75)}{.0616977} * \frac{CHIIRFR}{CAPY} + \underset{(4.83)}{.299798}$$

$$* \frac{FISIR}{CAPY} + \underset{(0.81)}{.159001} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9882$
D.W. = 1.541
SEE = .00720
Coefficient of variation = .01408

Interchanging $\frac{BEEIR}{CAPY}$ and BEECP, the quantity dependent form was estimated as:

$$(5)\quad BEECP = \underset{(30.98)}{187.790} - \underset{(-4.86)}{139.461} * \frac{BEEIR}{CAPY} - \underset{(-0.26)}{.0226654} * PORKCON$$

$$- \underset{(-0.97)}{1.09160} * VEALCON + \underset{(0.10)}{.662043} * \frac{CHIIRFR}{CAPY} + \underset{(2.38)}{35.8660}$$

$$* \frac{FISIR}{CAPY} - \underset{(-1.81)}{58.5241} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9925$
D.W. = 1.967
SEE = 1.28482
Coefficient of variation = .01293

Comparing these two equations, the statistical significance of the cross effects are lower in (5) especially for PORKCON, VEALCON, and $\frac{CHIIRFR}{CAPY}$. Also, the sign of $\frac{PCNDF}{CAPY}$ is positive in (4) and negative in (5).

These results indicate potential serial correlation and/or multicollinearity problems. Since most econometrics books offer detailed discussion of these problems, this article will focus on their corrective treatment.

In equation (5), the Durbin-Watson coefficient (D.W.) indicates that the null hypothesis of zero autocorrelation should be accepted at the one percent level. For equation (4), the D.W. is in the inconclusive region and suggests that autocorrelation could be a problem. The Cochran-Orcutt iterative method of generalized least squares was performed to correct for possible autocorrelation (see equation (6)). The results show that the rho parameter is significant at the 20 percent level.

$$(6)\quad \frac{BEEIR}{CAPY} = \underset{(4.30)}{.712855} - \underset{(-4.20)}{.00372542} * BEECP - \underset{(-1.72)}{.000716984} * PORKCON$$

$$- \underset{(-5.36)}{.020745} * VEALCON + \underset{(2.22)}{.122477} * \frac{CHIIRFR}{CAPY} + \underset{(4.25)}{.277566}$$

$$* \frac{FISIR}{CAPY} + \underset{(1.19)}{.206284} * \frac{PCNDF}{CAPY}$$

Final value of rho = .2076406
t-statistic for rho = 0.9727
$\bar{R}^2$ = .990
D.W. = 2.078
SEE = .00673
Coefficient of variation = 0.1316

There are several rules of thumb used to detect multicollinearity. One method is to scrutinize the partial correlation matrix for possible collinear relationships. Another detection mechanism is to use the "$R^2$ delete" process (Kmenta). Both procedures revealed high correlations between explanatory variables for beef demand.

There are several methods for correcting multicollinearity (Maddala and Willis and Perlack et. al.). The technique proposed in this study is ridge regression. Vinod summarizes several studies that attest to the superiority of RR over OLS. However, there has been disagreement as to the optimal RR technique. Ordinary ridge regression (ORR) estimates the following coefficients:

$$(7) \quad b^k = (X'X + kI)^{-1}X'Y$$

where:

Y = nxl vector of observations on the dependent variable,
X = nxm matrix of regressors,
I = mxm identity matrix,
k = small positive constant, and
$b^k$ = mxl matrix of coefficients for the ridge regression with k set to specific level.

