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TRAS A TIMBER VOLUME PROJECTION MODEL
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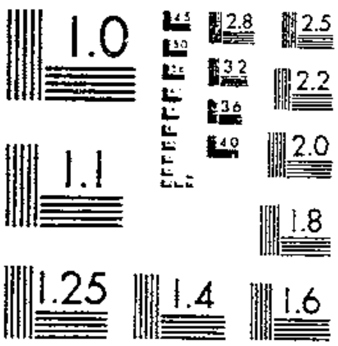

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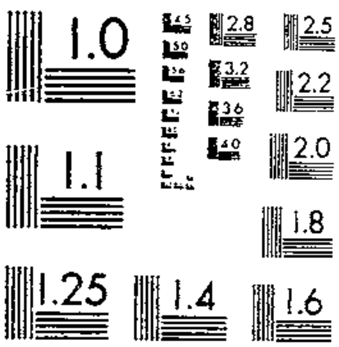

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

TRAS

A TIMBER VOLUME PROJECTION MODEL

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The theoretical basis of the TRAS timber volume projection model is presented along with the evolution of its major mathematical formulae and comparisons of projected and actual remeasured statistics.

CONTENTS

ACKNOWLEDGMENT

The procedures for projecting timber supplies described in this paper represent the efforts of many people. Austin A. Hasel and Robert B. Pope especially contributed substantially to the conceptual and procedural development. The authors gratefully acknowledge their invaluable contributions and the contributions of others.

A comprehensive computer program has been developed for these procedures and described in TRAS–A Computer Program for the Projection of Timber Volume, Agriculture Handbook No. 377. Source program and test decks can be supplied by the authors upon request.

INTRODUCTION

Timber growing is a long-term undertaking; the full implications of today's actions (or lack of action) are not apparent for several decades—often too late to correct today's mistakes. Periodically, the Forest Service of the U.S. Department of Agriculture has undertaken to review the timber supply outlook to help forest policy and program formulators avoid costly mistakes. These periodic timber reviews involve three major tasks:

1. Evaluation of changes that have taken place since the last timber review.
2. Appraisal of the current situation.
3. Projection of future supplies under various assumptions regarding timber removals, area of timberland, and levels of management.

The stand projection system described here was developed for use in accomplishing all three tasks. It was designed specifically to reconcile differences between surveys, to update surveys completed at different times to a common compilation date, and to make long-term projections of timber supplies.

THE MODEL

The basic model may be expressed as:

$$(1) \quad INV_2 = INV_1 + NG - TR$$

where:

INV_2 = timber inventory at the end of a specified year

INV_1 = timber inventory at the beginning of the year

NG = net annual growth

TR = timber removals

These components of the annual change in inventory are computed in numbers of trees per acre by 2-inch d.b.h. classes. For specified years during the projection period, numbers of trees are converted to total volume using the average volume per tree by 2-inch d.b.h. class and the area of commercial timberland.

Net Annual Growth

Net annual growth may be expressed as:

$$(2) \quad NG_i = PI_i - (M_i + GM_i + GR_i)$$

where:

PI_i = The annual potential increase in the number of trees in the i^{th} d.b.h. class assuming that no live trees from that class are harvested or die during the year.

M_i = Mortality—the number of live trees from the i^{th} d.b.h. class which die from natural causes during the year. Natural causes include wildfire, wind, disease, insects, etc.

GM_i = The growth on mortality—that part of the potential increase attributable to mortality trees in the i^{th} d.b.h. class. This is deducted from potential increase on the assumption that trees about to die have negligible growth during the last year.

GR_i = The growth on removals—one-half of that part of the potential increase attributable to those trees in the i^{th} d.b.h. class which are removed from the stand during the year in connection with timber harvesting or timber stand improvement activities. This assumes that trees removed grow at the normal rate and are removed uniformly throughout the year. This would allow half of the normal growth on such trees to occur on the average.

POTENTIAL INCREASE

Potential increase is calculated from an assumed relationship between number and size of trees. An examination of the frequency distribution of numbers of trees by size for a wide variety of forest areas led Meyer (1952) to conclude that the diameter distribution in any large forest area with a mixture of stand sizes and ages tends toward the inverse J-shaped form.

Figure 1 illustrates such a distribution of numbers of trees by diameter class plotted over the midpoints of the diameter classes. If the curve is shifted to the right by an annual diameter increment I, then the vertical segment PI represents the potential increase into the diameter class with midpoint N.

This potential increase, computed using a unique diameter increment for each diameter class, represents the number of trees in each class which would have to be removed annually to restore the original stand distribution.

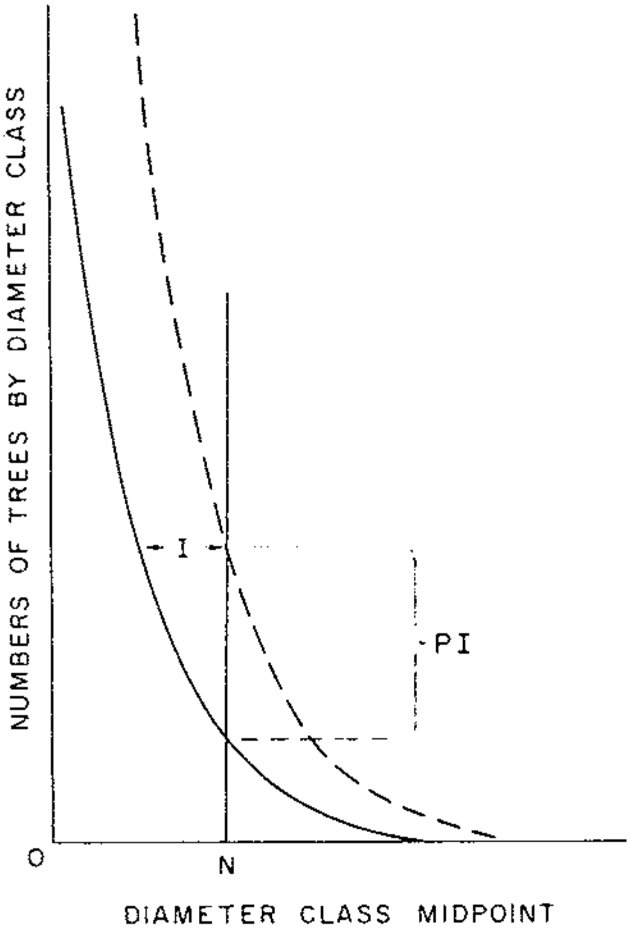

Figure 1.—Displacement of diameter class distribution due to diameter growth I.

Meyer (1952) describes the inverse J-shaped curve as an exponential function of the form:

(3) $YdX = k \times e^{-aX} \times dX$

where:

YdX = number of trees in a narrow diameter interval dX

X = diameter at breast height

e = base of natural logarithms

k and a = constants which characterize a certain frequency distribution.

