DECISIONS WITH COMPETING OBJECTIVES: AN APPLICATION TO SLUDGE DISPOSAL ALTERNATIVES IN MASSACHUSETTS*

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Economic theory generally involves choice between real-valued criterion vectors $x = (x_1, \ldots, x_n) \in X$, where X is the feasible set of such vectors. For example, in the theory of consumer demand, the criterion x_i can be interpreted as the quantity demanded of the i^{th} good. Under certainty, a rational consumer would choose a value of x that would maximize the ordinal utility u(x) subject to $x \in X$. If a function u(x) does not exist, in the paradigm of Marschak [1976], he is not *rational*, his choices are not *consistent*, and some of them will be regretted. Even greater focus is placed on the multidimensional nature of the criterion function by the alternative approach suggested by Lancaster [1966]. His approach views goods or services as being demanded because they yield want-satisfying characteristics to consumers. In general, a good or service has multiple characteristics, any one of which may be shared by more than one good or service (the traditional consumer model is a special case which views the number of characteristics as equal to the number of goods or services). Further, goods or services in combination may possess characteristics (attributes) different from those pertaining to the items separately.

In fact, what applied economists typically do at the public policy level, with few substantive exceptions, is to treat a single goal--usually economic efficiency--as the sole objective, gather data, and solve for the "optimum" vector x. Given the obvious relevance at this level of multiple, noncommensurable, and competing goals, it is remarkable that it has taken vector optimization or multiple criterion decision making techniques so long to become established and that few applications have occurred to date.

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In what follows, we provide an overview of portions of the literature on multiple goal decision making techniques and focus on the Surrogate Worth Trade-off method. The technique is then illustrated for an existing decision situation in Massachusetts.

Vector Optimization and Surrogate Worth Methods

Vector optimization, or mathematical programming with more than a single goal, was pioneered as early as 1951 by Kuhn and Tucker [1951] and Koopmans [1951]. It remained relatively undeveloped until the 1960's, however, when the expression "trade-off" became a part of the language of planners and managers of all sorts. In the past decade, the existing techniques have expanded to at least 20 different procedures for solving such problems.

The term "vector optimization" refers to the problem:

(1) $\text{Max } Z(x) = [Z_1(x), Z_2(x), \ldots, Z_p(x)]$

(2) $x \in X$

(3) $X_j \geq 0 \qquad j = 1, \ldots, n$

where:

$Z(x)$ is the p-dimensional criterion function (there are p goals or objectives),

x is an n-dimensional vector of decision variables, and

X denotes the constraints associated with the problem.

The region defined by (2) and (3) in E^n is referred to as the feasible region in decision space. Each solution to the problem implies a value for each objective (i.e., $Z_k(x)$, $k = 1, \ldots, p$). The p-dimensional objective function maps the feasible region in E^n (decision space) into the feasible region in objective space (E^p).

The expression "vector optimization" is a contradiction in terms, however, since without information about preferences which provide a means for combining incommensurable objectives, and hence all feasible solutions are not ordered, an "optimal" solution cannot be found to the problem. This complete ordering can be obtained only by introducing value judgments into the solution process.

Even with no knowledge of preferences, some of the feasible solutions to the vector optimization problem can be eliminated by the incomplete ordering associated with the p-dimensional objective function. If more of all objectives is preferred to less, then only non-inferior or Pareto optimal solutions are of interest.

A method for finding such non-inferior solutions can be illustrated by transforming the problem of (1) through (3) into:

(4) $\text{Max } S_k(x)$

subject to:

(5) $x \in X$

(6) $S_r(x) \leq f_r \quad \forall_{r \neq k}$

where, $S_k(x)$ is a single objective function, x is an n-dimensional vector of decision variables, X represents decision space, $S_r(x)$ designates objectives as constraints, and f_r depicts the upper bounds.

To determine the starting values for the f_r, a preparatory analysis should be conducted. This will involve maximizing $S_r(x)$ for all r = 1, ..., p subject to $x \in X$. Therefore, p optimization problems will be solved with each optimal solution resulting in primal values for x. By substituting each set of primal x values into the (p - 1) objectives in (6), various levels for each objective can be obtained. For each objective, the resulting solutions will have p values. The lowest value obtained for the corresponding objective will yield a starting point for setting f_r. This preliminary exploration will eliminate any randomness in selecting initial f_r levels. Furthermore, this opening approach guarantees that the beginning upper bound levels will not result in infeasible solutions, but instead will generate non-inferior solutions.

