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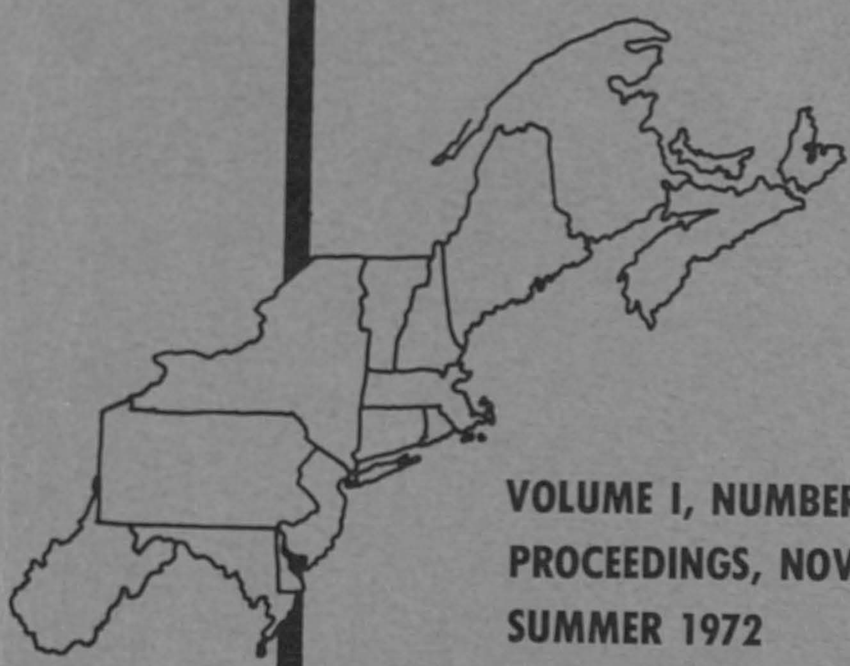

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A MODEL OF MILK PRODUCTION AND DISPOSITION: SUPPLY, UTILIZATION, AND GOVERNMENT SUPPORT IN THE DAIRY SECTOR[1/]

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INTRODUCTION

In this paper I will present a model of milk production and disposition which has been used with great success in analyzing the effects of alternative levels of price support on the dairy sector. The purposes of the model are twofold: (1) to generate short-run predictive estimates of milk production, consumption by major product groupings, government support purchases, and farm level gross income from sale of dairy products; (2) to provide a facility for analysis of alternative support prices, or other policy instruments. It is one of a collection of models of the dairy industry being developed by Farm Production Economics Division at Purdue University. This particular model serves a central role since it forms both the linkage from the reported aggregate statistics to a series of regional production models and also the linkage from a model of the current institutional framework to models which can analyze alternative institutional frameworks (e.g., spatial equilibrium or competitive position models). Primary emphasis will be on the model but representative results will be presented to show validity and possible uses of the model.

MODEL STRUCTURE

The model is a three phase recursive system as follows:

Phase I - Regional Milk Supply Projection (regression analysis),
Phase II - Market Allocation (quadratic programming),
Phase III - Regional Blend Price Computation (blend pricing formula).

1/ Journal Paper Number 4785, Purdue University Agricultural Experiment Station

Phase I - Estimation of Milk Supply Regressions

Nine regional milk supply functions were estimated.[1] Several alternative distributed lag models relating current milk production to lagged milk prices were run.[2] The equations chosen were of the form:

(1) $QS_{it} = F_i(QS_{it-1}, P_{it-1}, PG_t, PB_t, W_t\ T)$

The subscript i refers to regions (i=1, ..., 9); the subscript t refers to years, QS_t is quantity of milk produced; P_{t-1} is the lagged average wholesale price received for milk; PG_t, PB_t, W_t, and T refer to price of 16% dairy ration, price received for beef, farm wage rate without board and room, and a year date code, respectively.

