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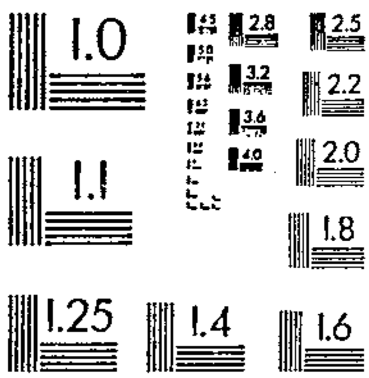

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The Distribution of Daily Changes in Commodity Futures Prices

J. Douglas Gordon

The Distribution of Daily Changes in Commodity Futures Prices, by J. Douglas Gordon, National Economics Division, Economic Research Service, U.S. Department of Agriculture. Technical Bulletin No. 1702.

Abstract

Agricultural futures prices over the entire life of a commodity futures contract tend to be more variable during growing seasons than after harvest. Thus, many economic models used in analyzing and forecasting price changes of commodity futures contracts are inaccurate. The variability of futures prices over shortrun, 2-month segments within a single season is more constant and, thus, fits within many standard economic models used for analyzing and forecasting.

Keywords: Futures prices, efficient markets, random walk, seasonality

Preface

This report presents results of continuing research on the behavior and performance of agricultural commodity futures markets and market prices. The work has been performed under the leadership of David Harrington, Chief, Farm Sector Economics Branch, National Economics Division, Economic Research Service, and Richard Heifner, senior economist with the Farm Sector Economics Branch. The author thanks Richard Heifner for several careful readings of the manuscript and many valuable suggestions. He also thanks Gerald Plato, Allen Paul, Jitendar Mann, John Kitchen, David Harrington, and Roger Conway for helpful comments.

This report supersedes *The Distribution of Shortrun Commodity Price Movements*, Technical Bulletin No. 1536, issued March 1976. Some of the techniques applied in that bulletin have also been used here.

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Summary

Agricultural futures prices over the entire life of a commodity futures contract tend to be more variable during growing seasons than after harvest. Thus, many economic models used in analyzing and forecasting price changes of commodity futures contracts are inaccurate. The variability of futures prices over shortrun, 2-month segments within a single season is more constant and, thus, fits within many standard economic models used for analyzing and forecasting.

Here are some recommendations when using models for analyzing and forecasting commodity futures contracts:

- If using option pricing models, consider including changing variance in the formulas. The assumption of constant variance means that the models will underprice options in high variance seasons and overprice options in times of low variance of price changes.

- If using time-series models, first correct for the nonstationary variance of percentage price changes before applying the models to futures market prices. Researchers using models that assume stationary covariance must adjust the futures market data to eliminate nonstationary variance of percentage price changes before fitting those models.

- Consider seasonality of variance when using econometric forecast and forecast accuracy models. The forecast and actual outcome prices may each be drawn from distributions with changing variance if they are taken at different times of the year. That is, some of the outcome prices may come from a low variance distribution, while others may come from a high variance distribution.

The Distribution of Daily Changes in Commodity Futures Prices

J. Douglas Gordon*

Introduction

Agricultural commodity prices vary substantially from year to year, and even from day to day. The size of the price movements and their distribution have always concerned the agricultural sector. Many producers use the prices on the futures markets in determining their production and storage plans. If there is a sharp drop or rise in price, producers may find that their expectations are based upon the wrong prices. If the price movement reflects something other than a change in underlying supply and demand, producers may believe the move is due to supply or demand fundamentals and be misled in their production plans. Knowledge about price trends and serial independence of price changes is important to persons who buy or sell commodity futures and options and to those concerned about the performance of these markets.

This study examines the distribution of futures price changes in order to test whether the assumptions about futures prices underlying many economic models are appropriate. If price changes are not independent, then futures prices will not follow assumed behavior. If price changes are independent, but not covariance stationary, they do not follow assumed behavior, but with appropriate transformations to the data, the assumptions of many economic models can be maintained.

