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*The Impact of Rural Residential Water Demand  
on Reservoir Size Requirements:*

*An Econometric and Simulation Analysis*

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### *Abstract*

This study suggests that demand management through pricing policies can be used to solve water supply problems. A demand function for water was developed using cross-sectional water data. The demand function was used in a simulation analysis to determine reservoir capacity needed to supply water for a rural community.

### *Acknowledgements*

The authors are indebted to the Accounting Division of the Office of the Public Service Commission of the Commonwealth of Kentucky for assistance in supplying information on water use and rates. The data on price and gallons of water sold were obtained from the annual reports sent to the Commission by the water districts. The work on which this report is based was supported in part by funds provided by the Office of Water Research and Technology of the United States Department of Interior as authorized under the Water Resources Research Act of 1964, as project A-052-KY, and in part by the Kentucky Agricultural Experiment Station.

*The Impact of Rural Residential Water Demand  
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Today more than at any other time in history, there is a need for a general economic policy regarding the allocation of residential water. "It is true that we must have a certain amount of water to live, but we also need food, shelter, and clothing. There is indeed no reason, in principle, why water should be treated any differently from these goods, yet the very possibility that even the source of water may be too expensive relative to the benefits it provides, is an idea that is alien to the industry" (Warford, p. 94). Traditionally, water utility managers have adjusted water quantities rather than prices as changes in demand occurred. Utility managers have viewed as essential the total quantity of water demanded by consumers. Hence, utility managers have not emphasized demand management. Historically, water has been available at low cost and economists have not become involved in water demand management. Water investment decisions have been delegated by the consuming public to the political and not the economic sphere.

The central argument of this paper is that since the price elasticity of demand for water is non zero, the demand for water can be influenced by the pricing policies of water suppliers. Demand for water by residents of specified rural areas in Kentucky is statistically estimated. The estimated relationship is used in an engineering simulation model and reveals the effect of different pricing levels on the size of water reservoir required by a hypothetical water district.

Few studies have been conducted on residential water demand. Wong argues that the major reason for the paucity of available literature is the absence of an economic policy on municipal water demand. He further expresses the difficulties associated with making econometric studies in water demand. The major difficulty is the lack of reliable price and consumption data on water use. Demand studies in water use are comparatively new and as yet there is not an extensive theory upon which to rely. Past studies differ substantially as to the empirical specification of the demand function. Moreover, water districts seldom conform to the same geographic boundaries in which data for the independent variables have been collected.

By allowing water to be supplied by the requirements type of forecasting, the range of choices has been constrained. Judith Rees states: "It would be impossible to rectify shortages of all goods by increasing the supply, as the economy's resources are not indefinitely expandable. There appears to be no rational grounds for allowing water supplies to be extended to meet all foreseeable 'needs', when the supply of most other commodities is only increased by foregoing alternative goods. It is possible that the construction of additional water supply capacity is diverting resources away from uses valued more highly on the margin by consumers" (p. 28).

#### *The Theoretical Model*

Figure 1 depicts a marginal cost pricing model for residential water (Warford). This model assumes that the water utility industry is an increasing cost industry. Cost data for residential water production are not readily available. However, Grima and Warford

suggest reasons for expecting the costs of residential water to increase over time. For example, as water consumption increases, it might become necessary to exploit less accessible sources. As supply firms tap water further away, pumping and related costs should increase to overcome the friction of distance. New plant and equipment might be needed to handle the extra capacity. In Kentucky, many communities build water supply reservoirs which constitute a substantial initial investment.

Figure 1 illustrates a situation in which demand is increasing over time and price is set equal to short run marginal costs (SMC). The SMC schedules become vertical at output levels OA and OB, signifying full capacity. Another schedule (LMC) represents the long run marginal costs.<sup>1</sup> If demand is initially represented by  $D_1$ , then the existing supply should be rationed among consumers by a price set at  $P_1$ . As demand increases it pays to raise price along the vertical SMC schedule until price equals LMC. At this point the price consumers are willing to pay for a marginal unit of output equals the sum of the capacity and operating costs incurred in producing that marginal unit. Once this point has been reached further increases in demand will justify increased investment. For example, if demand shifts to  $D_3$ , the appropriate increase in capacity will be equal to AB. Any increase in demand over  $D_2$  will justify an increase in output provided by the increment in capacity equal to ABDE while the cost will equal ABDC. The net benefits derived will equal area CDE. Once capacity has been installed the appropriate price charged will equal SMC. An optimal situation for the water utility requires that capacity be utilized and that price should equal both short and long run marginal costs.

