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ECONOMIES OF SIZE IN PUBLIC SCHOOLS: A COMPARISON OF INDIANA AND NORTH DAKOTA DATA

David L. Debertin

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Abstract

Data from two states are used for an analysis of the extent to which savings can be achieved through school consolidation. The relationship between per pupil expenditures and enrollment is examined. It is shown that per pupil expenditures for instruction are linked to teachers salaries and pupil/teacher ratio via a simple mathematical relationship. Differences in per pupil expenditures for instruction among school districts for both North Dakota and Indiana are due primarily to variation in pupil/teacher ratios decreases as enrollments increase. There was no evidence of a similar relationship for Indiana schools. However, teachers salaries in both North Dakota and Indiana increase as enrollments increase. For expenditures other than instruction, very little evidence was found for potential savings due to increases in enrollment.

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Public elementary and secondary education represents the largest single expenditure by units of state and local governments. Resource allocation questions for public schools are extremely important, since 30 percent of all tax dollars raised at the state and local level are used for elementary and secondary education. Politicians, school administrators and other decision-makers who deal with school finance problems in rural and urban areas face a key policy issue: *Can a savings of tax dollars be achieved through reorganization of existing administrative units and consolidation of school plants?*

During the past 5 years, two studies which focus on this issue were conducted. The first study [4] was undertaken in North Dakota, a very sparsely populated state. The second study [3] took place in Indiana, a state that encompasses a number of densely populated urban areas. Major differences exist between educational systems in the two states. There had been minimal consolidation of school inputs and reorganization of administrative units at the time the North Dakota study was conducted.

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*Dr. Debertin is Assistant Professor of Agricultural Economics at the University of Kentucky. The author is indebted to Thor Hertsgaard of North Dakota State University and to John M. Huie and J.B. Kohlmeyer of Purdue University for many of the ideas that formed the basis for the analyses in this paper.

A comprehensive program of administrative reorganization and consolidation of school plants was virtually complete at the time the Indiana study was undertaken.

The analysis presented in this paper explores the extent to which reductions in expenditures can be achieved through consolidation and reorganization efforts. Determinants of per pupil expenditure levels are identified for North Dakota and Indiana schools. A comparison is made of average cost curves for North Dakota and Indiana school districts. Major policy recommendations stemming from results of studies conducted in both states are presented.

An Economies of Size Model

Economists have long been preoccupied with the estimation of unit cost curves for firms. It is only natural to wonder if economies of size for public services such as schools also exist.[1] Average cost curves estimated in the North Dakota and Indiana studies reveal potential economies of size for school districts.

Theoretical Relationships

Expenditures for instruction (primarily teacher's salaries) represent the bulk (75-80 percent) of a school district's operating budget. It is postulated that the average cost or per pupil expenditures for instruction (PPE) is a decreasing function with respect to

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[1]See, for example, works by Hansen [7], Riew [12], and Cohn [1].

school system size or enrollment (ENRO). Moreover, consistent with traditional firm theory, a curvilinear relationship is hypothesized. The proposed economic model for instructional costs is:

$$(1)\quad PPE = \beta_0 + \beta_1 ENRO + \beta_2 \ln ENRO$$

It is postulated that PPE is a decreasing function of ENRO:

$$(2)\quad \frac{\partial PPE}{\partial ENRO} = \beta_1 + \beta_2\, ENRO^{-1} < 0$$

since one expects $\beta_1 > 0$ and $\beta_2 < 0$. Alternatively, the hypothesis may be stated as:

$$(3)\quad \left| \beta_2\, ENRO^{-1} \right| > \left| \beta_1 \right|$$

In addition, it is postulated that the function (PPE) will decrease at a decreasing rate:

$$(4)\quad \frac{\partial^2 PPE}{\partial ENRO^2} = -\beta_2\, ENRO^{-2} > 0$$

Determinants of Per Pupil Expenditures for Instruction

Average (per student) cost curves were estimated by regression per pupil expenditures for instruction (PPE) on district enrollment (ENRO). The estimated equation for 262 North Dakota districts was:

$$(1)\quad PPE = 557 + \underset{(.005)}{.024}\, ENRO - \underset{(15)}{121} \ln ENRO$$

$$R^2 = .23$$

(standard errors of regression coefficients appear in parentheses)

The equation for 269 Indiana districts was:

$$\text{(2)} \quad PPE = 237 + \underset{(.0345)}{.0002\ ENRO} + \underset{(0.35)}{63.34\ \ell n ENRO}$$

$$R^2 = .15$$

Enrollment (the combined impact of ENRO and ℓnENRO) was negatively related to per pupil expenditures for instruction for North Dakota schools, although explained variation was only a small proportion of total variation. For Indiana schools, enrollment was positively related to per pupil instructional expenditures. An examination of relationships between per pupil expenditures for instruction, average teachers salaries, and the pupil/teacher ratio reveals why these relations hold.