In order to evaluate the performance of cash and futures markets in allocating resources, we must first discover the nature of the commodity price movements themselves. All futures markets and several cash commodity markets report prices daily. The large quantity of available data in these markets makes evaluation of pricing problems much easier than in other markets. An examination of futures price movements cannot identify all possible pricing and resource allocation problems. It can, however, point out certain problems and show in which areas others might be occurring.

Commodity futures prices are also important in economic models. Often those models make strict assumptions about the nature of futures prices. The validity of these assumptions may have important consequences in the decision whether to use simple or more complex versions of these models. For example, commodity option pricing models often use variants of the Black-Scholes model to determine the option premium (*2, 3*).[1] These models typically assume that the variance of changes in the logarithm of daily price in the underlying futures market is constant. If variance differs with time, the price of the premium will be underestimated in some periods and overestimated in others.

In the past 20 years, there have been several studies of the behavior of futures market prices. See the collections edited by Cootner and Peck for examples (*7, 33, 34*). A few studies of the agricultural commodity futures markets have concluded that futures prices do not fully represent supply and demand fundamentals and that (a) a simple model based on these fundamentals will often outperform the futures market and (b) futures prices exhibit some systematic movements that would be absent from a fully efficient market, so that profitable trading rules can be developed to take advantage of this nonrandomness (*23, 24, 36*). If the futures markets are efficient, past prices cannot be profitably exploited. The efficiency of futures market prices can be tested in several ways.

This report examines the distribution of changes in daily futures prices for several commodities (corn, cotton, wheat, live cattle, live hogs, orange juice, rough rice, and soybeans). First, the author tested the independence of the observations. If the price changes are not mean independent, there may be imperfections in the market which may cause changes in price to reflect factors other than changes in supply and demand.

*The author is an economist with the Commodity Futures Trading Commission. He was assigned to the Economic Research Service, U.S. Dept. of Agriculture, when he conducted this research.

[1]Italicized numbers in parentheses refer to literature cited in the References section at the end of this report.

The author then tested changes in the logarithm of daily prices for normality. Many economic models using futures prices rely on assumptions that price changes are drawn from a normal distribution with finite and constant mean and variance. If the markets reject the normality hypothesis, the author examined them to see whether they can be described by an infinite variance distribution or by a mixture of normal distributions with changing variance. These results indicate what type of price model economists should use when including futures prices in economic analyses.

Market Efficiency

A major concept in the theory of market price behavior is the efficient markets hypothesis which uses prices to study market performance. This theory provides a general structure for analyzing several aspects of market performance. An efficient market is one in which prices reflect all available information. If a commodity futures market is efficient, it accurately represents current and expected future supply and demand for that commodity. Fama summarized the efficient markets theory and classified tests of the hypothesis by the amount of information necessary to perform the test (*11*):

> Strong form tests are concerned with whether individual investors or groups have monopolistic access to any information relevant for price formation. . . In the less restrictive semi-strong form tests, the information subset of interest includes all obviously publicly-available information, while in the weak form tests, the information subset is just historical price or return sequences.

If some tests reject the efficient markets theory, the market lacks some of the properties of an efficient market; in some cases, this lack of certain properties means that the futures market will not give an accurate estimate of the present or future conditions in the cash commodity market.

The efficient markets theory implies that, on the average, excess returns cannot be earned in an efficient market. In a futures market, net investment approaches zero, because the margin can be held in the form of a U.S. Treasury bill, paying interest to the speculator or hedger. The expected return from the investment in the market is also zero, unless individuals who are risk averse dominate the market.

Thus, in an efficient futures market with risk neutrality, today's futures market price should equal the expectation today of the price of the commodity to be delivered at the same point in the future. That is:

$$E_t(P_{t+n}|P_t) = F_{t+n,t} \quad (1)$$

Where:

E_t is the expectation at time t (today).

P_{t+n} is the price of the commodity delivered n days from now in fulfillment of the futures contract.