In Kentucky, only municipal water utilities build and operate water supply reservoirs. The rural water districts which do not have alternative sources of water contract to purchase water from the municipalities. It is hypothesized that the price at which water is sold depends upon the cost of the reservoir; that is, the more expensive the reservoir the higher the price charged for water. Also, the amount of excess capacity depends upon the cost of the reservoir. A water utility with a more expensive reservoir will build less excess capacity and accept the greater risk of running low on water. In the model  $SMC_1$  and  $SMC_2$  represent the short run marginal cost functions for two municipalities (figure 2).

Municipalities initially build water reservoirs to ensure adequate supply for their own customers. It is a common practice in Kentucky for municipalities to build excess capacity into the reservoir as a precaution against running low on water. Many rural residential water districts in Kentucky rely on this excess capacity for their water requirements. The concept of how this agreement is hypothesized to occur is illustrated in figure 2. Municipality 1 sells water to its customers at price  $P_1$ , a price high enough to cover operating costs. At price  $P_1$  the municipality will sell OA units of water to its urban customers, leaving an excess capacity of OB - OA which could be sold to rural water districts at price  $P_2$ . Municipality 2 will sell OC units of water to its urban customers at price  $P_3$ . This leaves an excess capacity of OD - OC which could be sold to rural water districts at price  $P_4$ . The information obtained from this model can be used to delineate supply functions for two rural water districts. With the

addition of supply functions for several water districts, it is possible to identify the rural residential demand for water.

Water system managers generally regard their system's excess capacity as having essentially zero marginal costs. Thus, utilizing a position as a spatial monopolist the water utility will set the price charged to rural water districts at the point where marginal revenues equal zero. This is the output level at which the price elasticity of the demand function is unitary and profit is maximized.

Figure 3 depicts a model of three demand functions for residential water use. The function  $D_1$  represents the demand for essential water such as drinking, cooking, washing clothes, personal hygiene, and waste removal. The demand function for such purposes because water has no close substitute has a very steep slope, and the consumer is willing to pay a very high price to consume small quantities of water. The slope of  $D_2$  is slightly less steep than  $D_1$  and represents demand for water of lesser importance to the household, such as for lawn watering and water using appliances. Demand function  $D_3$  is nearly horizontal and indicates the demand for water of least importance, such as leakages, careless use in sprinkling, and other waste. For these uses, the consumer is willing to pay only a very low price. The three demand functions can be summed horizontally as in figure 3b. Additional demand functions were added (figure 3c) to depict a continuum of specific water uses ranging from water for drinking to water for waste. An aggregate demand function for residential water use can be constructed by horizontally summing the series of individual demand functions creating a hyperbolic demand function (Grima).

### *The Econometric Model*

A simple single-equation model is a valid mathematical representation of the demand function for residential water use. The theoretical model clearly reveals that while the price of water is affected by the supply of water (amount of excess capacity in the reservoir), the supply of water available to rural water systems is not affected by the price. This is because the excess capacity of the municipal reservoir was designed merely as a safety valve, not for the purpose of selling water to rural water systems. When price is plotted on the vertical axis, supply functions faced by rural users become vertical functions of zero elasticity. Therefore supply functions which evolve from reservoirs of alternative sizes trace out an 'average' demand function for rural residential water (Sheperd, figure 4).

The general stochastic form of the demand model is:

$$(1) \quad Q_d = f(P, I, V, E, N, u)$$

where

$Q_d$  = quantity of water used in gallons/year/dwelling unit by the average family in each water district,

$P$  = average water bill in dollars/thousand gallons for the average family in each water district,

$I$  = mean income in thousands of dollars/year/dwelling unit,

$V$  = value of dwelling unit in thousands of dollars,

$E$  = evaporation in inches for June through September,

$N$  = number of persons/dwelling unit, and

$u$  = stochastic error term assumed to be  $N(0, \sigma^2)$ .

