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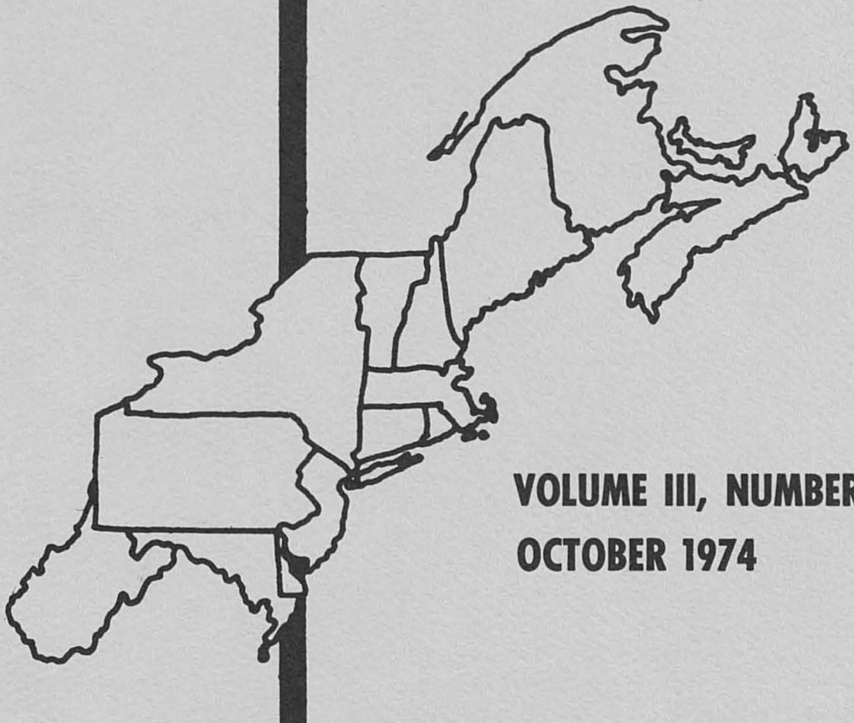

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AN APPLICATION OF MATHEMATICAL PROGRAMMING TO FLOODPLAIN LAND USE PLANNING*

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Introduction

The motivation of this study is to provide and solve a decision model for land use planning for an existing regional situation. The chosen framework views the task of floodplain management as a constrained optimization problem capable of solution by standard mathematical programming techniques. The framework is sufficiently flexible to permit incorporation of, in addition to floodplain zoning, such non-structural flood control measures as flood proofing, insurance, and flood warning systems. Further, the framework can be modeled so as to include political restraints and a broad range of socially desirable goals. In a larger regional context, the framework permits the internalization of the value of the externality1/ commonly associated with development of floodplain lands.

For present purposes, we focus on the floodplain planning (zoning) function, recognizing that the sophisticated flood control program includes a combination of structural and non-structural measures. The framework we develop below provides normative solutions to temporal land use patterns for a community on the Connecticut River floodplain. These objectives are pursued as follows. The second section provides the necessary background for the framework, including a review of the existing (limited) optimization models for floodplain land use and a brief description of the region under investigation. The empirical model occupies the succeeding section. Sample computer results are

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1/ That is, development along a particular reach of a floodplain may increase damages both above and below the development site. For these individuals, such costs would be considered external and hence would be ignored. In the larger regional context, however, these costs are internal and hence our model permits the regional decision-maker to internalize these costs with respect to the zoning decision.

recorded and interpreted in section four. The importance of the sensitivity analysis is also suggested in this part. The final section outlines major conclusions, suggests their significance, and admits the primary limitations of the framework.

Background

The literature suggesting the need for non-structural floodplain planning is extensive.2/ Likewise there is no dearth of literature pertaining to water resource systems optimization. Recent examples include: [1], [2], [10], and [9]. Unfortunately, the same cannot be said for floodplain land use planning involving other non-structural measures. To date the only major efforts which have been directed at providing a suitable methodology for floodplain planning are [6] and [3, 4].

