START
The Structure of Agricultural Investment: Comparing a Flexible Accelerator with Stochastic Coefficients

Roger Conway
James Hrubovcak
Michael LeBlanc
ABSTRACT

Two approaches, a flexible accelerator model and a stochastic coefficients alternative, are used to estimate the structure of aggregate agricultural investment. Structural estimates of the adjustment rates for each model are similar. The stochastic coefficients model, however, performs better in an out-of-sample forecast.

Keywords: Investment, flexible accelerator, stochastic coefficients, adjustment coefficient

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SUMMARY

This analysis estimates two alternative models of agricultural investment. The flexible accelerator approach is selected because of its comparatively well-developed theoretical foundation. Its power lies in the flexibility of the adjustment coefficient where, unlike most other partial adjustment models, the speed of adjustment depends on economic phenomena and therefore varies through time.

In addition to the flexible accelerator, the authors propose a more general alternative which allows economic phenomena to induce variability for all the parameters of the model rather than restricting it to the adjustment coefficient. Its application in this analysis is inspired, in part, by Lucas's argument that optimal decision rules vary with changes in policies.

Because there is no statistical procedure to identify the "true" model, one cannot test whether the flexible accelerator or the stochastic coefficients model is the correct model. Instead, the authors adopt an instrumentalist approach and compare a 5-year, out-of-sample forecast for each model. The results indicate that for nearly any sensible criterion the stochastic coefficients model outperforms the accelerator model.
The Structure of Agricultural Investment: Comparing a Flexible Accelerator with Stochastic Coefficients

Roger Conway
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INTRODUCTION

Capital investment is one way society exchanges the present for the future. Increases in net investment typically expand society's productive capacity and act as a medium for technological change. It is not, therefore, surprising that the determinants of aggregate investment have received considerable empirical attention. A/ Agriculture is one of the least studied sectors of the economy, despite being the research focus of a large number of economists. The empirical neglect of aggregate investment by agricultural economists may be a manifestation of their interest in farm policies which are generally output-, rather than input-, oriented. During the past two decades, only two published studies have examined aggregate agricultural investment (12, 19). B/ The Penson, Romain, and Hughes study is well conceived but examines only tractor investment. The Lamm study is more comprehensive but too general.

The objective of this analysis is to estimate a logically consistent model which provides insight into the structure of agricultural investment decisions. Although many approaches are possible (standard neoclassical, cashflow, securities value), the general framework used for this analysis is based on a Lucas-type accelerator (15, 29). We selected this approach because of its comparatively well-developed theoretical foundation. The power of the Lucas accelerator lies in the flexibility of the adjustment coefficient where, unlike most other partial adjustment models, the speed of adjustment depends on economic phenomena and therefore varies through time.

In addition to the flexible accelerator we propose a more general alternative which allows economic phenomena to induce variability for all the parameters of the model rather than restricting it to the adjustment coefficient. The stochastic coefficients model estimated in this analysis was first developed by Swamy and Tinsley (23). Its application here is inspired, in part, by Lucas's (14) argument that optimal decision rules vary systematically with changes in the

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A/ For a comprehensive summary of the investment literature through 1971, see (6) and (11).
B/ Underlined numbers in parentheses cite sources listed in the References section.
structure of the data series relevant to the decisionmaker. Therefore, any change in policy will alter the structure of these decision rules. A decrease in the rental rate resulting from an increase in the investment tax credit, for example, might change the structure of the investment relationship. Indeed, Lucas (14, p. 24) argues that "the standard, stable parameter view of econometric theory and quantitative policy evaluation appears not to match several important characteristics of econometric practice, while an alternative general structure, embodying stochastic parameter drift, matches these characteristics very closely." A stochastic coefficients model is an alternative empirical approach that permits one to capture any instabilities in economic relationships without excessive prior informational requirements.

**FLEXIBLE ACCELERATOR**

Economists have sought a theoretical framework for the partial adjustment or accelerator model since Nerlove's early applied work (17, 18). Many economists recognized the gap in econometric theory where an elaborate theoretical structure, determining the level of an input, was combined with an ad hoc theory of adjustment. Eisner and Strotz developed a more rigorous theory of adjustment by casting the farm business, or firm, in a dynamic optimization framework (6). The present value or net worth maximized by the firm depends on the optimal level of inputs selected by the firm and on the path of the current capital stock to the optimal level.

Lucas, Gould, and Treadway extended the work of Eisner and Strotz (9, 15, and 29). Although the models of Lucas, Gould, and Treadway differed in their complexity, they had the same underlying structure postulated by Eisner and Strotz. Each specified an objective function incorporating factor adjustment costs and a production function. They assumed the firm maximizes net worth over a given time period, and interpreted adjustment costs as either foregone profits because of shortrun rising prices in the capital supplying industry or as increasing costs associated with integrating new equipment into production (reorganizing production and training workers). These costs varied with the speed of capital adjustments. It is also assumed that the values of the expected input and output prices did not change. This "myopic" static or stationary-expectations assumption is required to define the dynamic maximization problem (16). Because expectations were static, the firm adjusted to a fixed target considered to be the longrun equilibrium of neoclassical theory. Given these assumptions, a firm that maximizes its present value changes capital stock in a manner similar to that suggested by the accelerator model.