It was previously argued that a single-equation model is adequate

to capture the structural relationship influencing the demand for water from rural water systems (figure 3). Expected signs on the model parameters are  $-Q_p, Q_I, Q_V, Q_E, Q_N > 0$ . The partial derivative with respect to price ( $-Q_p$ ), represents the negative slope of the demand function, while the partial derivatives with respect to other parameters,  $Q_I, Q_V, Q_E, Q_N > 0$ , represent shifters of the demand function in the price-quantity plane. Hence model criteria are the first partial derivative with respect to price should be negative,  $Q_p < 0$ , indicating that the demand function is downward sloping. Remaining coefficients are demand 'shifters' and are treated as constants when finding the derivative. The second partial derivative with respect to price should be positive,  $Q_p^2 > 0$ , i.e. the demand function is concave from above. The function is hyperbolic and asymptotic with respect to the price and quantity axes.

A power function satisfying these criteria is:<sup>2</sup>

$$(2) \quad Q_d = \alpha_0 P^{\alpha_1} I^{\alpha_2} V^{\alpha_3} E^{\alpha_4} N^{\alpha_5} u$$

Model (2) can be estimated by OLS after performing a log transformation on both sides of the equation, i.e.

$$(3) \quad \ln Q_d = \ln \alpha_0 + \alpha_1 \ln P + \alpha_2 \ln I + \alpha_3 \ln V \\ + \alpha_4 \ln E + \alpha_5 \ln N + \ln u$$

This functional form fulfills the criteria established above. The usual OLS assumptions are assumed to have been met. The estimated equation for the log-linear model is:

$$(4) \quad \ln Q_d = 3.20 - .92 \ln P^{**} - .14 \ln I + .14 \ln V \\ (.05) \quad (.22) \quad (.15) \\ + .29 \ln E^* + .33 \ln N \\ (.16) \quad (.33)$$

$$R^2 = .68 \quad F = 61.93^{**} \quad n = 150$$

\*\*Significant at .01 level    \*Significant at .10 level  
(standard errors are in parentheses)

The price elasticity for rural residential water use is estimated to be  $-.92$ , near unity. That is, a 10 percent increase in price will generate a 9.2 percent decrease in quantity demanded. This finding empirically supports the theoretical contention that the demand for rural residential water would have a price elasticity near unity. A comparison of this elasticity with those from other studies indicates that it is larger than most (Wong; Chiogioji and Chiogioji). This study involved price data which exhibited a higher mean and standard deviation than most previous studies (table 1). Thus, the price elasticity is expected to be relatively high. The finding supports the contention that the demand for water is relatively elastic and implies that price does have an effect on water consumption and can be used as an effective water management tool.

The income elasticity for rural residential water use was not significantly different from zero at the .10 alpha level. This elasticity is substantially lower (absolute value) than previous estimates (Wong; Chiogioji and Chiogioji). Since income data used in this study were collected from the same source as many previous studies (i.e. the Population Census), the near-zero income elasticity seems to indicate that rural residents react differently than do their urban counterparts. This finding is not surprising since water utilities charge rural districts high rates restricting rural water consumers to the portion of their demand function with greatest slope. At this

portion of the demand function consumers are restricting themselves to essential consumption. Thus one would expect the income elasticity to be near-zero. Also this difference may be due to the lower average income with smaller variance in rural areas. The lower mean income decreases the purchase and use of water-complementary appliances in rural areas. Rosenstiel found that Kentucky's rural residences use little water for non-essential uses such as lawn watering, leakages, and waste. The income elasticity for rural residential water use would probably be low since these non-essential uses have the greatest effect on the income elasticity.

Examining the values of the standard errors in equation (4) indicates that price is the only independent variable which contributes significantly to the explanation of the dependent variable. For this reason, income, value of residence, evaporation, and persons per household were deleted from the original model.<sup>3</sup> Thus, an alternative to the estimated function (4) is given as:

$$(5) \quad \ln Q_d = 4.51 - .92 \ln P^{**} \\ (.05)$$

$$R^2 = .67 \quad F = 298.39^{**}$$

\*\*Significant at .01 level (standard error in parentheses)

By comparing this model with the previous log-linear model, it can be seen that the price elasticity has not changed. This suggests that price is nearly orthogonal (uncorrelated) with the other variables in the model.