Since production is dependent upon lagged prices in the model and dairy production has little influence in the determination of current grain prices, beef prices, and wages, the system may be treated as recursive rather than simultaneous. Table 1 presents the results of the regressions selected as best representing each region. Appendix table 1 shows the short and long-run elasticities associated with each estimated coefficient. The regression analysis was highly successful inasmuch as hypothesized magnitudes and signs of elasticities were obtained. For the 9 regional regressions chosen, 46 percent of all estimated coefficients were significantly different from zero at $\alpha = .01$, and additional 20 percent were significant at $\alpha = .05$, and 34% were of borderline significance or non-significant.

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[1] Regional definitions are as follows: Region 1: Northeast + Ohio + West Virginia + Virginia; Region 2: Corn Belt + Michigan; Region 3: Minnesota + Wisconsin; Region 4: The Carolinas + Georgia + Florida; Region 5: Tennessee + Alabama + Mississippi + Louisiana + Arkansas + Oklahoma + Texas; Region 6: The Dakotas + Nebraska + Kansas; Region 7: Montana + Wyoming + Colorado + New Mexico + Utah + Nevada; Region 8: California + Arizona; Region 9: Washington + Oregon + Idaho.

[2] A design of 24 linear regressions for each region using alternative variables for grain price, beef price, wages, and time was estimated by OLS for the period 1946-70, and 1960-70. The 1946-70 period gave uniformly better results. Best fitting equations met the criteria of (1) proper signs, (2) highest $\bar{R}^2$, and (3) smallest standard errors.

Table 1. Results milk supply response regressions by region, 1946-1970.

Region	Constant term	Q_{t-1}	P_{t-1}	PG_t	PB_t	T	W_t	Standard error of estimate[1]	$\bar{R}^2$
1	3492.5	.8299***	623.1**	8.4	-102.8**	--	--	.6271	.9007[2]
2	10146.6	.8998***	517.3	-41.6	- 89.8**	-50.8	--	.5751	.9403
3	6256.3	.9322***	438.4*	-83.5***	- 35.3	--	--	.5872	.9477
4	244.5	.7118***	112.2***	- 3.0	- 7.2*	7.2	--	.0734	.9510
5	10148.2	.6042***	351.2***	-15.6*	- 25.3**	-60.3***	--	.2015	.9622
6	3714.9	.6737***	151.2*	- 2.9	- 19.7**	--	1252.1***	.1506	.9639
7	1020.9	.7512***	59.8***	- 3.1**	- 3.6**	- 3.4**	--	.0336	.8932
8	9.7	.9778***	255.4**	- 6.2	- 15.9*	--	--	.1775	.9821
9	88.3	.8889***	163.91***	- 1.3	- 10.5**			.0888	.7462

Legend:

Form of equations:

$$Q_t = b_0 + b_1 Q_{t-1} + b_2 P_{t-1} + b_3 PG_t + b_4 PB_t + b_5 T + b_6 W_t + e$$

Where:

Q_t = Regional Milk Production, 1946-1970, Million lbs.

Q_{t-1} = Regional Milk Production, lagged 1 year, Million lbs.

P_{t-1} = Regional average wholesale price of milk, 1946-1970, $ per cwt.

PG_t = Regional price of 16% dairy ration, 1946-1970, $ per ton.

PB_t = Regional price of beef, 1946-1970, $ per cwt.

W_t = Regional farm wage without board and room, 1946-1970, $ per hour.

T = Date code, 1958 = 100, annual increment = 1.00

* = Significantly different from 0 at α = .10, 1 tailed t test.

** = Significantly different from 0 at α = .05, 1 tailed t test.

*** = Significantly different from 0 at α = .01, 1 tailed t test.

1/ In billion pounds

2/ Corrected for degrees of freedom.

