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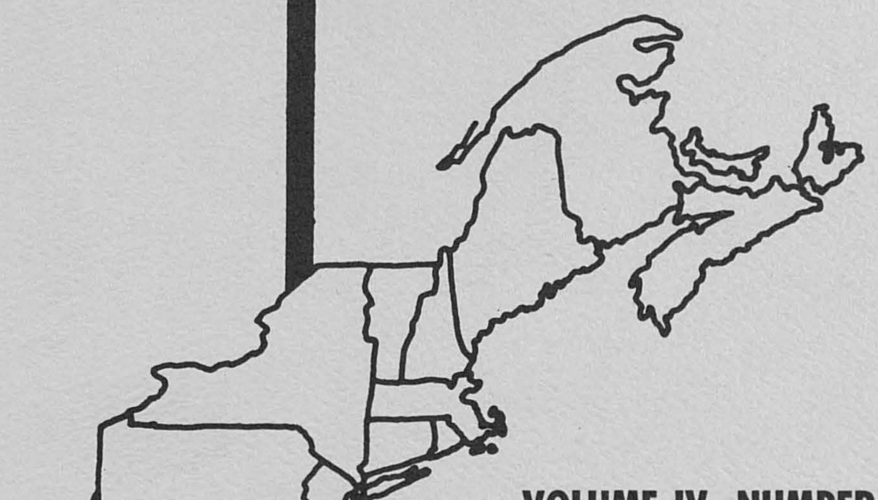

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SOLID WASTE MANAGEMENT FOR RURAL AREAS -- A QUADRATIC PROGRAMMING APPROACH*

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The expanding volume of solid wastes generated in the United States, with its accompanying health hazards and pollution dangers, has led to concern among public agencies and private citizens alike. It has been estimated by [11], for example, that the combined effect of increasing population and increasing per capita consumption in the United States has been an increase from 70 million to 175 million tons of solid waste generated annually during the period 1940 to 1970. In Massachusetts, an estimated 7.5 million tons of non-agricultural wastes were generated in 1970, and this volume is projected by [13] to double by the year 2000. Nationally, public expenditures for solid waste collection and disposal services for most communities are exceeded only by spending on the two categories of education and roads.

The solid waste problem has been recognized as one of the many severe problems faced by municipal authorities in large urban centers. However, the problem is also becoming increasingly acute in rural areas. The quality of refuse collection services in the smaller communities has been found to be inferior to that of the larger communities.1/ For example, [6, p. 1598] notes, "Failure to recognize solid waste problems in rural areas may be one reason why open dumps, open dump burning, and littering occur and are making many rural areas lose their advantage over cities in environmental quality."

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1/ In most cases, the present disposal methods practiced by smaller communities did not conform with the legal minimum requirements [5].

Recent regulatory measures imposed by state and federal jurisdictions for environmental protection have effectively ruled as unsatisfactory present methods of disposal by the majority of communities.2/ Most of these disposal methods can be classified as merely open dumps. In most instances, the sanitary landfill is the least expensive alternative that will meet state and federal requirements and regulations. However, the transition from open dumps to approved sanitary landfills has been slow. Communities have either attempted to upgrade their present dumps or have continued to operate their illegal disposal operations. Community compliance with the new regulations is more difficult as well as more expensive and has increased the managerial and technical requirements needed in operating a disposal facility.

Legislation has been passed at the federal and state levels, with grant support, to help the development of satisfactory disposal practices and to plan for all aspects of solid waste management.3/ This legislation recognizes that adoption of acceptable solid waste management practices may exceed the economic and technological capability of many communities. Thus, it may be necessary to consider groups of communities acting as a region in sharing capital, operating and maintenance costs of a common system to achieve significant economies of scale in refuse disposal.4/ While it can be easily demonstrated that the per unit cost of final disposal of each town's refuse may be significantly reduced through a joint disposal effort, under a regional solid waste management system, the number of disposal facilities would decline and one would expect the communities to experience an increase in total transfer cost due to increased distances of travel. As suggested by [2, p. 49], then, a regionalized solid waste management system will be economically justified when increases in transfer costs are more than offset by decreases in disposal costs. Thus, the economic feasibility of regional disposal will depend upon the trade-off between economies of scale of landfill operation and increased cost of transfer.

