University of Kentucky Staff Paper 475 October, 2013

Introduction to Benefit/Cost and Project Analysis

by

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Abstract

These are a series of color slides and exercises for providing key elements of an Introduction to Benefit/Cost and Project Analysis. There are five parts or sections. Part 1 provides an Introduction to Compounding and Discounting. Part 2 outlines a method for determining project worth. Part 3 deals with the choice of the discount rate. Annuity factor calculations are outlined in part 4. Part 5 discusses the calculation of Benefit/Cost ratios, Net Project Worth and the Internal Rate of Return.

Key words:

Benefit/Cost ratios
Project Analysis
Cost-Benefit Analysis
Discount Rate
Annuity factors

Introduction to Benefit/Cost and Project Analysis

- 1. Introduction to Compounding and Discounting
- 2. Project Worth
- 3. The Choice of the Discount Rate
- 4. Annuity Factors
- 5. Benefit Cost, Net Project Worth and the Internal Rate of Return

Part 1

Introduction to Compounding and Discounting

When investing money today, we expect a return on our investment.

Having $1 today is worth more than $1 available some time in the future.

Why do we prefer money today
to money tomorrow?

Money today has earning power.

A specific amount of money today
is equivalent to a larger sum
in the future.

A larger sum compensates for:
inflation,
foregone present consumption, and
risk.

If you invest money in a project,
you are foregoing the opportunity
to use your money until it is recovered.

1. Inflation makes the money repaid in the future have less worth.
2. Foregoing present consumption denies you the possibility to buy goods and services presently or invest in an alternative investment.
3. There is some risk that the money will not be recovered.

When investing your money, you expect to be compensated for your willingness to wait until the money is recovered.

The amount paid in compensation is called

Interest

An investment of $100 today
at an interest rate of 10 %
per annum
will be worth $110 a year from now.
$121 in two years, $161 in three etc.
This is called compounding.

Cash Today

Cash Tomorrow

The Compounding Formula is:

$$X\,(1+i)^{t} = Y$$

cash today

interest rate

time

cash tomorrow

Cash Today

Cash Tomorrow

If the Amount of Cash is $100 (X=$100), after 1 Year

$$\$100(1+.10)^{1} = \$110$$

at an interest rate of 10%

Cash Today

Cash after 1 year

If the Amount of Cash
is $100 (X=$100),
after 2 Years

$$\$100(1+.10)^{2} = \$121$$

at an interest rate of 10%

Cash Today

Cash after 2 Years

If the Amount of Cash
is $100 (X=$100)
After 3 years

$$100(1+.10)^{3} = \$133.10$$

at an interest rate of 10%

Cash Today

Cash after 3 years

The Larger Sum in the Future:

- Accounts for Inflation
- Present Consumption More Highly Valued than Future Consumption
- Risk

Present Value X

Future Value Y

Future Value =
Compounded Value of a Present Amount at a Predetermined Interest Rate

Cash Today

Cash after 3 years

Or,

Future Values Can be Reduced to Present Values by Discounting.

Present Value

Future Value

EXAMPLE OF DISCOUNTING

Suppose we are considering an investment of $1000 that will return $600 after 1 year, and $750 after 2 years. Is the return adequate to justify the investment?

In order to reduce future sums of money to present values, we must use discounting.

To discount we will first have to choose a discount rate. The discount rate has to be such that it represents the return we could obtain by using the $1000 in an alternative investment, as we will learn later. If we were to buy stocks, for example, we could earn an 8% return.

Using 8% as the discount rate, we can discount the $750 occurring after 2 years and the $600 occurring after 1 year to decide how much these future sums of money are worth to us today. If they are worth more than $1000, it is agood investment.

Discounting, we obtain:

$750 after 2 years -------- $ 642

$600 after 1 year -------- $ 555

$ 1197

The future sums of money are worth more then $1000 today!

Discounting allows us to:

1. Estimate net benefits given up from other investment project possibilities.
2. Convert costs and benefits of alternative investment projects into equivalents that can be compared at the present time.

In compounding, we used the interest rate.

In discounting, we will use the discounting rate.

The relationship between interest rates and discount rates will be explained in the next lesson. You will learn how the discount rate is determined.

DISCOUNTING FORMULA:

$$X = \frac{Y}{(1+r)^t}$$

Future Value

Present Value

r is the discount rate.

Present Value

Future Value

Future Value

$$\$100 = \frac{\$133.10}{(1 + .10)^3}$$

Present Value

This is the discounting formula.

t=3 years; r = 10 %

Present Value

Future Value

Discount Factor is

$$\frac{1}{(1+r)^{t}}$$

Discount Rate r, time t (in years)

Present Value

Future Value

Discount Factor is

$$X = Y \frac{1}{(1 + r)^t}$$

Discount Rate r, time t (in years)

Present Value X

Future Value Y

Discount Factor is

$$\frac{1}{(1 + .10)^{3}}$$

3 years; r = 10 %

Present Value

Future Value

Discounting:

The determination of present values of future amounts of money.

Present Value X

Future Value Y

To find X (present value):

Y (future value) = $110, r = .10

Time period = 1 year

X (present value) =

$$\$110 \quad \frac{1}{(1+.10)} = \$110 \times (.909) \quad = \$100$$

Increase the future value to $500.

Y (future value) = $500, r = .10

Time period = 1 year

X (present value) =

$$\$500 \frac{1}{(1+.10)} = \$500 \times (.909) = \$454.50$$

Increase the discount rate to 18%.

Y (future value) = $500, r = .18
Time period = 1 year

X (present value) =

$$\$500 \frac{1}{(1+.18)} = \$500 \times (.847) = 423.50$$

Decrease the discount rate to 3%.

Y (future value) = $500, r = .03
Time period = 1 year

X (present value) =

$$\$500 \quad \frac{1}{(1+.03)} = \$500 \times (.970) = \$485.00$$

You should note that:

The higher the discount rate,
the lower the present value.

The lower the discount rate,
the higher the present value.

As the time interval increases,
the present value decreases.

EXAMPLE:

We will calculate now the present value of $500 occurring at different future dates, at a discount rate of 10%.

Y (future value) = $500

r = .10

Varying lengths of time

t =2,...,50

Then the Present Values (X)

can be calculated as:

$$\$500 \frac{1}{(1+.10)^t}$$

$500 $\frac{1}{(1+.10)^2}$ = 500(.826) =$413.00
t=2

$500 $\frac{1}{(1+.10)^3}$ = 500(.751) =$375.50
t= 3

$500 $\frac{1}{(1+.10)^{10}}$ = 500(.385) =$192.50
t=10

$500 $\frac{1}{(1+.10)^{20}}$ = 500(.148) =$74.00
t=20

$500 $\frac{1}{(1+.10)^{30}}$ = 500(.057) =$28.50
t=30

$$\$500 \quad \frac{1}{(1+.10)^{40}} = 500(.022) = \$11.00$$

t=40

$$\$500 \quad \frac{1}{(1+.10)^{50}} = 500(.008) = \$4.00$$

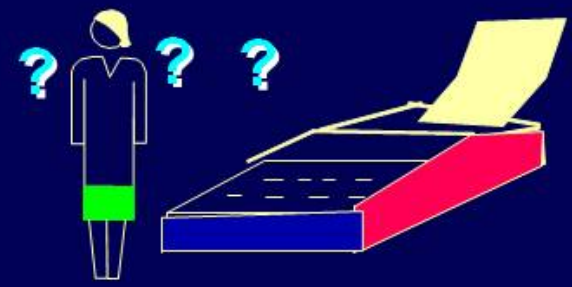
t =50

Exercise

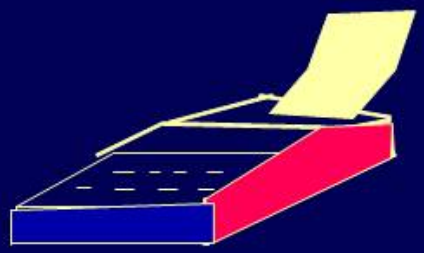
Complete the following table.

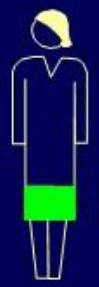
Rate of Interest (Discount)	End of Year	Compounding Factor $_t$ $(1+.20)$	Discount Factor $_t$ $1/(1+.20)$
20 %	1	1.2	?
20 %	2	1.44	?
20 %	3	1.728	?

Can you complete this table?



Rate of Interest (Discount)	End of Year	Compounding Factor $(1+.20)^t$	Discount Factor $1/(1+.20)^t$
20 %	1	1.2	.8333
20 %	2	1.44	.6944
20 %	3	1.728	.5787





Fill in the following table using a 17% discount rate

Rate of Interest (Discount)	End of Year	Compounding Factor $(1+.17)^t$	Discount Factor $1/(1+.17)^t$
17 %	1	_____	_____
17 %	2	_____	_____
17 %	3	_____	_____

Fill in the following table using an 8 % discount rate.

Rate of Interest (Discount)	End of Year	Compounding Factor $_t$ $(1+.08)$	Discount Factor $_t$ $1/(1+.08)^t$
8 %	1	_____	_____
8 %	2	_____	_____
8 %	3	_____	_____

In this lesson, we learned how to compound present values and discount future values.

Once we determine the discount rate to be used, we can discount.

The next lesson deals with how to determine the discount rate to be used.

Part 2

Project Worth

The

decision to invest

depends on

the magnitude of the:

1. benefit / cost ratio
2. net present value
3. internal rate of return

Discounting the incremental costs and benefits allows us to calculate the benefit/cost ratio and the net present value. Either measure will guide us on whether to invest in a certain project.

The Benefit / Cost Ratio:

Discounted Incremental Benefits (DIB)

and

Discounted Incremental Costs (DIC)

for the Project under Consideration

To form a Benefit/Cost Ratio:

Discounted Incremental Benefits (DIB)

Discounted Incremental Costs (DIC)

To form a Benefit/Cost Ratio:

$$\frac{\text{Discounted Incremental Benefits (DIB)}}{\text{Discounted Incremental Costs (DIC)}}$$

Form a Benefit/Cost Ratio:

$$\frac{\text{DIB}}{\text{DIC}} = \frac{\text{Benefit}}{\text{Cost}}$$

Form a Benefit/Cost Ratio:

$$\frac{\text{DIB}}{\text{DIC}} = \frac{\text{Benefit}}{\text{Cost}} \quad \text{Ratio}$$

Invest When

$$\frac{\text{DIB}}{\text{DIC}} = \frac{\text{Benefit}}{\text{Cost}} \quad \text{Ratio}$$

Exceeds (>) 1

(benefits exceed costs)