#### *The Simulation Model*

Estimates of demand parameters developed in this study were used

in an engineering simulation model to illustrate effects of alternative prices on required reservoir capacity. The demand function provides an important contribution to the efficacy of the simulation model because the demand function will make it possible to estimate quantities consumed by users at different price levels. Without the demand function one can only estimate water withdrawal from a reservoir without considering the price of water. It has been shown that water use is a function of the price of water. Hence analyses that ignore the price-quantity relationship may be in substantial error. The demand function also increases the possible applications of the simulation model by linking the demand side (which has traditionally been considered fixed) with the supply side of water management problems.

A hypothetical situation was used to illustrate the importance of price in designing reservoir capacity. The following assumptions were made:

- (1) The drainage basin for the reservoir was 4 square miles,
- (2) The water district consisted of 4,000 households,
- (3) There were 2.8 persons per household, and
- (4) The minimum low flow rate (evaporation, seepage, etc.) was 3.4 inches per year.

The outflow of water from the reservoir was equal to the quantity of water demanded ( $Q_d = 90.92 P^{-.92} \times 4000$ ) plus the low flow rate. To increase the accuracy of the simulation analysis the demand function was adjusted for monthly differences in demand. This was accomplished by using data obtained by Dowell on the percentage of annual distribution of water demand for Lexington, Kentucky (p. 25). The

annual quantity of water demanded was allocated monthly on the basis of the percentages of annual demand contained in table 2.

Inflows of water into the reservoir were simulated based on the Thomas-Fiering Normal Model. The key flow equation within the model is:

$$(6) \quad X_t = \bar{X}_t + \frac{r_t s_t}{s_t - 1} (X_{t-1} - \bar{X}_{t-1}) + S_t (1 - r_t^2)^{\frac{1}{2}} \epsilon$$

where

$X_t$  = monthly streamflow in month  $t$ ,

$\bar{X}_t$  = mean monthly streamflow in month  $t$ ,

$r_t$  = correlation coefficient between flows in month  $t$  and  $t-1$ ,

$s_t$  = standard deviation of monthly flow in month  $t$ ,

$\epsilon$  = a standard normally distributed random deviate, and

$t$  = time (monthly).

Equation (6) states that the flow in month  $t$  depends upon the flow in the previous month plus a random component. All parameters were estimated using 31 years of time-series data from a drainage basin in Kentucky. From the equation, a 50-year simulated run of inflow data were generated. Fifty years was taken as the design life of the reservoir.

Inflow and outflow equations were then incorporated in the following equation:

$$(7) \quad S_t = S_{t-1} + X_t - D_t \quad 0 \leq S_t \leq S_{\max}$$

where

$S_t$  = reservoir storage at the end of month  $t$ ,

$X_t$  = inflow during month  $t$ ,

$D_t$  = outflow during month  $t$ , and

$t$  = time (monthly).

Equation (7) states that the amount of water in storage at the end of the month is equal to the amount of water in storage at the beginning of the month plus the difference in the inflow and outflow during the month.

Reservoir storage required to meet the monthly demand ( $D_t$ ) for all months during a 50-year period was determined by initially assuming a reservoir of a given capacity to be full. Equation (7) was applied month by month to the reservoir for a 50-year period based on the demand model and inflows generated by equation (6). If the value of  $S_t$  became negative at any time, the assumed reservoir capacity was increased and the process repeated. The final reservoir capacity was the minimum capacity that kept  $S_t$  from becoming negative during the 50-year period.

Since each 50-year simulated streamflow record generated by equation (6) represents only one of an infinite number of possible streamflow records, the reservoir capacity determined by the above procedure is only one estimate of a variable containing several random components. The resultant variation in reservoir capacity was evaluated by replicating the entire simulation process 100 times. This produced 100 estimates for required reservoir capacity. The capacity that met the demand requirement for the entire 50-year period 99 percent of the time was selected as the final estimated reservoir capacity. This capacity was determined by fitting the Extreme Value distribution (Type 1, maximum) to the estimated reservoir capacities and then determining from this distribution the capacity that was adequate in 99 percent of the cases.

Three different price levels for the demand function were used in

the simulation analysis. The results of the simulations (table 3) provide empirical evidence that the design capacity of a reservoir can be substantially affected by the price charged for the water used from the reservoir. When the price was quadrupled from \$.50 to \$2.00 per 1000 gallons, the storage requirement was reduced by a factor of 2.9 from 2773 acre-feet to 960 acre-feet. A doubling of the price from \$2.00 to \$4.00 per 1000 gallons further reduced storage required to only 747 acre-feet.