Phase II - The Quadratic Allocation Model

Figure 1 shows a simplified flow chart of the milk allocation model. The predetermined regional supplies are allocated between regional fluid demands, national demands for manufactured products, and government support purchases to attain a simultaneous equilibrium of prices and quantities in each use. Fluid price differentials, processing costs, and processing transformations are explicitly entered in the flow network. Linear sloping demand functions are used for fluid milk (in each region), and for each of the 5 manufactured products, at the national level. The demand functions utilize exogenous shifters representing population change and secular trend of per capita consumption. The five major decision variables in the model (shown as circles in the flow chart), are the two support prices (butter and NFDM), and two quantities (cheese and evaporated milk) purchased by government and the ratio in which butter and nonfat dry milk (NFDM) are produced by the industry. Three internal consistency checks (shown by diamond shapes) are the price checks on cheese, evaporated milk, and other products. For each, the price computed in the model should equal the assumed government support price (i.e., for cheese, evaporated, or whole milk).

The allocation model is solved as a perfectly competitive quadratic programming model.1/, 2/ The institutional departures from perfect competition (e.g., classified pricing and government support purchases) were included as price or cost constraints upon the model.

Estimation of Demand Functions. Demand functions for each region and product were constructed by establishing linear demand functions through estimated price-quantity points. First approximations to per capita fluid consumption and price elasticities for fluid milk were taken from a study by Bullion [1]. Elasticities and per capita consumptions for manufactured products were obtained from MED-ERS-USDA. Prices were adjusted to the farm level for fluid milk and the processor level for manufactured products. Final demand aggregate functions were adjusted to balance with reported aggregate utilizations and prices in the base years (1969-70 and 1970-71). Each demand function incorporated two

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1/ The problem meets the conditions of linearity of demand and/or supply functions implicit in the quadratic formulation. Similarly with downward sloping demand functions it meets the Kuhn-Tucker conditions.

2/ For a description of the algorithm see [3]: Program HIPHI: Optimizing a Quadratic Function Subject to Linear Inequality Constraints, by David H. Harrington and Steven S. Hoffman, Dept. of Agricultural Economics, Purdue University.

additive shifters--one for population change, and one for secular trend of per capita consumption.[1/]

Product Transformations. Ranges of possible "recipes" for transformation of milk to manufactured products were taken from Agricultural Economic Report No. 165 (ASCS-USDA) [7], and Handbook of Milk Marketing Statistics [2]. Implicit transformations calculated from reported utilization and production figures for 1969 fell within the ranges specified by the theoretical physical transformation formulae. This served to verify the reasonableness of the coefficients.

