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Abstract: A simple, utility theoretic, demand model which nests the both the functional form of income and prices is presented. This model is used to calculate the income elasticities of twenty-one food items over the course of the last century.

Keywords: Functional Form; Income Elasticity; PIGLOG; Quadratic Utility

JEL Classification: C3; C5

1 Introduction

The purpose of this paper is to emphasize the impact of functional form on estimates of income elasticity for twenty-one foods over the course of the last century. We use a theoretically consistent empirical model of household food consumption that: (1) nests the functional form of the income terms in demand equations; (2) nests the functional form of the price term in demand equations. We will then show that existing models, which integrate prices and income either linearly or in logarithmic form, tend to overstate the size and the variability of the income elasticity for most of the twenty-one foods.

2 Data

In order to answer the question posed above, we will employ three different time series data sets. The first is data on per capita consumption of food items and their corresponding prices. Currently, this data set consists of annual time series observations over the period 1909-1995. Per capita consumption of twenty-one food items and corresponding average retail prices for those items were constructed from several USDA and Bureau of Labor Statistics sources. The second data series are demographic factors that help explain the evolving pattern of demands. These demographic factors include the first three central moments (mean, variance, and skewness) of the age distribution and the proportions of the U.S. population that are Black and neither Black nor White. The third data series involves the U.S. income distribution. The Bureau of the Census publishes annually quintile ranges, intra-quintile means, the top five-percentile lower bound for income, and the mean income within the top five-percentile range for all U.S. families.

3 Modeling the demand for food

We start with a theoretically consistent reduced form econometric model of n_q-vector of demands for food items with conditional mean given by, $E(q \mid p,m,d) = h(p,m,d)$, where q is an n_q-vector of food quantities, p is an n_q-vector of food prices, m is income and d is a k-vector demographic characteristics. Let x denote the scalar variable for total consumer expenditures on all nonfood items. Assume that each of the prices for individual food items and income are deflated by a price index measuring the cost of nonfood items. Consider the Gorman Polar Form (Gorman 1961) for the (quasi-) indirect utility function generated by a quadratic (quasi-)utility function,

$$v(p,m,d) = \frac{(m - \alpha(d)'p - \alpha_0(d))}{\sqrt{p'Bp + \gamma_0}},$$

where $\alpha(d)$ is an n_q-vector of functions of the demographic variables, $\alpha_0(d)$ is a scalar function of the demographic variables, B is an $n_q \times n_q$ matrix of parameters and γ_0 is a scalar parameter. For identification purposes, we choose the normalization $\gamma_0 = 1$.

Applying Roy's identity to this (quasi-) indirect utility function generates a system of demands.

$$E(q \mid p,m,d) = \alpha + \frac{(m - \alpha(d)'p - \alpha_0(d))}{(p'Bp + 1)} Bp\,. \tag{1}$$

Next, we define Box-Cox transformations for m and p by $m(\kappa) = (m^{\kappa} - 1)/\kappa$ and $p_i(\lambda) = (p_i^{\lambda} - 1)/\lambda$, for $i = 1, \ldots, n_q$, with $p(\lambda) \equiv [p_1(\lambda), \ldots, p_n(\lambda)]'$, and replace m and p with $m(\kappa)$ and $p(\lambda)$, respectively, in (1). Applying Roy's identity to the resulting (quasi-) indirect utility function then gives a demand system that can be written in expenditure form as,

$$E(e \mid p,m,d) = P^{\lambda} m^{1-\kappa} \left[\alpha(d) + \frac{m(\kappa) - \alpha(d)'p(\lambda) - \alpha_0(d)}{p(\lambda)'Bp(\lambda) + 1} Bp(\lambda) \right], \tag{2}$$

where $e = [p_1 q_1 \cdots p_n q_n]'$ is the n_q-vector of (deflated) expenditures on the food items q and $P = diag[p_i]$.