The use of this function in calculating the potential increase can be greatly simplified by recognizing that a frequency distribution of this form represents a geometric series, which means that the quotient (q) between numbers of trees in successive diameter classes is a constant. If trees are grouped by 2-inch d.b.h. classes, the potential increase for the i^{th} diameter class may be calculated using the following adaptation of Meyer's equation:

(4) $PI_i = N_i \; (\overline{q}^{D_i/2} - 1)$

where:

N_i = number of trees in the i^{th} d.b.h. class

$\overline{q}$ = the average stand structure quotient for all d.b.h. classes

D_i = the average annual increase in diameter for trees in the d.b.h. class

For computation, equation (4) may be written:

(5) $PI_i = N_i \; [(\text{antilog} (\log \overline{q} \times RG_i) - 1]$

where:

$RG_i = D_i/2$ = average annual radial growth

Calculation of annual potential increase by this method is illustrated in table 1. In this example, column (2) shows N_i, the actual number of trees in each 2-inch diameter class. Column (3) shows the logarithms of these numbers of trees. Since the highest d.b.h. class includes all trees larger than 21.0 inches d.b.h. and is not a true 2-inch class, the logarithm of the number of trees in that class is omitted. Values for this class are normally extrapolated from smaller classes. The risk of error from this source is minimized by using enough diameter classes to make the number of trees in the highest class a negligible part of the total.

Column (4) shows the values for a straight line fitted to the logarithms in column (3) by the method of least squares (fig. 2). The equation for this line is:

(6) $\text{Log } N_i = 2.24833981 - 0.14257272 \, D_i$

where:

D_i = the midpoint of each diameter class

Column (5) shows the computed stand table (N_i') using the antilogarithms of the logarithms in column (4). In column (6), $\overline{q}$, which is the same for all diameter classes, is computed from the slope coefficient of equation (6), −0.14257272.

$$\overline{q} = 1.0 \Big/ \left(10.0^{-0.14257272}\right)^2 = 1.92817$$

Note that $\overline{q}$ is the common multiplier for all adjacent values in column (5). For example:

$N_2' = N_4' \times \overline{q}$

$91.8744 = 47.6485 \times 1.92817$

Table 1.—*Calculation of annual potential increase in numbers of trees per acre using an average stand structure quotient ($\overline{q}$) for softwoods in North Carolina, 1963*

(1) d.b.h.	(2) N_i	(3) Log N_i	(4) Log N_i'	(5) N_i'	(6) $\overline{q}$	(7) RG_i	(8) PIR_i	(9) PI_i
2	89.0485	1.949627	1.963194	91.8744	1.92817	0.088	0.059480	5.2966
4	40.7564	1.610196	1.678049	47.6485	1.92817	.092	.062266	2.5377
6	22.4646	1.351499	1.392904	24.7118	1.92817	.096	.065060	1.4615
8	13.2075	1.120821	1.107758	12.8162	1.92817	.098	.066459	.8778
10	7.5287	.[illegible]5720	.822613	6.6468	1.92817	.099	.067160	.5056
12	4.1771	.620875	.537467	3.4472	1.92817	.100	.067861	.2835
14	2.1087	.324015	.252322	1.7878	1.92817	.105	.071372	.1505
16	1.0008	.000347	−.032824	.9272	1.92817	.106	.072076	.0721
18	.4708	−.327164	−.317969	.4809	1.92817	.110	.074895	.0353
20	.1877	−.726536	−.603114	.2494	1.92817	.108	.073484	.0138
22+	.1760	--	--	--	1.92817	.112	.076307	.0134
Total	181.1268							11.2478

Column (8) shows the average annual radial growth for each diameter class (RG_i). The potential increase rate (PIR_i) is computed as:

$$PIR_i = \overline{q}^{\,RG_i} - 1.0$$

$$PIR_i = [\text{antilog}(\log \overline{q} \times RG_i)] - 1.0$$

For the 2-inch class in the example shown in table 1, this is:

$$PIR_2 = [\text{antilog}(0.285145 \times 0.088)] - 1.0$$

$$= \text{antilog}\,[0.025093] - 1.0$$

$$= 1.05948 - 1.0$$

$$= 0.05948$$

The potential increase for the 2-inch class (PI_2) is then computed as:

$$PI_2 = N_2 \times PIR_2$$

$$= 89.0485 \times 0.05948$$

$$= 5.2966$$

Modification of Meyer's Method

Four modifications of Meyer's method of computing potential increase were made during the course of developing projection procedures. These are as follows:

1. A unique stand structure quotient (q_i) was computed for each 2-inch d.b.h. class rather than an average for all classes ($\overline{q}$).

The potential increase rate (PIR) is then computed as:

$$PIR_i = q_i^{\,RG_i} - 1.0$$

where:

q_i = the number of trees in the $(i-1)^{th}$ diameter class divided by the number in the i^{th} class

RG_i = the average annual radial growth for the i^{th} diameter class

Of all the modifications of Meyer's method this one is by far the most important in bringing projections closer to actual stand development. It makes the assumptions regarding the relationship between numbers of trees and size much less specific. Meyer's method assumes an exponential relationship between numbers and size of