If k = 0 then the result is the BLUE OLS estimate $b^0$. Hoerl and Kennard (1970a) explain the justification for using ridge regression as being the increased accuracy of its estimates in terms of the mean square error (MSE) criterion. Although RR estimates are not BLUE, there always exists an RR estimate that has a smaller MSE than the corresponding OLS estimate.1/

After extensive trials with different k levels, ridge regression did not reduce the variances of all the estimators for the price dependent form of the beef demand equation (equation 4). This indicates the absence of serious multicollinearity. However, using RR on the corresponding quantity dependent form (equation (5)), a k was found that reduces the estimates of the MSE's. This RR equation can be written as:

$$(8) \quad BEECP = \underset{(30.99)}{187.584} - \underset{(-5.09)}{136.668} * \frac{BEEIR}{CAPY} - \underset{(-0.20)}{.0169629} * PORKCON$$

$$- \underset{(-0.94)}{0.991758} * VEALCON + \underset{(0.02)}{.108115} * \frac{CHIIRFR}{CAPY} + \underset{(2.42)}{35.0786}$$

$$* \frac{FISIR}{CAPY} - \underset{(-2.03)}{61.2490} * \frac{PCNDF}{CAPY}$$

The same estimation procedure was applied to pork and broilers. OLS regressions for these commodities in PD and QD form are shown in equations (9) to (12).

$$(9)\quad \frac{PORIR}{CAPY} = \underset{(14.52)}{.526149} - \underset{(-10.35)}{.00646933} * PORKCON + \underset{(3.75)}{.329547} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(5.95)}{.233815} * \frac{CHIIRFR}{CAPY} + \underset{(2.39)}{.268913} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9241$
D.W. = 1.9139
SEE = 1.82053
Coefficient of variation = .02552

$$(10)\quad PORKCON = \underset{(14.84)}{75.6725} - \underset{(-10.35)}{133.401} * \frac{PORIR}{CAPY} + \underset{(3.24)}{43.5059} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(4.17)}{29.0461} * \frac{CHIIRFR}{CAPY} + \underset{(3.22)}{47.3882} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9241$
D.W. = 1.9139
SEE = 1.82053
Coefficient of variation = .02565

$$(11)\quad \frac{CHIIRFR}{CAPY} = \underset{(3.65)}{.969997} - \underset{(-4.93)}{.0177695} * CHICKCON + \underset{(3.21)}{.424229} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(2.75)}{.267363} * \frac{EGGIR}{CAPY} + \underset{(2.20)}{.261054} * \frac{PORIR}{CAPY} - \underset{(-2.95)}{.807180} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9862$
D.W. = 2.3833
SEE = .02234
Coefficient of variation = .02302

$$(12)\quad CHICKCON = \underset{(16.32)}{59.9653} - \underset{(-4.93)}{38.7579} * \frac{CHIIRFR}{CAPY} + \underset{(1.84)}{13.8519} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(2.28)}{11.5451} * \frac{EGGIR}{CAPY} + \underset{(1.00)}{6.33701} * \frac{PORIR}{CAPY} - \underset{(-8.10)}{52.5019} * \frac{PCNDF}{CAPY}$$

$\bar{R}^2 = .9897$
D.W. = 1.7902
SEE = .52979
Coefficient of variation = .01643

For the pork price and quantity dependent demand (equation (9) and (10)), the significance levels of the individual estimators are of consistent magnitudes. The D.W. coefficients allow the null hypothesis of zero autocorrelation to be accepted at the 10 percent level. While it does not preclude the existence of multicollinearity, the zero-order correlation of $\frac{PORIR}{CAPY}$ and PORKCON with the other independent variables seem to negate the presence of serious multicollinearity. Consequently, as opposed to the beef demand equation, no corrective procedures were applied.

For the broiler demand equations, both dependency forms indicated potential autocorrelation. The Cochrane-Orcutt procedure produced results shown in equation (13) and (14). In both cases the rho parameter was significant at the five percent level implying the presence of autocorrelation in the OLS regression.

$$(13)\quad \frac{CHIIRFR}{CAPY} = \underset{(4.14)}{.689559} - \underset{(-6.06)}{.0138269} * CHICKCON + \underset{(5.65)}{.411028} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(2.92)}{.204832} * \frac{EGGIR}{CAPY} + \underset{(4.46)}{.325219} * \frac{PORIR}{CAPY} - \underset{(-2.71)}{.461411} * \frac{PCNDF}{CAPY}$$