The initial attempt at providing a methodology for assisting in the planning of floodplains was made by [6]. His formulation utilized an iterative computer routine for examining and comparing large numbers of discrete combinations of structural measures and floodplain land uses. Relative land values in alternative uses served as the measure of the opportunity cost of allocating land to particular uses. The objective of the comparison of all discrete combinations was simply to search for that combination which minimizes the sum of flood protection cost, opportunity cost, and residual damages.

The efforts of [3] represent the first application of operations research methods to problems of floodplain land use planning. He develops and demonstrates a recursive linear programming framework where the activities represent acres devoted to particular land uses. Land prices serve as a proxy for the expected present value of the stream of economic rents associated with a particular land use of an acre of land. The objective function is to select activities so as to maximize this stream of economic rents.3/ The solution is constrained by the availability of land resources in various locations and by various land use constraints.

The framework developed for the present investigation is similar to the model suggested by Day. Ours is a mathematical (linear) programming framework designed to provide (conditional) normative decisions regarding choice of land use alternatives over a 25 year planning horizon for a particular community on the Connecticut River floodplain. The community under examination is briefly described in the remainder of this section.

2/ Examples include [12, 13], [14], and [4].

3/ Flood proofing and other non-structural measures, of course, influence the magnitudes of these objective function coefficients.

The floodplain in Farmington comprises almost 3,000 acres of which approximately 2,176 are undeveloped. Despite the threat of floods, demand for urban development on the floodplain is rather strong. Part of the industrial park is situated on the floodplain; and demand for industrial and other land uses remains strong, in part due to the location of Route 4 and the proximity of adequate public services.

The floodplain was divided into three basic regions on the basis of the demand for the various land uses. That is, each of these three regions is presumed homogeneous with respect to the demand for (and price of) land for the various uses. Each of these basic regions is further subdivided into three zones on the basis of flood frequency. These zones provide information regarding the probabilities of a flood occurring in a particular year and suggest the relative risk involved if development is permitted. The lowest frequency (risk) zone corresponds to the "maximum flood of record" (1955), the second zone reflects land which is expected to be flooded once every hundred years, and the highest risk zone is land which is flooded, on the average, every fifty years.

Decision Framework

The decision model applied to the problem of land use planning for the community of Farmington is set out below. The activities, the objective function (and coefficients) and the constraints are treated seriatim.