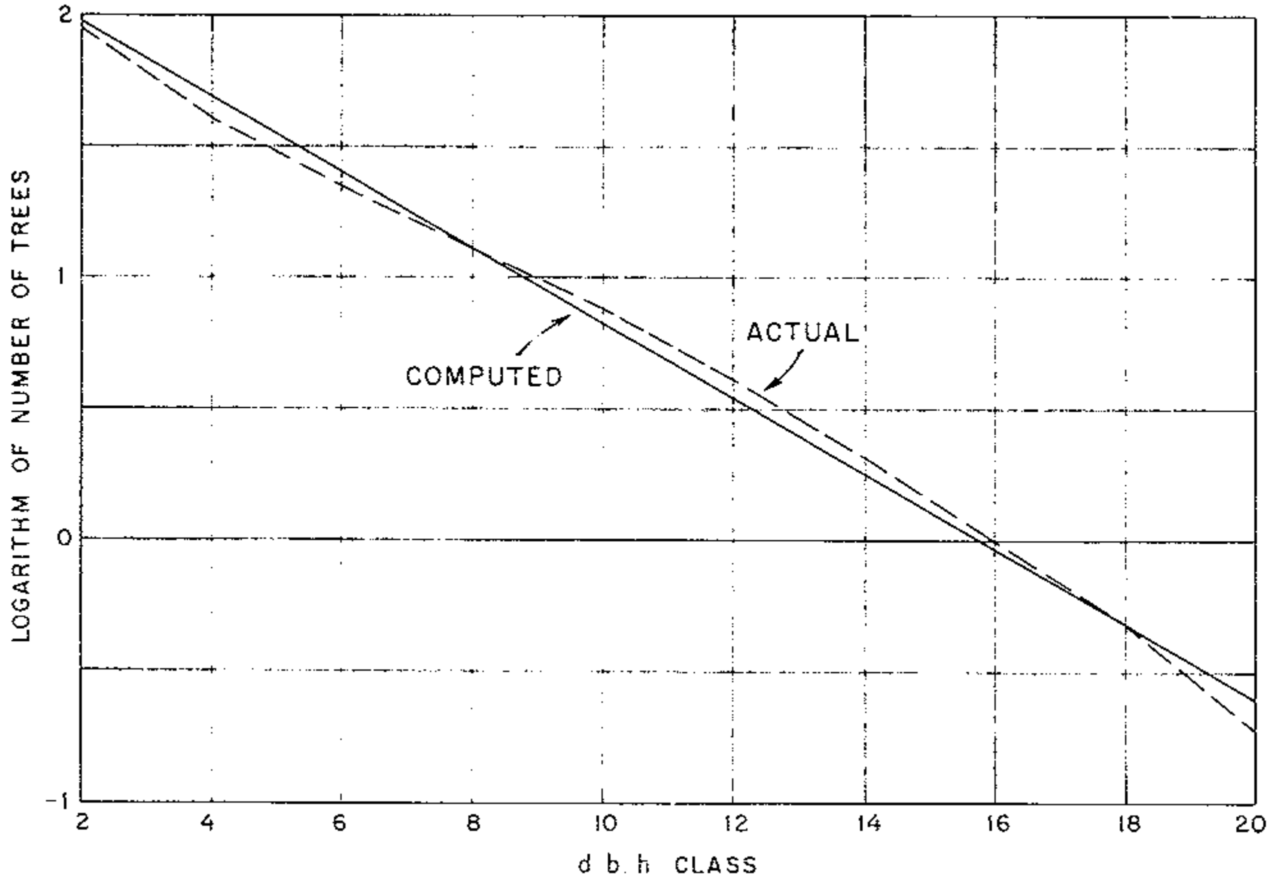

Figure 2.–Relationship between d.b.h. and the logarithm of numbers of softwood trees, North Carolina, 1963.

trees and a constant average stand parameter $\overline{q}$ for all diameters. The above modification allows q to vary by diameter class, which gives the projection system the flexibility to deal with substantial variation from a uniform inverse J-shaped curve. It attributes such variations to a variable stand structure quotient q_i which can be calculated from numbers of trees in pairs of adjacent diameter classes.

2. The ingrowth rate (IGR) for each d.b.h. class was calculated and potential increase derived from the resulting ingrowth. With this method potential increase is the ingrowth into the diameter class minus the ingrowth into the next larger class or outgrowth. The formula for ingrowth rate (IGR) is:

$$IGR_i = \left(q_i^{\ RG_i} - 1.0\right) \times \left(\frac{q_i}{q_i - 1.0}\right)$$

Meyer originally computed potential increase directly for each diameter class using the average radial growth for all trees in the class. The use of ingrowth rates estimates ingrowth at the lower limit of each class and allows radial growths to be averaged around the lower limits where the transition of trees from the lower class to the higher actually occurs. This adds flexibility to deal with variation in diameter growth between adjacent classes. The importance of this depends upon the degree of such variation.

3. *Accumulative* numbers of trees by stand size class were used rather than number of trees in each class. Numbers of trees are accumulated from higher to lower d.b.h. classes, so that each stand size class includes all of the trees larger than the lower limit of the class. For example, the 20-inch class includes all trees 19.0 inches and larger, and the 18-inch class includes all those 17.0 inches and larger. The 2-inch class includes all trees 1.0 inch and larger, or all the trees in the stand.

When exponentially distributed stand tables are thus accumulated, the accumulative numbers of trees become an exponential function of the lower limits of the diameter classes. When the curve describing this function is shifted to the right, ingrowth rather than potential increase into each diameter class may be read over each lower limit.

A stand structure quotient (Q_i) is computed for each successive pair of "cumulative number of trees" and an ingrowth rate calculated as in the q_i methods.

The use of cumulative stand table makes the largest stand size class comparable to all other classes and a valid observation in the projection procedures. For example, trees 19.0 inches and larger can be treated in the same manner as those 1.0 inch and larger which eliminates at least some of the difficulties associated with the largest 2-inch d.b.h. class.

4. A *parabolic function* to describe the relationship between number and size of trees was developed as an alternative to the exponential function. Meyer (1952) noted that a curve of the exponential type fits the actual data for a large variety of situations, including virgin forests and well managed forests as well as any large forest area with a reasonable representation of all age classes, such as for a county or state. The modifications described above serve only to further increase the flexibility of the exponential model. In the case of even-aged stands, however, or areas with very few age classes, numbers of trees are clearly not an exponential function of size. The parabolic function was developed for such cases.

While the use of accumulative numbers of trees permits such stands to be approximated by an éx-ponential mode, it is more consistent to adopt a functional model that agrees more closely with the actual data. The accumulative stand tables for even-aged stands usually fit a parabolic curve more closely than an inverse J. Ingrowth may be computed using a parabolic model in much the same way as with an exponential model.

The TRAS computer program (Larson and Goforth, 1970) includes both an exponential model, the Q method, and a parabolic function model, the Non-Linear Interpolation or NLI method. These correspond to the third and fourth modifications respectively of Meyer's method already described. Both involve accumulative stands by 2-inch diameter classes in the computation of ingrowth. With both options, potential increase is the difference between ingrowth and outgrowth for each diameter class.

The Q Method.—The calculations required to compute potential annual increase in numbers of trees using the Q method are illustrated in table 2. Ingrowth into each diameter class is computed using the following equation:

$$(7)\quad ING_i = AN_i\,[\text{antilog}\,(\log Q_i \times RG_i) - 1]$$

where:

ING_i = ingrowth into the i^{th} 2-inch diameter class.

AN_i = accumulative number of trees larger than the lower limit of the i^{th} 2-inch diameter class.

Q_i = $AN_{i-1} \div AN_i$

RG_i = average annual radial growth around the lower limit of the i^{th} 2-inch diameter class.

Table 2.—*Calculation of annual potential increase in numbers of trees per acre using the Q Method, softwoods, North Carolina, 1963*

D.b.h. class	N_i	D_i	AN_i	Q_i	Log Q_i	RG_i	$INGR_i$	ING_i	PI_i
		1.0+	181.1268					11.1468	
2	89.0485								5.2538
		3.0+	92.0783	1.967	0.29380	0.092	0.064	5.8930	
4	40.7564								2.9163
		5.0+	51.3219	1.794	.25382	.096	.058	2.9767	
6	22.4646								1.3030
		7.0+	28.8573	1.778	.24993	.098	.058	1.6737	
8	13.2075								.7034
		9.0+	15.6498	1.844	.26576	.099	.062	.9703	
10	7.5287								.4181
		11.0+	8.1211	1.927	.28488	.100	.068	.5522	
12	4.1771								.2406
		13.0+	3.9440	2.059	.31366	.105	.079	.3116	
14	2.1087								.1574
		15.0+	1.8353	2.149	.33224	.106	.084	.1542	
16	1.0008								.0783
		17.0+	.8345	2.199	.34223	.110	.091	.0759	
18	.4708								.0417
		19.0+	.3637	2.294	.36059	.108	.094	.0342	
20	.1877								.0192
		21.0+	.1760	2.066	.31513	.112	.085	.0150	
22+	.1760								.0150
Total	181.1268								11.1468

Calculations of potential increase (PI) for the 6-inch class in table 2 are as follows:

$$AN = 0.1760 + 0.1877 \ldots\ldots + 22.4646 = 51.3219$$

$$Q = 92.0783 \div 51.3219 = 1.794$$

INGR = Ingrowth Rate = [antilog (0.25382 X 0.096)] −1 = 0.058

$$ING = 0.058 \times 51.3219 = 2.9767$$

$$PI = 2.9767 - 1.6737 = 1.3030$$

The NLI Method.—The application of the parabolic function method is illustrated in table 3. As with the Q method, the stand is accumulated to show the number of trees larger than the lower limit of each diameter class, D_i. D_i' is the lowest limit of each diameter class *plus one year's diameter growth* for that class. The parabolic function is fitted to three successive stand size classes simultaneously for the equation:

(8) $AN_i = b_0 + b_1 x + b_2 x^2$

where:

AN_i = the cumulative number of trees in the i^{th} stand size class (trees larger than the lower limit of the i^{th} 2-inch diameter class)

$b_0 = AN_{i-1}$

b_1 and b_2 = coefficients

$x = D_i' - D_{i-1}'$

Equation (8) is then solved for AN_i', the cumulative number of trees in the i^{th} stand size class after one year's growth, by substituting the difference between D_{i-1}' and D_i for X. This is the mathematic equivalent of plotting AN_i for three successive stand size classes over the corresponding D_i' values and then reading the resultant parabolic curve at the middle D_i value.