The method proceeds by incrementally increasing one of the f_r (let us call it $f_{r'}$) contained in (6) until an infeasible solution results, while holding all other f_r fixed. At each level of $f_{r'}$, primal solution values for x result. These x quantities are substituted into the $S_r(x)$ and maximized $S_k(x)$ to obtain objective function values. Parametric variation of the right-hand side yields points in the non-inferior set where each level of the objectives represents a non-inferior solution in functional space. Another f_r (let us call it $f_{r''}$) is then increased parametrically until an infeasible solution results, holding all other f_r fixed. Again, the corresponding x primal values are substituted into the objectives acting as constraints and a maximized function to determine points in the non-inferior set. This procedure is repeated for all f_r bounds until an infeasible solution is generated. The k^{th} objective from (4) is then introduced into the constraint set (6) and the $r \neq k$ objectives are introduced into (4) seriatim.

Of a host of vector optimization methods (recently surveyed and compared by Cohon and Marks [1975], the "surrogate worth trade-off" method was selected for use here on the basis of the quantity of information it supplies as well as computational efficiency. This relatively new method asserts that the decision making process is facilitated by an assessment of the trade-off values of marginal gains and losses among any two objectives relative to the level of attainment for all the objectives. The method was introduced by Haimes and Hall [1974] and later by Haimes, Hall and Freedman [1975]--the reader is referred to these sources for a detailed treatment of the method. For present, we merely note that the surrogate worth trade-off method provides a means of assisting the decision-maker to find a "preferred" non-inferior solution, essentially by determining the tangency between the locus of points of the transformation curve contained in objective space and his perception of the social indifference curve (representing societal utility levels). For simplicity, for example, this can be accomplished by evaluating each non-inferior solution (containing objective and decision space solutions) according to the following criteria:

1. $0 < W \leq 10$, when δ_{kr} marginal units of $S_k(x)$ are preferred over one marginal unit of $S_r(x)$, given fixed levels of all objectives,

2. $W = 0$, when δ_{kr} marginal units of $S_k(x)$ are equivalent to one extra unit of $S_r(x)$, given fixed levels of all objectives,

3. $-10 \leq W < 0$, when δ_{kr} marginal units of $S_k(x)$ are not preferred to one extra unit of $S_r(x)$, given fixed levels of all objectives.

Each non-inferior solution is assigned an ordinal W value with W = 0 representing the "preferred" solution.

<u>Application</u>

The methodology described above is demonstrated for the situation currently prevailing in Boston. Decisions are being made on the method of disposal of Boston's sewage sludge. Ocean disposal is the present mode, and incineration and land application are the alternatives receiving the greatest attention in the current decision process. The preliminary studies and reports pertaining to the Boston situation (primarily Havens and Emerson [1974] and Ecol Science [1976]) emphasize cost and environmental aspects associated with various disposal systems, and ultimately support the incineration alternative.

Some rather obvious shortcomings of these studies, however, have been suggested by the New England Consortium on Environmental Protection [1976]. Among these, three seem particularly crucial. First, these alternatives were treated in pretty much an "all-or-none" context, rather than permitting the possibility of a "hybrid" mix of several. Second, criticisms about the quantification of indices of likely environmental effects were advanced. An extension of these charges might be that regardless of how the indices were constructed, no attempt was apparently made to include formally monetary cost and environment in the same decision analysis. Third, the Consortium was disappointed that degrees of uncertainty were not formally recognized in the choice process and that value of the uncertainty-reducing information to result from adoption was not considered (indeed, the wealth of experience with various forms of R and D strategies appears to have been ignored). Again, an extension of their arguments might be that uncertainty was not formally and explicitly considered alongside environment and dollar costs in making recommendations on the alternative disposal modes.

For the present application, then, we focus on these shortcomings--viz., we provide a choice framework with three activities (tons of sludge disposed by ocean, incineration, and land application) and three objectives (dollar costs, expected environmental costs, and uncertainty with respect to these environmental costs). For this framework, hybrid activities are admissible and the multiple goals are treated explicitly by surrogate worth methods.