#### *Conclusions*

1. Based on data collected from selected rural water districts in Kentucky, the demand per household for rural residential water is estimated as  $Q_d = 90.92 P^{-.92}$ . Thus, the demand for residential water, as in other economic goods is a decreasing function of price and is hyperbolic. This finding contests the "water is different" philosophy and supports the use of demand analysis in water management.
2. The analysis reveals that the elasticity of demand for residential water is not as inelastic as has been believed. The evidence obtained on price data in this study indicates that the price elasticity for residential water is near unity (-.92). This implies that price can be used as an effective tool for controlling the demand for water.
3. Demand shifters (income, value of residence, evaporation, and persons per residence) do not appear to have a significant impact on rural residential water use. These results should be further investigated and included in policy variables related to demand

management when forecasting and designing capacity for water systems.

4. Based on the simulation analysis, it is concluded that price may be used to reduce reservoir capacity requirements and thus make it possible to meet demands with smaller water supply reservoirs or possibly delay the enlargement of an existing reservoir. Total cost for water supply may be reduced by charging more for the water and building a smaller reservoir. The simulation results suggest that there may be an "economically most efficient" reservoir size for a community. The trade-off between higher priced water and smaller, less costly, reservoirs needs to be investigated.

### *Footnotes*

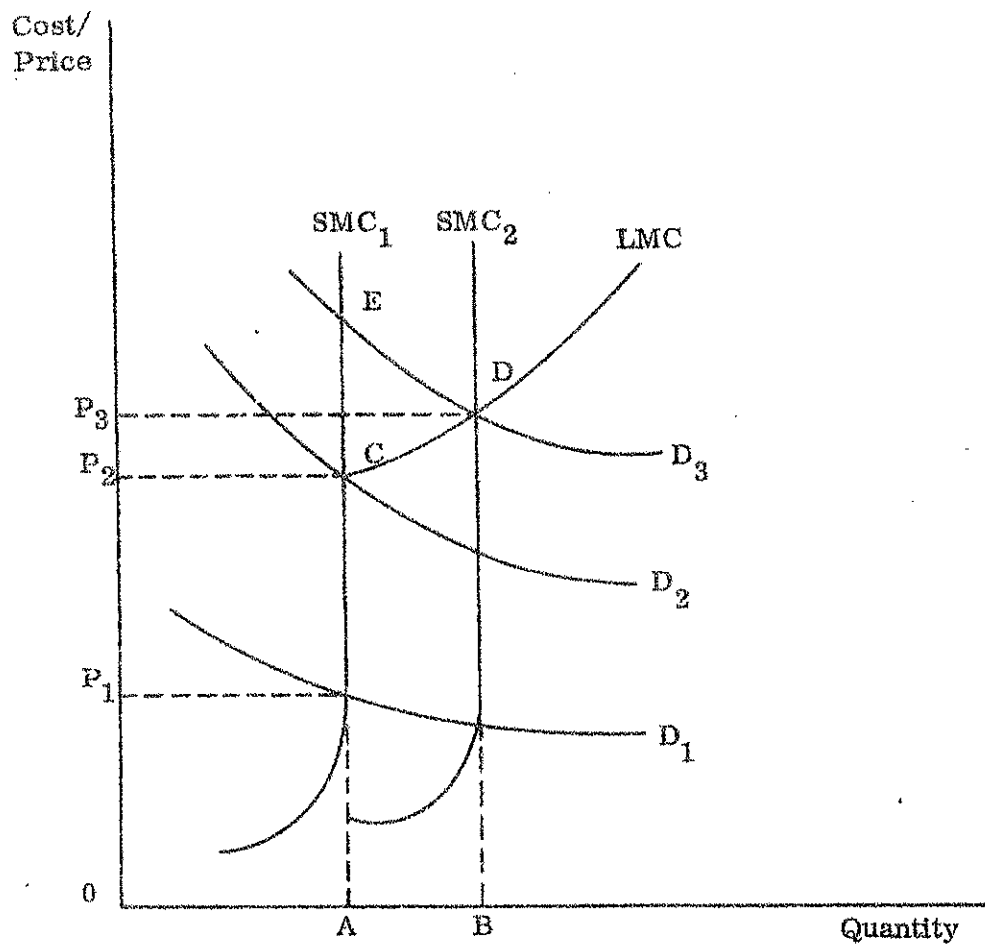
1. Marginal costs are expressed as a flow at the social rate of discount and are defined to include any externalities. Long run marginal costs represent capacity plus operating costs and are higher than short run marginal costs which represent operating costs up to OA. The fact that long run marginal costs are rising may therefore be due to rising capacity or operating costs or both.
2. A linear model with the same parameters was estimated with a  $R^2 = .14$ . Price was the only parameter significant at the .10 significance level. This yields empirical evidence that the demand function is curvilinear.
3. When the equation was estimated in a stepwise regression, the coefficient of determination increased from  $R^2 = .67$  to  $R^2 = .68$  as is evident when comparing equations (3) and (4).