1. Activities

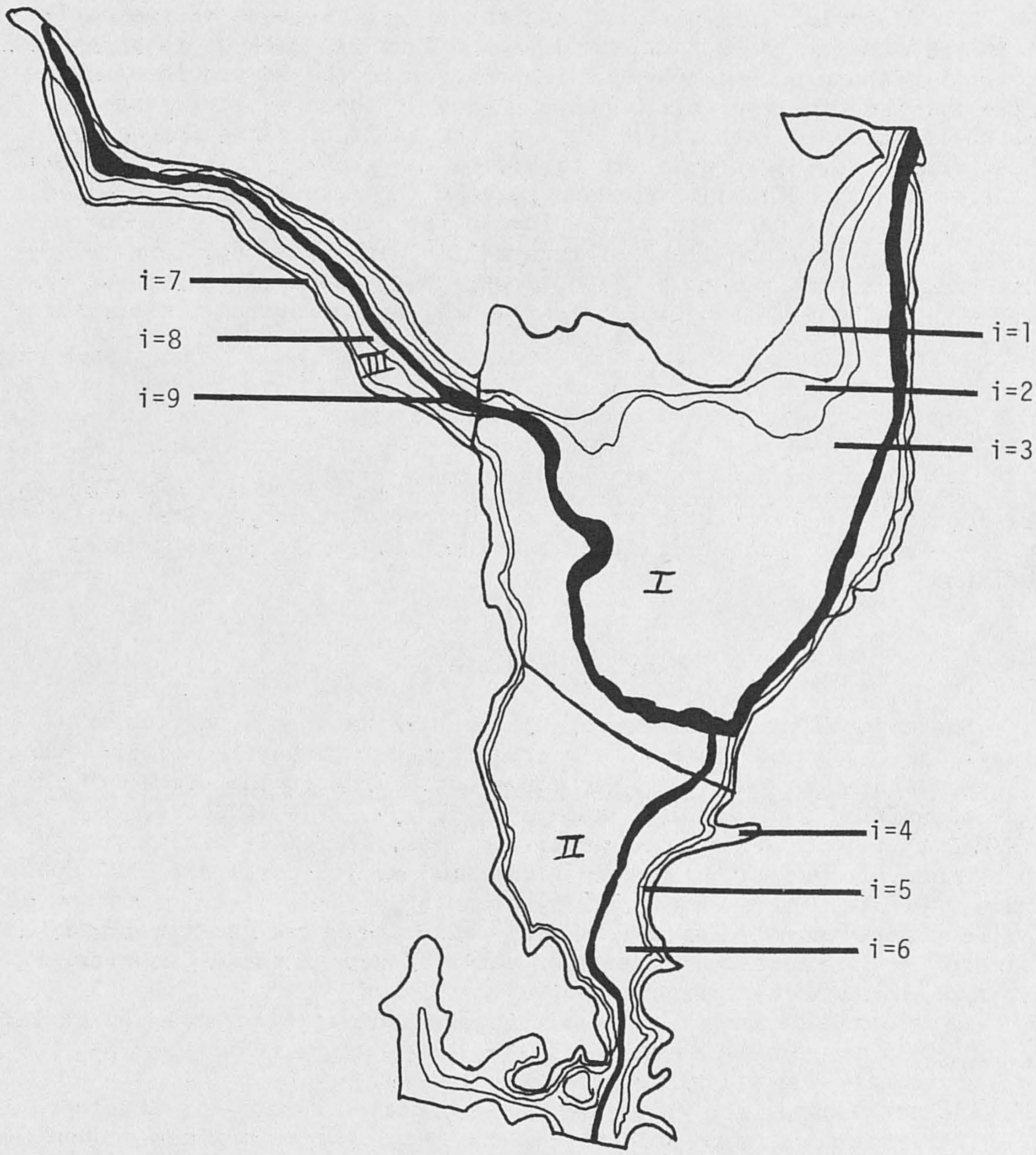
The control variables represent acres of land in a particular area or zone (i) which ought to be restricted to a particular land use (j) in planning period (t). The zones considered in the model are the nine identified in Figure 1; hence $i = 1,\ldots,9$. The land uses considered ($j = 1,\ldots,6$) are, respectively: single family dwellings, apartments, industrial use, commercial use, agricultural use, and open space. Finally, five ($t = 1,\ldots,5$) planning periods (each representing a five year interval) are considered. With these designations, then, a value of x_{ijt} denotes the number of acres of area i to devote to land use type j in the t^{th} planning period.

2. Objective Function

The objective is to select x_{ijt} so as to achieve maximum economic rent over the planning period. Since this magnitude is of course unknown, land price was selected to serve as a proxy. That is, in the

FIGURE 1

AREAL DESIGNATIONS FOR THE FARMINGTON FLOODPLAIN



absence of serious market imperfections the price of land reflects the buyer's expected net returns attributable to the land.4/

One rather serious problem remains, however. At least for the area under investigation, land purchasers are either unaware of any expected flood damages or make decisions independently5/ of this awareness. That is, for this region land prices for otherwise equivalent parcels are similar whether located on the floodplain or adjacent to it. Thus, for our purposes, we subtract from price (present value of expected economic rents) the present value of expected flood damages if land in area i is devoted to land use j in t.6/ The objective function coefficients are, then, given by:

$$(1) \quad c_{ijt} = P_{ijt} - D_{ijt},$$

where P_{ijt} denotes land price per acre and D_{ijt} is expected present value of damages per acre.