The purpose of this paper is to provide a framework that will incorporate both aspects of solid waste management into the same decision model, and to demonstrate the usefulness of the framework by applying

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2/ In a survey taken by Raytheon Service Company in February 1972, of the 351 communities in Massachusetts, more than ninety percent process solid waste in an illegal and unsatisfactory manner.

3/ The major legislature on the federal level was the passage of the Solid Waste Disposal Act of 1965. In Massachusetts, the Bureau of Solid Waste Management was established with authority to set up regional districts, designate sites and to contract for equipment and facilities.

4/ In a study undertaken by the Office of Solid Waste Management Programs for a four county area in West-Central North Carolina, it was shown that a regional disposal effort could reduce cost while eliminating some 23 open dumps [9].

it to an existing regional situation. A mathematical programming model is developed which seeks to minimize total regional cost (transfer plus landfill operating cost) of solid waste disposal. The optimum solution will generate a location pattern of a regional disposal site(s) and specify a size(s) and date(s) of construction and operation.

This framework is demonstrated for a region in Northern Berkshire County, Massachusetts. The region consists of the communities of Adams, Cheshire, Clarksburg, Florida, North Adams, Savoy and Williamstown (see Figure 1). The study area was first delineated in a report prepared by [13]. The political boundaries of the region were outlined primarily by the extent and type of roads available. The total population is expected to increase twenty percent by the year 2000, with most of the increase occurring in the larger communities of Adams and Williamstown. Primarily on the basis of the recent trends in solid waste generation, an increase in solid wastes of seventy-five percent is projected for year 2005. At present, each town operates its own disposal facility -- an open dump. Regionalization of disposal would eliminate all or some of such dumps and perhaps provide a more efficient service.

RELATED RESEARCH

A number of models of regional solid waste management appear in the literature. For example, [7] provides an application of the fixed-charge model for selecting disposal alternatives. Associated with each disposal alternative is a fixed cost, an operating cost and associated with each route is a transfer cost. Their objective is to minimize the sum of a variable (disposal plus transfer) cost related to the level of activity and the fixed-charge cost required to initiate the activity, i.e.