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Source: (Adapted from Warford, 1966, p. 97).

Fig. 1. --Marginal cost pricing.

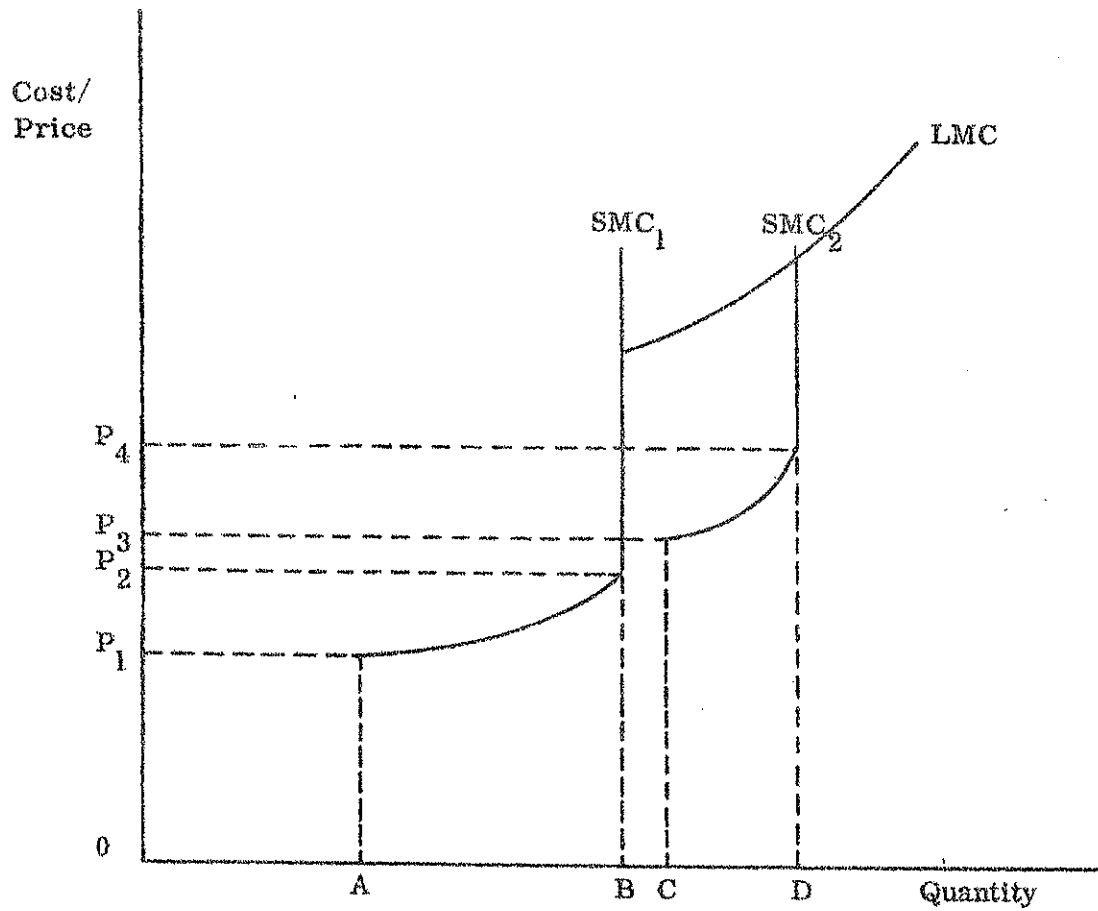
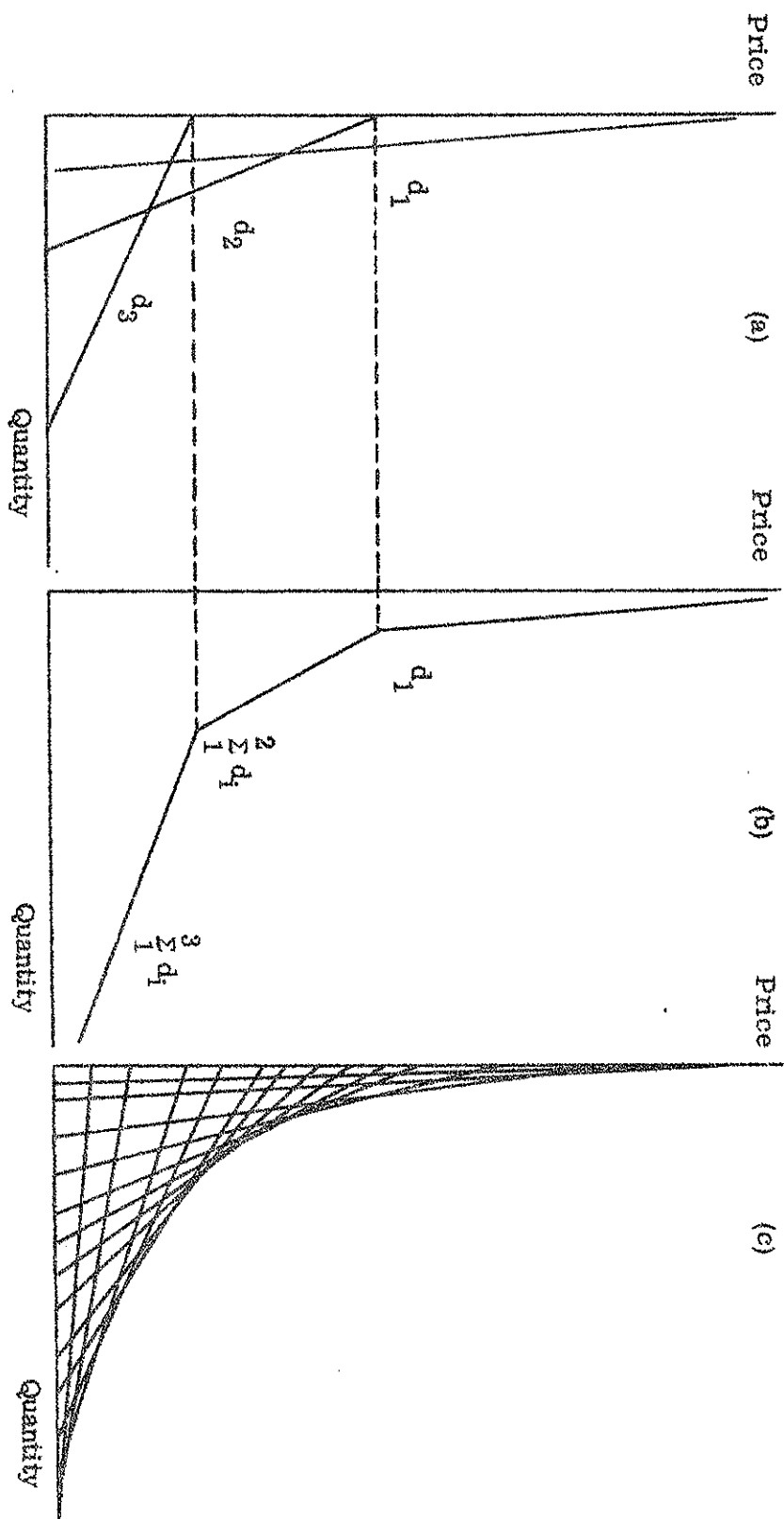
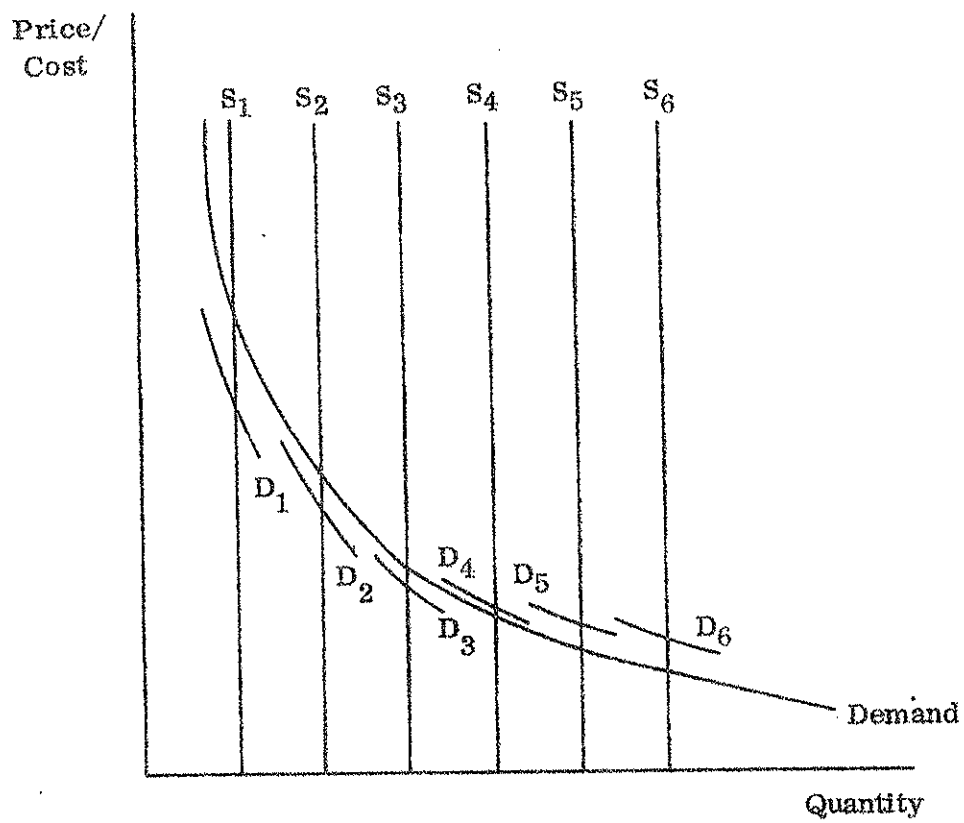