Denoting by x^*_{ijt} the 6 x 1 column vector $[x_{i1t}\ x_{i2t}\ \ldots\ x_{i6t}]'$, we define the 9 x 1 vector $x_t = [x^*_{1jt}\ x^*_{2jt}\ \ldots\ x^*_{9jt}]'$. The activities can then be arrayed in matrix form where each column is a time partition of all activities, viz., $x = [x_1\ x_2\ \ldots\ x_5]$ for the five period case. Similarly, the matrix c is formed by simply replacing each element (x_{ijt}) in x by its associated objective function coefficient (c_{ijt}). Finally, applying the stacking operator ϕ7/ to both c and x, we obtain $\phi x = X$ and $\phi c = C$, and the objective function is given as:

$$(2) \quad \text{Maximize } Z = C'X,$$

subject to the following set of constraints.

3. Restraints

The constraints operating on this decision criterion can be expressed as:

4/ This ignores problems of consumer surplus and alternative buyer motives, of course. Support is given to this measure, however, by [4] and [5].

5/ A rationale couched in a lexicographic utility framework for explaining why improbable events of large losses might be ignored by "rational" individuals is provided by [7].

6/ The methodology for computing these expected damages is developed in [14].

7/ The operator ϕ simply stacks columns (time partitions in this case) one upon the other. In this case $\phi x = X = [x_1\ x_2\ \ldots\ x_5]'$. The operator ϕ therefore transforms the 54 x 5 matrix x into a 270 x 1 column vector X.

(3) $AX \leq B$,

where A is the 60 x 270 matrix of (input-output) coefficients in the restraint set and B denotes the 60 x 1 column vector of restrictions. In our formulation, A is block diagonal, composed of five time-partitioned submatrices A_t, i.e. $A = I_5 \otimes [A_1\ A_2\ \ldots\ A_5]$, where $\otimes$ is the Kronecker product. Each A_t is 12 x 54, where the columns relate to the respective time partitioned decision variables x_t. The B vector is partitioned conformably--the first 12 restrictions apply to period one, the second to period two, and so forth.

The matrices A_t define the restraints. Denoting by a_{kijt} the element in the k^{th} row of the column associated with the activity x_{ijt}, the first nine rows of each A_t are defined by:

$$a_{kijt} = \begin{cases} 1, & \text{if } k = i \\ 0, & \text{otherwise} \end{cases}.$$

The corresponding elements of B are simply the acres available in region i = 1,...,9 for assignment to a land use in period t.8/ This set of constraints restricts the solution on the basis of land availability.

The tenth row of each A_t recognizes that demand for residential purposes is not unlimited.9/ For this row,

$$a_{ijt} = \begin{cases} 4.4, & \text{for } j = 1, \\ 12.5, & \text{for } j = 2, \text{ and} \\ 0, & \text{otherwise} \end{cases}$$

and the corresponding element(s) of B are P_t, where 4.4 is the average number of persons per acre in single family dwellings, 12.5 is the number for apartments, and P_t is the population projected to demand living space on the floodplain in period t.10/

The eleventh and twelfth row of each A_t serve the same function for commercial and industrial demand, respectively. For row eleven, then,

$$a_{ijt} = \begin{cases} 1, & \text{if } j = 4, \\ 0, & \text{otherwise} \end{cases}$$

and for twelve

8/ For t > 1, the corresponding elements are reduced by the magnitudes of the decision variables previously determined.

9/ The restraint amounts to a linear approximation of a non-linear demand relation. A caveat is provided in the Conclusions below.

10/ See [11] for further support and development of these figures and concepts.

$$a_{ijt} = \begin{cases} 1, & \text{if } j = 3, \\ 0, & \text{otherwise} \end{cases}$$

Likewise, the right-hand sides are respectively projected demands for commercial and industrial uses in period t.