The optimal adjustment paths for the quasi-fixed inputs are derived by incorporating a shortrun restricted profit function into a longrun dynamic optimization framework (1, 2). The assumptions of competitive input and output markets are maintained, as we assumed that these competitive real prices are known with certainty and remain stationary over time.

In the usual Marshallian framework, the relative fixity of inputs usually causes adjustments to a new equilibrium position to occur slowly. Immediate adjustment is prevented because certain inputs cannot be changed until a period of time.

---

3/ Nerlove documents the partial adjustment model's popularity in (16).
4/ This assumption probably could be relaxed if a more general approach to the formation of expectations were allowed. For a description of a coherent subjective Bayesian conceptualization of rational expectations, see (22).
has elapsed after the original decision to alter the inputs. Excluding uncertainty, increased costs for the firm for adjusting production lead to slower rather than faster adjustment. Using Eisner's and Strotz's model, production factors become more or less fixed as a function of the cost of varying the input sooner rather than later.

It is assumed that a quasi-fixed input can be varied at a cost \( C(K) \), where \( \dot{X} \) equals \( dK/dt \), and:

\[
\dot{K} = I - \delta K,
\]

where \( I \) is the gross addition to capital stock and \( \delta \) is the rate of exponential depreciation. Also, the cost of adjustment is defined as:

\[
C(K) = qI + qD(K),
\]

where \( q \) is the purchase price of the quasi-fixed asset; \( D(K) \) is a twice differential function; and \( D'(K) > 0 \). Adjustment costs at the initial time \( t=0 \) are:

\[
C(0) = q\delta K.
\]

This formulation assures constant marginal costs of replacement with increasing marginal costs of net change. Costs are expressed in units of the asset price of the quasi-fixed factor.

Net receipts can, therefore, be written as:

\[
R(t) = PG(W,K) - C(K),
\]

where \( G(W,K) \) is the unit-output-price (UOP) restricted profit function, \( P \) is the unit price of output, \( K \) is a quasi-fixed capital input, \( W \) is a vector of normalized (output price) input prices.\(^5\)

If the firm requires a rate of return, \( r \), a weighted average of the rate of return to equity and the cost of external financing, then the present value of net receipts at time \( t=0 \) is:

\[
V(0) = e^{-rt} \int_0^\infty R(t)dt.
\]

The firm's longrun dynamic problem stems from choosing time paths for variable inputs, \( X(t) \), and the quasi-fixed input, \( K(t) \), to maximize \( V(0) \) given \( K(0) \) and \( X(t), K(t) > 0 \). That is, because \( G \) assumes shortrun optimizing behavior conditional on \( P, W, \) and \( K \), the optimization problem facing the firm is finding among all the possible \( G(W,P) \) combinations, the time paths of \( X(t) \) and \( K(t) \), maximizing the present value of net receipts.

A solution to (5) can be obtained by using either the Euler equation or Pontryagin's maximum principle. If we assume price expectations and normalized profits and adjustment costs on output price, then the Hamiltonian necessary for applying the maximum principle is:

\(^5\) The restricted profit function is the locus of shortrun maximized profit of a firm as a function of output price, input prices, and quantities of fixed factors (13). The profit function is nonincreasing and convex in \( W \) (normalized input prices) and nondecreasing in \( P \) and \( K \).
\[ H(X,K,K,y,t) = e^{-rt}(G(W,K(t)) - C(K)) + yK(t), \]  

where \( y \) is a costate variable, the dynamic equivalent of a Lagrangeian multiplier of static optimization problems and \( C \) is the normalized adjustment cost. Costate variables generally vary through time and are assumed to be nonzero continuous functions of time. Necessary conditions for the maximization of \( H \) require:

\[ G'(W,K) - rC'(K) + C''(K)K = 0, \]  

where a prime denotes the first derivative taken with respect to \( K \). We assume these necessary conditions are sufficient to obtain a maximum. That is, the marginal profit associated with the quasi-fixed input equals its marginal cost of adjustment. Equation (7) has a stationary solution \( K^*(P,W,r) \), which is obtained by setting \( \dot{K} = \ddot{K} = 0 \):

\[ C'(X^*(K^*), K^*) - rC'(0) = 0 \]  

The variable \( K^* \) is the steady-state or long-run profit-maximizing demand for the quasi-fixed input obtained by solving equation (8).