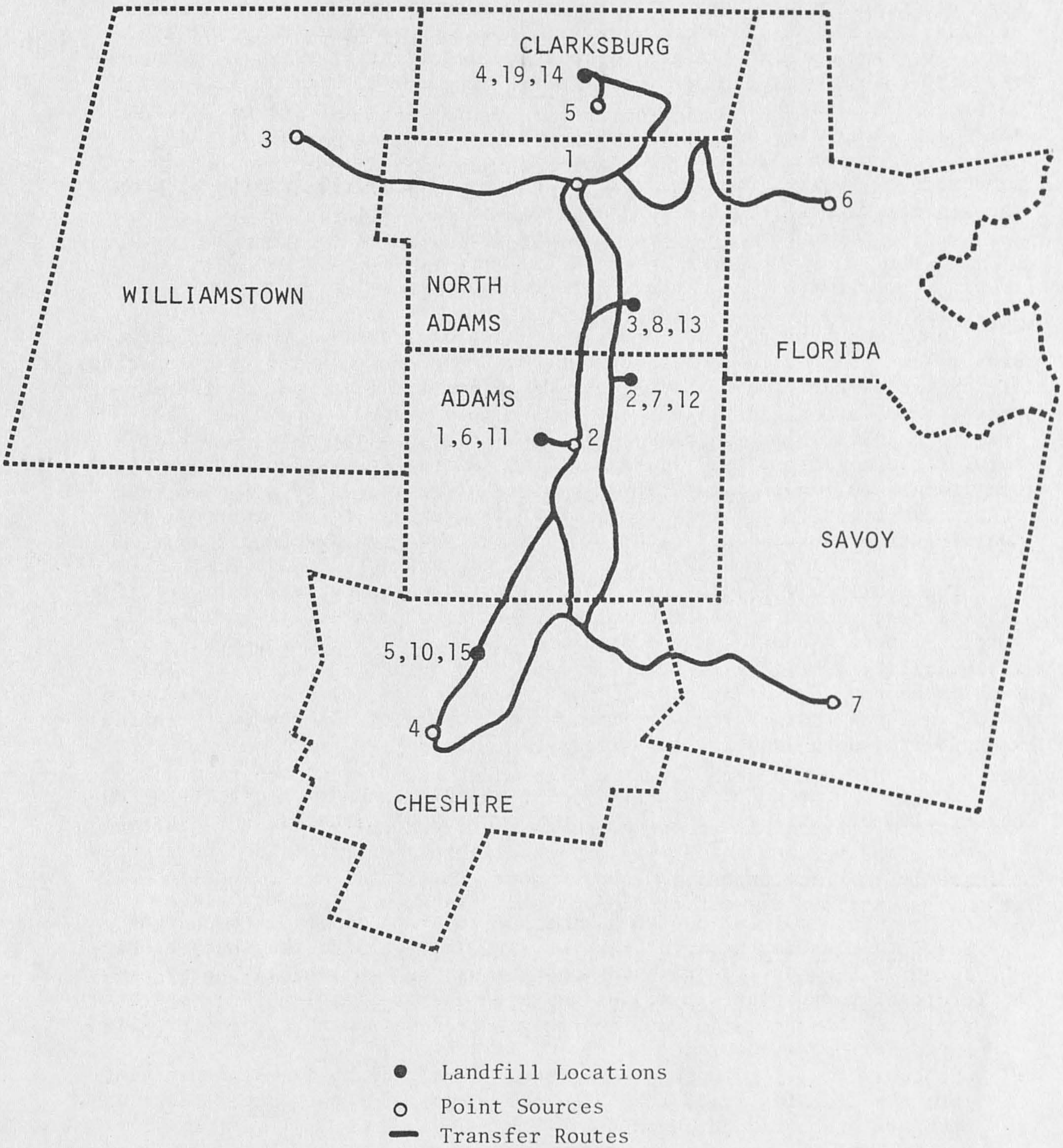
$$\text{Minimize} \sum_{j=1}^{J} \delta_j F_j + \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij} X_{ij}$$

where F_j represents the fixed cost for disposal activity j and δ_j takes on values of zero or one. The C_{ij} represent the variable cost of allocating X_{ij} tons from origin i to j. The objective function is minimized subject to constraints that require that all community waste be disposed and that the maximum capability of the disposal facility j not be exceeded. Similarly, [10] set up a fixed-charge model to establish the location of transfer stations between origins of solid waste and landfill sites.

Two obvious limitations of this class of fixed-charge models is that a time dimension is not included in the model (so that decisions are made without regard to expected trends in solid waste generation) and that

Figure 1

Identification of Point Sources, Landfill Locations, and Transfer Routes



disposal costs must be assumed linear in terms of volume, which is not an appropriate assumption in most cases.5/

Other approaches appear in the literature as well. The Stollsteimer model, as applied by [2], develops a procedure for simultaneously determining the numbers, sizes and locations of facilities that minimize the combined transfer and processing costs. Economies of scale are assumed to exist and disposal costs are assumed not to vary among disposal site locations. A major limitation of this model is that there is no assurance that the solution is the least cost one, since not all possible relevant alternatives are examined (that is, only a pre-specified set of alternatives are enumerated and compared). Other studies -- e.g., [1], [14], [16] -- do not introduce transfer and disposal of solid waste into the same model. That is, the location of disposal facilities was based upon the minimization of transfer cost or route distance only.