Equation 2 forms the basis of our analysis of the effect of functional form on the income elasticities of food groups over the course of the last century. The fundamental questions addressed will concern the estimated values of the Box-Cox parameters. In particular, how these departures from the PIGLOG $(\kappa = 0, \lambda = 0)$ and quadratic utility form $(\kappa = 1, \lambda = 1)$ affect the estimates of the income elasticities of the twenty-one food items.

4 Instruments for the Moments of the U.S. Income Distribution

The demand model described above is nonlinear in income. Therefore, the demand equations do not aggregate directly across individuals to average income at the market level. The advantage of using the Gorman class of Engel curves is that to generate a theoretically consistent, aggregable model of demand, only a limited number of statistics concerning the income distribution are needed. The demand model proposed in this paper requires two moments of the income distribution, specifically those associated with $m^{1-\kappa}$ and m.

For the income distribution defined by the density function $f(m)$, $m \in \Re_+$, we want to calculate the simplest possible information theoretic density for income conditional on the information that income falls within a given range, say, $m \in (\ell_{i-1}, \ell_i]$, such as the i^{th} quintile with given probability $\Pr\{m \in (\ell_{i-1}, \ell_i]\} = \pi_i$, and with conditional mean income $E\{m \mid m \in (\ell_{i-1}, \ell_i]\} = \mu_i$. To do so, we choose two equal subintervals in each range, so that the probability density function has a jump at the midpoint of that range, $\overline{\ell_i} = (\ell_i + \ell_{i-1})/2$ as well as each boundary point, ℓ_i. On $(\ell_{i-1}, \ell_i]$ this density function satisfies,

$$f(m)=\frac{\pi_i}{\frac{1}{2}(\ell_i-\ell_{i-1})}\times\begin{cases}\dfrac{(\frac{3}{4}\ell_i+\frac{1}{4}\ell_{i-1})-\mu_i}{\frac{1}{2}(\ell_i-\ell_{i-1})}, & m\in\left(\ell_{i-1},\,\frac{1}{2}(\ell_i+\ell_{i-1})\right]\\[2ex] \dfrac{\mu_i-(\frac{1}{4}\ell_i+\frac{3}{4}\ell_{i-1})}{\frac{1}{2}(\ell_i+\ell_{i-1})}, & m\in\left(\frac{1}{2}(\ell_i+\ell_{i-1}),\,\ell_i\right]\end{cases}.$$

The formal derivation of this income density (among others) and its properties are derived in Lafrance, Beatty, Pope and Agnew.

5 Empirical Results

We estimate equation 2 using a two stage SUR procedure using nonlinear least squares. Of crucial importance are the point estimates for the Box-Cox terms on income κ and the Box-Cox term on prices λ.