The results are linked to the partial adjustment or flexible accelerator literature because the short-run demand for the quasi-fixed factor can be generated from equations (7) and (8) as an approximate solution in the neighborhood of \( K^*(t) \) (12). The approximate solution is the linear differential equation:

\[ K = B(K^*(t) - K(t)), \]  

where:

\[ B = -0.5(r - [r^2 - 4H''(K^*)/C'(0) + 0.5]). \]  

This derivation allows the adjustment coefficient, \( B \), to depend on economic forces: the discount rate, the cost of adjustment, the production relationship embodied in the profit function, and the profit-maximizing behavior of the firm. If, however, the discount rate is constant and the adjustment cost function \( C(K) \) is linear, then the adjustment coefficient is a constant, and equation (9) reduces to the classical fixed accelerator model.

Before estimating the theoretical framework, the adjustment equation must be expressed as a difference equation, and functional forms for the profit and cost of adjustment functions must be selected. The accelerator equation is respecified in a discrete form by first assuming that short-run production is conditional on capital stocks at the beginning of the period. Therefore, capital stock adjustments during the period do not affect production until the following period. Second, the adjustment relationship specified in equation (9) is replaced by:

\[ K(t) - K(t-1) = B(K^*(t) - K(t-1)). \]  

A quadratic approximation is used for the profit function because it facilitates estimating the model without placing a priori restrictions on the elasticities of substitution (8). The quadratic structure generates linear input demand functions and simple expressions for demand and substitution elasticities. In addition, the optimal path for capital is globally, rather than locally, valid because the underlying differential equation is linear (28).
The UOP profit function is specified as a quadratic function of normalized prices and the level of capital available at the beginning of the current period:

\[ p = a + b_w w + b_k K + 0.5(b_{ww} w^2 + b_{kk} K^2) + b_{WK}WK, \]  

(12)

where a's and b's are parameters.

Although no reason exists to expect that a quadratic adjustment cost function is correct in all circumstances, Could found it to be a good approximation (9). A quadratic approximation to the cost of adjustment is:

\[ C(K) = qI + q(0.5dK^2), \]

(13)

where D(0) = 0.

To complete the empirical model, one must derive the optimal level of capital stock and describe the adjustment process where the current level of capital moves toward the optimal level. Adjustment costs, hypothetically, are external to the shortrun maximization decision. The necessary conditions for optimal capital adjustment are derived by applying equation (7). The resulting equation,

\[ b_k + b_{kk}K + b_{WK}W - u - rqdK + qdK = 0, \]

(14)

is a second order differential equation where \( u=q(r + \delta) \) is the normalized rental rate associated with the quasi-fixed factor. The steady-state solution comes from setting \( K = K = 0 \), producing:

\[ K^* = -(b_k + b_{WK}W - u)/b_{kk}, \]

(15)

where \( K^* \) is the optimal level of capital stock.

Therefore, the adjustment equation is

\[ B = -0.5(r - [r^2 - 4b_{kk}/qd]^{0.5}). \]

(16)

To obtain the estimated form for the flexible accelerator, substitute the steady-state solution for capital and the rental rate of capital for the difference equation (11), appending a stochastic error possessing classical properties:

\[ K(t) - K(t-1) = -0.5(r - [r^2 - 4b_{kk}/qd]^{0.5})(-b_k + b_{WK}w - u^*/b_{kk} - K(t-1)) + e_t, \]

(17)

where \( u^* \) is the rental rate of capital and \( e_t \) is the classical stochastic error term.6/  

STOCHASTIC COEFFICIENTS

A first-order variant of the generalized autoregressive integrated moving average (ARIMA) stochastic coefficients process model developed by Swamy and Tinsley (23) is also used to estimate the flexible accelerator investment model. This model is a generalization of other stochastic coefficients models, such as

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6/ A detailed description of the rental rate cost of capital is in the appendix.
the Kalman filter and Cooley-Prescott procedure. In vector notation, the time-
varying model is written as:

\[ y_t = X'_t \beta_t. \]  

(18)

To implement empirically this model, some structure must be imposed on \( \beta_t \) because there are only \( T \) observations. In this report, the coefficients in (18) are driven about a fixed vector of mean values, \( \bar{\beta}_t \), by a stationary stochastic vector, \( \epsilon_t \). Thus,

\[ \beta_t = \bar{\beta}_t + \epsilon_t \]
\[ \epsilon_t = \phi \epsilon_{t-1} + u_t, \]  

(19)

where \( u_t \) is a vector of white noise innovations,

\[ u_t \sim \mathrm{ws}(0, \Delta_u). \]

The variance-covariance matrix \( \tilde{\beta}_t \) is

\[ E(\beta_t - \bar{\beta})(\beta_t - \bar{\beta})' = \Gamma, \]  

(20)

and the unconditional variance of the dependent variable,

\[ \var(Y_t) = \sum_{i,j=1}^{k} \Gamma_{ij}, \]

where

\[ \vec(\Gamma) = [I - \Phi \Phi]^{-1} \vec(\Delta_u), \]

and \( \vec(\Gamma) \) is the column stack of the matrix \( \Gamma \).