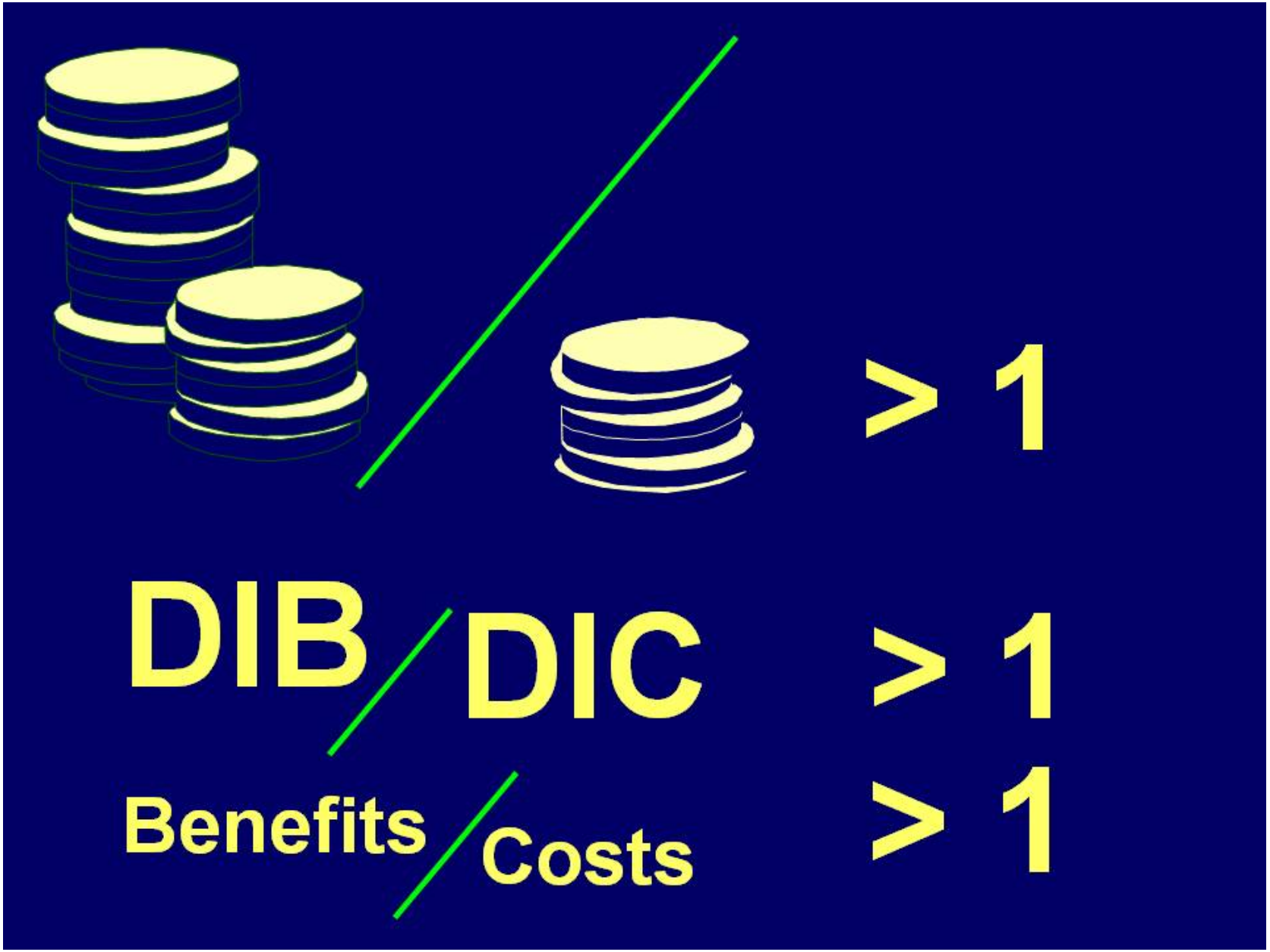
Benefit /Cost Ratio > 1

means that:

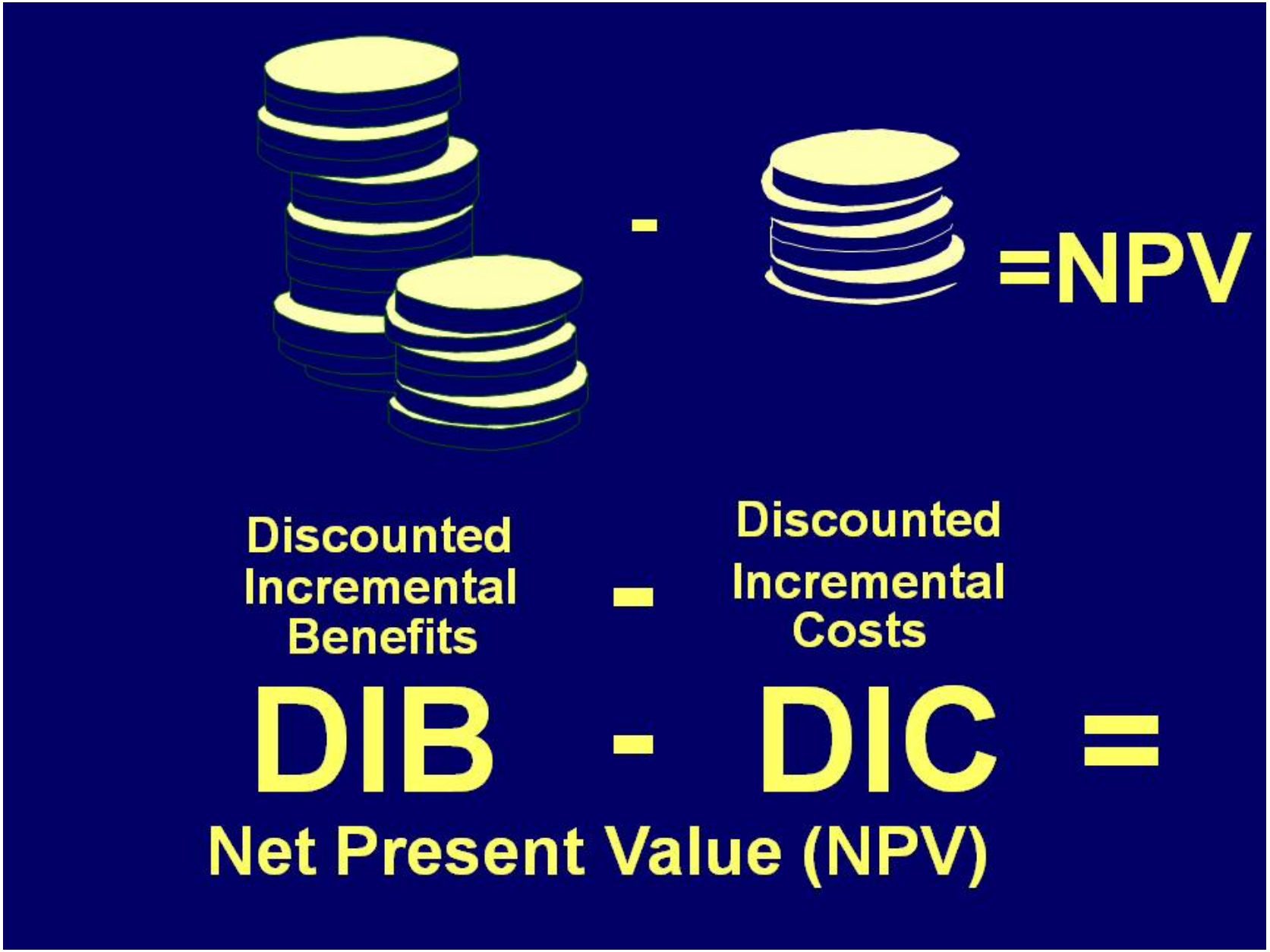
Discounted Incremental Benefits

are larger than

Discounted Incremental Costs


DIB / DIC > 1
Benefits / Costs > 1


DIB > DIC
Benefits > Costs


-
=NPV
Discounted Incremental Benefits
-
Discounted Incremental Costs
DIB - DIC =
Net Present Value (NPV)

-

=NPV

Benefits Exceed Costs

Invest When

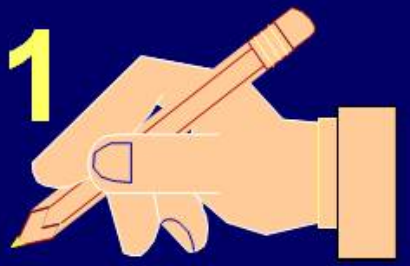
Net Present Value (NPV)

> 0

Decision Rules:

1. DIB/DIC >1

   B/C Ratio > 1



2. NPV > 0

   DIB - DIC > 0

Ranking of alternative investment opportunities

If we are trying to rank alternative investment opportunities to see in which we should invest first, we can use the NPV and the IRR.

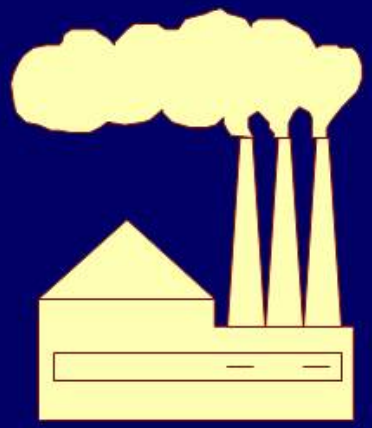
Both rules show that:

Discounted Incremental Benefits must Exceed Discounted Incremental Costs

Their theoretical base is:

Best Economic Policy:

Maximize Rate of Growth in Net Income



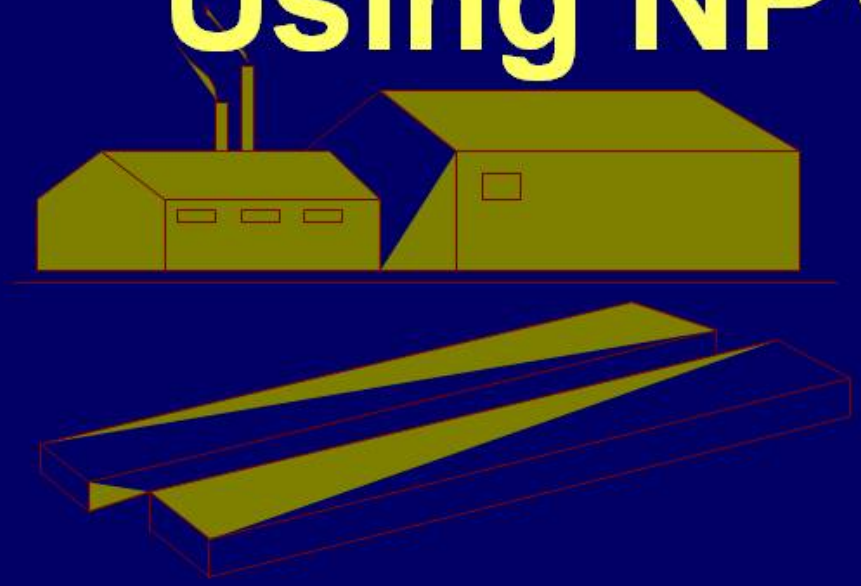
It means that we try to obtain a

Change in Income
that maximizes net returns
from the investment

or

with the investment we will have more net benefits than without it.

Ranking Projects Using NPV



Use Criterion

NPV > 0

Projects can be ranked based

on the size of NPV,

only if they are:

"Mutually Exclusive"

Examples of "mutually exclusive" projects:

1. Projects that are similar in size and technology at different locations

Example:

Decide to locate an investment in region A or B.

Examples of "mutually exclusive" projects:

2. Projects with similar goals but different sizes or technologies

Example:

Alternative irrigation investments with different technologies that need to be compared.

Ranking Projects using the IRR (Internal Rate of Return)

The IRR is based on the concept of the *Break-Even Point*.

Break-Even Point:

NPV = 0

DIB = DIC

Benefits = Costs

Internal Rate of Return (IRR):

the discount rate at which

the NPV = 0

Internal Rate of Return (IRR):

the highest rate of interest we could afford to pay to obtain all the resources necessary for the project and still break even.