Fig. 2.--Kentucky water pricing model.



Source: (Adapted from Grima, 1972, p. 93).

Fig. 3. ---Curvilinear demand function.



Source: (Adapted from Shepherd, 1963, p. 163).

Fig. 4.--Demand adjusted by completely inelastic supply functions.

Symbol	Name	Mean	Standard Deviation	Range	Units and Source
Qd	Quantity of Water Used	56.39	50.71	2.87-521.48	(1)
P	Price	2.27	1.48	:27-14.49	(2)
I	Mean Income of Household	6.59	1.51	3.52-11.28	(4)
V	Value of Dwelling Unit	11.68	3.05	5.00-18.90	(3)
E	Pan Evaporation June-September	23.10	4.20	14.77-26.89	(5)
N	Persons per Household	2.87	.29	2.3-3.4	(6)

- (1) Thousands of Gallons/Year/Household, Kentucky Public Service Commission, 1972.
- (2) Dollars/Thousand Gallons, Kentucky Public Service Commission, 1972.
- (3) Thousands of Dollars, Population Census, 1970.
- (4) Thousands of Dollars, Housing Census, 1970.
- (5) Inches, Climatological Data - Kentucky, 1972.
- (6) Housing Census, 1970.

Table 2. Annual Distribution of Water Demand  
for Lexington, Kentucky, 1966

Month	Percent	Month	Percent
January	7.1	July	9.9
February	7.3	August	9.5
March	7.9	September	9.5
April	7.7	October	8.1
May	8.0	November	7.3
June	10.1	December	7.6

Source: (Dowell, 1967)

Table 3. Simulation Results

Price \$/1000 Gallons	Quantity Demanded		Quantity Demanded
	Gallons/Year/ Household	Storage Acre-Feet	Gallons/Person/ Day
.50	169,730	2,773	166
2.00	47,740	960	47
4.00	25,320	747	25