Optimization of (2) subject to (3) plus the usual non-negativity conditions is straightforward. Since A is block diagonal, the separate time partitions can be solved separably.11/ The optimal solution to this basic formulation is provided in the section below.12/ A number of variations which recognize open space externalities, political considerations, and uncertainty with respect to the key parameters (sensitivity analysis) are briefly treated in the final section.

Empirical Results

The normative temporal land use patterns suggested by the constrained optimization of (2) are depicted in Table 1. The following comments are in order.

First, not unexpectedly, the low risk zones (i = 1, 4, 7) of each of the three basic regions were quickest to develop. Area one was completely developed by the end of period 2, area four developed immediately and area seven developed during the third and fourth periods. It is notable that no apartments appeared in the final solution. When alternative formulations were employed which recognized local political and other restraints, however, some apartment construction entered the optimal solution.

The shadow prices (dual values) associated with several of the restraints are depicted in Table 2. Their magnitudes are well within the expected range. For example, if the industrial demand for land in period one were an acre greater, this would contribute to the value of the objective function (2) an estimated 5,140 dollars.

Several preliminary caveats should be expressed at this juncture. First, we present above only the basic (skeletal) model for purposes of demonstrating the technique. Variations are hinted at below.

11/ This formulation simplifies computations and is easily understood by regional planners. More sophisticated approaches (e.g. dynamic programming) are available, of course. These involve substantially greater computational expense and are relatively undiscernible to most regional planners.

12/ The elements of both B and C are rather space consuming and are therefore not included in this paper. These data are available upon request.

Table 1

Solutions to the Basic Framework*
(x_{ijt} in acres)

Area	Land-Use Activity	Periods One	Two	Three	Four	Five
One	Single Family	80	21	--	--	--
	Industrial	37	39	--	--	--
	Commercial	15	15	--	--	--
	Open Space	75	--	--	--	--
Two	Single Family	--	45	--	--	--
	Open Space	45	--	--	--	--
Three	Single Family	--	--	--	80	80
	Commercial	--	--	--	--	15
	Open Space	1162	1162	1162	1082	987
Four	Industrial	18	--	--	--	--
Five	Industrial	--	--	4	--	--
	Commercial	--	--	--	2	--
	Open Space	6	6	2	--	--
Six	Open Space	470	470	470	470	470
Seven	Single Family	--	14	--	--	--
	Industrial	--	16	50	--	--
	Open Space	80	50	--	--	--
Eight	Single Family	--	--	80	--	--
	Commercial	--	--	15	13	--
	Open Space	108	108	13	--	--
Nine	Open Space	80	80	80	80	80

*Single family, industrial, and commercial development values are incremental acres devoted to the respective land uses. The open space values for each period represent total acres remaining in open space.

Table 2

Shadow Prices for the Basic Framework
(dollars)

Resource	Periods One	Two	Three	Four	Five
Population	1,557	1,330	1,204	868	868
Commercial	26,199	25,199	21,699	17,713	9,804
Industrial	5,140	4,140	1,481	0	0

Second, there is a significant degree of uncertainty in some of the key parameters. The sensitivity of the solution to this uncertainty is treated in the final portion of this section.

Earlier, we suggested that the framework was sufficiently flexible to treat political, legal, and other restraints as well as alternative community goals. The investigation underlying this paper, indeed, examines solutions under a large number of such alternative specifications. We include here two such alternative structures for illustrative purposes only.13/

One such set of supplemental constraints stems from community leaders' desires that in each period for every acre of urban development on the floodplain ($j = 1,\ldots,4$) there be retained at least 4 acres of open space ($j = 5, 6$). In terms of (3), this translates into the augmentation of each A_t by a restraint where:

$$a_{ijt} = \begin{cases} 1, & j = 1,\ldots,4 \\ -0.25, & j = 5, 6 \end{cases}$$

and the corresponding element of B for each t is zero. Conversations with community groups and planners reveal a host of such additional constraints on the basic model, many of which will of course differ among communities.