Both the conditional expected value and variance of the dependent variable vary with observations on the conditioning variables. One may decompose the variance in the dependent variable among its contributing factors. Permitting the independent variable to influence the variance of the dependent variable is important because an independent variable may possibly have a relatively large impact on the variance of the dependent variable even though it has a relatively minor impact on the mean of the dependent variable. This decomposition is analogous to allocation of the multiple R2 among the explanatory variables in a conventional regression equation, as shown in Theil (25).

One may average over the sample period to make \( \var(y_t) \) unit free:

\[ 1 = 1/T \sum_{t=1}^{T} \sum_{i=1}^{k} \sum_{j=1}^{k} x_{it}x_{jt} \Gamma_{ij}/\sum_{i=1}^{k} x_{it}' \Gamma x_{it}, \quad i,j = 1, \ldots, k. \]  

(21)

When the coefficient process is stationary, both \( \Delta_u \) and \( \phi \) will collapse to scalar characteristics of the intercept coefficients. One may obtain t-tests of the individual components by using an asymptotic approximation of the covariance matrix of the estimated column stack, \( \vec(\Delta_u) \), to test the significance of the uncertainty allocations to slope coefficients.

Generally, the first regressor, \( X_{1t} \), is a unit vector intercept with a stochastic component of its coefficient which serves as the analogue of the additive disturbance familiar to fixed coefficient specifications. The stochastic coefficients model will have a total residual, \( u_t \), that is a weighted sum of the stochastic
elements of the coefficients of the intercept regressor and the time-varying regressors, where $u_t = X_t e_t$.

The residual, $u_t$, does not necessarily increase when using a stochastic coefficients estimation approach. Should ordinary least squares be a consistent estimator of the means of the coefficient vector, $\bar{b}$, then estimates of $u_t$ (where $t=1, \ldots, T$) from the two estimators will converge as the sample size increases (27).

A frequent problem when estimating regression models with time series data is the existence of serial correlation and/or heteroskedasticity. Researchers employing a fixed coefficients model need to check for these problems and correct for them. However, to test for these problems is not simple, and these tests may produce incorrect or conflicting results. A virtue of stochastic coefficients estimation is that it solves these difficulties by explicitly permitting vector serial correlation and heteroskedasticity to exist and then correcting for them. This may be seen in equation (19), noting that a two-variable representation may be written as follows:

$$y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + (u_{0t} + u_1 X_{1t} + u_2 X_{2t}).$$

(22)

Therefore, equations (19) and (22) allow for both the existence of vector serial correlation and heteroskedasticity (because $X_t$ varies from period to period). A fixed coefficient model assumes a priori that the $\phi$'s other than $\phi_{00}$ are zero and that $u_{1t}$ and $u_{2t}$ are equal to zero with probability 1. In most cases, these restrictive assumptions are unwarranted.

One can obtain an estimable functional form for the stochastic coefficients model alternative to the flexible accelerator model by simplifying the nonlinear adjustment relationship of equation (16). If $C(K)$ is linear and the discount rate is constant, then equation (17) is rewritten:

$$K(t) - K(t-1) = B\left(-\left(b_k^* + b_{wk}^* - u^* / b_{kk}^*\right) / b_{kk}^* - K(t-1)\right),$$

(23)

or more simply,

$$K(t) - K(t-1) = b_k^* + b_{wk}^* W + h_u^* u + B K(t-1),$$

(24)

where $b_k^* = -B b_k / b_{kk}$, $b_{wk}^* = -B b_{wk} / b_{kk}$, and $h_u^* = -B / b_{kk}$.

An iterative procedure produces estimates reported here. Because we use arbitrary values of the unknown parameters $\Delta_u$ and $\phi$ as the starting values in the initial iteration, the limiting distribution of the estimates obtained after one iteration will depend on these arbitrary starting values. This is not true of the estimates obtained after two iterations because these estimates are efficient and consistent (24).

The empirical model based on equation (24) is as follows:

$$K_t - K_{t-1} = b_{kt} + b_{wkt} W_t + b_{ut} u_t + B_t K_{t-1},$$

(25)

where the coefficients of this linear regression is driven about a fixed vector of mean values, $\bar{b}$, by a stationary stochastic vector, $e_t$. For example,

$$b_{wkt} = \bar{b}_{wk} + e_t,$$

(26)

where $e_t = \phi e_{t-1} + u_t$. 

7
The analysis uses aggregate time series data for 1923 through 1983. Changes in the stock of farm machinery are explained by the ratio of prices paid for farm inputs to prices received for farm outputs, the implicit rental rate of capital inputs, and the lagged capital stock.