Final value of rho = - .5454534
t-statistic for rho = -2.6032
$\bar{R}^2$ = .992
D.W. = 2.406
Coefficient of variation = .01579

$$(14)\quad CHICKCON = \underset{(14.86)}{55.1317} - \underset{(-5.81)}{54.9668} * \frac{CHIIRFR}{CAPY} + \underset{(3.31)}{21.4603} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(2.51)}{12.1366} * \frac{EGGIR}{CAPY} + \underset{(2.16)}{14.3434} * \frac{PORIR}{CAPY} - \underset{(-6.35)}{41.9466} * \frac{PCNDF}{CAPY}$$

Final value of rho = - .4294952
t-statistic for rho = -1.9024
R BAR 2 = .989
D.W. = 2.280
SEE = .49487
Coefficient of variation = .01535

The correlation matrix for the broiler demand variables indicates a high degree of collinearity. The $R^2$ delete tests confirm this observation. Using rdge regression did reduce the MSE's substantially for the quantity dependent form but not the price dependent form. This RR equation was estimated as:

$$(15)\quad CHICKCON = \underset{(16.85)}{59.8061} - \underset{(-5.07)}{38.5285} * \frac{CHIIRFR}{CAPY} + \underset{(1.87)}{13.6374} * \frac{BEEIR}{CAPY}$$

$$+ \underset{(2.19)}{11.0075} * \frac{EGGIR}{CAPY} + \underset{(1.07)}{6.61706} * \frac{PORIR}{CAPY} - \underset{(-8.17)}{51.8968} * \frac{PCNDF}{CAPY}$$

Forecasts for 1977 and 1978 from these demand equations are shown in Table 1. The quantity dependent demand equations are more accurate in terms of percentage deviations from actual 1977 and 1978 data. In the case of broilers, the forecasts are quite pronounced in favor of the quantity dependent forms.

Four sets of demand equations were chosen for use in the livestock model. For the first set, PD-OLS, equations, (4), (9), and (11) were selected. This is a group of OLS equations in price dependent form. The other price dependent set, PD-MIX, is composed of equations (6), (9), and (13). Quantity dependent equations were algebraically transformed into corresponding price dependent equations. The third group of demand equations, PDQD-OLS, consisted of equations (5), (10), and (12). The other quantity

Table 1

Single equation forecasts for 1977 and 1978

| Equations | Method | 1977 forecasts | 1977 actual | Percent deviation from actual | 1978 forecasts | 1978 actual | Percent deviation from actual |
|---|---|---|---|---|---|---|---|
| 4 | OLS | 1.865 | 1.636 | 14.00 | 2.215 | 2.010 | 10.45 |
| 6 | GLS | 1.769 | 1.636 | 8.13 | 2.104 | 2.010 | 4.48 |
| 5 | OLS | 132.978 | 124.612 | 6.71 | 129.234 | 120.4 | 7.34 |
| 8 | RR | 132.888 | 124.612 | 6.64 | 129.221 | 120.4 | 7.33 |
| 9 | OLS | 1.982 | 1.888 | 4.98 | 2.209 | 2.131 | 3.71 |
| 10 | OLS | 63.912 | 60.987 | 4.80 | 63.577 | 61.6 | 3.21 |
| 11 | OLS | 1.408 | 1.585 | 11.17 | 1.378 | 1.756 | 27.43 |
| 13 | GLS | 1.422 | 1.585 | 10.28 | 1.438 | 1.756 | 22.11 |
| 12 | OLS | 39.921 | 41.300 | 3.37 | 40.819 | 44.4 | 8.07 |
| 14 | GLS | 39.411 | 41.300 | 4.57 | 43.343 | 44.4 | 2.20 |
| 15 | RR | 39.912 | 41.300 | 3.36 | 40.822 | 44.4 | 8.05 |