On the basis of the data supplied by the preliminary studies of Havens and Emerson and Ecol Science, net economic benefits, environmental impact, and uncertainty functions ($S_r(x)$) were estimated. The data and details of the estimation process are provided in Lindsay [1976] and are available upon request. The net economic benefits function was exponential, while the others were treated as linear. With these estimations, we can illustrate the choice framework of (4) through (6) as:

$$(7)\quad \text{Max } Z = 2.61\, x_1 - 32x_1 \exp(.00000019x_1) + 16.73x_2 - 53x_2 \exp(.00000027x_2) + 2.61x_3 \exp(.00000025x_3)$$

subject to:

$$(8)\quad x_1 + x_2 + x_3 = 96{,}000$$

$$(9)\quad .0010028x_1 + .0005671x_2 + .00041x_3 = f_2$$

$$(10)\quad .00024x_1 + .0001744x_2 + .000119x_3 = f_3$$

where the objective function (7) represents net economic benefits.[1/] Constraint (8) is one of the obvious general-type constraints (i.e., that all 96,000 dry tons per year be disposed), and (9) and (10) are objective constraints of the type (6). The restraint (9) depicts the level of environmental impact for all systems as a constrained objective (f_2 is the upper right-hand bound representing this level), and (10) depicts the level of variability of environmental impact about the expected level (f_3 is the bound on this level). The activities x_1, x_2, and x_3 are dry tons of sludge utilized by ocean disposal, land application, and incineration systems, respectively.

As discussed earlier, parametric variation of f_2 and f_3, holding other values constant, will yield a set of non-inferior solutions--a set of solutions with the property that no improvement in one objective level can occur without at least one other objective deteriorating. Table 1 summarizes objective and decision space information characterizing three dozen such solutions for this application. The first solution, for example, calls for land application of 21,000 dry tons per year, incineration of 75,000 dry tons per year, would entail an estimated net cost of over 3.9 million dollars, an expected enviornmental impact of 42.66 index units, and 12.59 index units of variability. It also suggests that a small unit degradation of the environment would lower net costs by 615 dollars and a small unit allowed increase in variability would lower net costs by 110 thousand dollars.

[1/] The choice of this goal as the "primary" objective is unimportant since, as Haimes, Hall and Freedman [1975, p. 43] note, the dual variable (δ_{ij}) associated with constrained objective j equals the inverse of the dual attached to the primary objective used as a constraint ($\delta_{ij} = \delta_{ji}^{-1}$).

Table 1
Non-Inferior Solutions

Solutions Decision Space*			Solutions Objective Space			Shadow Prices		
X_1	X_2	X_3	f_1	f_2	f_3	δ_{12}	δ_{13}	Solution Number
0	21	75	-3,934,736	42.66	12.59	615.40	109,988.00	1
0	26	70	-3,904,394	43.44	12.86	2,414.53	100,240.00	2
0	32	64	-3,869,722	43.39	13.20	4,576.20	88,530.60	3
0	36	60	-3,847,684	45.02	13.42	6,018.22	80,721.80	4
0	42	54	-3,816,072	45.96	13.75	8,183.03	69,001.60	5
0	48	48	-3,786,353	46.90	14.08	10,350.40	57,271.40	6
0	54	42	-3,758,487	47.84	14.42	12,520.30	45,530.70	7
0	60	36	-3,732,525	48.79	14.75	14,692.20	33,783.80	8
0	64	32	-3,716,184	49.41	14.97	16,141.10	25,948.50	9
0	70	26	-3,693,273	50.36	15.30	18,317.20	14,184.30	10
0	75	21	-3,675,647	51.14	15.58	20,132.00	4,376.30	11
0	77	19	-3,668,900	51.46	15.69	20,858.60	450.47	12
16	32	48	-3,659,372	53.87	15.13	5,499.90	79,423.00	13
24	24	48	-3,599,837	57.36	15.65	11,715.90	43,738.40	14
16	48	32	-3,581.733	56.39	16.02	11,264.30	48.229.60	15
32	16	48	-3,542,933	60.84	16.18	671.80	101,458.00	16
16	64	16	-3,517,264	58.90	16.91	17,044.90	16,973.90	17
24	48	24	-3,482,730	61.13	16.99	11,715.90	43,738.40	18
32	32	32	-3,457.871	63.36	17.07	6,409.90	70,392.90	19
32	48	16	-3,385,931	65.87	17.96	12,164.10	39,266.50	20
48	16	32	-3,344,573	70.33	18.12	1,578.20	92,437.70	21
32	64	0	-3,327,131	68.38	18.84	17,934.30	8,079.70	22
48	24	24	-3,303,257	71.58	18.56	3,596.70	82,548.90	23
36	60	0	-3,294,124	70.13	19.10	16,711.40	13,665.30	24
48	32	16	-3,265,211	72.84	19.00	7,306.20	61,440.00	25
42	54	0	-3,245,773	72.74	19.50	14,879.30	22,031.50	26
48	48	0	-3,198,949	75.35	19.89	13,050.30	30,381.10	27
54	42	0	-3,153,611	77.97	20.28	11,224.10	38,715.80	28
64	16	16	-3,155,069	79.81	20.05	2,470.90	83,495.00	29
60	36	0	-3,109,811	80.58	20.68	9,401.18	47.034.20	30
64	32	0	-3,081,384	82.32	20.94	8,188.80	52,564.70	31
70	26	0	-3,040,023	84.94	21.33	10,376.50	49,496.70	32
75	21	0	-3,006,698	87.12	21.66	4,859.30	67,748.70	33
80	16	0	-2,974,412	89.30	21.99	3,349.90	74,630.00	34
85	11	0	-2,943,144	91.48	22.32	1,842.10	81,502.50	35
90	6	0	-2,912,912	93.65	22.65	336.70	88,362.40	36