The U.S. Department of Agriculture (USDA) supplied the ratio of prices paid to prices received. The prices paid index included allowances for interest, taxes, and wage rates in addition to production items, such as feed, seed, and fertilizer (32). The prices received index was an aggregate index of prices received for all farm products.

We estimated implicit rental rates for trucks, tractors, and long-lived farm equipment and then aggregated into a single rental rate for farm machinery. Rental rates for each of the three categories are functions of the price of assets, service lives, rates of capacity depreciation, the tax treatment of assets in each category, and the discount rate.

A single price index series for all three farm machinery categories is from the Commerce Department's Bureau of Economic Analysis (BEA) capital stock study (35). The service lives for each equipment category amounted to 85 percent of Bulletin F depreciation lives (37). The service lives for trucks, tractors, and long-lived equipment were 5, 9, and 13 years, respectively. We determined the rate of economic depreciation for each category by using the double declining balance depreciation method where the capacity of assets in the ith category in year t is represented as:

\[ n_i(t) = \left[ 1 - \frac{2}{L_i} \right]^{t-1}, \quad i = 1, 2, \ldots, m, \quad (27)\]

for \( 1 \leq t \leq L_i \), and \( a_i(t) = 0 \) for \( t > L_i \).

The tax treatment of each category, based on the allowable tax depreciation method and tax life, resulted in the greatest amount of tax saving over the service life of the asset. Before 1955, tax depreciation allowances were limited to the straight line rate, and tax lives were set equal to averages of Bulletin F lives. From 1955 to 1980, assets in each category were depreciated under the sum-of-year's-digits method. In 1962, the minimum allowable tax lives were shortened. The tax life of long-lived equipment fell from 15 to 10 years, but the tax life for trucks increased to take full advantage of the investment tax credit. In 1975, the Asset Depreciation Range (ADR) system was introduced and the allowable tax lives were again reduced. The tax lives of tractors and long-lived equipment fell from 10 to 8 years. In 1981, the Economic Recovery Tax Act (ERTA) introduced the Accelerated Cost Recovery System (ACRS). Trucks were depreciated over 3 years, while tractors and long-lived equipment were depreciated over 5 years.

We interpreted the marginal ex ante Federal income tax rates developed for this analysis as the expected tax rates an investor or firm would pay on an additional dollar of income before undertaking any new investment. These ex ante rates were estimated for sole proprietorships from 1962-79 Treasury Department data. Before the Revenue Act of 1964, the lowest marginal tax rate applied to all taxable income below $2,000. We assumed, however, that the appropriate marginal tax rate corresponded to the lowest tax bracket. Post-1979 estimates of marginal income tax rates use the actual statutory tax brackets but employ USDA data for onfarm and off-farm income to develop proxies for IRS taxable income.
We assumed that all capital purchases were completely debt financed. Nominal interest rates equaled rates charged by Federal land banks on new farm loans (30). The nominal interest rates were adjusted for the tax deductibility of interest charges and inflation for computing the real required after-tax rate of return or the real discount rate.

We developed an aggregate index of the stock of trucks, tractors, and long-lived equipment from USDA estimates of farm capital purchases (31) and converted the nominal dollar investment series into constant dollar estimates by deflating with price indices from the BEA capital stock study. The constant dollar investment series was then depreciated with the appropriate service lives to estimate a constant dollar machinery stock using the perpetual inventory method.

RESULTS AND INTERPRETATION

The estimated forms for the flexible accelerator and stochastic coefficients model appear in equations (17) and (25), respectively. A nonlinear maximum likelihood procedure employing a Davidon-Fletcher-Powell solution algorithm is used to estimate the flexible accelerator model (7). The stochastic coefficients model is estimated using a first-order variant of a generalized ARIMA stochastic coefficients process model (23). Estimated parameters and associated asymptotic statistics are shown as follows:

<table>
<thead>
<tr>
<th>Flexible accelerator</th>
<th>Value</th>
<th>Asymptotic standard error</th>
<th>Asymptotic t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_k</td>
<td>12,3107</td>
<td>58,020.8</td>
<td>2.12</td>
</tr>
<tr>
<td>b_{wk}</td>
<td>-343.014</td>
<td>20,670.4</td>
<td>-16.6</td>
</tr>
<tr>
<td>b_{kk}</td>
<td>3,926</td>
<td>2.409</td>
<td>1.65</td>
</tr>
<tr>
<td>d</td>
<td>-3,499.94</td>
<td>1,685.26</td>
<td>-2.08</td>
</tr>
</tbody>
</table>

Stochastic coefficients 1/

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Asymptotic standard error</th>
<th>Asymptotic t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_k*</td>
<td>3,645.78</td>
<td>387.48</td>
<td>9.41</td>
</tr>
<tr>
<td>b_{wk}*</td>
<td>-4,583.02</td>
<td>658.74</td>
<td>-6.96</td>
</tr>
<tr>
<td>b_u*</td>
<td>1,990.27</td>
<td>938.45</td>
<td>2.12</td>
</tr>
<tr>
<td>B</td>
<td>.0242</td>
<td>.018</td>
<td>2.06</td>
</tr>
</tbody>
</table>

Note that \( b_{k*} = -Bb_{k}/b_{kk} \); \( b_{wk*} = -Bb_{wk}/b_{kk} \); \( b_u* = -B/b_{kk} \).