DECISION FRAMEWORK

The framework provided below is capable of assisting regional decision makers in choosing, on a regional basis, numbers, sizes, and locations of sanitary landfill operations, timing of construction, and assignment of wastes from communities within the region to selected landfills. The framework involves making these decisions in such a way as to minimize total regional costs (landfill operations and transportation) subject to constraints on landfill capacity, quantity of wastes to be disposed, and other conditions (e.g., legal or political) specific to the region. This decision framework can be solved by quadratic programming procedures.

The decision variables are allocations of community waste to specific landfills of chosen sizes in each time period. Operationally, (X_{ijt}) denotes shipment of solid wastes (tons per time period) from community i to a landfill(s) operating at location and scale j in time period t. The problem becomes one of selecting these variables (X_{ijt}) so as to minimize total regional cost of transferring and disposing of all community wastes without exceeding landfill capacities.6/

For the purposes of this study, the objective function (which is representative of total regional cost) is composed of disposal and transfer

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5/ To be sure, [15] introduced a time horizon into their fixed-charge model to consider dynamic behavior, and incorporated the concave character of the cost functions of operations such as landfilling by discrete approximation. Since each discrete approximation is a separate variable, however, the approach is rendered intractable for virtually any practical situation.

6/ One could easily incorporate regional, legal, or political constraints into the decision framework. For the purposes of this paper these constraints are not considered.

costs. The disposal cost portion is discussed first, then the transfer cost, and finally, the methodology for inclusion of a degree of uncertainty into the objective function.

Disposal Cost

For each landfill j operating in time period t, total disposal cost is approximated with a quadratic function of the form $TC_{jt} = AX_{jt} - BX^2_{jt}$. For each level of output, average total cost or unit cost is simply total cost divided by output. The average total cost curve, then, is linear[7/] and can be represented as:

$$(1) \quad C_{jt} = a_{jt} + b_{jt} \sum_{i=1}^{I} X_{ijt},$$ [8/]

where a_{jt} and b_{jt} represent the intercept and slope of the unit disposal cost curve, respectively.

Transfer Cost

Because the location of the sanitary landfill(s) determines transfer distance, it is logical to consider the cost of transfer a key aspect of the model. This may be accomplished by defining unit transfer and landfill costs as:

$$(2) \quad C_{ijt} = a_{ijt} + b_{jt} \sum_{i=1}^{I} X_{ijt},$$

where, $a_{ijt} = a_{jt} + T_{ijt}$,[9/]

a_{jt} = intercept of unit disposal cost linear approximation,

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7/ Within the disposal or operating range of each landfill j diminishing returns are not presumed to exist. Thus, unit cost can be represented with a downward sloping linear cost function. However, the literature [13] suggests that diminishing returns may set in sharply once the maximum of the disposal range is attained.

8/ Note that since unit costs of operating landfill j in period t depend upon aggregate quantities disposed, we sum X_{ijt} over i. The same logic applies to (2).

9/ The term a_{ijt} in general would include the internal community collection cost preparatory to shipping to landfill j. In our application it is assumed that this cost is invariant to choice of landfill and is therefore excluded.

T_{ijt} = transfer cost of shipping waste from i to j,

C_{ijt} = combined unit cost of disposal and transfer.

The objective function of the quadratic programming model can now be written as:

$$(3) \quad \text{Minimize } Z = \sum_{ijt} \theta_t \left(a_{ijt} + b_{jt} \sum_i X_{ijt} \right) X_{ijt}$$

where θ_t is the discount factor and thus reflects the minimization of total discounted cost of disposal and transfer.

This function can be written in matrix notation (in order to simplify the inclusion of uncertainty which follows below) as:10/

$$(4) \quad \text{Minimize } Z = AX + X'BX$$

where A is an (IJT) row vector, X is conformable, and B is an (IJT) dimensional block diagonal matrix of slopes,

$$B = \begin{bmatrix} M_{11} & 0 & \cdots & 0 \\ 0 & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & 0 \\ 0 & \cdots & 0 & M_{TT} \end{bmatrix}, \text{ and the IJ dimensional } M_{tt} \text{ is } M_{tt} = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & & \cdot \\ \cdot & & \ddots & \cdot \\ \cdot & & & 0 \\ 0 & \cdots & 0 & D_J \end{bmatrix}$$

and D_j is an I dimensional square matrix of slopes, where each element is b_{jt} of equation (3).