*In thousands of dry tons per year.

This set of non-inferior solutions is now available to the decision-maker as additional information from which a selection can be made.

Surrogate Worth Trade-Off

As mentioned earlier, the surrogate worth method attempts to provide the interaction between the analytic framework and the decision-maker. Three hypothetical decision-makers were referred to a modified Table 1 (the decision space information was covered) and asked questions of the following type. On a scale of -10 to +10, what worth (W) would you attach to an annual decrease in cost of, say, δ_{12} dollars relative to a worsening of environmental impact by one index unit, given current levels of cost (f_1), environmental impact (f_2), and variability of environmental impact (f_3); similarly for δ_{13} given f_1, f_2 and f_3.

Generally, when $W_{12}(f_2^* \; f_3^*)$ and $W_{13}(f_2^* \; f_3^*)$ are both zero at $(f_2^* \; f_3^*)$ then a preferred solution is obtained. However, in practice this will usually not occur and the use of regression analysis will have to be employed to determine new non-linear solutions. For the attached worth values (W_{12} and W_{13}), for each decision-maker, and indifference band was determined and a linear regression was employed to determine the preferred solutions within the indifference band. The estimated regressions for each decision-maker are:

$$W_{12}^{1}(f_2 \; f_3) = 17.76 - 9.78f_2 + 31.01f_3$$

$$W_{13}^{1}(f_2 \; f_3) = 35.30 - 0.05f_2 - 2.21f_3$$

$$W_{12}^{2}(f_2 \; f_3) = 298.62 - 50.90f_2 + 148.03f_3$$

$$W_{13}^{2}(f_2 \; f_3) = 86.61 - 4.40f_2 + 8.53f_3$$

$$W_{12}^{3}(f_2 \; f_3) = -23.75 - 1.98f_2 + 8.16f_3$$

$$W_{13}^{3}(f_2 \; f_3) = -237.03 + 44.25f_2 - 130.15f_3$$

The value R^2 for each equation was in the range .95 to .99. Each set of equations, corresponding to a decision-maker (denoted by superscript), was solved simultaneously to determine $(f_2^* \; f_3^*)$ and these are presented in Table 2.

Graphically, the approach may be depicted in Figure 1 where KL depicts the objective space trade-off function with SS' representing hypothetical social welfare indifference curves (the shape reflects the usual assumption of diminishing marginal utility). The closer SS' moves to the origin the higher the level of utility resulting for society (f_1 values are negative--costs exceed benefits). At point E, the ordinal value of W is greater than zero. Movement along KL from E to approaching D results in W monotonically declining towards zero. Similarly, movement from F towards D results in W monotonically approaching zero from below. The tangency point represents the societal "preferred" non-inferior solution for f_1, f_2 and f_3 as expressed by the particular decision-maker's choices.