1/ Mean values. Conditioned on second iteration estimates of \( \phi \) and \( A_u \).
The asymptotic t-statistics associated with both models are reasonable. Estimates of the underlying structural parameters are, however, quite different. In the flexible accelerator model, the input/output price ratio is the major determinant of optimal capital stock. The real rental rate has virtually no effect on the optimal stock. The underlying structural parameters derived from the stochastic coefficients model, however, attribute important roles to both the input/output price ratio and the real rental rate.\(^7\)

Following is the estimated coefficient of variation of coefficients for the stochastic coefficients model:

<table>
<thead>
<tr>
<th></th>
<th>Span 1923–83 (^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_k^*)</td>
<td>0.0299</td>
</tr>
</tbody>
</table>

\(^1\) The coefficient of variation of coefficients is equal to 100 times the ratio of its standard deviation to its mean.

The rental rate coefficient has the greatest variability, followed by the input/output price coefficient and lagged capital stock. The intercept has the least variability.

The decomposition of normalized variance of agricultural investment is as follows:

<table>
<thead>
<tr>
<th>Item</th>
<th>(b_k^*)</th>
<th>(b_{wk}^*)</th>
<th>(b_u^*)</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.44E–03</td>
<td>4.75E–04</td>
<td>-7.73E–05</td>
<td>5.73E–03</td>
</tr>
<tr>
<td>Input/output price</td>
<td>4.75E–04</td>
<td>2.35E–03</td>
<td>4.70E–04</td>
<td>7.02E–03</td>
</tr>
<tr>
<td>Rental rate</td>
<td>-7.75E–05</td>
<td>4.70E–04</td>
<td>1.26E–04</td>
<td>7.35E–05</td>
</tr>
<tr>
<td>Lagged capital stock</td>
<td>5.73E–03</td>
<td>7.02E–03</td>
<td>8.73E–04</td>
<td>9.67E–01</td>
</tr>
<tr>
<td>Net contribution</td>
<td>7.57E–03</td>
<td>1.03E–02</td>
<td>1.40E–03</td>
<td>9.81E–01</td>
</tr>
</tbody>
</table>

\(^7\) The implied mean structural parameters estimated by the stochastic coefficients model are \(b_{kk} (2E–05)\), \(b_{wk} (-458.3)\), and \(b_k (364.6)\).
The highest proportion of the variance of agricultural investment is attributable to the lagged capital stock variable. The contribution of the input/output price variable is the second highest followed by the intercept and the rental rate of capital. This analysis suggests that attributing all of the variance in the dependent variable to the intercept term as is implicit in a constant coefficient model is inappropriate. Furthermore, this relative ranking contrasts with the ranking based on asymptotic t-statistics where the intercept had the greatest influence on the dependent variable mean value followed by the input/output price ratio, rental rate, and lagged capital stock.

Figures 1 through 4 contain the time path results of estimating equation (25). Both the input price and rental rate coefficients show increased variability between 1935 and 1946 and 1967 through 1983. The earlier period of variability was, of course, during the Depression and war years while the post-1966 period signaled the beginning of sharper business cycle turns than hitherto experienced during the 1950's and early to mid-1960's.

Both the flexible accelerator and the stochastic coefficients model generate adjustment coefficients which vary through time. Variation in the flexible accelerator's adjustment coefficient depends on the variation in the discount rate and the normalized price of machinery. It is more difficult, however, to identify the causes of the variation in the stochastic coefficient model's adjustment parameter. Several explanations exist for parameter variation. First, the "true" coefficients may be generated by a nonstationary or time-varying random process. Second, omitted variables that exhibit nonstationary behavior and are not orthogonal to the included variables may induce variability in the parameters (5). Third, it is conventional econometric practice to use proxy variables in place of unobservable explanatory variables. In most cases, proxy variables imperfectly capture changes in the economic behavior of the "true" variable. Furthermore, the relationship between the proxy and the true variable may change over time. Fourth, aggregation over microunits can induce variation. It is highly restrictive to assume the aggregation weights of microeconomic units do not change over time (38), (39). Fifth, coefficient variation may result from imposing an incorrect functional form (20). Finally, fixed coefficient econometric models may not be consistent with the dynamic economic theory of optimizing behavior. Changes in economic or policy variables will result in a new environment that may, in turn, lead to new optimal decisions and new microeconomic and macroeconomic structures (14). Because one or more of these explanations may generate the variation in parameters, we cannot identify the specific reason or reasons determining why the adjustment coefficient in the stochastic coefficient model is nonconstant. Allowing for alternative sources of parameter variation to occur in the stochastic coefficients model is a major conceptual difference separating it from the flexible accelerator model.