The number of trees in each diameter class, after one year's growth, N_i' is computed as:

$$N = AN_i' - AN_{i+1}'$$

Potential increase (PI_i) is then computed as:

$$PI_i = N_i' - N_i$$

Using the 6-inch d.b.h. class from table 3 as an example, the calculations are:

$$b_2 = \frac{\dfrac{AN_{i+1} - AN_i}{D_{i+1}' - D_i'} - \dfrac{AN_i - AN_{i-1}}{D_i' - D_{i-1}'}}{D_{i+1}' - D_{i-1}'}$$

$$= \frac{\dfrac{28.8573 - 51.3219}{7.196 - 5.192} - \dfrac{51.3219 - 92.0783}{5.192 - 3.184}}{7.196 - 3.184}$$

$$= 2.264988$$

$$b_1 = \frac{AN_i - AN_{i-1}}{D_i' - D_{i-1}'} - b_2 (D_i' - D_{i-1}')$$

$$= \frac{51.3219 - 92.0783}{5.192 - 3.184} - 2.264988\,(5.192 - 3.184)$$

$$= -24.845108$$

$$b_0 = AN_{i-1} = 92.0783$$

$$x = D_i - D_{i-1}'$$

$$= 5.0 - 3.184$$

$$= 1.816$$

$$AN_i' = b_0 + b_1 x + b_2 x^2$$

$$= 92.0783 - (24.845108)(1.816) + 2.264988(1.816)^2$$

$$= 54.4292$$

$$N_i' = AN_i' - AN_{i+1}'$$

$$= 54.4292 - 30.6464 = 23.7828$$

$$PI_i = N_i'$$

$$= 23.7828 - 22.4646 = 1.3182$$

Obviously, a parabola cannot be fitted which has the highest stand size class in the middle. For that class the same coefficients are used as for the next-to-highest class. The x value for the highest class is:

$$x = D_i - D_{i-2}'$$

$$= 21.0 - 17.22 = 3.78$$

Thus two points are read from the last parabola fitted rather than just one.

In table 3, potential increase is computed as the difference between the N_i and N_i' stand tables. Ingrowth into each diameter class can be similarly computed as:

$$ING_i = AN_i' - AN_i$$

The ingrowth into the 4-inch class is then:

$$ING_i = 98.2509 - 92.0783$$

$$= 6.1726$$

Table 3.—*Calculation of annual potential increase in numbers of trees per acre using the NLI method, softwoods, North Carolina, 1963*

(1) D.b.h. class	(2) N_i	(3) D_i	(4) AN_i	(5) RG_i	(6) D_i'	(7) AN_i'	(8) N_i'	(9) PI_i
2	89.0485	1.0+	181.1268	---	1.184	---	---	5.2538
4	40.7564	3.0+	92.0783	0.092	3.184	[1] 98.2509	43.8218	3.0654
6	22.4646	5.0+	51.3219	.096	5.192	54.4292	23.7828	1.3182
8	13.2075	7.0+	28.8573	.098	7.196	30.6464	13.9434	.7359
10	7.5287	9.0+	15.6498	.099	9.198	16.7030	7.9808	.4522
12	4.1771	11.0+	8.1211	.100	11.200	8.7221	4.4382	.2611
14	2.1087	13.0+	3.9440	.105	13.210	4.2839	2.2778	.1691
16	1.0008	15.0+	1.8353	.106	15.212	2.0061	1.0877	.0869
18	.4708	17.0+	.8345	.110	17.220	.9184	.5174	.0466
20	.1877	19.0+	.3637	.108	19.216	.4010	.2182	.0305
22+	.1760	21.0+	.1760	.112	21.224	.1827	.1827	.0067
Total	181.1268							11.4264

[1] Computed values in this table are taken from a computer printout and values obtained by addition or subtraction may contain rounding differences.

Testing the Growth Model

Stand tables from remeasured Forest Survey plots in South Carolina show a generally exponential distribution. Ten-year projections were made both with the Q and with the NLI methods using only those trees still living when remeasured plus the annual ingrowth for the period between surveys. This excluded the effect of mortality and removals from the comparison of the two models. The Q method underestimated the remeasured basal area by 0.9 percent while the NLI method overestimated by 1.8 percent. Although the two models agreed closely on projected total basal area, the Q method was better at predicting the remeasured stand distribution. The average error of projected numbers of trees by diameter class was 2.9 percent with the Q method and 5.1 percent with the NLI method.

In a second test, a number of Englemann spruce plots in Idaho measured every 5 years over a period of 20 years provided an opportunity to compare the two methods using stand tables with a more even-aged distribution.

Only trees surviving the entire 20-year period were included in the test, thus eliminating the effects of mortality and ingrowth.[1] Differences between 5-year projections and the remeasurement of 7,189 sample trees showed a remarkable agreement between projected and actual for both methods (table 4).

In addition to the eight 5-year projections shown in table 4, two 20-year projections were made using the Q_i and NLI methods. Projected compared with actual terminal stands after 20 years of growth are shown in figure 3. The basal area of the initial stand was 1291.5 square feet; these same trees 20 years later measured 2150 square feet, nearly a twofold increase. The basal area of the projected stand using the Q_i method was 2036.6, 5.3 percent less than actual, compared to 2208.3 square feet using the NLI method, or 2.7 percent more than actual.

As with the 5-year projections, the Q_i method underestimated and the NLI method overestimated the terminal stand, with the NLI projection agreeing more closely with the actual.

As can be seen from figure 3, the North Idaho stand has a diameter distribution more characteristic of even-aged stands than the exponential all-age stand in South Carolina. Except for trees in the 18- and 20-inch class, which accounted for only 2 percent of the stand basal area, projected numbers of trees agree very closely with actual using the NLI method. The Q_i method underestimated numbers of trees 10 inches and larger and overestimated the number of 2-inch trees.

[1] 109 ingrowth trees were inadvertently left in the test data. Ingrowth was computed based on [illegible] used in making the projections.