Table 2

Evaluation of forecasts: 1965-1976

| Models and selected endogenous variables | Root-mean-square error (RMS) | Theil's inequality coefficient |
|---|---|---|
| PD-OLS: | | |
| BEEIR | .05796 | .02019 |
| BEECP | 3.219 | .01414 |
| PORIR | .06901 | .02356 |
| PORKCON | 522.4 | .01856 |
| CHIIRFR | .09968 | .03745 |
| CHICKCON | 209.0 | .01372 |
| PD-MIX: | | |
| BEEIR | .05947 | .02070 |
| BEECP | 3.214 | .01412 |
| PORIR | .07007 | .02391 |
| PORKCON | 522.1 | .01855 |
| CHIIRFR | .1012 | .03804 |
| CHICKCON | 209.2 | .01373 |
| PDQD-OLS: | | |
| BEEIR | .5858 | .02046 |
| BEECP | 3.212 | .01412 |
| PORIR | .07042 | .02405 |
| PORKCON | 523.0 | .01859 |
| CHIIRFR | .1025 | .03858 |
| CHICKCON | 207.9 | .01365 |
| PDQD-MIX: | | |
| BEEIR | .05864 | .02048 |
| BEECP | 3.212 | .01412 |
| PORIR | .07043 | .02405 |
| PORKCON | 523.0 | .01859 |
| CHIIRFR | .1027 | .03867 |
| CHICKCON | 207.8 | .01364 |

dependent collection PDQD-MIX, included equations (8), (10), and (15). The equations in the MIX sets were primarily chosen for their ability to forecast 1977 and 1978.

Using these four classifications of demand equations, the livestock model was then tested in terms of prediction accuracy. These results are shown in Table 2 in terms of the estimated root-mean-square errors and Theil inequality coefficients ($U_2$) (Theil). The conclusions drawn from these findings are:

1. Genrally, PD models have lower RMS' and inequality coefficients for price variables than PDQD models.
2. Likewise, in most cases, PDQD models have lower RMS's and inequaility coefficients for quantity variables than PD models.
3. For the PD modles, the RMS's and inequality coefficients are similar in magnitude for the OLS and MIX forms.
4. The results from (3) also apply to the PDQD models.

## Summary

Applying genralized least squares and ridge regression did improve the individual equation's forecasting ability over OLS procedures. However, when used in context of the livestock model, forecast evaluation outcomes did not differ for MIX and OLS sets.

Of greater importance is the finding that retail prices are more accurately predicted with a price dependent framework and consumption quantities more precisely forecasted via a quantity dependent structure. Consequently, forecasting objectives, whether it be prices or quantities, should be considered before choosing the form of the demand function.

The two dependency forms have different own and cross effects. Table 3 illustrates the own price and principal cross flexibilities for the four models. Some of the differences in these flexibilities can be attributed to statistical problems like autocorrelation, multicollinearity, etc. However, significant differences still exist after employing corrective procedures. This result is contrary to the initial hypothesis that these two demand forms would produce similar structural outcomes. It is true that statistical problems may still be present and that further refinements are necessary. Conceivably a more efficient method of testing dependence forms is a causality test. Further research in this area is needed to shed more light on the question of the proper dependency form.

Table 3

Own price and cross-price flexibilities of various models (evaluated at geometic means)

| Item | BEECP | PORKCON | CHICKCON |
|---|---|---|---|
| BEEIR: | | | |
| PD-OLS | - .9331 | - .1162 | - .0802 |
| PDQD-OLS | -1.5268 | - .0230 | - .0090 |
| PD-MIX | - .7963 | - .1005 | - .1225 |
| PDQD-MIX | -1.5588 | - .0176 | - .0015 |
| PORIR: | | | |
| PD-OLS | - .3097 | - .9250 | - .3058 |
| PDQD-OLS | - .5050 | -1.0708 | - .4148 |
| PD-MIX | - .3097 | - .9250 | - .3058 |
| PDQD-MIX | - .5050 | -1.0708 | - .4148 |
| CHIIRFR: | | | |
| PD-OLS | - .3548 | - .2139 | -1.1630 |
| PDQD-OLS | - .4911 | - .1576 | -1.6881 |
| PD-MIX | - .2963 | - .2646 | - .9041 |
| PDQD-MIX | - .4911 | - .1412 | -1.6984 |