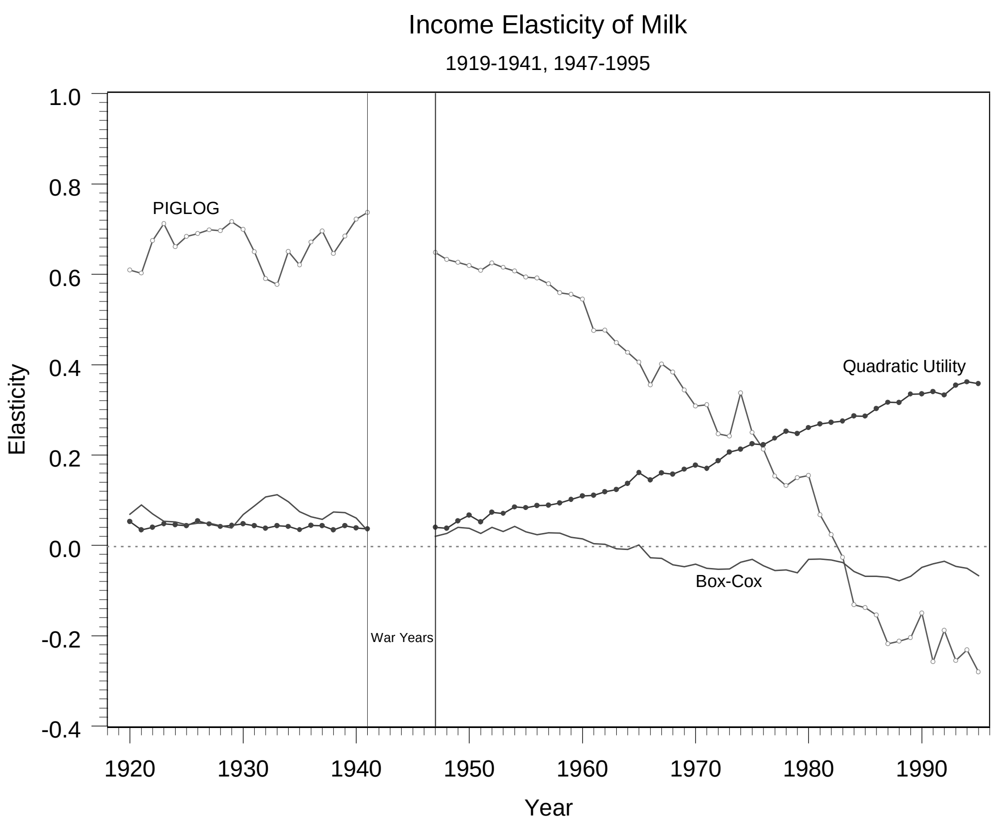
Table 1 shows us that the Box-Cox coefficients on income and prices are both significantly different from zero. Additional hypothesis tests show that each coefficient is significantly different from one, jointly different from zero and jointly different from one. All of these tests had p-values numerically equal to zero.

Table 1. Estimates of the Box-Cox Parameters

Box–Cox Coefficient	Point Estimate	Standard Error	P-Value
Income (κ)	.818649	.016988	0.0000
Prices (λ)	.794752	.018073	0.0000

The main result of this paper can clearly be seen in Figure 1. If the system of food demands were to be estimated using $\kappa=0,\lambda=0$, which results in a PIGLOG specification, one would erroneously conclude that the income elasticity of milk has declined precipitously over the course of the last century. Conversely if the system were to be estimated using the $\kappa = 1$, $\lambda = 1$, which results in a quadratic utility specification, one would conclude that the income elasticity of milk had in fact increased over the course of the last century. Either of these models might lead a researcher to conclude that there has been some form of structural change in the demand for milk over the course of the last century. However, the model proposed in this paper shows that the income elasticity of milk has only changed slightly over the period moving from slightly positive to slightly negative.

Figure 1. Income Elasticity of Milk.



In general, both the PIGLOG and quadratic utility specifications tend to overstate the size of the income elasticities of food. In addition, the PIGLOG and quadratic utility models imply that the income elasticity of food has varied considerably over the last century. Table 2 reports summary statistics of the income elasticities of food over the entire sample period. We see that for fifteen of twenty-one foods the standard deviation of the income elasticity of the approximate PIGLOG and the quadratic utility models are greater than the standard deviation for the model where κ and λ are estimated. In addition we note that the range of the income elasticities is greater for the PIGLOG and quadratic utility models than the case in which κ and λ are estimated in most cases.

Table 2. Summary Statistics of the Income Elasticities.