If

IRR > r

If the IRR is greater than the Discount Rate used for the analysis, the project is more profitable than the average available investment opportunity in the economy.

Therefore, Invest!

The IRR tells us what, if we were to borrow for all our costs, would be the *highest* rate of interest we would pay and still *break even*.

The IRR represents profitability and can help us compare projects with different goals, sizes, and technological structures.

The IRR represents profitability and can help us compare projects with different goals, investment size, and technological structures.

Thus, the IRR is a tool for comparing and ranking projects (except "Mutually Exclusive" projects)

For example, we have the following 4 project results all calculated with a 12% discount rate:

Year	1	2	3	4	5	6
cash flow project A	-10,000	800	1,000	20,000	20,000	
	NPV = 16,479			IRR = 53		
cash flow project B	-1,000	800	1,500	1,500		
	NPV = 1,765			IRR = 95		
cash flow project C	-20,000	10,000	10,000	10,000	10,000	10,000
	NPV = 14,328			IRR = 41		
cash flow project D	-19,000	9,500	9,500	9,500	9,500	10,000
	NPV = 13,865			IRR = 41		

Choose between projects

A, B, C, D.

Then choose between the

"mutually exclusive" C and D.

For example, we have the following four project results all calculated with a 12% discount rate:

Year	1	2	3	4	5	6
cash flow *project A*	-10,000	800	1,000	20,000	20,000	
		NPV = 16,479		IRR = 53		
cash flow *project B*	-1,000	800	1,500	1,500		
		NPV = 1,765		IRR = 95		
cash flow *project C*	-20,000	10,000	10,000	10,000	10,000	10,000
		NPV = 14,328		IRR = 41		
cash flow *project D*	-19,000	9,500	9,500	9,500	9,500	10,000
		NPV = 13,865		IRR = 41		

CORRECT ANSWER:

Between A, B, C, D, we choose B.

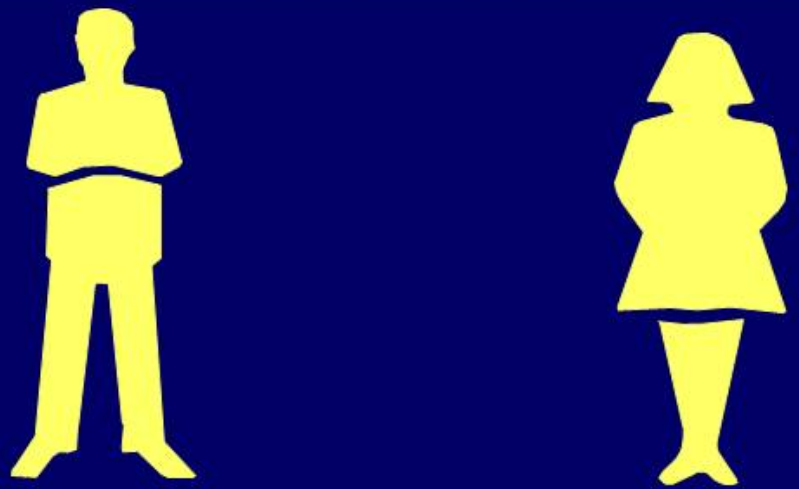
Between C and D (*mutually exclusive*), we choose C.

In this lesson, we learned about:

1. Determining incremental benefits and costs (cash flow).
2. Estimating the economic life of an investment project.
3. Using the NPV, B/C, and the IRR to decide in which investment project opportunity to invest.

In the following lessons, you will learn to

1. Calculate net present value.
2. Determine the benefit/cost ratio.
3. Estimate the internal rate of return.



Part 3

The Choice of the Discount Rate

IN THIS LESSON, YOU WILL

LEARN THE REASONING

IN CHOOSING A

DISCOUNT RATE.

For public investments, the discount rate is given by the planning agency of a country to ensure that all projects are evaluated in the same way.

Private businesses have to determine their own discount rate.

For public investments, the discount rate is given by the planning agency of a country to ensure that all projects are evaluated in the same way.

Private businesses have to determine their own discount rate.

How is this discount rate determined?

In this lesson, we will develop the rationale for setting the discount rate at a particular level.

Discounting

reduces future values to present values

The Discount Rate:

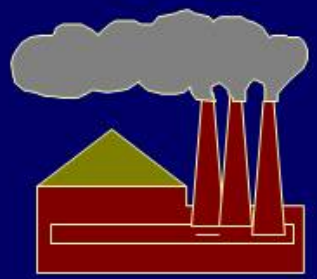
The Discount Rate:

- The rate of return that would produce a particular future value when a certain amount of money is presently invested.

The Discount Rate:

- The rate of return that
  would produce a particular Future value
  when a certain amount of
  money is presently invested
- The rate of return forgone
  by investing in the project

The discount rate represents an average rate of return and is the average net profit rate in the economy.



The discount rate is the price paid to use funds for the investment (i.e., interest rate for borrowed funds) or the returns received for delaying the use of funds (i.e., opportunity cost for owned funds)

Principle of Efficiency:

The discount rate should equal the market rate of interest

In the real world, there is no single market rate of interest.

Furthermore, interest rates fluctuate over time.

Market rates of interest reflect

1. The overall profitability of the economy.
2. Transactions costs in the financial sector. This is why borrowers pay higher rates than savers receive.
3. Anticipated inflation according to the duration of the deposit or the length of the loan.
4. Lender risk because the lender believes the borrower might not be able to repay the loan.

Real Discount Rate:

discount rate adjusted to eliminate the effects of inflation

Since inflation rates are normally positive, real discount rates are normally lower than discount rates that have not been adjusted for the effects of inflation.

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discount rate adjusted to eliminate the effects of inflation

Since inflation rates are normally positive, real discount rates are normally lower than discount rates that have not been adjusted for the effects of inflation.

? What is the anticipated inflation rate?

By using a real discount rate,

all costs and benefits are estimated in real terms (net of inflation).

The choice of the appropriate discount rate

is always the subject of debate

FOR A PRIVATE BUSINESS :

EXAMPLE

IF I WERE NOT TO INVEST IN THE PROPOSED

INVESTMENT PROJECT

OPPORTUNITY, I COULD EARN 7% AFTER TAXES

BY INVESTING THE MONEY IN

DEMAND DEPOSITS, BONDS, STOCKS, etc.

In appraising the investment,

I will use a 7% discount rate.

This is the opportunity cost

of my funds.

In many countries, there is a

planning commission

that decides on the discount rate
to be used for all public investments.

In many countries, there is a

planning commission

that decides on the discount rate
to be used for all public investments.

Private investors must choose a discount rate.

Remember that discount rates reflect

1. Profitability
2. Transactions costs
3. Anticipated inflation
4. Lenders' risk

In calculating Net Present Value ,

the *higher* the discount rate that is used,

the *lower* the Net Present Value
for an investment project.

The higher the discount rate, the fewer the projects meeting the Net Present Value > 0 criterion

The higher the discount rate, the fewer the projects having an internal rate of return higher than the discount rate.

The higher the discount rate,
the fewer the projects having an internal rate of return higher than the discount rate.

With high discount rates, proposed projects which generate large benefits early in the project life will rank higher than projects which generate most of their benefits late in the project life.

Lower discount rates will mean that more projects will meet the investment criteria.

If the agreed-upon discount rate is lower than the market rate of interest, the project is an inefficient undertaking.

A discount rate lower than the market rate of interest can be used by the public sector.

This is because the public sector can raise at least a portion of the needed capital coercively. This is done by raising taxes or through control of the money supply.

Thus, the public sector is partially shielded from market forces.

By choosing an artificially low discount rate, the government may reduce the wealth of the society.

Others have argued that discount rates should approximate market rates of interest only for projects with relatively short lives.

They suggest that government projects should have discount rates below the market rate of interest.

In financial analyses, the discount rate is approximated by choosing the market rate of interest at which the enterprise is able to borrow money.

This rate may differ from the rate used in "economic analysis," as you will see in the set of lessons dealing with the appraisal of public investments.

Sometimes, the discount rate is chosen on the basis of the borrowing rate the country must pay in order to finance the project.

Therefore, it is international financial events that influence the selection of projects,rather than the contribution of projects to national income.

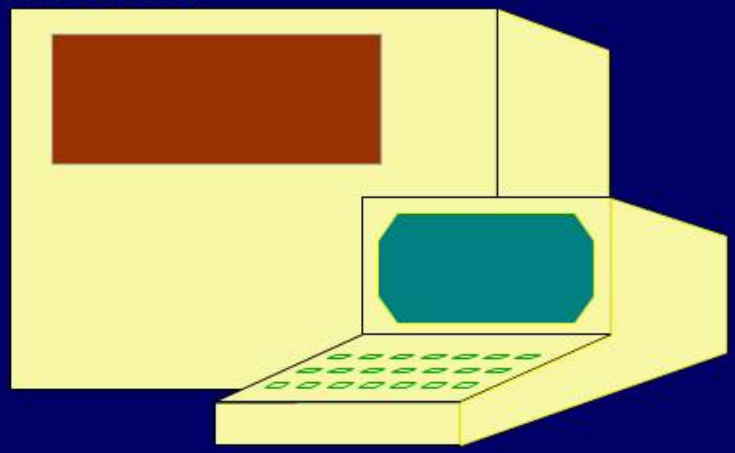
If public and private investments are comparable, then discount rates should also be comparable.

It may be justified to undertake an inefficient public sector investment if significant benefits occur late in the project life.

However, the inefficiency of such an investment should be recognized.

IN THIS LESSON, YOU LEARNED THE APPROPRIATE CONSIDERATIONS IN CHOOSING A DISCOUNT RATE.

IN THE NEXT LESSON, YOU WILL LEARN TO USE THE **DISCOUNT FACTORS** CORRESPONDING TO A GIVEN DISCOUNT RATE. FURTHERMORE, YOU WILL LEARN THE USE OF **ANNUITY FACTORS** AS A SHORT-CUT IN CALCULATIONS.



Part 4

Discount Factors and Annuity Factors

In this lesson, you will
learn how to discount benefits
and costs over the life of a project.
Discounting is done using
a table of

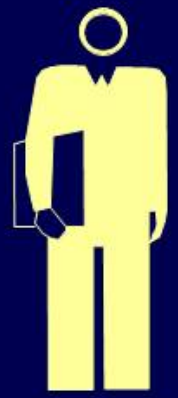
Discount Factors

Take a look at the discount tables found in the Introduction.

Find the table of discount factors that correspond to a discount rate of 12%. listed for the different years.

You may use a hand calculator or spreadsheet.

Verify the discount factors used in this lesson.