Unlike the other estimated structural parameters, the magnitude of the mean adjustment coefficient is similar for both models (fig. 4). On average, the adjustment coefficient for the flexible accelerator is greater (0.048) than for the stochastic coefficients approach (0.027). However, a comparison of the time paths shows the stochastic adjustment coefficients to be far more volatile than those of the flexible accelerator, especially during 1944-46. The coefficient of variation for the stochastic adjustment coefficients is nearly 3.5 times greater than the flexible accelerator's (1.71 compared to 0.49). The alternative models generate adjustment coefficients which are most similar during 1931-41 and again during 1968-83. While some turning point correspondence exists between the two models, there are periods when the two models exhibit dramatically different results (for example, 1946).
Figure 3
Rental Rate Parameter

Figure 4
Adjustment Coefficient
The estimated adjustment coefficient derived from the flexible accelerator model was relatively constant from 1952 to 1972. Rates increased abruptly, however, in 1973 and again in 1979. These abrupt increases resulted from sharp decreases in the real interest rate and the normalized price of machinery, which is noteworthy because the accepted explanation attributes a large increase in investment during these years to large increases in agricultural income. Investment, it is argued, increased either because cashflow problems were reduced or farmers sought to avoid taxes by taking advantage of credits and accelerated tax provisions. Results from the flexible accelerator model would suggest a possible alternative explanation. Namely, the increase in investment can be attributed to the drive to increase profits.

One important difference between the two models shown here is that the time path for the flexible accelerator represents any changes in the data used to construct the coefficient (see equation (10)). The stochastic coefficients model, on the other hand, allows the possibility of behavioral differences as economic agents optimize over time.

One may also note that there are periods when the stochastic coefficients adjustment variable time path is negative. Because of factors such as risk and uncertainty, imperfect information, weather shocks, or dramatic Government programs changes, the agricultural sector may overadjust or, more generally, change the adjustment of actual to desired capital stock over time. Therefore, the adjustment coefficient may be greater than one and, on occasion, even negative. This view of the time-varying adjustment coefficient is consistent with Griliches' valuable insight (10) that restricting a time-dependent adjustment coefficient between zero and one cannot, in general, be derived from the properties of the solution to the optimal adjustment path toward an uncertain, continuously changing equilibrium level. Griliches' contention concerning the inappropriateness of those restrictions on the adjustment coefficient when the equilibrium value is uncertain appears to be supported by the variation in the estimates of the adjustment coefficient shown here. Resler, Barth, Swamy, and Davis also reported similar empirical results (21). The stochastic coefficients model shows a sharper decline in the coefficient value than the flexible accelerator from 1979 through 1983, indicating a more rapid decline in replacement investment.

Because there is no statistical procedure to identify the "true" model, one cannot test whether the flexible accelerator or the stochastic coefficients model is the correct model. Instead, one may adopt an instrumentalist approach (see (3)). Both the stochastic coefficients and flexible accelerator models were reestimated with the last 5 years excised to compare out-of-sample predictions. Boland notes that predictive superiority is a sufficient condition for favoring one model over another (3). Table 1 compares both models' out-of-sample forecasts for the years 1979 through 1983. We chose 1979 as the cutoff year because it was the beginning of a dramatic decline in machinery net investment, a situation that provides a good test of forecast superiority. Absolute percentage error results showed the stochastic coefficients model was clearly superior for 4 of 5 years. Root mean square error (RMSE) statistics also confirmed this result (flexible accelerator, 1.94 and stochastic coefficients, 1.11). The results indicated that for nearly any sensible risk function the stochastic coefficients model outperformed the accelerator model. After missing the 1979 investment figure by a wide margin, the stochastic coefficient forecast continually improved. By 1983, the absolute error was only $20 million, a relative error of less than 1 percent. Only in 1979 did the flexible accelerator forecast more accurately than the stochastic coefficients model. After 1979, the flexible accelerator forecasted quite poorly and failed to capture negative net investment during 1980-83.
CONCLUSIONS

The stochastic coefficients model pinpointed the lagged capital stock variable as having the predominant share of influence on the variance of agricultural investment. Evaluation of the time path of the lagged capital stock coefficient showed greater volatility under the stochastic coefficients model than under the flexible accelerator. The fixed coefficients model cannot detect this. A comparison with the flexible accelerator suggests that the ability of the stochastic coefficients model to represent adequately several types of nonstationary processes and to adapt quickly to changing economic conditions enables it to give better predictions than the fixed slope coefficients model.