Uncertainty

The preceding framework permits one to formally recognize the uncertainty regarding the unit disposal cost estimates. To incorporate this uncertainty we can rewrite equation (4) as:

10/ It will be noted that the use of a quadratic objective function allows one to incorporate the concave character of the total cost functions, as would be the case for sanitary landfilling, directly into the model. A quadratic programming procedure, applied to regional solid waste management, has not been found in the literature except for the approximation of a concave cost function [15]. For an applied treatise on the use of quadratic programming, see [8].

$$(5) \quad \text{Minimize } Z = AX + X'(B + \phi\Omega)X$$

where Ω is the variance-covariance matrix of the estimators of the element B and ϕ is a scalar which expresses an aversion to risk.

The objective function will be minimized subject to a number of restraints such as landfill capacity and quantity of wastes to be disposed, which are basic to all regional situations. First, there will be a prespecified maximum level of operation associated with each landfill j,11/ expressed as:

$$(6) \quad \sum_{i=1}^{I} X_{ijt} \leq S_j \qquad \forall_{j,t}$$

This constraint states that each landfill (j) has a pre-specified capacity (S_j) which cannot be exceeded in any time period by the refuse generated by the communities allocated to that particular landfill in period t. These constraints apply to each landfill and time period, hence there are in general JT of these constraints. While equation (6) insures that a landfill is not used per unit time at a rate or level in excess of its capacity, equation (7) insures that over time the aggregate capacity of the landfill is not exceeded:

$$(7) \quad \sum_{i=1}^{I} \sum_{t=1}^{T} X_{ijt} \leq S_j^* \qquad \forall_j$$

where S_j^* is the capacity of the j^{th} landfill. There are J of these inequalities. Finally, to insure that each origin i is allocated a landfill, the following constraint is needed:

$$(8) \quad \sum_{j=1}^{J} X_{ijt} \geq W_{i,t} \qquad \forall_{i,t}$$

This third constraint states that each waste generating origin i has a quantity of refuse in t (W_{it}) that must be disposed at some landfill j (or group of landfills) for each period t, hence there are IT of these constraints.

11/ This scale cannot be altered within a single run except via post-optimization procedures. In an earlier attempt, an integer programming approach was employed which later proved unsuccessful. Problems resulted when landfills could not be constrained, within program limitations, to operate at a near maximum or pre-specified size.

As a final comment, these constraints may be adjusted to reflect local, political, or other considerations. For example, at a certain landfill location, its use could be restricted to a lower level of operation. This may be accomplished to reducing S_j and/or S_j^*.

Regional Application

As suggested earlier, this decision framework is applied to a seven community region in Massachusetts (Figure 1), each representing a solid waste generating origin (i). Each origin can be considered a point source, where one assumes all solid waste originates.12/ Within the area there exist five potential landfill locations, each of which may be developed at one of three different scales. Thus, when location and size are taken into consideration there are fifteen possible landfills (j). Decisions are made every five years over a planning period of twenty-five years. In the notation above, then, for this application I = 7, J = 15, and T = 5.

The elements of the objective function include: (i) transfer cost of the solid waste, (ii) disposal, and (iii) inclusion of uncertainty into the framework. Transfer cost per ton (T_{ijt}) is the product of transfer costs in dollars per ton-mile and the distance from each community point source to landfill location j. These distances are measured along transfer routes (and are developed in [12]).