Discount Table for a 50-year Project

Year	12% Discount factor
1	$1/(1+.12)^{1}$
2	$1/(1+.12)^{2}$
3	$1/(1+.12)^{3}$
4	$1/(1+.12)^{4}$
5	$1/(1+.12)^{5}$
.	.
.	.
.	.
10	$1/(1+.12)^{10}$
.	.
.	.
50	$1/(1+.12)^{50}$

Calculate to 3 decimal Places

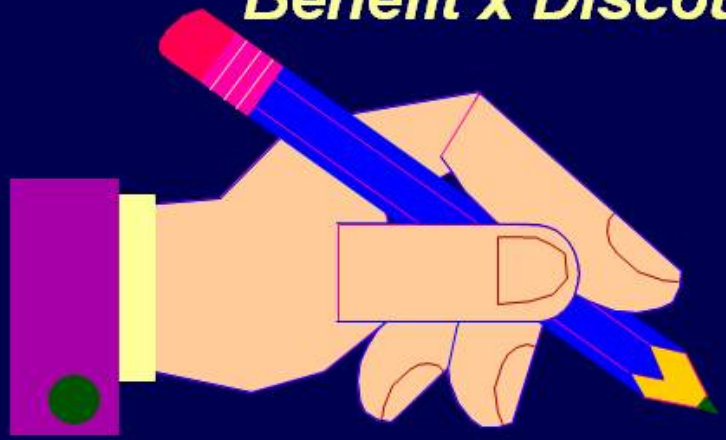
Discount Table for a 50-year Project

Year	12% Discount factor	
1	$1/(1+.12)^{1}$	= .892
2	$1/(1+.12)^{2}$	= .797
3	$1/(1+.12)^{3}$	= .711
4	$1/(1+.12)^{4}$	= .635
5	$1/(1+.12)^{5}$	= .567
.	.	
.	.	
.	.	
10	$1/(1+.12)^{10}$	= .321
.	.	
.	.	
.	.	
50	$1/(1+.12)^{50}$	= .003

Now let's see how to calculate present value

To calculate the present value of future money
multiply future costs and benefits
by the discount factor
for every year of the project
1990-1994

Cost x Discount Factor = Present Value
Benefit x Discount Factor = Present Value



Let's use this technique to calculate present value of cost

Assume a 5-year project with the following costs...

Cost x Discount Factor = Present Value

Year	Cost
1	100
2	110
3	120
4	130
5	140

On the table find the discount factors corresponding to a discount rate of 12% for years 1-5.

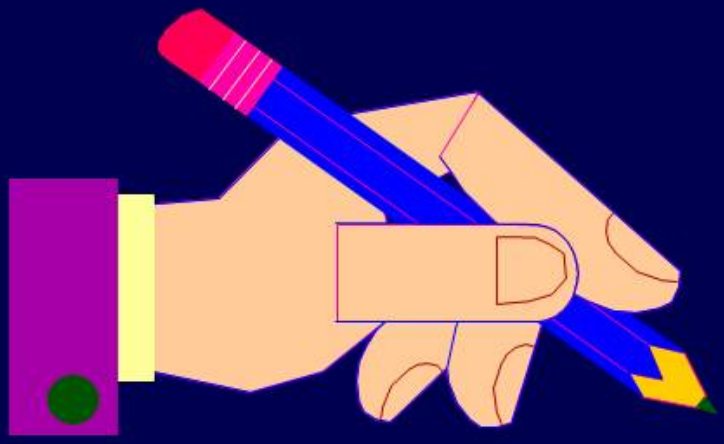
Multiply the discount factor by costs for each project year.

Year	Cost	Discount Factor
1	100	.892
2	110	.797
3	120	.711
4	130	.635
5	140	.567

The result is the

present value of costs...

Costs x discount factor = present value of costs

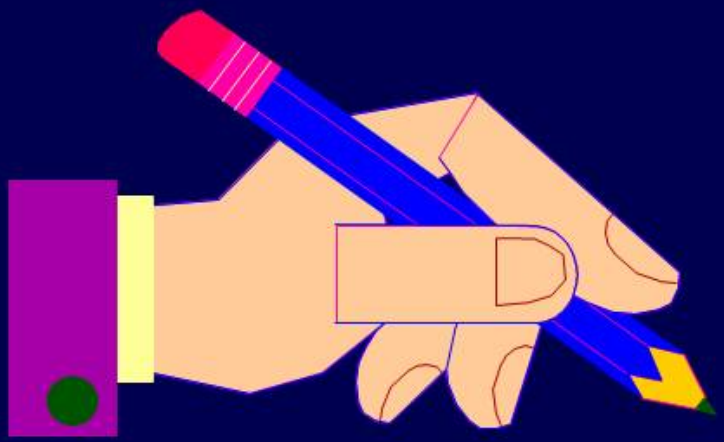


Year	Cost	Discount Factor	Present Value of Cost
1	$ 100	.892	$ 89.20
2	110	.797	87.67
3	120	.711	85.32
4	130	.635	82.55
5	140	.567	79.38

Cost x Discount Factor = Present Value of Cost

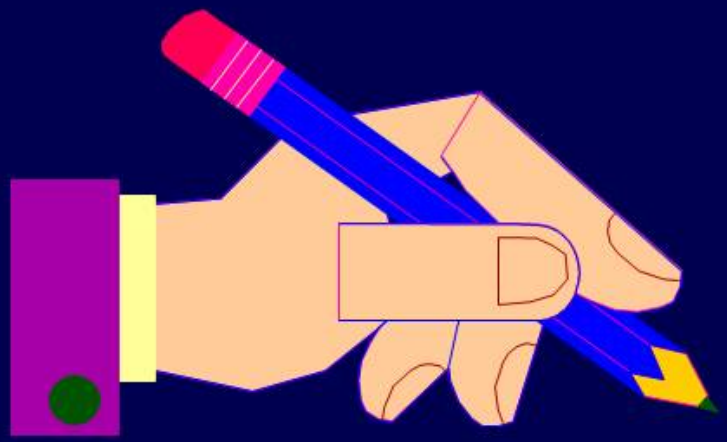
We can also calculate the present value of benefits

Assume that the project has the following benefits over its 5-year life.



Year	Benefits
1	$150
2	160
3	170
4	180
5	190

Multiply the discount factor corresponding to a discount rate of 12% to obtain the present value of benefits.

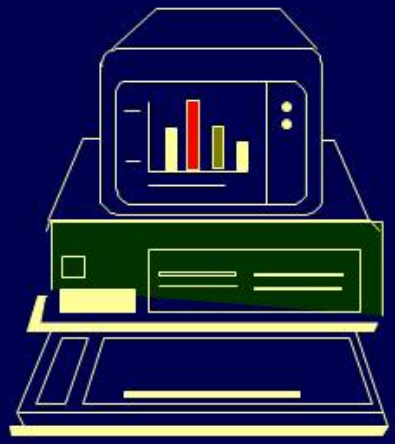


Year	Benefits	Discount Factor
1	$150	.892
2	160	.797
3	170	.711
4	180	.635
5	190	.567

Calculate the present value

of benefits by multiplying

benefits x the discount factor at 12 %



Year	Benefits	Discount Factor	Present Value of Benefits
1	$150	.892	$ 133.80
2	160	.797	127.52
3	170	.711	120.87
4	180	.635	114.30
5	190	.567	107.73

Benefits x Discount Factor =
Present Value of Benefits

The technique of finding the present value of the costs and benefits is called

Discounting.

The term

Discounted

means future costs and benefits
for which present values
have been obtained

Costs x discount factor = discounted costs

Benefits x discount factor = discounted benefits

The net present value (NPV) is the difference between the *present value of benefits* and the *present value of costs.*

Year	Present Value of Benefits	Present Value of Costs	Net Present Value
1	$ 133.80	$ 89.20	$ 44.60
2	127.52	87.67	39.85
3	120.87	85.32	35.55
4	114.30	82.55	31.75
5	107.73	79.38	28.35
Over 5-year project life			
Total	604.22	424.12	180.10

An Annuity Factor

is derived as follows:

At a discount rate of 10%
the discount factors for years 1-5
are:

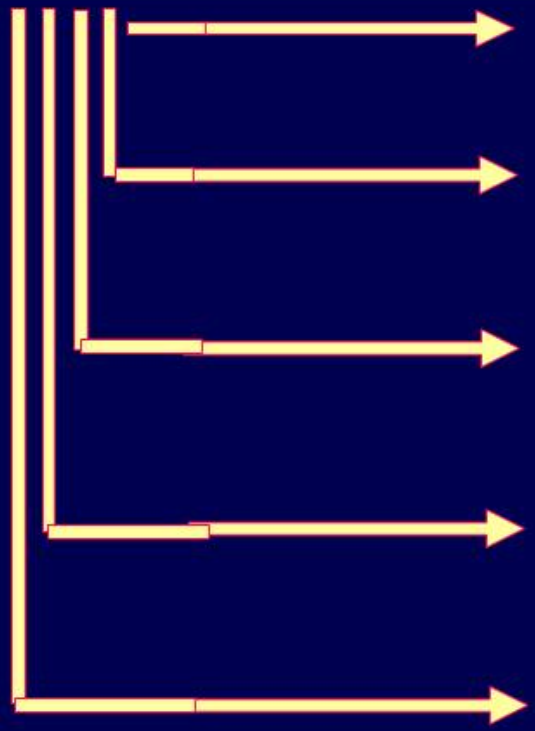
Year	10 % Discount Factor	Present Worth of Annuity Factor
1	.909	
2	.826	
3	.751	
4	.683	
5	.621	

Now we will calculate

annuity factors

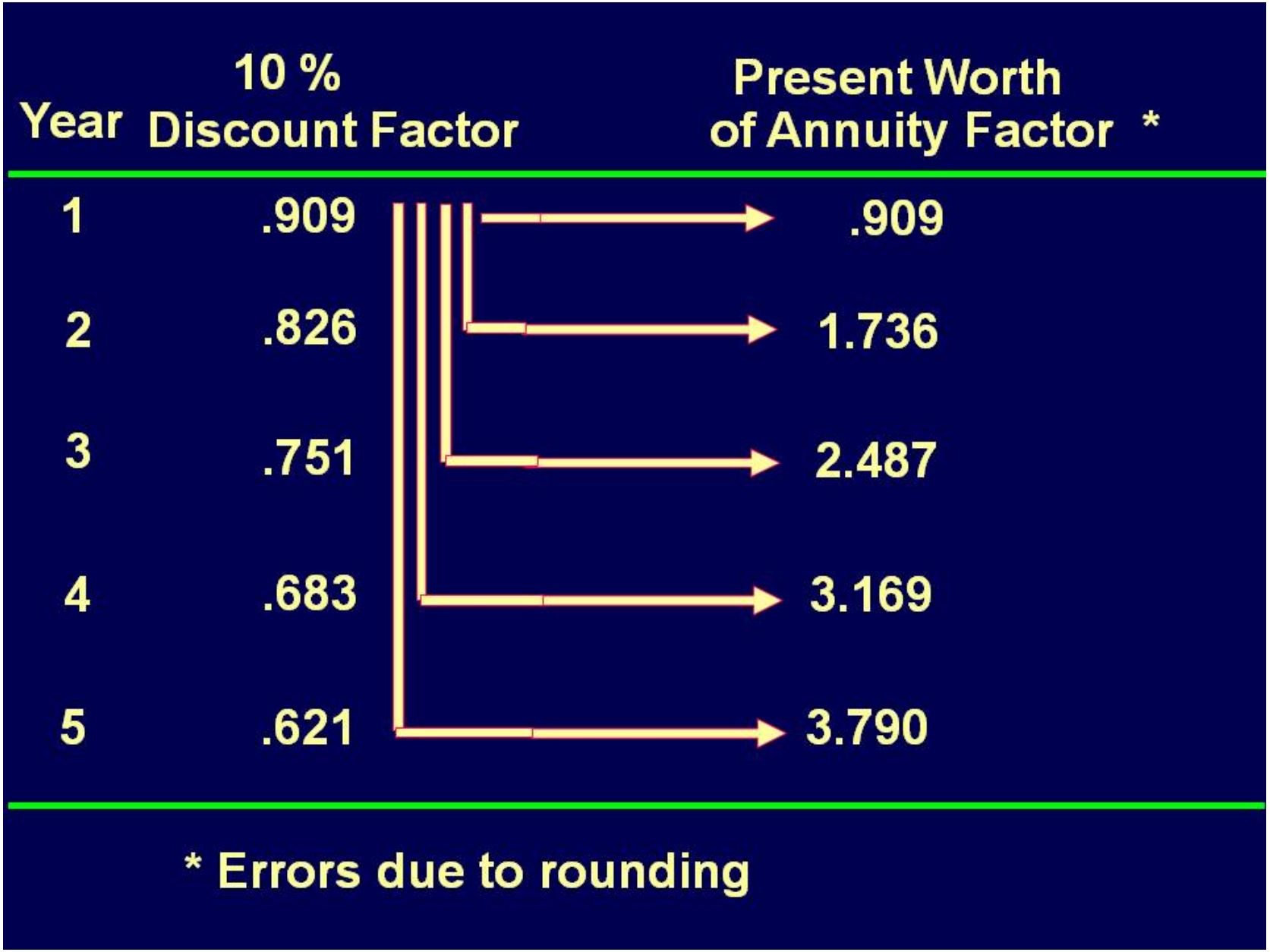
based on the discount factors:

Year	10 % Discount Factor	Present Worth of Annuity Factor
1	.909	
2	.826	
3	.751	
4	.683	
5	.621	



- *The annuity factor for year 1 is the same as the discount factor for year 1.*
- *The annuity factor for year 2 is the sum of the discount factors for years 1 and 2.*
- *The annuity factor for year 3 is the sum of the discount factors for years 1, 2, and 3.*

and so on ...


Year
10 % Discount Factor
Present Worth of Annuity Factor *
1
.909
.909
2
.826
1.736
3
.751
2.487
4
.683
3.169
5
.621
3.790
* Errors due to rounding

Verify the value of annuity factors at 10%
by adding the discount factors.

An Annuity Factor

is the sum of all discount factors up to the year in question.

Annuity factors are used when the value of costs or benefits is the same in consecutive years

An Exercise

Now calculate the annuity factors corresponding to a discount rate of 12%.

Verify that the values correspond with the ones presented here

Year	12% Disc. Factor	12% Annuity Factor
1	$1/(1+.12)^1 = .892$	.892
2	$1/(1+.12)^2 = .797$	.892+.797=1.690
3	$1/(1+.12)^3 = .711$	.892+.797+.711=2.401
4	$1/(1+.12)^4 = .635$	.892+.797+.711+.635=3.037
5	$1/(1+.12)^5 = .567$	.892+.797+.711 +.635+.567=3.604
.	.	.
.	.	.
.	.	.
10	$1/(1+.12)^{10} = .321$	.
.	.	.
.	.	.
.	.	.
50	$1/(1+.12)^{50} = .003$	.

50-year project

An Exercise

Calculation: Use discount factors corresponding to a 10% discount rate for a 5-year project

with yearly costs of $100, and yearly benefits of $200.

Determine:

1. Present (discounted) values each year and
2. The sum of present values.

Year	Costs	10% Disc. Factor	Disc. Costs	Benefits	Disc. Benefits
1	$100	.909		$200	
2	100	.826		200	
3	100	.751		200	
4	100	.683	?	200	?
5	100	.621		200	

Year	Costs	10% Disc. Factor	Disc. Costs	Benefits	Disc. Benefits
1	$100	.909	$90.90	$200	
2	100	.826	82.60	200	
3	100	.751	75.10	200	
4	100	.683	68.30	200	?
5	100	.621	62.10	200	

Year	Costs	10% Disc. Factor	Disc. Costs	Benefits	Disc. Benefits
1	$100	.909	$90.90	$200	$181.80
2	100	.826	82.60	200	165.20
3	100	.751	75.10	200	150.20
4	100	.683	68.30	200	136.60
5	100	.621	62.10	200	124.20
Total			???		???

Year	Costs	10% Disc. Factor	Disc. Costs	Benefits	Disc. Benefits
1	$100	.909	$90.90	$200	$181.80
2	100	.826	82.60	200	165.20
3	100	.751	75.10	200	150.20
4	100	.683	68.30	200	136.60
5	100	.621	62.10	200	124.20
Total			$379.00		$758.00

758.00 - 379.00 = $379.00

758.00 - 379.00 = 379.00 (NPV)

Net Present Value

Exercise:

Now take the annuity factor for year 5 of 3.790

Multiply by the value of benefits and costs each year

3.79 x 100 = $379 total discounted costs

3.79 x 200 = $758 total discounted benefits

Values are the same as those calculated with discount factors

What follows is
an exercise with net benefits
(benefits minus costs).

Find the net present value (NPV) or
(NPW) net present worth, as it is
sometimes called.

Suppose we have the stream of net benefits:

Year	Net Benefits	NPW
1	$5,000	
2	5,000	
3	6,000	
4	6,000	
5	6,000	

Find the Net Present Worth using annuity factors at a 10 % discount rate.

Suppose we have the stream of net benefits:

Year	Net Benefits		NPW
1	$ 5,000		
2	5,000	$5,000 x 1.735 =	$8,675
3	6,000		
4	6,000	$6,000 x (3.790-1.735)	
5	6,000	=	$12,330
Total		$8,675 + $12,330 =	$21005

Find the Net Present Worth using annuity factors at a 10 % discount rate.

EXERCISE

Calculate the NPV of the following Net benefits, using annuities, at a Discount rate of 11%.

YEAR	NET BENEFITS	NPV
1	- 100	
2	- 100	
3	- 100	
4	200	
5	250	
6	300	
7	300	
8	300	
9	300	
10	300	
11	300	
12	300	
13	300	
14	300	

YEAR	NET BENEFITS		NPV
1	-100		
2	-100		
3	-100	2.44 x -100 = -244	-244
4	200	3.1 -2.44 = .66 x 200 = 132	132
5	250	3.69 -3.1= .59 x 250 = 147.5	147.5
6	300		
7	300		
8	300		
9	300		
10	300		
11	300		
12	300		
13	300		
14	300	6.98 -3.69= 3.29 x 300 = 987	987
			1022.5

Summary

* Discounting refers to finding the present value of future costs and benefits.

* Present values are obtained by multiplying discount factors or annuity factors by costs and benefits over the life of the project.

In this lesson, you learned how to calculate, using discount factors, the **Net Present Value** of an investment.

You learned to use annuity factors when the costs and/or benefits (or net benefits) remain constant in consecutive years.

This short-cut will save time when doing calculations by hand.

In the next lesson you will learn to use measures of project worth in *appraising* an investment.

Part 5

Calculating the Net Present Value, Benefit/Cost Ratio, and Internal Rate of Return.

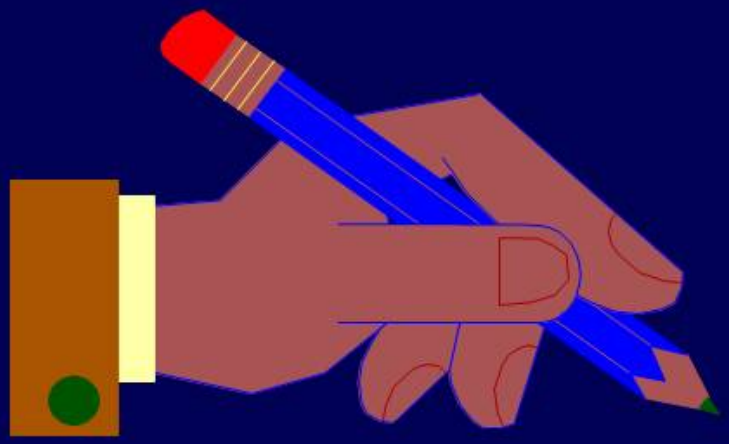
In this lesson, we will learn how to calculate the Net Present Value, the Benefit/Cost (BC) Ratio, and the Internal Rate of Return (IRR).

NPV, B/C and IRR are the key criteria for deciding whether to invest in a proposed investment project or for ranking alternative investment opportunities based on their profitability.

Calculation of Net Present Worth

or "Net Present Value"

What is Net Present Worth?

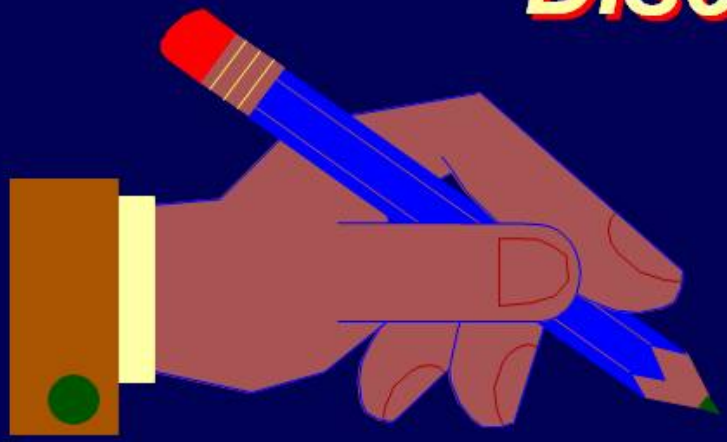


Net Present Worth (Net Present Value)

The difference between discounted incremental benefits and discounted incremental costs.

This difference is also known as

Discounted Cash Flow



Let us do the calculations with an example.

Example:

A small factory for canning vegetables has the following costs and benefits, projected over the next three years:

YEAR	1	2	3	(1000$)
COSTS	80	90	100	
BENEFITS	100	110	120	

Purchasing additional machinery, will expand its operations. Costs and benefits with the investment are expected to be:

YEAR	1	2	3	(1000$)
COSTS	230	170	120	
BENEFITS	150	210	270	

Determine incremental benefits and costs.

Benefits

Year	With	Without	Incremental
1	$150	$100	$50
2	$210	$110	$100
3	$270	$120	$150

Costs

Year	With	Without	Incremental
1	$230	$80	$150
2	$170	$90	$80
3	$120	$100	$20

Subtract incremental costs from incremental benefits for each year to determine cash flow.

Year	Incremental Benefits	Incremental Costs	Cash Flow
1	$50	$ 150	
2	$100	$80	
3	$150	$20	

Subtract incremental costs from incremental benefits for each year to determine cash flow.

Year	Incremental Benefits	Incremental Costs	Cash Flow
1	$50	$150	- $100
2	$100	$80	$20
3	$150	$20	$130

Choose discount rate
(in this example, 10%)
and multiply cash flows by the corresponding discount factor.

Year	Cash Flow	x	Discount Factor	=	Discounted Cash Flow
1	- $100				
2	$20				
3	$130				

Choose discount rate
(in this example, 10%)
and multiply cash flows by the corresponding discount factor.

Year	Cash Flow	x	Discount Factor	=	Discounted Cash Flow
1	- $100		.909		
2	$20		.826		
3	$ 130		.751		

Choose discount rate
(in this example, 10%)
and multiply cash flows by the corresponding discount factor.

Year	Cash Flow	x	Discount Factor	=	Discounted Cash Flow
1	- $100		.909		- $90.90
2	$20		.826		$16.52
3	$130		.751		$97.63

Net Present Value = $97.63 + 16.52 -90.90 = $23.25

The NPV is positive (NPV = 23.25).