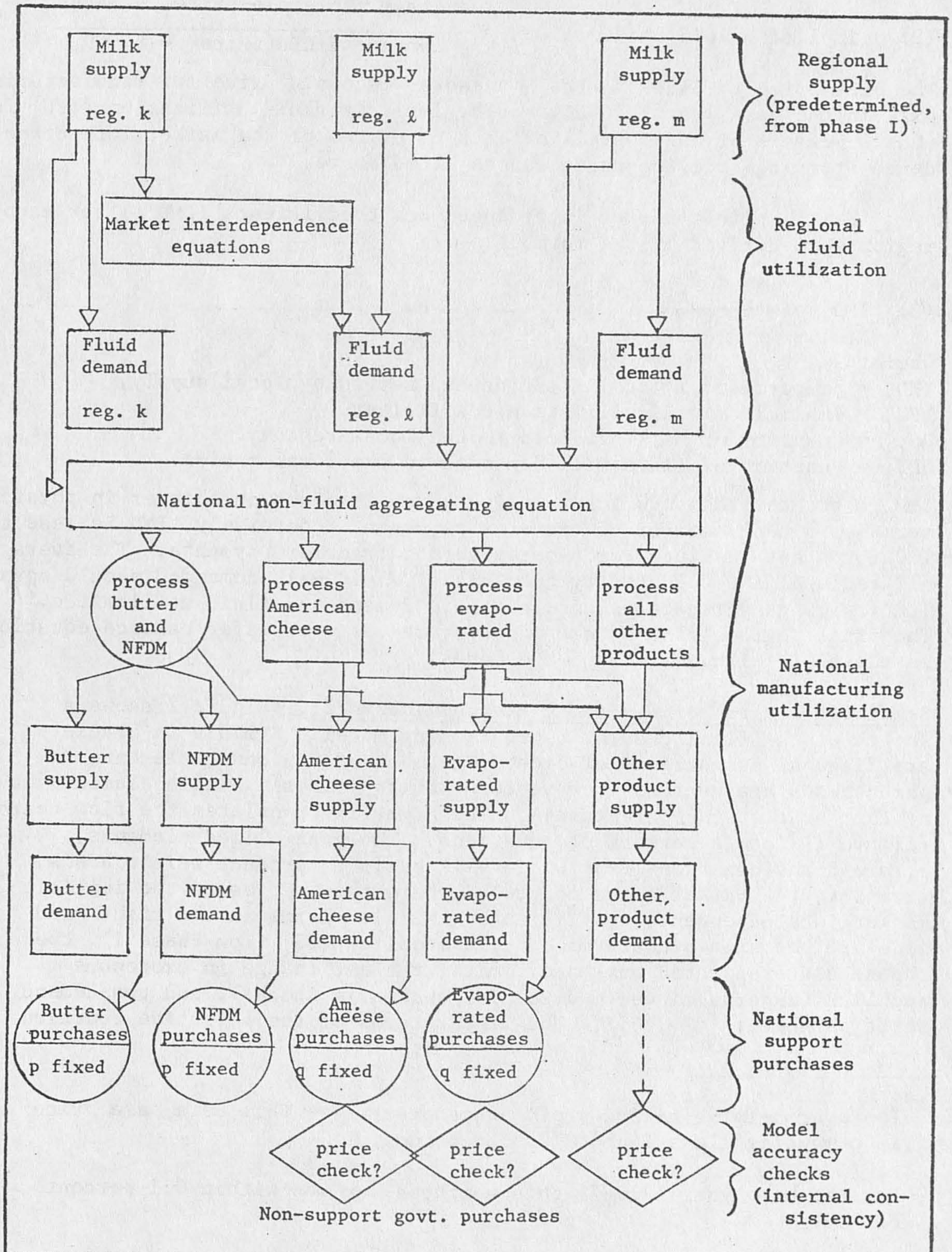
One major industry decision variable in the model is the ratio of production of NFDM to butter. Physical transformations give no guide here--they only set a maximum ratio of approximately 2 to 1. The actual ratio depends on the separate utilizations of the fat and SNF components of milk. This ratio has maintained reasonable stability with a slight trend toward narrowing in recent years. Modification of the price ratio of butter and NFDM can also be expected to influence this ratio--higher prices for NFDM will widen the ratio. Accordingly, estimates were modified from utilization projections made by ESAD-ERS-USDA to account for both the trend and alteration of this ratio caused by changing price ratios.

Institutional Constraints on Pricing. Three institutional departures from perfect competition included in the model are: classified pricing, government price supports, and fluid market interdependence between regions. Classified pricing is incorporated in the model by setting the regional fluid-manufacturing price differentials as a "cost" of supplying fluid milk. In this respect they are treated the same as the processing cost differentials employed for manufactured products. Government support activities are included for butter and NFDM, for which unlimited quantities may be purchased at the support price level. Other government purchases, American cheese and evaporated milk are not primarily price support activities hence the quantities to be purchased are fixed in the model. The fixing of these quantities allows an internal consistency check inasmuch as consumer prices should equal government purchase prices for these products.

Market Interdependence. The interdependence of fluid markets arises from several sources. Chief among these are: (1) the physical flows of milk between regions, (2) the payment flows between regions (e.g., super-pools, standby pools, etc.), and (3) the necessity for local processing of some manufactured products. The latter prevents deficit supply areas from selling all their production at the fluid price. As an estimate of market interdependence the implicit coefficient of fluid utilization for each regional milk supply was calculated.

[1/] Shifters estimated by ESAD-ERS-USDA.

Figure 1. Flow Chart of Milk Allocation Model



The following formula was tested and used.

$$(2)\quad \text{implicit fluid utilization}_i = \frac{\text{average wholesale price}_i - \$4.69}{\text{average fluid price}_i - \$4.69}$$

The base price of \$4.69 is the Minnesota-Wisconsin price for manufacturing milk in the base year 1970-71.[1/] The implicit fluid utilization (IFU) is thus a measure of the overall effects on prices of the market interdependence stemming from the three causes cited above.