P_t is today's price of the spot commodity at the delivery point.

$F_{t+n,t}$ is today's price on the futures exchange of the commodity to be delivered n days from now.

If risk aversion dominates the market, equation (1) does not hold. In that case, the speculators receive a premium in order to persuade them to carry the risks transferred from the hedgers. For grains, where hedgers generally hold short positions, risk aversion implies that $E(P_{t+n}|P_t) > F_{t+n,t}$. When one "holds short positions" under risk aversion, the price expected in the future is greater than the futures price. This rewards those holding long positions (those speculating that price will rise between now and delivery). Keynes (*23*) gave the name "normal backwardation" to the phenomenon of current futures price lying below the expected cash market price in the maturity month.[2]

When there are both hedgers and speculators holding both long and short positions in the futures market, the direction of a risk premium, if any, is uncertain. Several authors have examined futures markets for risk premiums with mixed results (*17, 32*).

If today's price fully reflects all known supply and demand information for the commodity and if there is no risk premium, then today's expectation of tomorrow's futures price for the contract maturing in month i should equal today's futures price for that contract, implying that the change in price from today to tomorrow

[2]The term "backwardation" is often used to describe the case where futures prices for contracts maturing in the distant future are below prices of the near futures contract. In this report, that will be called an inverted price structure. The term backwardation will be used to represent the situation where there is a risk premium in the market of the type described above.

is random with mean zero.[3] If there is a risk premium, then the price change from today is random with mean greater or less than zero depending on whether the hedgers are net short or long in the market, respectively. Also, there should be no information about tomorrow's futures price to be gained from knowing yesterday's expectation, or any expectation prior to that. In other words:

$$E_t(F_{i,t+1}|F_{i,t}, F_{i,t-1}, \ldots) = E_t(F_{i,t+1}|F_{i,t}) = F_{i,t} \qquad (2)$$

or

$$E_t(F_{i,t+1}|F_{i,t}, F_{i,t-1}, F_{i,t-2}, \ldots) - F_{i,t} = 0 \qquad (3)$$

where:

$F_{i,t}$ is the futures price for contract $_i$ at date $_t$.

$F_{i,t+1}$ is the futures price for contract $_i$ on the following day.

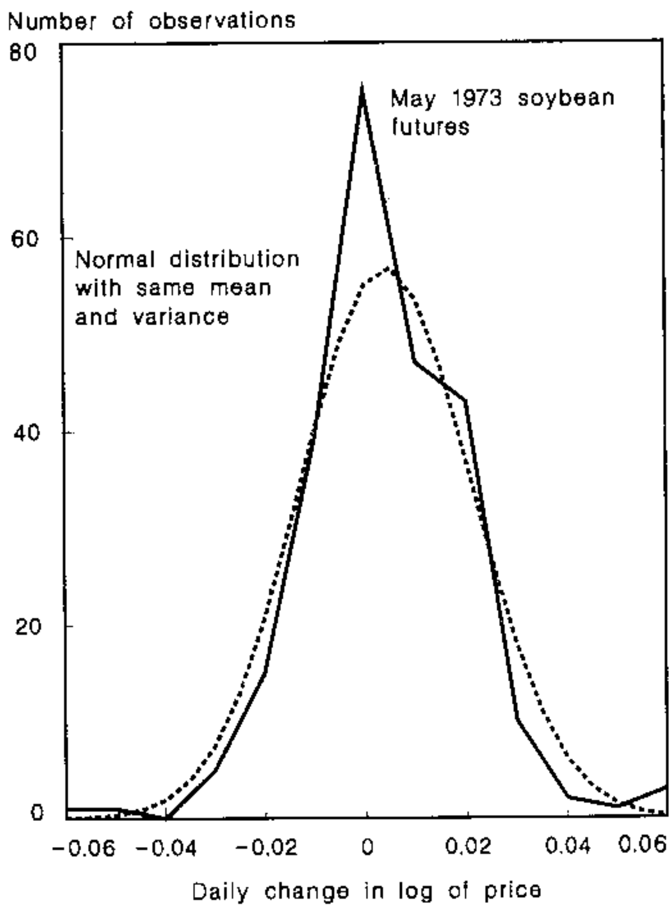
An efficient futures market, according to the efficient markets theory, should follow a martingale process which is serially mean independent with an expected value of 0. The variance and higher moments are not restricted. With a risk premium, the distribution of changes in the logarithm of price will follow a sub- or supermartingale, where the expected value is above or below 0 depending on whether the long or short side of the market collects the premium.