This investment is acceptable.

ACCEPT INVESTMENT IF NPV IS POSITIVE.

Steps in Calculating the Benefit/Cost Ratio

In this lesson, we will

discuss the six necessary steps

for determining the

Benefit/Cost Ratio

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

1. Determine incremental benefits and costs.

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors.

4. Add discounted benefits over the life of the project.

5. Add discounted costs over the life of the project.

6. Divide discounted benefits by discounted costs.

1. Determine incremental benefits and costs

To determine incremental benefits and costs, we need to determine the present costs and benefits of the enterprise, without the investment project.

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Next, we need to determine the costs and benefits of the enterprise, as if the investment had been implemented (with the project).

1. Determine incremental benefits and costs

To determine incremental benefits and costs, we need to determine the present costs and benefits of the enterprise, without the investment project.

Next, we need to determine the costs and benefits of the enterprise, as if the investment had been implemented (with the project).

The difference in benefits and costs "with" the project and the "without" the project are the incremental benefits and costs.

The costs and benefits associated with the investment project for each year of the project life might be obtained from a variety of sources.

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For example, by studying other industries with similar projects, and then adjusting for particular characteristics of our project, estimates of cost and benefits can be obtained.

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Estimates can also be obtained by studying budgets from experiment stations, the extension service, or from other technical experts.

Example:

A small factory for canning vegetables has the following costs and benefits, projected over the next three years:

YEAR	1	2	3	(1000$)
COSTS	80	90	100	
BENEFITS	100	110	120	

Purchasing additional machinery, will expand its operations. Costs and benefits with the investment are expected to be:

YEAR	1	2	3	(1000$)
COSTS	230	170	120	
BENEFITS	150	210	270	

Benefits

Year	With	Without	Incremental
1	$150	$100	$50
2	$210	$110	$100
3	$270	$120	$150

Costs

Year	With	Without	Incremental
1	$230	$80	$150
2	$170	$90	$80
3	$120	$100	$20

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

2. Choose the Discount Rate.

For example

10 percent

See the lesson

"choice of the discount rate".

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

Determine incremental benefits and costs.

Benefits

Year	With	Without	Incremental
1	$150	$100	$50
2	$210	$110	$100
3	$270	$120	$150

Costs

Year	With	Without	Incremental
1	$230	$80	$150
2	$170	$90	$80
3	$120	$100	$20

Year	Incremental Benefits	Discount Factor	Discounted Incremental Benefits
1	$50	.909	$ 45.45
2	100	.826	82.60
3	150	.751	112.65

Year	Incremental Costs	Discount Factor	Discounted Incremental Costs
1	$150	.909	$ 136.35
2	80	.826	66.08
3	20	.751	15.02

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

4. Add discounted incremental benefits over the life of the project.

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$ 45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total		

4. Add discounted incremental benefits over the life of the project.

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$ 45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total	$240.70	

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

5. Add discounted incremental costs over the life of the project.

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$ 45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total	$240.70	

5. Add discounted incremental costs over the life of the project.

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$ 45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total	$240.70	$217.45

1. Determine incremental benefits and costs

2. Choose the Discount Rate

3. Multiply incremental benefits and costs by the discount factors

4. Add discounted benefits over the life of the project

5. Add discounted costs over the life of the project

6. Divide discounted benefits by discounted costs

6. *Divide discounted incremental benefits by discounted incremental costs.*

Year	Discounted Incremental Benefits		Discounted Incremental Costs
1	$ 45.45		$ 136.35
2	82.60		66.08
3	112.65		15.02
Total	$240.70	÷	$217.45

6. *Divide discounted incremental benefits by discounted incremental costs*

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$ 45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total	$240.70	$217.45

÷ = 1.11

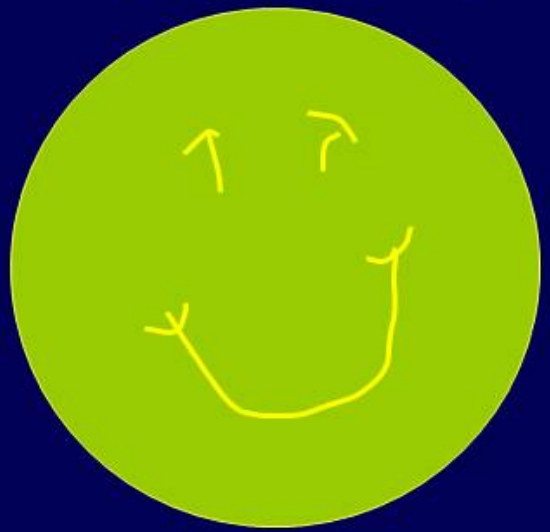
The Benefit/Cost Ratio is 1.11.

Decision criteria:

B/C < 1 (Reject project)

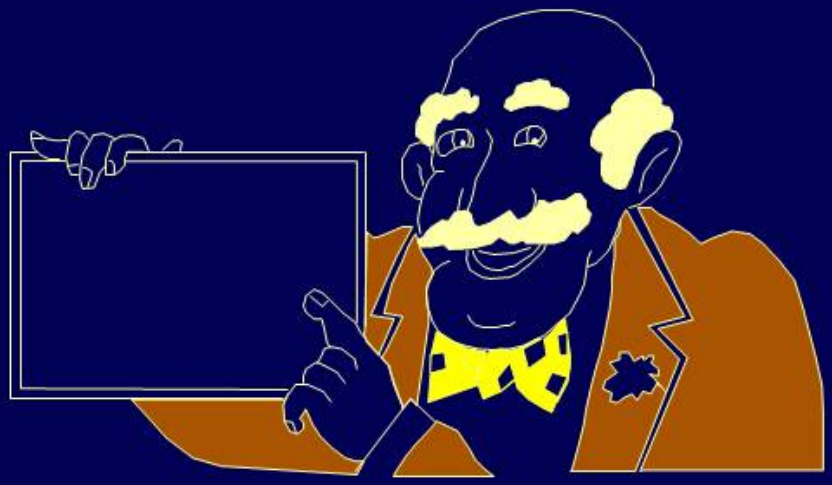
Decision criteria:

B/C > 1 (Accept project)



Calculating the Internal Rate of Return

To calculate the Internal Rate of Return, we need to calculate the net present value at varying discount rates, until the net present value at some discount rate equals zero.



To calculate the Internal Rate of Return, we need to calculate the net present value at varying discount rates, until the net present value at some discount rate equals zero.

The discount rate at which the net present value is zero is called the Internal Rate of Return.

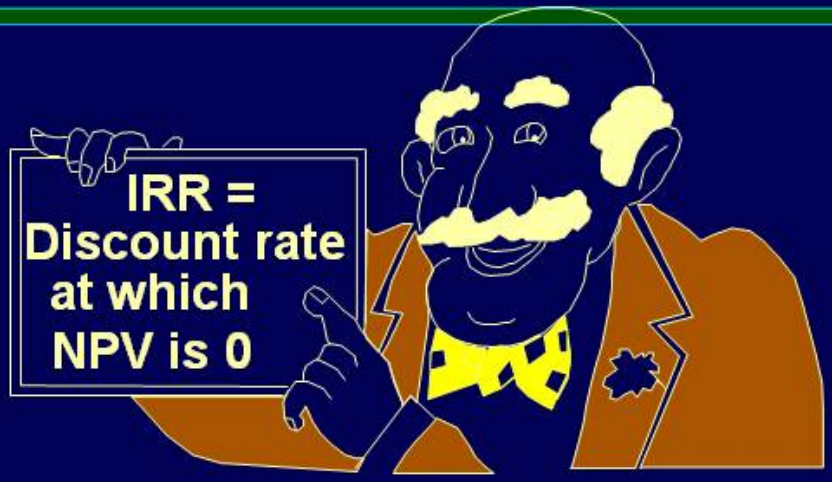
It is difficult to find this exact discount rate. Therefore, we first attempt to find a discount rate for which the net present value is a small positive number, and then, to find a discount rate for which it is a small negative number.

It is difficult to find this exact discount rate.

Therefore, we first attempt to find a discount rate for which the net present value is a small positive number, and then, to find a discount rate for which it is a small negative number.

We then interpolate to find the exact

Internal Rate of Return.



GOING BACK TO OUR EXAMPLE...

Calculation of Net Present Value at 10% Discount Rate.

Year	Incremental Benefits	Discount Factor	Discounted Incremental Benefits
1	$ 50	.909	$ 45.45
2	100	.826	82.60
3	150	.751	112.65

Year	Incremental Costs	Discount Factor	Discounted Incremental Costs
1	$ 150	.909	$ 136.35
2	80	.826	66.08
3	90	.751	15.02

Net Present Value at 10% Discount Rate.

Year	Discounted Incremental Benefits	Discounted Incremental Costs
1	$45.45	$ 136.35
2	82.60	66.08
3	112.65	15.02
Total	$240.70	$217.45

NPV = 23.25

Net Present Value at other Discount Rates:

Same Procedure, using different discount factors

You will now calculate the

Net Present Value

for the following discount rates:

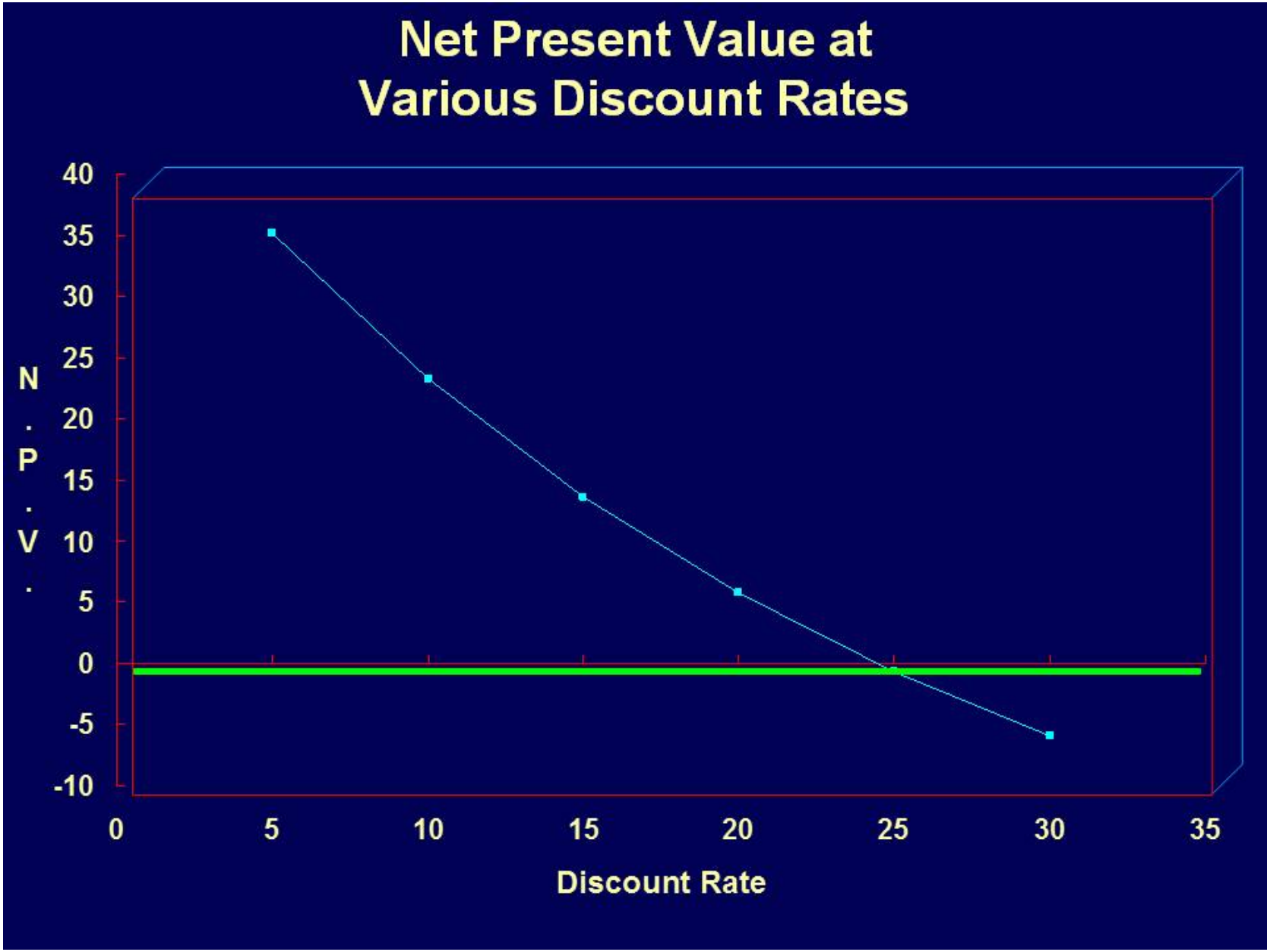
Net Present Value (N.P.V.) at other Discount Rates:

Discount Rate	N.P.V.
5	
10	
15	
20	
25	
30	

Net Present Value (N.P.V.) at other Discount Rates:

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

On the following diagram we will plot the net present values associated with each discount rate.


Net Present Value at
Various Discount Rates
N
.
P
.
V
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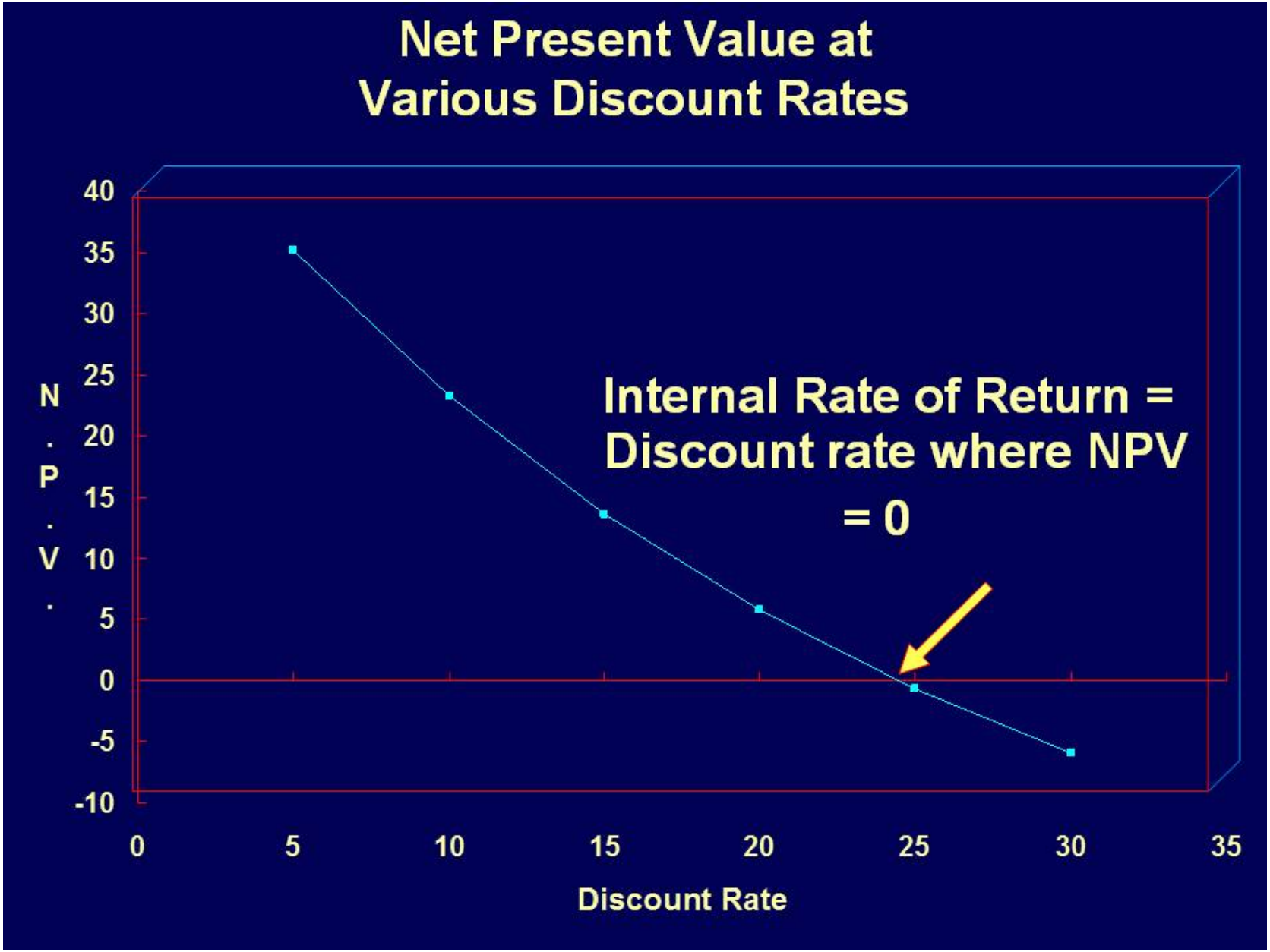
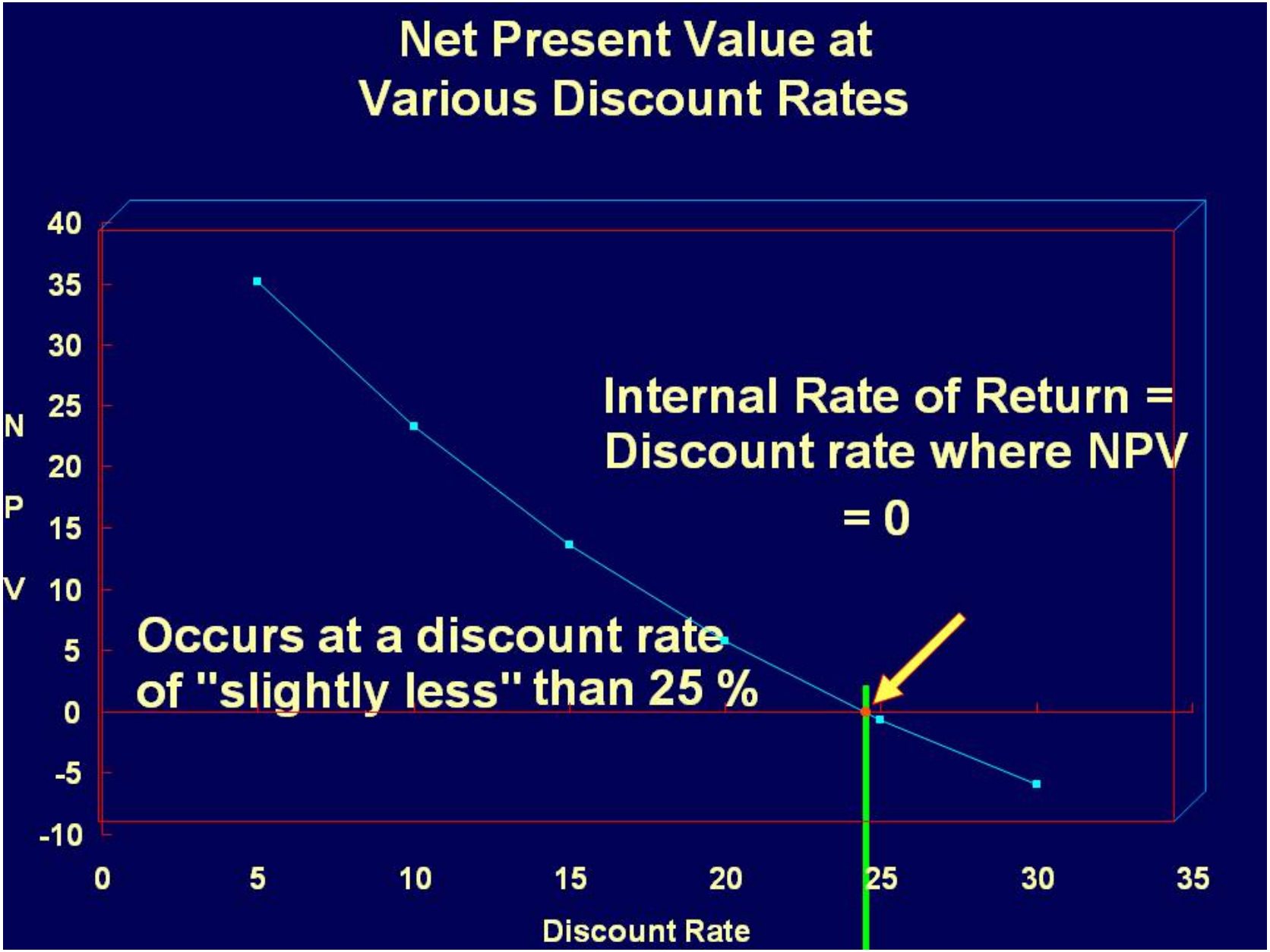
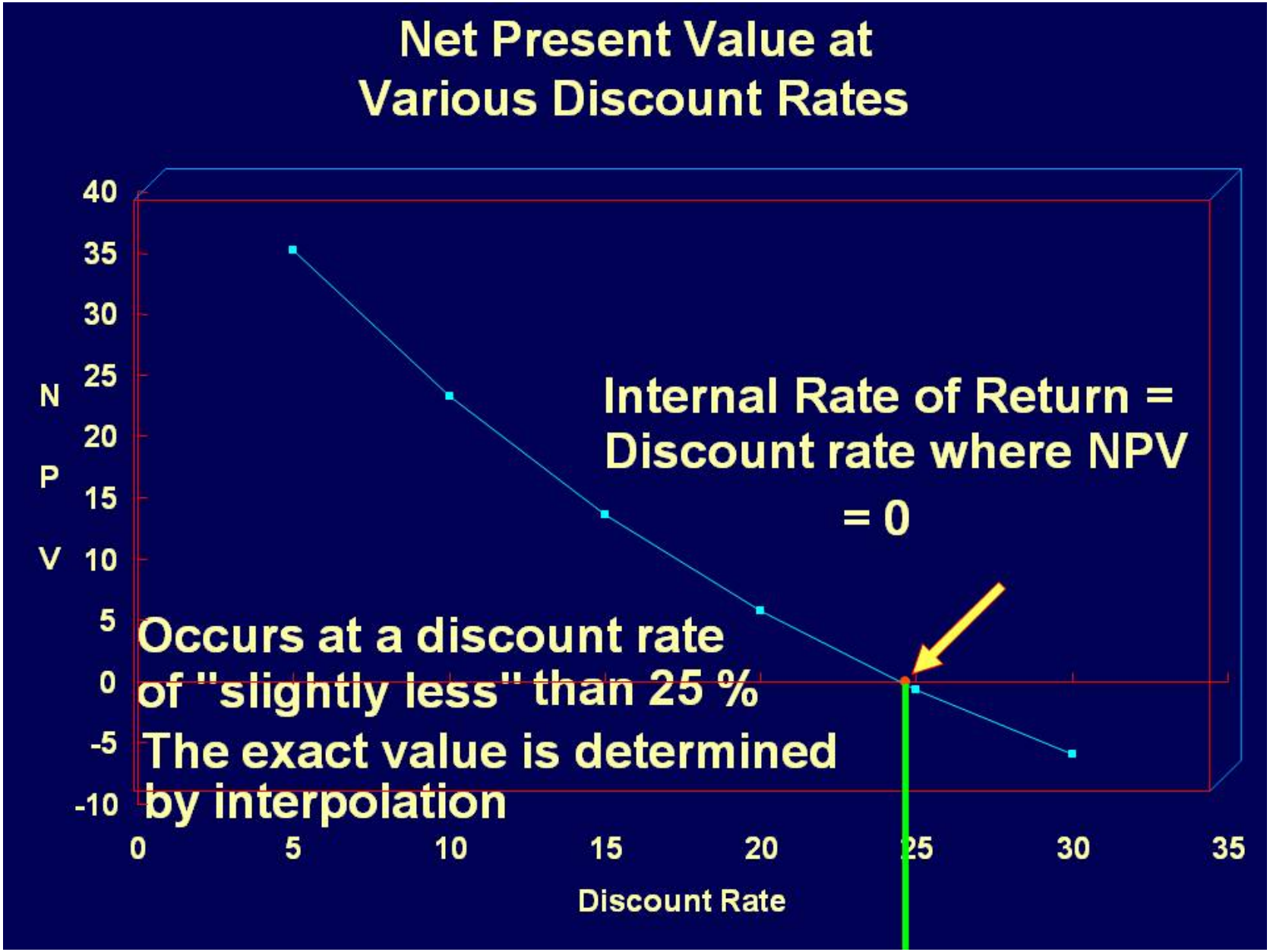
40
35
30
25
20
15
10
5
0
-5
-10
0
5
10
15
20
25
30
35
Discount Rate