Next implicit market interdependence coefficients (IMD's) for each region were derived by the formula:

$$(3)\quad IMD_i = \frac{IFU_i(QS_i)}{QDF_i}$$

where:

IMD_i = proportion of local fluid demand served by local supply,
IFU_i = implicit fluid utilization coefficient,
QS_i = quantity of milk produced in the local region,
QDF_i = quantity of fluid milk demanded in the local region.

IMD is greater than 1.0 for a region which is a net outshipper in physical terms or a net receiver of transfer payments. Conversely, IMD is less than 1.0 for a net inshipper or a net giver of transfer payments. The average of all regional IMD weighted by quantity of fluid milk demanded should equal 1.0 implying aggregate balance of fluid demand and fluid utilization.[2/] The final regional IMD's were incorporated as a transfer balance equation in the fluid milk portion of the model.

Schematic Diagram of Quadratic Programming Model. Figure 2 shows a sketch of the final quadratic programming model. Symbols in brackets are diagonal submatrices of order 9 (number of regions), elements in parentheses are vectors (row vectors having primes), single elements have neither brackets nor parentheses. The A matrix translates the flow chart (figure 1) into a network of equations. The generalized elements, a, are transformations from whole milk basis to product weight basis, except in the market share controls--where they represent the implicit market interdependence coefficients (IMD's). Terms in the right hand side are the nine predetermined milk supplies (QS_i from Phase I), the cheese and evaporated purchase limits, the net change in exogenous supplies (exports minus imports plus change in stocks), and two demand shift parameters (Θ_1, Θ_2). The linear terms of the objective function

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1/ The overwhelming majority of market orders use this as a base price in computing Class I and Class II prices.

2/ For the base year 1970-71 this weighted sum was within 0.1 percent of 1.0.

Figure 2. Schematic quadratic allocation model.

	9 local fluid utilizations	9 market interdependence	9 local fluid demands	9 local manufacturing utilizations	PROCESS				MFG. PROD. DEMAND					Ex.-Im. + Δ Stocks				SUPPORT PURCHASES				Pop. Shifter	Per Capita Cons. Shifter		RHS
					Butter & NFDM	Cheese	Evap.	"Other"	Butter	NFDM	Cheese	Evap.	"Other"	Butter	NFDM	Cheese	Evap.	Butter	NFDM	Cheese	Evap.				
9 local production transfers	[1]	$\begin{bmatrix}0\\1\end{bmatrix}$		[1]																					(QS)
Intermarket balance		(1,-1)																							0
9 local fluid transfers	[-1]	$\begin{bmatrix}-1\\0\end{bmatrix}$	[1]																						(0)
9 market share controls		$[-a_1]$	[1]																						(0)
Mfg. aggregating eq.				(-1)'	1	1	1	1																	0
Butter transfer					-a				1					±1				1							0
NFDM transfer					$(-\alpha_1)$					1					±1				1						0
Cheese transfer						-a					1					±1				1				≤	0
Evap. transfer							-a					1					±1				1				0
"Other" transfer					$(-\alpha_2)$	-a		-1					1												0
Cheese purch. cont.																				1					$\bar{G}_c$
Evap. purch. cont.																					1				$\bar{G}_e$
Butter — Ex-Im + Δ stocks														1											Δ_b
NFDM — Ex-Im + Δ stocks															1										Δ_n
Cheese — Ex-Im + Δ stocks																1									Δ_c
Evap. — Ex-Im + Δ stocks																	1								Δ_e
Pop. Shifter																						1			θ_1
Per Capita Cons. Shifter																							1		θ_2
Linear terms of objective function	(d)'	(d)'	$(i_f)'$	(0)'	(ρ_b)	ρ_c	ρ_e	ρ_r	i_b	i_n	i_c	i_e	i_r	0	0	0	0	(G_b)	(G_n)	(G_c)	(G_e)	0	0		
Condensed quadratic matrix of objective function	-----	-----	$[q_f]$	-----	-----	-----	-----	-----	$[q_b$	q_n	q_c	q_e	$q_r]$	-----	-----	-----	-----	-----	-----	-----	-----	(ϕ_p)	(ϕ_c)		