	Mean/*Standard Deviation*			Minimum/Maximum		
	$\kappa = 0, \lambda = 0$	$\kappa = 1, \lambda = 1$	$\kappa = \hat{\kappa}, \lambda = \hat{\lambda}$	$\kappa = 0, \lambda = 0$	$\kappa = 1, \lambda = 1$	$\kappa = \hat{\kappa}, \lambda = \hat{\lambda}$
Milk	0.38397	0.14482	0.0061179	-0.28006	0.026664	-0.078287
	0.32437	*0.10915*	*0.052534*	0.73674	0.36186	0.11162
Butter	0.36399	-0.32342	-0.34457	-1.24634	-1.05828	-1.17185
	0.52119	*0.32191*	*0.39083*	0.92355	0.050147	0.14193
Cheese	1.34438	0.058931	0.0045295	1.08009	-0.1195	-0.22536
	0.16952	*0.074345*	*0.079614*	1.82451	0.16945	0.12749
Frozen Dairy	1.05307	0.074519	0.33203	0.7374	-0.28875	0.082154
	0.10546	*0.16223*	*0.13094*	1.36101	0.38462	0.73254
Powdered Milk	0.88573	0.25009	0.2605	0.45098	-0.083651	0.026205
	0.16333	*0.28085*	*0.11744*	1.16342	0.98386	0.55959
Beef and Veal	0.82642	0.20637	0.18126	0.59517	0.10475	0.098776
	0.061152	*0.049699*	*0.03963*	0.9775	0.33554	0.26238
Pork	0.90688	0.29355	0.24526	0.71011	0.11237	0.1525
	0.056958	*0.14166*	*0.041185*	1.01364	0.57878	0.34626
Other Red Meat	1.3058	0.14157	0.053631	0.78382	-0.030233	-0.18173
	0.25213	*0.098816*	*0.063446*	1.97565	0.34024	0.15522
Fish	1.05314	0.60535	0.47961	0.83872	0.24857	0.27846
	0.083945	*0.22384*	*0.11687*	1.26699	0.91825	0.65558
Poultry	0.85615	0.35384	0.36388	0.62402	0.082109	0.11068
	0.077121	*0.15192*	*0.12361*	1.04591	0.58804	0.54282
Fresh Citrus	0.90935	0.46889	0.4435	0.69823	0.12647	0.19297
	0.090211	*0.26023*	*0.16941*	1.09875	0.9994	0.80564
Fresh Noncitrus	0.45747	0.60384	0.33679	-0.090433	0.1965	0.068466
	0.26233	*0.2648*	*0.10125*	0.91827	1.08615	0.50325
Fresh Vegetables	0.53421	0.29957	0.15007	0.36878	0.099527	0.10807
	0.08771	*0.12981*	*0.025032*	0.72089	0.47776	0.20416
Potatoes	0.71104	0.0091859	-0.018652	0.49281	-0.09943	-0.23709
	0.087866	*0.05029*	*0.13224*	0.92539	0.096552	0.15203
Processed Fruit	0.74486	0.30097	0.29877	0.50233	0.17093	0.21824
	0.07678	*0.091024*	*0.055681*	0.89166	0.60043	0.52287
Processed Vegetables	0.6469	0.37785	0.32865	0.4048	0.24987	0.20279
	0.11392	*0.049383*	*0.057818*	0.85371	0.50071	0.47374
Fats and Oils	1.14403	0.25932	0.23624	0.88275	0.15149	0.17844
	0.094695	*0.044004*	*0.027624*	1.31305	0.33565	0.3475
Eggs	0.94212	0.20666	0.029444	0.70294	0.06077	-0.245
	0.091881	*0.11908*	*0.1225*	1.15332	0.43316	0.16394
Flour and Cereals	0.27848	0.031499	-0.15582	-0.15982	0.0054256	-0.41041
	0.27454	*0.011072*	*0.17565*	0.66572	0.063042	0.090112
Sugar	0.76212	0.28129	0.24308	0.62273	0.064675	0.16882
	0.068516	*0.11589*	*0.051112*	0.95909	0.43781	0.32794
Coffee and Tea	0.92895	0.32466	0.27907	0.64916	0.116	0.19995
	0.083933	*0.15473*	*0.04908*	1.1348	0.60719	0.38547

6 Conclusion

The demand model proposed in this paper is a straightforward but powerful generalization of currently used models. Using this approach we test and reject the restrictions that existing models implicitly place on the Box-Cox parameters on income and prices. Our results show that the existing models of food demand significantly overstate the size and variability of the income elasticity of most food groups.

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