Often the term "random walk" is used to refer to a martingale process with the additional restriction that each random price movement must be drawn from a distribution with finite and constant variance. That hypothesis will be called the strict random walk hypothesis in this report. The distribution of changes in the logarithm must be "covariance stationary" under the strict random walk hypothesis.[4] This property, if it holds, allows one to examine a stationary process with parametric tests. Under the martingale hypothesis, the variance need not be finite or constant, so the appropriate tests are nonparametric, that is, distribution free.

Several authors have found that daily changes in the logarithm of price are best described by an infinite variance distribution. Mann and Heifner (*27*) examined the distribution of shortrun commodity price movements using distribution-free tests of independence. They also fitted symmetric stable distributions to the distribution of price changes. The theory of stable distributions began with Levy (see Feller (*13*) for a short history). Mandelbrot suggested this family of distributions to describe distributions that were leptokurtic, that is, sharply peaked distributions with more weight in the tails than the normal distribution (*25*). (See figure 1 for an example of a leptokurtic curve.) Fama and Roll proposed this family of distributions to represent the distribution of shortrun price changes (*11*, *12*).

Figure 1

Example of a Leptokurtic Distribution



[3]Normally, the change in the logarithm of price is the random variable of interest. For small changes, the percentage price closely approximates the change in logarithm of price. An advantage of working with ratios or logarithms of ratios is that they are independent of the units of measurement, allowing direct comparison between commodities and generally making further adjustments for price inflation unnecessary. Logarithms also have greater appeal than the absolute change because prices may vary upwards by an arbitrarily large amount, yet they have a lower bound of zero. In the analyses of this report, all price changes are changes in the logarithm of price unless otherwise noted.

[4]A covariance-stationary process is one in which, for process X_t, $E(X_t) = \mu$, and $Cov(X_{t+h}, X_t) = \gamma(h)$; $\gamma(0)$ is the variance which is constant. That is, the covariance depends only on the distance in time, h, between the observations (*8*). This process is also called a weakly stationary process.

Distributions of price changes with "thick" tails may also be due to nonconstant variance. Anderson has presented evidence that for several commodities the distribution of changes in the logarithm of price has changing variance (*1*). Variance changes because of the "maturity" effect explained by Samuelson (*38, 39*) and because of seasonality in the flow of information.

Because commodity price changes may be drawn from distributions with infinite variance or from mixtures of distributions with changing variances, the results of parametric tests of randomness are likely to be misleading. There are several nonparametric tests of independence.

Tests of Randomness and Distributional Form

If futures market prices are efficient in the sense that they contain all relevant information, then the changes in daily price must be serially mean independent. If the changes in price are not serially mean independent, then there is a profitable trading rule that can be devised from the deterministic element of the price changes. This implies that the futures price does not reflect all that is known about the value of the commodity, consequently the futures market is not economically efficient. Tests of randomness of price changes are weak-form tests of efficiency.[5] The results of several studies of changes in the logarithm of price have suggested that distributional assumptions (particularly the assumption that the changes are normally distributed) are not legitimate (*1, 12, 27*).

There are several tests of randomness. Kendall and Stuart suggest three criteria in choosing a test (*21*):

(a) If possible, the test should be distribution free.
(b) Since we may wish to test fairly long series, the calculations should be kept to a minimum.
(c) Although we may not be able to specify an alternative hypothesis with precision, we may have some idea of its nature and can select a test which is likely to have high power against the alternative. For example, if we suspect a trend, we may find it useful to employ a different test from one used to test against periodicity.