Table 4.–Percent difference between actual and projected basal area by method for four 5-year remeasurement periods, Englemann spruce, North Idaho

Method	Remeasurement period			
	1	2	3	4
Q	−2.34	−1.75	−1.10	−0.81
NLI	+0.39	+0.41	+0.59	+0.64

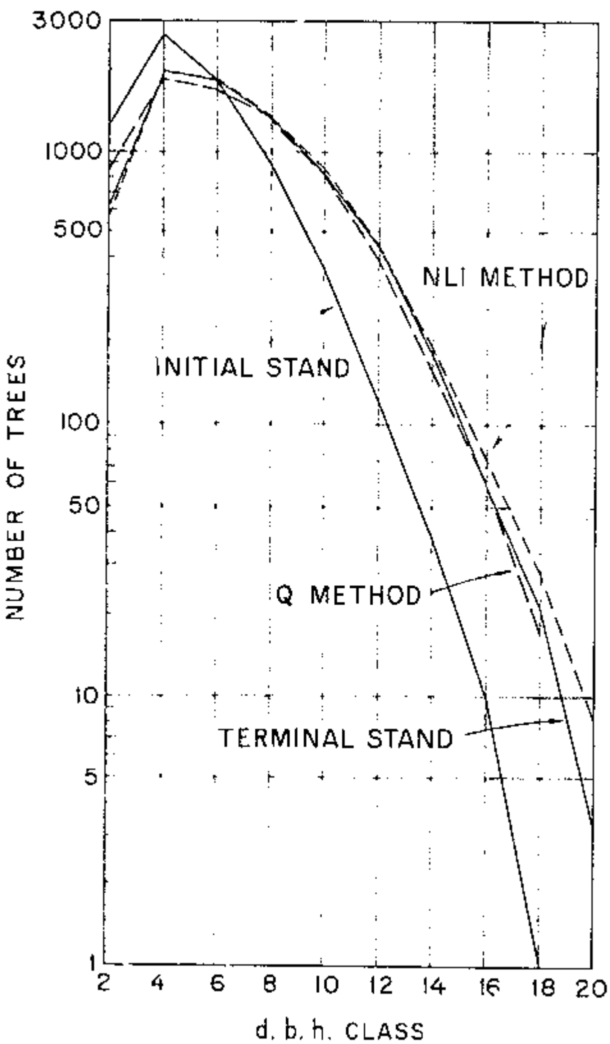

Figure 3.–Initial, terminal, and projected numbers of trees, Engelmann spruce in North Idaho.

On the other hand, as the actual stand approaches the inverse J-shaped curve, the NLI method can be expected to overestimate the potential increase as it did with the South Carolina data. Even though the NLI method will usually project higher potential increases for exponential stands, an underestimate of radial growth due to sampling error can sometimes cause the NLI projection to agree more closely with the actual remeasured data than the Q method.

The higher estimates of potential increase projected by the NLI method for exponential stands result from the unique relationship between parabolic and exponential curves fitted to the same exponentially distributed data. How much higher the NLI method estimates of potential increase for exponential tree distributions will be is related to the stand structure quotient or Q. To measure this effect, nine constant Q stands were generated all having the same total number of trees and the same radial growth over all diameters. All stands were projected 1 year by both the Q method and the NLI method. Following are the percentages by which the NLI method estimates of potential increase exceeded those obtained with the Q method at different values of Q:

Constant Q	Percent
1.25	0.8
1.50	2.7
1.75	5.2
2.00	8.0
2.25	11.0
2.50	14.2
2.75	17.4
3.00	20.7
3.25	24.0

It is apparent that the NLI method should not be used with stands that tend to exhibit an exponential relationship between numbers of trees and d.b.h., especially on stands with high Q values. The use of the NLI method in the South where Q values average around 2.0 would result in about an 8 percent overestimation of potential increase.

Potential Increase and Stand Structure

The rate of potential increase depends on two factors, the radial growth and the stand structure. Thus, with radial growth held constant, the potential increase will vary with changes in the stand structure quotient Q.

In order to illustrate the relationship between stand structure and potential increase, the potential increase was calculated for two stands with the same radial growth and numbers of trees 5.0 inches and larger, but having Q value of 1.9 and 2.0, respectively. The results are shown below:

	Stand 1	Percent	Stand 2
Stand structure quotient Q =	1.900	+5.3	2.000
Average annual radial growth	.125	0.0	.125
Number of trees 5.0 inches and larger	160.900	0.0	160.900
Potential increase:			
Number of trees 5.0 inches and larger	13.441	+8.3	14.563
Square feet of basal area	5.596	+1.6	5.688
Net cubic foot volume	105.500	−2.8	102.500
Net board foot volume	410.000	−9.0	373.000

Thus, with the same radial growth and the same number of trees 5.0 inches and larger, a 5.3 percent increase in the stand structure quotient, Q, results in an 8 percent greater potential increase in terms of numbers of trees. But, because the higher Q value puts a larger proportion of the potential increase in smaller trees, the potential increase in terms of basal area was only 1.8 percent higher. For the same reason, potential increase in terms of cubic foot volume was less by 2.8 percent, and for sawtimber volume in board feet, less by 9.0 percent.

This characteristic of potential increase requires some care in interpreting the results of the stand table projection procedure described here. It should be recognized that potential increase includes two components, one reflecting the annual increase in tree diameter and another reflecting the change in stand structure during the preceding year. If, for example, mortality and harvesting changes Q from 1.9 to 2.0, there is an 8.3 percent rise in potential increase due entirely to the change in stand structure the previous year.

The net growth calculated from the potential increase as described above is actually an allowable or available harvest with specific assumptions regarding the annual change in stand structure; an increase in Q implies an increase in rotation age. Stands with higher Q values have a larger proportion of trees in the smaller diameter classes where they will take longer to reach maturity. Thus, in making projections for entire forest properties or other large timber supply entities, the effect of changing assumptions regarding rotation age is an integral part of the procedure. Further, the procedure permits handling a multiplicity of rotation ages. All that is required is a target stand table which may represent a single rotation age or a composite of any number of rotation ages. Procedures for computing target stands are considered further in the section on applications.

Sapling Ingrowth

The change in the total numbers of trees in the stand, in the absence of removals and mortality, is specified to this model by the number of trees that reach 1 inch d.b.h. during the year, the sapling ingrowth. Sapling ingrowth in potential increase computations is thus not a function of radial growth as is the ingrowth for trees 3 inches and larger. It must be supplied as an independent input based on assumptions regarding regeneration rates. If sapling ingrowth is equal to the ingrowth into the 3-inch and larger stand, the number of trees in the 2-inch class will remain constant. Thus, if the average annual change in numbers of trees is known, which is often the case where resurveys have been made, the sapling ingrowth can be estimated by adding the average annual change in numbers of 2-inch trees and the ingrowth into the 4-inch plus stand, which is based on radial growth for 4-inch trees.[2] As a matter of fact, TRAS provides the option of using either sapling ingrowth or average annual change in the 2-inch class.

Any change in the sapling ingrowth assumption affects projected net growth by changing both the total number of trees in the stand and the future potential increase rates. The potential increase rates change because of the reflection in the future Q value of the changed relationship between the number of small and large trees.