Table 1--Machinery net investment forecasts

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>Stochastic coefficients error</th>
<th>Flexible accelerator error</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>0.920</td>
<td>-1.246</td>
<td>-2.166</td>
<td>1.745</td>
</tr>
<tr>
<td>1980</td>
<td>-0.816</td>
<td>-1.672</td>
<td>-0.856</td>
<td>1.614</td>
</tr>
<tr>
<td>1981</td>
<td>-1.343</td>
<td>-1.798</td>
<td>-0.455</td>
<td>0.669</td>
</tr>
<tr>
<td>1982</td>
<td>-2.301</td>
<td>-2.124</td>
<td>0.177</td>
<td>0.167</td>
</tr>
<tr>
<td>1983</td>
<td>-2.131</td>
<td>-2.151</td>
<td>0.020</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Billion dollars (1972)
REFERENCES


Appendix

We developed a formula for implicit rental rates from the equality between the purchase price of the asset and the present value of the future rents generated by the asset (4). Assuming constant new asset price expectations and allowing for alternative depreciation patterns, the basic relationship is:

\[ q_i = \int_0^L_1 e^{-rt}u_i n_i(t) \, dt \quad i = 1, 2, \ldots, m, \quad (28) \]

where \( q_i \) is the purchase price of the \( i \)th asset when new, \( L_1 \) is the service life, \( u_i \) is the rental rate expressed in terms of an undepreciated unit of capital, \( n_i(t) \) is the capacity of the asset available in year \( t \) of its service life, and \( r \) is the discount rate.

Equation (28) ignores all tax considerations. When capital income is subject to an income tax, the term on the right side of equation (28) is modified to include the effects of the tax. The modified term includes the present value of the rents generated by the asset, and the present value of the tax savings produced by the investment tax credit and the tax depreciation deductions. Assuming the firm's marginal tax rate remains constant at \( T \), equation (28) respecified to accommodate the tax system becomes:

\[ q_i = (1 - T)u_i N_1 + \theta_i q_i + T(1 - h_{1i})Z_1 q_i \quad i = 1, 2, \ldots, m, \quad (29) \]

where \((1 - T)u_i N_1\) is the present value of the future rents, \( \theta_i q_i \) is the present value of the investment tax credit, and \( T(1 - h_{1i})Z_1 q_i \) is the present value of the future tax depreciation deductions.

If price expectations and the marginal tax rate are constant, the rental rate remains constant over the life of the asset. The productive capacity of the asset, however, declines over the life of the asset so that:

\[ N_i = \int_0^{L_1} e^{-rt}n_i(t) \, dt \quad i = 1, 2, \ldots, m, \quad (30) \]

where \( r \) is the discount rate, the real after-tax rate of return required by the firm.

Although the firm pays taxes on the rents generated by each asset, the firm can deduct the decline in the value of the asset as an expense. If the present value of the depreciation deductions claimed for tax purposes is equal to the true decline in capacity for each asset, the tax system does not distort the asset mix.

If \( z_i(t) \) is the fraction of the price of the \( i \)th asset deducted from income in year \( t \) of the assets tax life (\( M_1 \)), the present value of the tax depreciation is \( TZ_1 q_i \), where:

\[ z_i = \int_0^{M_1} e^{-(r+p)z_i(t)} \, dz_i(t) \quad i = 1, 2, \ldots, m, \quad (31) \]
and \( p \) is the rate of inflation. However, in years when the tax depreciation base declined by the amount of the investment tax credit, the real value of the tax depreciation deduction is \( T(1 - h_i)Z_i q_i \), where \( h \) is the percentage of the credit which reduces the depreciation base.

In addition to the depreciation deductions, firms may also be eligible to claim an investment tax credit. If firms claim the credit at the end of the first year of the asset's service life, the present value of the credit is \( \theta_i q_i \), where:

\[
\theta_i = e^{-(r+p)\theta_i} \quad i = 1,2,\ldots,m. \tag{32}
\]

A more realistic rendering of the discount rate shows it as a weighted average of the longrun real after-tax interest rate (external financing) and the longrun real after-tax return to equity (internal financing). Because nominal interest charges are deductible from taxable income, the real cost of external or debt financing \( r_d \) is:

\[
r_d = \frac{r_n(1-T) - p}{1 + p}, \tag{33}
\]

where \( r_n \) is the nominal interest rate. After combining the real costs of both equity and debt financing, the real cost of the capital or real after-tax discount rate is:

\[
r = fr_d + (1-f)r_e, \tag{34}
\]

where \( f \) is the fraction debt financed, \( r_d \) is the real after-tax cost of debt financing, and \( r_e \) is the real after-tax return to equity \((26)\).

Given the market price of the asset, equation \((27)\) is rewritten as:

\[
u_i = q_i \frac{[1 - \Theta_i - T(1 - h_i)Z_i]}{N_i(1 - T)} \quad i = 1,2,\ldots,m, \tag{35}
\]

which is the real rental rate the firm must charge to earn the required real after-tax rate of return.
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