[5]Although the tests are weak-form tests of efficiency, they have strong implications for the market. Grossman and Stiglitz showed that in equilibrium there will be no incentive to arbitrage if the market is efficient (*18*). At that point extra information would have no economic value. Consequently, we can only expect markets to closely approach the behavior assumed by the efficient market theory.

A turning-point test provides one test of randomness. It examines the number of times a move upward (or downward) is reversed and compares that number with a theoretically calculated value. A series where each observation was greater than the previous one would have no turning points and would fail this test for randomness. The turning-point test is strong against cycles and runs up and down in price but is weak against trend. These features make such a test useful for analyzing futures price series. There may be an underlying trend in the series due to the existence of a risk premium. In that case we would not wish to reject efficiency without further examination of the market. There are other distribution-free tests that are stronger against trend.

Turning-Point Test

If in any sequence of three observations, the middle observation is above or below the other two, there is a turning point. The number of turning points, p, expected in a random series n periods long is

$$E(p) = \frac{2}{3}(n - 2) \tag{4}$$

where:
E is the expected value.
p is the number of turning points.
n is the number of periods.

The variance of the number of turning points, p, is

$$Var(p) = \frac{16n - 29}{90} \tag{5}$$

The distribution of the number of turning points, p, rapidly approaches normality as the number of observations increases (*21*).

The Difference-Sign Test

If there is a risk premium in the futures market prices, there may be an underlying trend in the prices. If hedgers are net short in the market and market participants are risk averse, the longs should collect a risk premium from the shorts, in payment for assuming the risk that prices will fall.

Futures prices before contract maturity would then be below the futures price in the delivery month, and should gradually rise as the time to maturity falls. A submartingale would describe this process. That is, the equality in equations (1), (2), and (3) would be replaced with $>$. If hedgers are net long in the market and market participants are risk averse, the shorts will expect a risk premium from the longs, in payment for assuming the risk that prices will rise. This situation can be described by a supermartingale. If the market is efficient and risk neutral, there should be no trend in the futures prices.

The difference-sign test, a test of the number of day-to-day moves in one direction, provides a simple test for trend. If the number of daily upward moves is substantially greater than the number of downward moves, the series has a significant positive trend. Conversely, if the number of daily upward moves is substantially less than the number of downward moves, the series has a significant negative trend.

In a random series the expected number of positive differences is $\frac{1}{2}(n - 1)$. The variance is $\frac{1}{12}(n + 1)$ (*21*).

The X^2 Normality Test

The X^2 goodness-of-fit test for normality provides a simple test of the hypothesis that changes in the logarithm of price are distributed normally. First, the normal distribution is divided into sections (in this study into deciles). Next, the observations are grouped into deciles and the number of observations in a given decile is compared with the expected number. The test statistic is:

$$T = \sum_{j=1}^{10} \frac{(0_j - E_j)^2}{E_j} \tag{6}$$

Where 0_j is the number of observations in class j, $j = 1, 2, \ldots, 10$; E_j is the expected number of observations in class j when H_0 is true; and $E_j = p_j N$ where p_j is the probability of an observation being in class j, under the assumption that the distribution of the random variable X is normal. N is the number of observations (*6*).

The test statistic T is compared with the value of X^2 at the 95-percent confidence level with 7 degrees of freedom. This value is approximately 14.07. If the value of T is greater than 14.07, we reject the null hypothesis that the distribution of price changes is distributed normally with constant mean and variance.