A seldom recognized relationship exists between per pupil instructional expenditures (PPE), average annual teachers salary in the district (S), and the pupil/teacher ratio (P/T). For public school systems, expenditures for items other than teachers salaries constitute less than 5 percent of the costs usually allocated to the accounting category titled instruction. Assuming instructional costs to include only salaries paid to teachers, the following equations hold true:

$$\text{(3)} \quad PPE \equiv S \div P/T$$

Equation (3) holds because:

$$\text{(4)} \quad PPE \cdot P \equiv S \cdot T$$

Differenting (3) with respect to P/T yields:

$$\text{(5)} \quad \frac{\partial PPE}{\partial P/T} = -S/(P/T)^2$$

Per pupil expenditures for instruction vary inversely as the square of the pupil/teacher ratio. Equation (3) defines a family of rectangular hyperbolas in the PPE and P/T plane, and the distance of the hyperbola from the origin is dependent on the value of S. This simple relation is fundamental to an understanding of all of educational finance. Its importance can hardly be overemphasized. Policy implications include the following:

(1) *Economies in per pupil expenditures for instructional costs due to consolidation occur because the increased enrollment allows the school administrator to increase the pupil/teacher ratio. There is no direct effect of enrollment on per pupil instructional expenditures.*

(2) *Substantial reductions in per pupil expenditures for instruction will occur by increasing the pupil/teacher ratio only if the school district is initially operating at a relatively low pupil/teacher ratio. Hence, the administrator of a school system will achieve a substantially larger reduction in per pupil expenditures by increasing the pupil/teacher ratio one unit if the school is initially operating at a pupil/teacher ratio of 10/1 than if the school is initially operating at a pupil/teacher ratio of 20/1.*

(3) *Reductions in per pupil expenditures for instruction can also be achieved through decreases in average*

> *teachers salaries. Teachers salaries can perhaps be reduced if the school administrator is willing to hire as replacements inexperienced teachers with bachelors, rather than masters degrees. However, the bulk of the variation in per pupil expenditure for instruction among districts in both North Dakota and Indiana schools is attributable to variation in pupil/teacher ratios, not teachers salaries.*

Moreover, it is naive to regress PPE on S and/or P/T in addition to enrollment because of the identity relation. Such specifications are often seen in the literature [see, for example 12]. The exact impact of a selected change in either average salaries or pupil/teacher ratios on per pupil expenditures for instruction can be determined without running the regression! A further understanding of the effect of enrollment on per pupil expenditures for instruction can be obtained through analysis of the separate effects of enrollment on pupil/teacher ratios and average salaries.

Pupil/Teacher Ratios

A key relation is the impact of enrollment on pupil/teacher ratios. The following equation was estimated for 262 North Dakota school districts. Secondary course offerings (NC) was included on the right-hand side of the equation since in sparsely populated areas, it may not be possible to offer a large number of courses without operating a very low pupil/

teacher ratio. Nearly all North Dakota districts have only one or at most two plants. A curvilinear relation between pupil/teacher ratios and enrollment is proposed. Increases in enrollment can be expected to have the greatest impact on pupil/teacher ratios in the low enrollment districts.

$$(6)\quad P/T = -5.74 - \underset{(.00017)}{.00043}\ ENRO + \underset{(0.74)}{10.51}\ \ell n ENRO - \underset{(.035)}{.175}\ NC$$

$R^2 = .66$
(262 North Dakota districts)

A similar specification was employed on a district basis for Indiana schools. (Course offering data were not available for Indiana.)

$$(7)\quad P/T = 21.03 - \underset{(.00003)}{.00002}\ ENRO - \underset{(.61)}{.17}\ \ell n ENRO$$

$R^2 = .01$
(269 Indiana districts)

For North Dakota schools, the combined effect of ENRO and ℓnENRO is strongly positive. Small enrollments in sparsely populated areas force administrators of most North Dakota schools to operate at very low pupil/teacher ratios. Even a modest expansion of programs in North Dakota schools results in reductions in pupil/teacher ratios and substantive increases in per pupil expenditures. Most consolidated plants in Indiana, however, are already operating at relatively high pupil/teacher ratios. Any increase in enrollments usually results in the hiring of additional teachers, and the pupil/teacher ratio does not increase. Large enrollments in Indiana schools enable administrators to exercise a great deal of arbitrary control over pupil/teacher ratios. Even though in the short run, enrollment is fixed, substantial changes in pupil/teacher ratios are achieved by an alteration of teaching loads (number of class hours per teacher per day) and by the addition of special instructors for courses such as music, art, and remedial reading. Hence, the relationship between enrollment and pupil/teacher ratios is strong for North Dakota schools and very weak for Indiana schools.