Appendix 1

Variable Names and Description 1/

| Variable names | Description |
|---|---|
| BEEIR | Retail beef and veal price index (1967 = 1.0) |
| PORIR | Retail pork price index (1967 = 1.0) |
| CHIIRFR | Retail frying chicken price index (1967 = 1.0) |
| EGGIR | Retail egg price index (1967 = 1.0) |
| FISIR | Retail fish price index (1967 = 1.0) |
| PCNDF | Consumer price index for nondurables less foof (1967 = 1.0) |
| CAPY | Per capita personal consumption expenditures on nondurable goods and services |
| BEECP | Per capita beef consumption (pounds) |
| PORKCON | Per capital pork consumption (pounds) |
| CHICKCON | Per capita chicken consumption (pounds) |
| VEALCON 2/ | Per capita veal production (pounds) |

1/ Primary data sources for these variables were various commodity situation reports published by the USDA/ESCS.

2/ Veal production was used since veal consumption data was not available. Historically, veal trade and stock levels have been small as compared to consumption and production.

## Footnotes

[1]There are two major criticisms in using ridge regression. First, RR is a statistical technique devoid of economic reasoning in choosing the optimal k. The parameter k is chosen such that MSE $(b^k)<MSE(b^0)$ is satisfied for all or most of the independent variables. This is a "trial and error" process that begs the question: How small a k? Second, ORR assumes that k is the same for each independent variable. This problem has been rectified with the use of the generalized ridge technique (GRR) (Hoerl and Kennard (1970a, b)). The algorithm for GRR is not available at the U.S. Department of Agriculture. This permits usage of only the ORR technique for the following estimates.

## References


Friedman, Milton. "The Marshallian Demand Curve." Journal of Political Economy, 57 (1949):463-74.

Heien, Dale. "Price Determination Processes for Agricultural Sector Models." American Journal of Agricultural Economics, 59 (1977):126-32.

Heien, Dale, J. Matthews, and A. Womack. "A Methods Note on the Gauss-Seidel Algorithm for Solving Econometric Models." Agricultural Economics Research, 23 (1973):71-80.

Hoerl, Arthur E. and Robert W. Kennard. "Ridge Regression: Applications to Nonorthogonal Problems." Technometrics, 12 (1970):69-82.

Hoerl, Arthur E. and Robert W. Kennard. "Ridge Regression: Biased Estimation of Nonorthogonal Problems." Technometrics, 12 (1970):55-67.

Houthakker, H.S. "Additive Preferences." Econometrica, 28 (1960):244-57.

Intrilligator, Michael D. Econometric Models, Techniques, and Applications. (Englewood Cliffs: Prentice-Hall, Inc.), 1978.

Kmenta, Jan. Elements of Econometrics. (New York: The MacMillan Company), 1971.

Maddala, G.S. Econometrics. (New York: McGraw-Hill Book Company), 1977.

Phlips, L. Applied Consumption Analysis. (Amsterdam: North-Holland Publishing Company), 1974.

Samuelson, P. "Using Full Duality to Show that Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand." Econometrica, 33 (1965):781-96.

Theil, H. Applied Economic Forecasting. (Chicago: Rand McNally and Company), 1966.

Vinod, Hrishikesh D. "A Survey of Ridge Regression and Related Techniques for Improvements Over Ordinary Least Squares." The Review of Economics and Statistics, 60 (1978):121-31.

Willis, Cleve E. and Robert D. Perlack, et. al. "Multicollinearity: Effects, Symptoms, and Remedies." Journal of the Northeastern Agricultural Economics Council, 7 (1978):55-61.