Squared Ranks Test

The squared ranks test for equal variances tests whether two random samples have identical variances, though possibly different means (*6*). The samples are combined and the observations ranked; the ranks are then squared. The test statistic given by Conover is

$$T_1 = \frac{\sum_{i=1}^{n} [R(U_i)]^2 - n\bar{R}^2}{\left[\frac{nm}{N(N-1)} \sum_{i=1}^{N} R_i^4 - \frac{nm}{N-1} (\bar{R}^2)^2\right]^{1/2}}$$

Where:

$$\bar{R}^2 = \frac{1}{N} \left[\sum_{i=1}^{n} [R(U_i)]^2 + \sum_{j=1}^{m} [R(V_j)]^2\right];$$

$$\sum_{i=1}^{N} R_i^4 = \sum_{i=1}^{n} [R(U_i)]^4 + \sum_{j=1}^{m} [R(V_j)]^4;$$

$X_1, X_2, \ldots X_n$ is a random sample of size n from population 1;
$Y_1, Y_2, \ldots Y_m$ is a random sample of size m from population 2;
$U_i = |X_i - \bar{X}|$ $i = 1, \ldots, n$;
$V_j = |Y_j - \bar{Y}|$ $j = 1, \ldots, m$;
$\bar{X}, \bar{Y}$ are the means of samples X and Y;
$R(U_i)$ are the ranks of the U_i; and
$R(V_j)$ are the ranks of the V_j.

We reject the null hypothesis of identical variances if $T_1 > z_{1-\alpha/2}$, that is T_1 is greater than the 1-$\alpha/2$ quantile of the normal distribution with N – 2 degrees of freedom (for samples with one group (X or Y) of size greater than 10). The squared rank test is nonparametric. Because we are examining distributions other than the normal one, tests based on the normal distribution should be avoided. If the variance of price changes is constant among samples, we can estimate the parameters of the symmetric stable distribution that best fit the data.

Calculating the Parameters of Symmetric Stable Distributions

Stable distributions are those for which sums of random variables from the distribution also follow the same distribution. For example, from the central limit theorem we know that sums of randomly normally distributed variables also follow the normal distribution. The normal (Gaussian) distribution is the only stable distribution with finite variance.

> As cited by Mann and Heifner (*27*), "The stable Paretian family of distributions for a random variable u is defined (Gnedenko and Kolmogorov

(*14*), p. 164) by the logarithm of its characteristic function:

$$\text{Log } \varphi(t) = \log E(e^{iut})$$

$$= i\delta t - \gamma|t|^{\alpha}\{1 + i\beta\frac{t}{|t|} w(t, \alpha)\}$$

where

$$w(t, \alpha) = \begin{cases} \tan(\pi\alpha/2) & \text{if } \alpha \neq 1 \\ (2/\pi)\log|t| & \text{if } \alpha = 1 \end{cases}$$

$$0 < \alpha \leq 2$$

and $i = \sqrt{-1}$.

Mann and Heifner (*27*) also describe four parameters of stable distributions:

> α—the characteristic exponent which determines the height of the extreme tails of the distribution; δ—the location parameter; γ—the scale parameter; and β—an index of skewness. In most applications to price data, symmetric stable distributions are assumed so that $\beta = 0$.

If $\alpha = 2$ then the distribution is normal. If $\alpha < 2$ the variance and higher moments are not defined. If $\alpha \leq 1$ the mean is not defined. The Cauchy distribution is a stable distribution with $\alpha = 1$. The Holtsmark distribution is stable with $\alpha = 3/2$.

Mann and Heifner continue (*27*):

> The location parameter δ corresponds to the mean when $\alpha > 1$ or the median (for all α). For the Gaussian distribution, δ is efficiently estimated by the sample mean. For other stable symmetric distributions, efficiency is gained by disregarding some of the extreme observations and utilizing the mean of the remaining observations as an estimate. Fama and Roll (*11*, pp. 826-33) recommend using the mean of the central half of the observations.
>
> The scale parameter $\gamma = c^{\alpha}$ measures the dispersion of the distribution. For the normal distribution, c^2 equals one-half of the population variance. Fama and Roll (*11*, pp. 822-24) suggest using the distance between the .28 fractile and the .72 fractile to estimate c for symmetric stable distributions.