As an example of an adjustment to the objective function (rather than the constraint matrix), consider the open space (external) benefit associated with the agricultural use of land. To the extent that there exists a positive externality associated with the scenic or open space value of retaining land in agricultural use, a model which fails to incorporate this external benefit will tend to under-allocate land to

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13/ The other structures examined as well as the solutions to all models are available upon request.

the agricultural use from a societal standpoint. An alternative is to "internalize" this value by adding to C_{i5t} a portion of the value of open space C_{i6t}. From a societal standpoint, then, this recognition of an external benefit makes agricultural use more competitive with the urban uses and more in accordance with community desires.

Since many of the parameters used in the objective function (C) and right-hand side (B) are subject to particularly significant degrees of uncertainty, sensitivity analysis has been performed. This has been accomplished primarily via discrete changes14/ in the elements of C and B in accordance with estimated or assumed standard errors of the coefficients.15/

The results of this analysis showed remarkable solution insensitivity to the magnitudes of uncertainty with which we are dealing. Solutions typically varied very little in response to likely changes in expected flood damages and demands for residential, commercial, and industrial floodplain land.16/ The solutions underwent drastic changes, however, when A was altered to reflect political or social constraints or when C was revised to "internalize" externalities such as was described above.

Conclusions

This paper demonstrates the usefulness of mathematical programming procedures for assisting with community land use decision making. Major advantages of the formulation given by (2) and (3) include:

(a) It provides a formal means of testing solution sensitivity over the range of reasonable uncertainties surrounding key parameters.

(b) Shadow prices reveal useful information regarding values of relaxing certain constraints, such as open space restrictions, etc.

(c) It permits the formal recognition of externalities and the means of internalizing them for community decision making purposes.

14/ This can be accomplished in a formal mode using such means as: ranging, parametric programming, duality, and quick reoptimization or by examining results of the analysis while making small changes. Our analysis used a combination of procedures.

15/ Some of these distributions are expressed in [14].

16/ Until period four, only X_{533} showed any sensitivity at all to the ranges of parameters examined. As expected the solution became more sensitive with respect to choice of discount rate in periods four and five.

(d) Finally, a useful aspect of developing such an operations research framework lies in the process of developing the restraints and objective functional itself. That is, it forces one to clearly identify and quantify the critical variables in the decision process.

One of the primary limitations which should be recognized in applying such a framework is the assumption of a linear objective functional. That is, economic theory postulates a (negative) relationship between x_{ijt} and c_{ijt}, viz., $c_{ijt} = c_{ijt}\ (x_{ijt})$. Thus the objective functional, $c_{ijt}\ (x_{ijt})\ [x_{ijt}]$ is non-linear.17/ Our preference functional (2) is in fact a linear approximation to this relation where restraints 10 through 12 of each A_t in (3) define the bounds on the linear approximation. The non-linear function could of course be used for solution. The primary drawbacks would be the greater solution expense and the greater problems (of understanding and interpretation) for community planners with limited training in such procedures.

Another set of limitations is that the parameters are not known with certainty. This objection is for the most part obviated by the existence and judicious use of sensitivity analysis.

Finally, our recursive linear programming solution, in which each time partition is optimized separately, does not necessarily lead to an overall optimum for the entire time horizon. For the discrete-time class of problems, dynamic programming would provide this overall optimum. The number of state variables involved in this type of problem, however, renders this solution procedure intractable.

In summary, we feel that if the solutions are interpreted with the conditioning assumptions and limitations in mind, the framework offered above can be of use to many communities in the planning process.

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17/ See [8] for an example of empirical estimation of $c_{ijt} = c_{ijt}\ (x_{ijt}, E_{ijt})$, where E_{ijt} denote other exogenous factors such as parcel size and distance to the nearest central city.

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