Net Present Value (N.P.V.) at other Discount Rates:

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92


Net Present Value at
Various Discount Rates
N.P.V.
40
35
30
25
20
15
10
5
0
-5
-10
0
5
10
15
20
25
30
35
Discount Rate
Internal Rate of Return =
Discount rate where NPV
= 0


Net Present Value at
Various Discount Rates
N
P
V
40
35
30
25
20
15
10
5
0
-5
-10
0
5
10
15
20
25
30
35
Discount Rate
Internal Rate of Return =
Discount rate where NPV
= 0
Occurs at a discount rate
of "slightly less" than 25 %


Net Present Value at
Various Discount Rates
40
35
30
25
20
15
10
5
0
-5
-10
N
P
V
Internal Rate of Return =
Discount rate where NPV
= 0
Occurs at a discount rate
of "slightly less" than 25 %
The exact value is determined
by interpolation
0
5
10
15
20
25
30
35
Discount Rate

Now, we will learn to interpolate between the two discount rates--the one with the positive N.P.V. and the one with the negative N.P.V.

This procedure determines the

Internal Rate of Return.

Find the two discount rates
on either side of the N.P.V. = 0.

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

For the IRR,

we will use simple proportions to solve.

The distance between 5.79 and 0 is what proportion of the distance between 5.79 and -.64?

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

The distance between -0.64 and 5.79 =
0.64 + 5.79 = 6.43

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

The distance between -0.64 and 5.79 is

0.64 + 5.79 = 6.43

The ratio 5.79/6.43 = 0.90

Discount Rate	N.P.V.
5	$ 35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

Therefore, 0.90 is the proportion of the distance between 5.79 and zero.

The distance between 20 and 25 is 5.

Discount Rate	N.P.V.
5	$35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

The distance between 20 and 25 is 5.

.90 x 5 = 4.5

Discount Rate	N.P.V.
5	$35.20
10	23.25
15	13.64
20	5.79
25	- 0.64
30	- 5.92

Based on the proportion of 0.90, the discount rate corresponding to a zero net present value, lies 90% of the distance 5 :

$$0.9 \times 5 = 4.5$$

This means that the discount rate corresponding to a zero net present value is the discount rate of 20 + 4.5

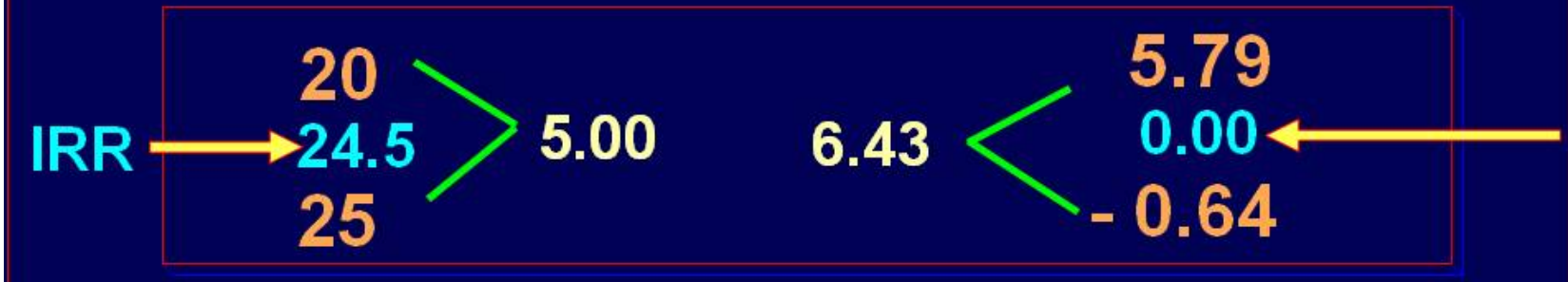
$$20 + 4.5 = 24.5$$

The distance between 20 and 25 is 5.

0.90 x 5 = 4.5

The internal rate of return is 20 + 4.5 = 24.5.

Discount Rate	N.P.V.
5	$35.20
10	23.25
15	13.64
20	5.79
IRR → 24.5	0.00 ←
25	- 0.64
30	-5.92

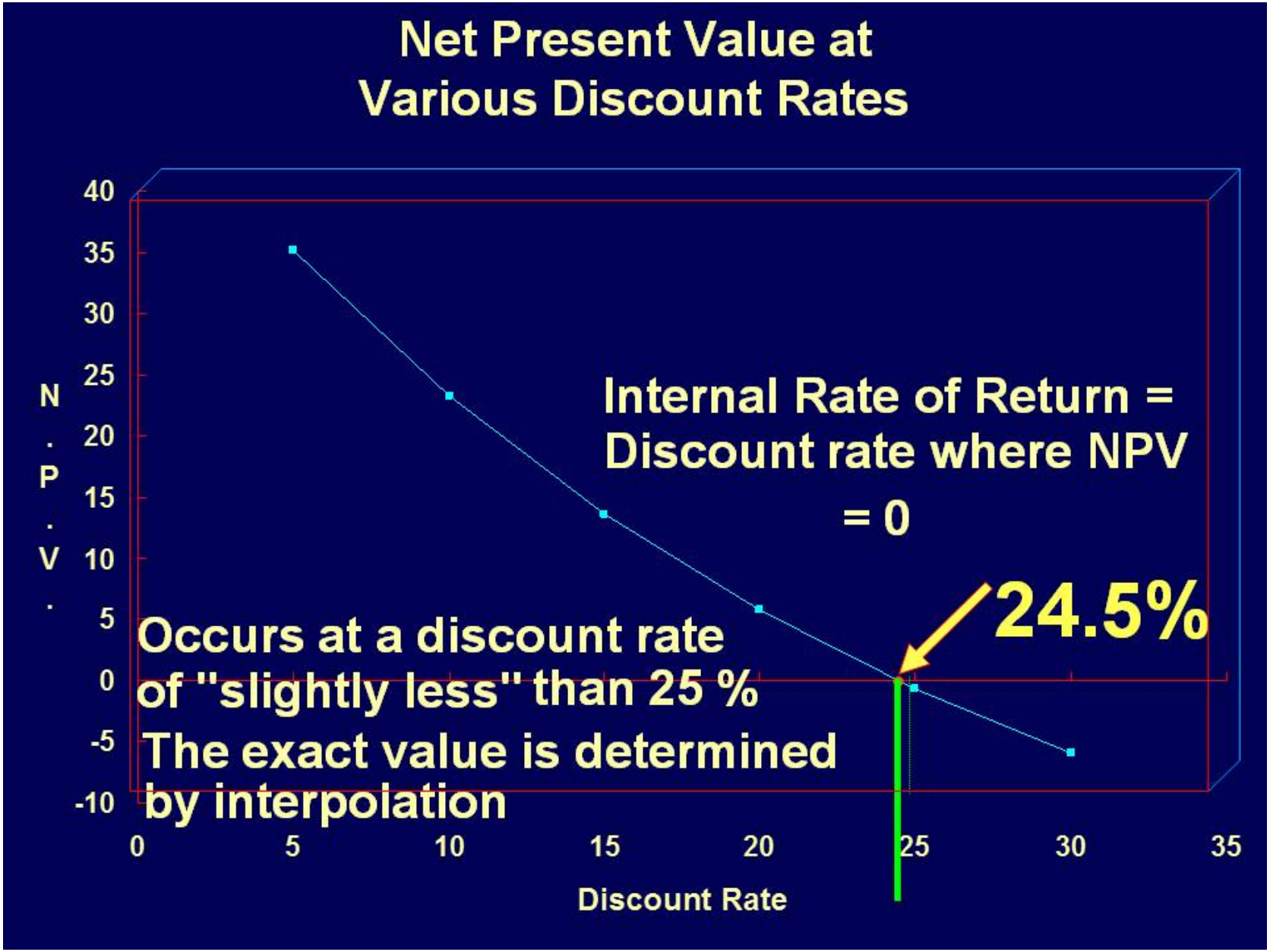


5.79/6.43 = 0.90

0.90 x 5.00 = 4.50

20 + 4.50 = 24.50

24.50 = IRR


Net Present Value at
Various Discount Rates
N.P.V.
40
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30
25
20
15
10
5
0
-5
-10
0
5
10
15
20
25
30
35
Discount Rate
Internal Rate of Return =
Discount rate where NPV
= 0
24.5%
Occurs at a discount rate
of "slightly less" than 25 %
The exact value is determined
by interpolation

The smaller the difference between the discount rate giving us a negative net present value and the discount rate giving us a positive net present value, the more accurate our interpolation and the more precise the

Internal Rate of Return

If the cash flow

does not have a

negative number,

the IRR = ∞.

infinity

The formula to use for the interpolation is:

$$\text{IRR} = \text{Lower Discount Rate} + \text{Difference Between Discount Rates} \times \frac{\text{NPV at lower discount rate}}{\text{Absolute diff. between the two NPVs}}$$

$$\text{IRR} = 20 + 5 \times \frac{5.79}{5.79 + [.64]} = 24.5$$

Check the Calculations!

$$IRR = 20 + 5 \frac{5.79}{5.79 + [.64]} = ?$$

By completing this lesson, you have all necessary tools to proceed with a simple investment project appraisal.