Forecast Accuracy

Another test of futures market efficiency is forecast accuracy. This test examines the ability of the futures price at some period before contract maturity to forecast the futures or cash market price at some point during the contract maturity month. The equation to be tested is

$$F_T = a + bF_{T-n}$$

Where:

> F_T is closing futures price on a specified day during the maturity month, F_{T-n} is closing futures price on a specified day n months prior to contract maturity, a is the coefficient of the constant term, and b is the slope coefficient.

The null hypothesis is the joint hypothesis that $a = 0$ and $b = 1$, in which case the forecast is said to be unbiased. To evaluate the forecast, the futures price in the maturity month is regressed against futures price n months earlier, and the F-statistic for the joint null hypothesis that $(a,b) = (0,1)$ is calculated. Also, the Durbin-Watson statistic can be calculated to test for the presence of first-order serial autocorrelation in the residuals (*35*).

The individual parameters a and b are also interesting. If $b = 1$ but a is significantly different from 0, we can interpret the intercept term as a risk premium. This premium may be either positive or negative, depending on whether the hedgers are net short or net long in the market.

Because the forecast accuracy test does not examine daily changes in futures price, it is not performed on the commodities in this study. The results of the test can be important in determining market efficiency. The distribution of daily price changes can affect the efficiency and interpretation of the results of this test. If the forecast and outcome prices are drawn from distributions that have different variances, the efficiency of the test will be reduced and calculated values of the error of the estimate will be biased downward, unless an adjustment is made for changing variance of the data. See Plato and Gordon (*35*) for a more detailed explanation of the consequences of the distribution of futures price changes for forecast accuracy. That manuscript also contains examinations of forecast accuracy for several commodities using the seemingly unrelated regressions technique.

Previous Empirical Studies

Mann and Heifner discuss the results of several studies of the nature of commodity price movements. Many of these earlier papers have been collected into books edited by Cootner (*7*) and Peck (*33, 34*). The literature on this topic has grown considerably in recent years. The results of these studies are mixed: some have found futures markets to be efficient, yet others have rejected one or more properties associated with market efficiency.

In 1976, Rutledge found that the volatility of futures prices of some commodities did not tend to increase as the contracts approached maturity (*37*). That article prompted a reply by Samuelson, who expanded upon his earlier model of increasing volatility of futures prices (*38, 39*).

Mann and Heifner fitted parameters of the symmetric stable family of distributions to changes in logarithms of prices for each contract (*27*). They found that such price changes were not normally distributed, and in fact, were best characterized by infinite variance distributions. Most contracts had values of α, the characteristic exponent, much less than 2.

Cargill and Rausser calculated autocorrelation functions and spectral density figures for several futures contracts (*4, 5*). They found that several contracts showed some autocorrelation or significant peaks in the spectral density function. The null hypothesis of this type of model requires that the futures prices follow a strict random walk. That is, the price changes are covariance stationary; they have mean zero and finite and constant variance. The authors noted that the parametric tests may reject the null hypothesis if a series is mean independent, but the variance of percentage price changes is infinite or nonconstant.

Rausser and Carter tested the forecasts of the soybean, soybean meal, and soybean oil futures markets against those of Autoregressive Integrated Moving Average (ARIMA) models developed from an econometric model of the soybean complex (*36*). They found that the ARIMA models provided superior forecasts of monthly average cash market prices compared with futures markets.

Miller examined the live cattle futures market for the maturity effect in the variability of futures prices (*31*). She found that there was a significant trend in the variability of the futures prices over the life of the contract. She also found much weaker evidence that spot prices followed a mean reverting process.

More recently, Anderson tested the volatility of several futures markets (*1*). He found that the markets exhibited a tendency toward greater price variability as the contracts approached maturity, but in many cases the main determinant of volatility in prices was seasonality in the flow of information.