include regional fluid differentials (d), demand intercepts (i_f, i_b, i_n, i_c, i_e, i_r), processing costs (ρ_b, ρ_c, ρ_e, ρ_r), and government support prices (G_b^r, G_n, G_c, G_e). The quadratic matrix is presented in condensed form. In mathematical form it is a diagonal matrix containing the slope coefficients[1/] of the demand functions (qf_i, q_b, q_n, q_c, q_e, q_r) and the demand shift coefficients (ϕ_ρ, ϕ_c) as bordering vectors. The circled elements ($-\alpha_1$, $-\alpha_2$, ρ_b, and G_b through G_e) are major control variables. The first three control the ratio of production of NFDM to butter and are controlled by industry reaction to the government support prices (G_b through G_e).

Calendar Years vs. Marketing Years. One last consideration in applying the model is that farm production statistics are on a calendar year basis and demand statistics are on a marketing year basis. To attain comparability with published series it is necessary to "blend" the results of two model (calendar) years to obtain all results on a marketing year basis.

Phase III - Blend Pricing

The final phase of the model is to link the utilizations and prices derived in Phase III with the production of Phase I for the subsequent year. This is the simplest step; it involves calculating regional blend prices based on the fluid and manufacturing prices and utilizations derived in Phase II. The standard blend pricing formula used was:

$$(4) \quad PB_i = \frac{P_{fi}Q_{fi} + P_{mi}Q_{mi}}{QS_i}$$

where Q_{fi} includes both local fluid utilization and net inflow of transfer payments (from the IMD's) and the sum of Q_{fi} plus Q_{mi} equals QS_i.

RESULTS

Validation Against ESAD Model

The method was validated on an independent set of projections made by traditional outlook methods for a report to ASCS-USDA by ESAD-ERS-USDA.[2/] When identical assumptions and relationships were used in the

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1/ For the perfectly competitive solution mode the true slope coefficients are divided by two to obtain the q coefficients.

2/ See reference [8]. Also published in The Congressional Record, March 9, 1972, pp. E2327-E2333.

model, the results of both the method employed by the ERS report[1/] and the 3-phase model give results which are identical to 1/4 of a percent, even for the most sensitive variables. This is to be expected because the mathematical structure underlying both methods is the same.

Other Uses of Model

As a further test of the model a second series of solutions were run for the same 5 support price situations used in the ESAD model. These solutions project the aggregate and regional effects on production, consumption, government purchases, acquisition costs, and product prices for 3 price support levels and 2 butter-NFDM price ratios over the next 5 years. These solutions differ from the validations runs in that milk production was projected from the Phase I regressions (rather than by expert consensus), and an implicit inconsistency in the ratios of production of NFDM to butter was respecified to remove the inconsistency. Table 2 shows some aggregate results obtained from this series of solutions.

CONCLUSIONS

The quadratic programming technique has (again) been shown to be applicable to complex problems involving linear supplies, demands, and transformations. The "old saw" that the results are destined to be inaccurate has been laid to rest. The model itself provides a useful adjunct to the traditional methods employed in the ERS-USDA study in that it expands the amount of information that can be easily obtained. It gives regional production and utilization information as well as regional prices, all of which are very difficult to obtain with the budgeting method. Finally, the whole model may be fully automated, thus giving the policymakers access to more information at less cost. Automation can free researcher time from routine calculation to concentrate on analysis of more alternatives, to build more realism into the models, or to develop more refined estimates of the technical relationships in the models.

[1/] Budgeting of changes to key aggregate variables and deriving other implicit changes to be consistent with them.

Table 2. Milk production, sales of dairy products, USDA purchases and acquisition costs, and gross farm income, 1971-72 to 1976-77.