Figure 1. Geometric Interpretation of W Value

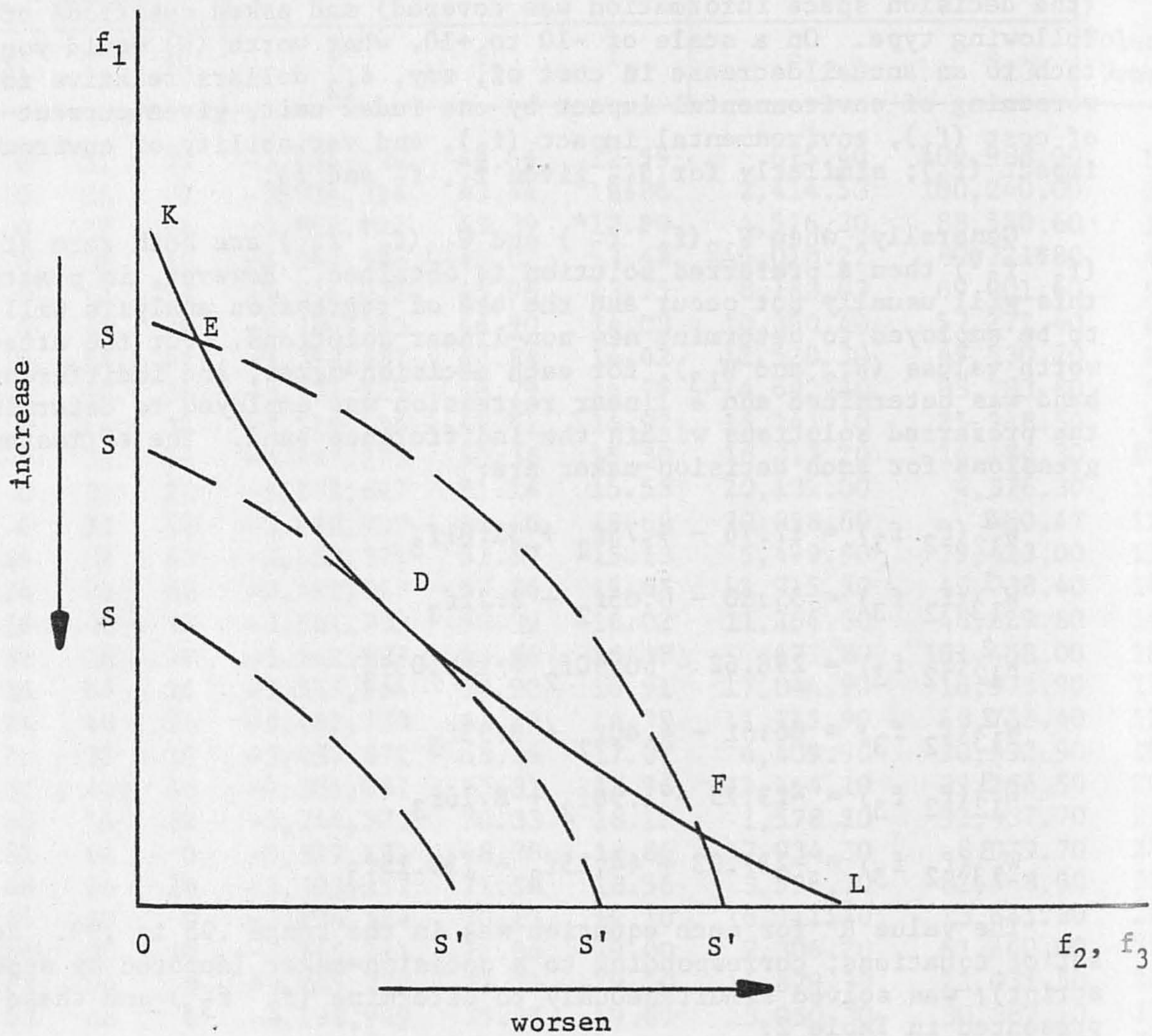

Table 2
Preferred Solutions

Decision-Maker	δ_{11}	δ_{12}	δ_{13}	f_1	f_2	f_3
1	-51.91	15,778.7	27,908.0	-3,720,182	48.95	14.86
2	-53.19	11,434.9	51,403.1	-3,772,190	47.30	14.24
3	-52.28	14,510.8	34,764.4	-3,734,583	48.60	14.70

Conclusions

With very few exceptions, economists and economic analysis have lagged behind in empirical decision analysis where more than a single objective (treated other than in lexicographic fashion) is involved. Most of the recent work, in this area, in economics has been in the form of optimum control or, quite recently, adaptive dual control, where strong assumptions are made regarding the actual preference function of the decision-maker (usually, known and quadratic). While these approaches will undoubtedly prove highly valuable in a variety of applications, there are many situations where optimum control methods are less suitable. The approach presented here also provides the attractive feature that it lets decision-makers make decisions, rather than presenting them with an "optimal" solution, implying a "take it or leave it" suggestion.

The analysis reported here is largely an attempt by the authors to "try out" the method for a current situation where decision-makers are actually puzzling over a solution to a problem. The results are illustrative only owing to data problems and the preliminary nature of the investigation. Ongoing efforts are addressed to collecting better data, improving the model specification (e.g., non-linear environment and risk functions), and making the problem dynamic. The more realistic version will also be subjected to a preposterior analysis to assess whether the technique appeared to be worth its costs.

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