Estimates of the elements a_{jt} and b_{jt} of the disposal cost function (1) were derived by application of Ordinary Least Squares (polynomial regression) for data provided by [3] and [13]. The estimations were quite precise13/ in some cases and less so in others. Fortunately, the chosen framework permits the incorporation of preferences with respect to uncertainty as measured by the variance of the unit disposal cost estimates. Initially, the model was run with no risk aversion assumed (ϕ = 0.0). Later, aversion to risk was varied up to fifty percent (ϕ = 0.50) to examine the influence on the optimal solution. Solution sensitivity to transfer cost estimates was evaluated as well, with such costs increased up to fifty percent for some runs. Finally, values for S_j and W_{it} are from [3].

EMPIRICAL RESULTS

As indicated above, the model was solved under alternative sets of assumptions reflecting reasonable levels of uncertainty. The optimal

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12/ For detail on the region and data, refer to [12].

13/ Indeed, for the larger landfills, with the flatter curvature, the standard errors of b_{jt} were less than one percent of the magnitude of b_{jt} while for the smaller operations, the standard errors of b_{jt} were in excess of ten percent of these values of b_{jt}. Again, details are in [12].

solution was identical in each case -- indicating an insensitivity with respect to any minor errors in parameter specification. This solution calls for a single landfill at Clarksburg (j = 4) to be constructed and operated for t = 1, 2, and 3. Since the useful life of this landfill was exhausted at this time, the site of the landfill is shifted to Adams (j = 1) in period 4. Finally, for period 5, the solution calls for the scale of this landfill to be expanded.

The shadow prices or dual values are presented in Table 1. The dual values can be interpreted as the values associated with relaxing a slack constraint or tightening a surplus constraint by one unit or in this case one thousand tons. For example, the shadow price of zero for constraint one in time period one suggests that the value of the objective function would not be reduced by increasing the disposal capacity of the landfill associated with the first constraint. This is the case since there is excess capacity in that landfill for this solution. The shadow price of 2014 for constraint (W_1) in time period one, for example, indicates that the value of the objective function (total costs) can be reduced by 2014 dollars for each 1000 tons by which origin one might be able to reduce its wastes needing disposal.

Table 1

Shadow Prices for Constraints in Dollars Per 1000 Tons

Constraint	Time Period (t) 1	2	3	4	5
S_1	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
S_{15}	0	0	0	0	0
W_1	2014	1345	750	480	1250
W_2	2455	1713	1081	191	998
W_3	2505	1758	1121	768	1501
W_4	3106	2279	1574	614	1367
W_5	1874	1207	645	680	1424
W_6	2649	1882	1226	899	1617
W_7	3320	2466	1729	782	1515

Conclusions

The purpose of this paper was to provide a framework that would incorporate decisions with respect to the transfer of solid waste and its final disposal into the same decision model. The framework employed was a quadratic programming model, which was able to incorporate the concave character of the cost functions such as would be exhibited for sanitary landfill disposal. The model was also capable of considering the trade-offs between increased transfer costs due to regionalization and economies of size in landfill disposal. Solutions to the model would yield the location pattern, scale(s) of operation and dates of construction of sanitary landfills. For the area under investigation, Northern Berkshire County, the results suggested that one large regional landfill be constructed to serve the entire region's disposal needs. The solution and subsequent sensitivity analysis indicated that the increased transfer costs associated with regionalized solid waste management were not significant enough to affect the numbers of regional landfills in the optimum solution.

The existing solid waste models are subject to some rather important limitations that are not inherent in the quadratic programming framework. An obvious limitation of the class of fixed-charge models is that (pragmatically) the disposal costs must be assumed linear, which is an inappropriate assumption in most cases. In general, for most of the optimization and location models to date, explicit introduction of transfer and disposal of solid waste into the same model was not considered. That is, the location of disposal facilities was based upon the minimization of transfer cost or route distance only. In the models that did explicitly consider both aspects, optimal solutions could not be guaranteed. Specifically, not all possible solution alternatives were examined. The algorithm used to solve the quadratic model could examine all outcomes thus assuring optimality. The main advantage of this framework is it's ability to solve regional solid waste problems with locations, sizes and timing characteristics of disposal facilities in one stage.

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