Teacher's Salaries

Per pupil instructional expenditures increase, *ceteris paribus*, as salaries increase. The following equation for 262 North Dakota districts was estimated for the 1968-69 school year. In addition to the logarithm of enrollment, the equation in the North Dakota study also employed the accreditation level of the school (ACC) as specified by the state department of public instruction in order to test whether schools with the highest accreditation paid the highest salaries. Enrollment in the linear form (ENRO) was found to be nonsignificant and was not included in the final regression specification.

$$(10) \quad S = 4201 + \underset{(100)}{243}\ ACC + \underset{(204)}{1131}\ \ell n ENRO$$

$$R^2 = .50$$

A similar equation for 269 Indiana districts was estimated. Accreditation data for Indiana districts do not exist.

$$(11) \quad S = 4842 + \underset{(93)}{1262}\ \ell n ENRO$$

$$R^2 = .41$$

Enrollment was positively related to teachers salaries in both states. In North Dakota schools, reductions in per pupil expenditures for instruction due to the effect of enrollment on pupil/teacher ratios are partially offset by increases in teachers salaries.

In the Indiana study, it was argued that salaries of public school teachers are positively related to enrollment and to socioeconomic characteristics of district residents including levels of per pupil assessed valuation (ASSV), family income (INCO), population density (DEN) and the percent of persons over 65 (OV65). A detailed conceptual justification for this regression model may be found in [5]. The following equation was estimated for 269 Indiana districts:

$$(12)\quad S = 3645 + \underset{(95)}{1098}\,\ell n\text{ENRO} + \underset{(.006)}{.045}\,\text{ASSV} + \underset{(.026)}{.089}\,\text{INCO} + \underset{(13.1)}{33.0}\,\text{OV65} + \underset{(.084)}{.149}\,\text{DEN}$$

$$R^2 = .54$$

While socioeconomic variables such as population density and family income explain variation in teachers salaries, the logarithm of total enrollment remained the primary explanatory variable. Salaries in districts with large enrollments are higher than in small enrollment districts because: (1) large enrollment districts have the most attractive salary schedules, (2) large enrollment districts tend to attract the most teachers holding graduate degrees, (3) large enrollment districts have been most successful in retaining teachers. This results in a high mean experience for teaching level in large enrollment districts. Salary schedules in Indiana are entirely based on teacher training and experience. Hence, teachers in the large enrollment districts tend to be high on the salary schedule [5].

Other Expenditures

In the North Dakota study, the following equations were estimated for per pupil expenditures for plant maintenance (PLTM), plant operation (PLTO) and textbooks and visual aids (TBVA). A major plant maintenance item is janitor's salaries, while plant operation consists primarily of utility costs. Expenditures on textbooks and visual aids exclude books purchased by students. The ratio of high school to elementary students (RATIO) is included in the PLTM equation since high schools often have more equipment to maintain than do elementary schools.

$$(13)\quad PLTM = 21.38 + \underset{(.0007)}{.0008\ ENRO} + \underset{(3.43)}{8.97\ RATIO} - \underset{(2.24)}{5.48\ \ell n ENRO} \qquad R^2 = .05$$

$$(14)\quad PLTO = 86.06 - \underset{(.0007)}{.0020\ ENRO} - \underset{(2.40)}{20.65\ \ell n ENRO} \qquad R^2 = .28$$

$$(15)\quad TBVA = 40.68 - \underset{(.0006)}{.0003\ ENRO} - \underset{(2.12)}{5.10\ \ell n ENRO} \qquad R^2 = .06$$

Substantive economies existed only for plant operation expenditures (heat, electricity and other utilities).

White and Tweeten [13, 14] have discussed in detail the impact of population density on transportation expenditures. A similar analysis was conducted for North Dakota schools. This analysis was in complete agreement with White and Tweeten's findings. Conclusions in both analyses supported the position that much of the apparent savings in instructional costs possible when plants are consolidated is offset by increases in transportation costs. This is particularly true in sparsely populated rural areas where transportation expenditures are a relatively large proportion of the total budget of a school.

Policy Implications

The analysis presented in this paper supports the position that consolidation of public school systems does not necessarily reduce per pupil expenditure levels. The bulk of savings in tax dollars possible when school plants are consolidated occurs through increases in pupil/teacher ratios. These savings may be fully offset by increased teacher's salaries and transportation costs.

Although consolidation of school plants does not necessarily result in a savings in tax dollars, there may be other potential justification for consolidation. It has not been conclusively shown that the higher salaries paid to teachers in the consolidated districts make it possible to retain experienced teachers, nor attract teachers holding advanced degrees [5]. Moreover, evidence indicates school districts paying high teachers salaries do not necessarily produce high achieving students [cf. 2, 3,4,6,8,9,10,11,13,14]. However, consolidation of school plants may make it possible for administrators to offer a wide variety of course offerings at the secondary level without operating at a low and costly pupil/teacher ratio. The additional cost of expanded laboratory and library facilities can be spread over a large number of students in the consolidated plants.

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