Item and alternative	Support price	Butter price	71-72	72-73	73-74	74-75	75-76	76-77
Milk Production	($)		(Billion pounds)					
I	4.93	--	118.90	120.65	119.98	119.34	118.76	118.19
II	5.20	--	--	120.93	120.60	120.03	119.47	118.90
III	5.50	--	--	121.22	121.21	120.68	120.12	119.55
Fluid Sales			(Billion pounds)					
I	4.93	--	51.34	50.97	50.59	50.21	49.83	49.45
II	5.20	--	--	50.68	50.30	49.92	49.54	49.17
III	5.50	--	--	50.36	49.98	49.60	49.22	48.65
Manufactured Product Sales			(Billion pounds, milk equivalent)					
I	4.93	--	55.63	56.14	56.79	57.50	58.16	58.77
II(a)	5.20	ratio 1/	--	54.72	55.37	55.95	56.63	57.30
II(b)	5.20	constant 2/	--	55.11	55.60	56.27	57.12	57.61
III(a)	5.50	ratio	--	53.45	54.09	54.74	55.39	56.37
III(b)	5.50	constant	--	54.29	54.95	55.64	56.33	57.02
USDA Purchases			(Billion pounds, milk equivalent)					
I	4.93	--	8.13	9.90	9.06	8.19	7.43	6.72
II(a)	5.20	ratio	--	11.89	11.38	10.70	9.95	9.19
II(b)	5.20	constant	--	11.50	11.16	10.40	9.46	8.68
III(a)	5.50	ratio	--	13.76	13.60	12.89	12.16	11.04
III(b)	5.50	constant	--	12.93	12.74	11.99	11.22	10.44
USDA Acquisition Costs			(Million dollars)					
I	4.93	--	401	488	447	404	366	331
II(a)	5.20	ratio	--	618	592	556	517	478
II(b)	5.20	constant	--	598	581	541	492	451
III(a)	5.50	ratio	--	757	748	709	669	628
III(b)	5.50	constant	--	711	701	660	617	574
Gross Farm Income from Dairy			(Million dollars)					
I	4.93	--	6848	6899	6898	6862	6830	6812
II	5.20	--	--	7221	7237	7204	7172	7139
III	5.50	--	--	7580	7616	7583	7550	7516

1/ Butter and NFDM prices at current ratio.

2/ Butter price constant at 71-72 price.

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[8] U.S. Dept. of Agriculture, ERS, Impacts of Alternative Dairy Price Support Levels, Report to ASCS-USDA, February 1972.

Appendix Table 1. Short-run and long-run elasticities of milk supply response regressions by region, 1946-1970.

Region	Q_{t-1} 1/	P_{t-1} 2/		PG_t 3/		PB_t 4/		T 5/	W_t 6/	
		SR	LR	SR	LR	SR	LR		SR	LR
1	.172	.096	.56	.000		-.057	-0.33	--	--	--
2	.089	.083	.93	-.122	-1.37	-.076	-0.85	-.200	--	--
3	.076	.059	.78	-.196	-2.58	-.024	-0.31	--	--	--
4	.296	.158	.53	-.056	-0.19	-.030	-0.10	.165	--	--
5	.388	.158	.41	-.103	-0.27	-.040	-0.10	-.533	--	--
6	.314	.082	.26	-.029	-0.09	-.057	-0.18	--	-.191	-.608
7	.244	.102	.42	-.091	-0.37	-.028	-0.11	-.127	--	--
8	.042	.146	3.47	-.061	-1.45	-.044	-1.05	--	--	--
9	.110	.159	1.45	-.022	-0.20	-.047	-0.42	--	--	--

$$\text{Elasticity}_j = \frac{\text{percent change in quantity of milk produced}}{\text{percent change in independent variable}_j}$$, all other variables held constant at their means.

1/ Elasticity of adjustment: proportion of adjustment to a once-and-for-all change completed in one year.

2/ Own price elasticity of supply.

3/ Cross-elasticity of supply with respect to grain prices.

4/ Cross-elasticity of supply with respect to beef prices.

5/ Annual percentage change in milk supply (trend).

6/ Cross-elasticity of supply with respect to wage rates.