The following is an example of how a 50 percent increase in sapling ingrowth would affect projected net growth after 10 years:

	A	B	Difference in percent
Sapling ingrowth in no. trees	8.57	12.86	+50
Net growth:			
Total stand in no. trees	5.12	8.11	+58
Basal area in square feet	2.33	2.81	+21
Volume in cubic feet	46.50	51.30	+10
Sawtimber in board feet	199.00	206.00	+ 4

Since in this illustrative projection very few trees would grow 4 inches in 10 years (average annual radial growth = 0.100 inch), most of the 10 percent increase in

[2] The formula for estimating sapling ingrowth from average annual change in the number of trees in the 2-inch class when removals and mortality occur is

$$I = O + \frac{N \times [AC + M + (N \times RR)]}{N - M}$$

where:

I = sapling ingrowth

O = outgrowth into 4-inch class

M = annual mortality in 2-inch class

N = number of trees in 2-inch class

RR = annual removal rate for 2-inch class based on inventory

AC = annual change in number of trees in 2-inch class cubic foot volume growth reflects a difference in the distribution of the trees by d.b.h. class, which results in a larger Q value and an increase in potential increase rate. All of the increase in sawtimber growth is the result of a shift in the distribution of trees by d.b.h. The differences in volume growth result from the fact that the number of trees in each diameter class is a discrete value, but the Q method calculations assume a continuous exponential distribution within each class.

An illustration of how a 50 percent increase in the average annual sapling ingrowth influences the net growth by d.b.h. class in numbers of trees per acre at the end of a 20-year projection is shown below:

D.b.h.	Sapling ingrowth = 8.57	Sapling ingrowth = 12.86	Difference in percent
2	−0.1354	0.3095	--
4	.7636	1.9656	+257
6	1.3946	2.2673	+63
8	1.2961	1.6476	+27
10	.8397	.9381	+12
12	.4531	.4743	+5
14	.2400	.2437	+2
16	.1246	.1251	+1
18	.0680	.0681	--
20	.0378	.0378	--
22+	.0351	.0351	--

The magnitude of the premature effect of changed sapling ingrowth on higher diameter classes depends on initial stand structure, radial growth, mortality rates and removal rates. In practice this effect is negligible. Realistic changes in sapling ingrowth are much smaller than the 50 percent increase used for illustration above. The effect tends to be absorbed in the higher diameter classes by mortality, removals, and fluctuations in the structure of those classes themselves. The effect, however, is inherent in all stand table projection methods because stand structure as well as radial growth influences potential increase. The effect moves upward one diameter class each projection cycle. It can be reduced by using diameter classes narrower than 2 inches. This will, however, increase both the volume of data needed and the variability of that data. The effective lower limit on diameter class width is the point at which trees could grow completely across a class in one cycle year.[3]

MORTALITY

The second most important component of net growth is mortality. Annual mortality for each diameter class is computed by multiplying a mortality rate by the number of live trees in the class at the beginning of the year.

[3] In any attempt to modify diameter class width, radial growth (RG) in all the equations must be changed to diameter growth divided by class width. This will be necessary when adapting TRAS to the metric system.

GROWTH ON MORTALITY AND REMOVALS

In the procedure described here, trees that die during the year are assumed to have no diameter growth. However, these trees are included when potential increase is computed so the growth on them must be deducted from potential increase. Growth on mortality is the potential increase rate multiplied by the mortality. Potential increase rate is potential increase divided by the number of live trees in the class at the beginning of the year.

A similar deduction is made to account for growth lost by harvesting. It is assumed that harvesting takes place uniformly throughout the growing season so that harvested trees attain half their growth on the average. So growth on removals is computed by multiplying the potential increase rate by half the removals.

Timber Removals

Annual change in numbers of trees is the difference between net growth and timber removals during the year. Timber removals are computed in two steps. The distribution of removals by diameter class is first computed from removal rates expressed either as a proportion of the inventory or a proportion of net growth. A preliminary removal estimate is computed based on these rates. This unadjusted removal estimate is compared with the projection assumption level of removals and the removals by d.b.h. adjusted to agree with the assumed total without changing the distribution of removals by d.b.h. class.

Removals assumptions may be entered as input to TRAS in one of six ways:

1. A list of the cubic volume of removals for each year of the projection.
2. The total average annual change in cubic volume which is subtracted from total net growth to obtain total cubic removals as a residual.
3. Stand tables for two future years which are connected with the beginning stand table by a parabolic function for each diameter class. Annual change in number of trees for each diameter class is computed from these curves each cycle and subtracted from net growth to obtain removals in numbers of trees by diameter class as a residual.
4. Up to eight components of removals expressed in standard product volume units each accompanied by a factor to convert to cubic volume. The sum of these products is the cubic removals estimate.
5. The total volume of removals assumed to equal the total volume of net growth.
6. Removals to be determined by removal rates alone (unadjusted removals).

Removal rates may be expressed either as a proportion of the number of live trees in each diameter class at the beginning of the year or as a proportion of the net growth of each class during the year. Unlike the mortality rates, removal rates include growth on removals. The formulae for the calculation of unadjusted removals for each diameter class are—

with removal rates based on inventory:

$$R = \frac{N \times RR}{1 + \left(\frac{PIR}{2}\right)}$$

or

with removal rates based on net growth:

$$R = \frac{RR\,(PI-M-GM)}{\left(1 + \frac{PIR}{2}\right) + \left(\frac{PIR \times RR}{2}\right)}$$

where:

R = removals

N = number of trees in class at beginning of year

RR = removal rate

PI = potential increase

PIR = potential increase rate $\frac{PI}{N}$

M = mortality

GM = growth on mortality

In summary, the annual change in numbers of trees by d.b.h. class is calculated from the following components:

$$INV_2 = INV_1 + (PI-M-GM-GR) - TR$$

where:

INV_1 = number of trees at the beginning of the year

INV_2 = number of trees at the end of the year

PI = potential increase

GM = growth on mortality

GR = growth on removals

M = mortality

TR = timber removals

Commercial Forest Area

Normally, all computations using the Timber Supply Model are done on a per-acre basis. Results may be expanded by supplying total area as an input item. The program provides for annual changes in forest area, but users should know in using this feature that the program has no provision for adding the volume of timber on areas going out of the forest area base to removals. If commercial forest area is to be varied during projections, it is preferable to make all projections on a per-acre basis and expand to total volume externally.

The problem in varying the area base during a projection is in assigning volume to the area moving in or out. If increases in area arise from conversion of cropland to forest, assuming no volume on these areas may be appropriate since very few trees would be large enough to contain measurable volume. On the other hand, if these additions are forested areas, such as transfers between ownership, the volume on these areas should be added to the inventory.

Input Modification

Input may be modified during the course of a projection in one of two ways, either by constant or varying annual changes, or through the use of constraint or feedback equations.

PROGRAMMED ANNUAL CHANGES IN INPUT

Most inputs to the Timber Supply Model may be modified over time by supplying the data as three points on a parabolic curve, the initial value, a midvalue, and the terminal value along with corresponding years. The model computes equation coefficients which are then used to modify the inputs annually.

CONSTRAINT EQUATIONS

The second method of modifying input is through the use of constraint equations in which the independent variable is not time, but some projection parameter calculated annually, such as square feet of basal area. The purpose of the constraint equations is to prevent the development of unrealistically high inventory basal areas during projections.

The program described in Agriculture Handbook No. 377 includes a set of constraint equations that reduces radial growth and ingrowth and increases the mortality rate as basal area per acre increases. The coefficients in the equations were derived from remeasured plot data in South Carolina. Since the equations are used only to calculate the rate of change, it is possible that they can be used, at least on an interim basis, in making projections for other States or larger areas throughout the South. It seems reasonable that the shape of the curve (the rate of change) lends itself to extrapolation more easily than the level of the curve. For local areas and for areas outside the South, these equations should not be used without careful checking and possible modification. Some modification of the equations can be made without changing the program merely by changing the initial control basal area, an independent input item. This has the effect of shifting the scale to increase or decrease the constraints.

COMPARISON OF PROJECTED AND ACTUAL STAND DEVELOPMENT

At best a projection system can only approximate what actually takes place in the forest. It would be impractical to attempt to take into account all of the factors that conceivably could influence stand development. The objective is a compromise between keeping the system simple enough to use conviently, and at the same time provide enough sophistication to achieve meaningful results.

As demonstrated with remeasured Englemann spruce plots in Idaho, changes in the distribution of trees by d.b.h. class can be rather reliably projected in the absence of cutting and mortality by utilizing the relationship between average annual increment and numbers of trees by d.b.h.

The remeasured plots in South Carolina provided an additional opportunity to test the reliability of the entire system including the influence of many factors. Some of these factors, such as mortality rates, removal rates, and sapling ingrowth, were under the control of the system. Many other factors, however, were not controlled and contributed to random variation in results. It was assumed, for example, that the input radial growth, mortality rates, removal rates, and volumes per tree remained constant throughout the projection period.

When the South Carolina data were projected as a complete system including mortality and removals, the average basal area per acre of projected stands agreed with remeasured stands within 2 percent (table 5). The Q method underestimated the terminal stand 1.4 percent, while the NLI method overestimated the terminal stand by 1.4 percent.

While based on total average basal area, there was no evidence to support the use of one method over the other, the agreement between actual and projected numbers of trees by size was closer using the Q method than using the NLI method (fig. 4). The NLI method significantly overestimated the numbers of 4- and 6-inch trees.

Since the composite stands in South Carolina display the typical reverse J-shaped distribution, the test corroborates the appropriateness of using the Q method which assumes that distribution.

Table 5.—*Actual and projected basal area per acre by 2-inch d.b.h. class in South Carolina, 1959 to 1969*

[Square feet]

D.b.h. (inches)	Initial	Terminal		
		Actual	Projected	
			Q	NLI
2	6.83	10.96	10.96	10.61
4	7.95	12.76	12.78	13.82
6	7.00	11.81	12.00	12.60
8	6.35	10.31	9.99	10.12
10	5.67	8.89	9.05	9.14
12	4.74	8.16	7.62	7.79
14	3.62	6.33	5.96	6.16
16	2.49	4.54	4.27	4.43
18	1.61	2.88	2.89	2.99
20	.99	1.88	1.85	2.13
22+	1.42	2.56	2.55	2.46
Total	48.68	81.11	79.96	82.27

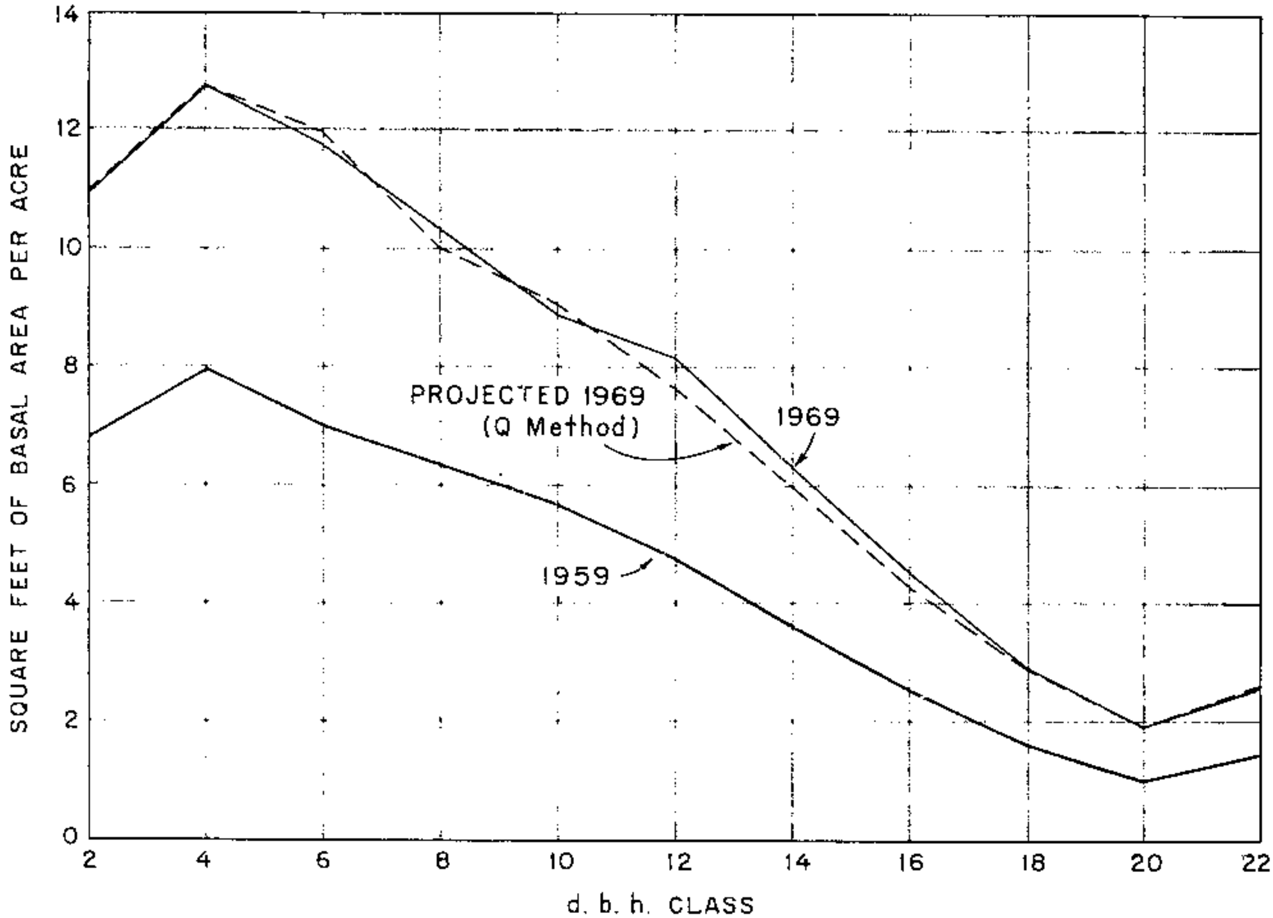

Figure 4.– Actual and projected basal area per acre in South Carolina, 1959 and 1969.

APPLICATIONS

The stand projection system was designed to perform three tasks required for national timber resource compilations and reviews. These are:

1. Reconciliation of differences between forest surveys.
2. Updating of surveys with varying completion dates to a common date for national compilation.
3. Making long-term projections with various assumptions regarding removals and management.

The stand projection growth model was selected because of its special suitability to working with aggregate stands for large areas. For national compilations, reconciliation and updating is done at the state level, and long-term projections at the national level. Aggregate stands at the state and regional level very closely approximate the J-shaped exponential model. Calculations of regression coefficients by 2-inch class provide for the minor deviations from the J-shaped curve that remain.

Reconciliation

Often, for a variety of reasons, differences between forest inventories do not represent the real trend. The stand projection procedure provides a way of making sure prior estimates are comparable to estimates based on recently completed surveys.

One especially troublesome problem in comparing current surveys with earlier ones is the discrepancy between change in inventory volume and the margin between net growth and removals. For example, both the first and second surveys may show a substantial surplus of growth over removals, when the inventory has increased very little, or perhaps decreased. One explanation, of course, is that by chance the two survey years were not representative of the years between surveys. However, in many instances an examination of production estimates between surveys did not reveal enough difference to account for all of the discrepancy.

Reconciliation of these discrepancies using the stand projection procedures rests on the assumption that total numbers of live trees by d.b.h. are probably the most reliable and comparable estimates between surveys, and that removals estimates are the least reliable. Mortality estimates are also among the least reliable, but for the most part they represent too small a component of change to be useful in reconciling differences.

An example of a reconciliation problem is shown in Agricultural Handbook No. 377. The first step is to reconcile projection input with current survey estimates. Even though projection inputs are derived from the same sample used to compile resource statistics, small discrepancies arising from rounding and pooling must be removed. This is done by adjusting the inputs iteratively until the output from a 1-year projection agrees with the resource statistics. Seldom are more than two or three iterations required.

The second step is to enter stand tables for one or two earlier surveys. (Actually in practice these two steps are conducted simultaneously.) An average annual change in numbers of trees is computed for each 2-inch d.b.h. class based on the two or three stand tables. If two stand tables are used, the annual change is constant, i.e., the difference divided by years between surveys. If three stand tables are used, a parabolic curve is fitted to the three points in time and used to compute the annual changes.

Starting with the initial stand, the annual change for each year is added to net growth to obtain a "must-have-been" removal estimate. This puts all the discrepancy into removals and assures that the change in inventory is equal to the difference between net growth and removals.

Input from earlier surveys, such as radial growth and mortality rates, are used where they are reliable enough to show a real trend. More often than not, however, it is preferable to use inputs, except for numbers of trees, based on the most recent survey if there is any risk of introducing errors arising from differences in definitions, specifications and measurement procedures on earlier surveys. Thus, differences reflect only changes in total numbers of live trees.

In analyzing trends between surveys it is usually convenient to assign all unexplained removals to a category of "other removals." Sometimes it is desirable to reduce the "other removals" estimates by varying some of the projection input parts, especially where "other removals" is unexplainably large and there is some evidence to support varying inputs. For example, there may be evidence to support a decrease in radial growth, an increase in mortality rates, or a change in the net volume per tree between surveys.[4]

[4] Procedures for preparing input for these various options are described in Agriculture Handbook No. 377 (Larson and Goforth, 1970).

The most likely source of discrepancy is an underestimation of timber removals. Because of incomplete returns from production surveys or the failure to adequately account for other removals from land clearing or logging residues, timber removals are often underestimated. On the other hand, overestimation of net growth often results from an underestimation of mortality. Both contribute to the discrepancy.

In some instances, stand tables for earlier surveys may not be available. If estimates of total inventory volume are available, past net growth and removals consistent with annual change in inventory can be reconstructed from projection input from the most recent survey by backdating. Net growth is computed and added to the net annual change in inventory to obtain the total volume of timber removals. Removal rates distribute the total volume of timber removals by 2-inch d.b.h. classes.

If no information is available previous to the current survey and comparable estimates are essential in computing regional and national estimates for past compilations, the current stand table may be backdated using estimates of timber removals based on production information. An iterative procedure must be used. The first backdate produces the first approximation of the initial stand. This stand then becomes the basis for updating. Differences between updated and actual stands are added or subtracted to the initial stand until the updated stand agrees with the actual stand within acceptable limits.

Updating

Another major use of the stand projection procedure is to update surveys with varying completion dates to a common date for regional and national compilations of data. It could also be used for updating inventories for smaller units, such as specific timber tracts on industrial ownership or National Forests on public ownerships.

Updating may be done in a number of different ways. Perhaps the most common is what might be called the "bookkeeping method." An estimate of total removal is supplied annually as input based on available production statistics. The total removal estimate is distributed by d.b.h. class using removal rates based on the recently completed survey. Either trend level removals or estimates for specific years may be used.

An alternative is to extrapolate the trend in the average annual change in numbers of trees by 2-inch classes. Where production data is unavailable or unreliable, this method may be preferable to the bookkeeping method, especially for short updates of 2 or 3 years.

Both the "bookkeeping" and "extrapolation" methods are apt to give unacceptable results for updating more than 5 years. For surveys older than 5 years, an alternative is to make an interim estimate of total volume using an extensive ground sample, such as the 3P sampling procedure described by Van Hooser and Biesterfeldt (1972). The interim estimate of total inventory volume would be used to compute an average annual change which is added to the net growth computed annually to obtain annual removals. The total volume of removals is distributed by d.b.h. class using removal rates as described above under backdating.

Still another method of updating using the stand projection method is to select only disturbed plots for remeasurement on the ground, and rely on the stand projection method to keep the undisturbed update based on no removals and normal mortality rates. A small sample of the disturbed area might be used to check for changes in average radial growth and mortality rates. Selection of disturbed plots would require some remote sensing procedure, such as annual photography from aircraft or space satellites.

Long-Term Projections

The purpose of long-term projections is somewhat different from that of updating. Aside from covering a longer period of time, the purpose of a projection is not to determine the most likely outcome, but to illustrate the consequences of a range of assumptions regarding the inputs. The usual procedure is first to make a base or bench mark projection with the simplest possible assumptions, such as holding all inputs constant, including perhaps, no constraints on growth factors in response to changes in stand density. Annual removals will be the result of constant removal rates applied to a changing inventory base.

Projection on a per-acre basis is usually desirable to make it easier to evaluate the output and to permit varying the area base in subsequent projections.

The first consideration following the base projection is the possible need for constraints on the growth factors. If annual removals are fairly close to annual growth, the average basal area will not change much and the need for constraints will not be so important. On the other hand, if net growth exceeds removals by a substantial margin, stand density will increase and rapidly approach the biological limit of what the area can logically support. Generally, the need for constraints does not become critical until the average basal area exceeds 100 square feet, but as the basal area approaches biological limits, constraints become critical in order to keep the projection within the bounds of a reasonable possibility. Yield tables provide some guidance in judging what constitutes the limits of a reasonable possibility.

If the base projection produces illogical results either removal must be increased or constraints must be introduced on radial growth, mortality rates and sapling ingrowth, as described in the section on input modification.

Once a base projection has been made that represents a reasonable possibility, any number of projections can be run demonstrating the consequences of a wide range of assumptions involving changes in such inputs as area of commercial forest land, level and distribution of removals, and changes in growth in response to management practice.

To the extent that degrees of probability can be attached to the assumptions, the projections may be treated as forecasts of available timber supplies.

The stand projection method is especially well adapted to making aggregate projections for large areas such as a state or region. The degree of aggregation depends on the purpose of the projections.

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