Several researchers have studied the distribution of commodity price changes from trade to trade. Mann studied intraday commodity price changes (*26*). He found that successive price changes were nonrandom, tending to be of the same size and of the same sign more often than one would expect. Martell and Helms also found price changes within a trading day to be nonrandom (*29*). They found a tendency toward reversals in price between trades. Elton, Gruber, and Rentzler found that intraday price changes for Treasury bill futures allowed arbitrage profits and consequently could not be considered efficient in the strict sense (*9*). These intraday studies suggest that profitable arbitrage opportunities exist in futures markets.

In 1980, Grossman and Stiglitz showed that competitive equilibrium with prices reflecting all available information is not consistent with profits from arbitrage (*18*). If there are no profits to be had from arbitrage, then there is no incentive to enter the market. Hellwig showed that in a dynamic framework, with agents conditioning their expectations on past rather than current market prices, markets can come arbitrarily close to being informationally efficient, while still yielding some arbitrage profits (*19*).

These theoretical results suggest that markets may allow profits from trade-to-trade, but that they can approximate informationally efficient ones when prices farther apart in time are examined. That is, price changes reflect the flow of new information into the market, but they also reflect the process of achieving equilibrium. Price changes farther apart will contain more of the new supply and demand information relative to variability caused by the open outcry bidding system.

In recent years, studies of the forecast accuracy of closing prices on specific days for prices at contract maturity have been performed for several commodity and financial futures markets. Forecast accuracy tests generally require the assumption that futures price changes are mean independent and often that they are drawn from a distribution with constant variance (that is, seasonality in the variance must be eliminated before the test is performed). A few studies of forecast accuracy have aggregated observations from overlapping time periods. That is, a March forecast for September is aggregated with a January forecast for July, a November forecast for May, and so forth, into a series of 6-month-ahead forecasts. The random events moving prices in March, April, and May will affect each of the forecasts mentioned above, leading to autocorrelation of the residuals of the regression. This can create problems in interpreting the results of forecast accuracy tests, unless the interdependence of the observations is taken into account.

There have been several studies of the accuracy of forecasts of the agricultural commodity futures markets. Tomek and Gray used a forecast accuracy test in their study of the futures market for Maine potatoes (*40*). They found that the market did not provide accurate and unbiased forecasts of maturity price, unlike the corn futures market, which was tested for purposes of comparison. Leuthold found that the live cattle futures market provided inaccurate and biased forecasts of maturity price (*23*). Martin and Garcia tested the live cattle and hog futures markets for forecast accuracy (*30*). They found that neither market could be considered efficient.

O'Brien and Schwartz examined the gold futures market for evidence of Keynesian backwardation (32). That is, they tested whether the futures prices were below the expected cash prices at contract maturity. Backwardation might imply that those holding long positions in the market were exacting a premium from the shorts in order to accept the risk of price variation. O'Brien and Schwartz found that backwardation did occur regularly in that market.

Marquardt tested the forecasts of the cattle, hogs, corn, wheat, and soybeans futures markets against those of several outlook letters (*28*). He found that the futures markets generally gave more accurate price forecasts than the outlook letters. Just and Rausser performed a similar semistrong test of efficiency in the futures markets for corn, wheat, soybeans, soybean oil, soybean meal, cotton, cattle, and hogs (*20*). They tested forecast efficiency of the futures markets in these commodities against the price forecasts of several commercial firms and the USDA. They also found that the futures markets generally outperformed the commercial and Government forecasts.

Statistical Analysis

This study examines daily closing prices for futures markets in corn, soybeans, wheat, rough rice, cotton, live cattle, hogs, and orange juice. The data cover the period from the contracts maturing in January 1979 through those maturing in May 1984. Each contract was analyzed separately over its lifetime, which consisted of between 200 and 350 price changes. The data used in the independence and distributional tests are changes in the logarithm of daily closing price (that is, approximately the same as percentage price change).[6]

Turning-Point Test Results

Table 1 summarizes the results of the turning-point test. The calculated test statistic was compared with that from the normal distribution at a 0.05 significance level (95-percent confidence level). For each of the eight commodities, the author calculated the number of contracts with significantly nonrandom price changes. A more detailed table of the turning-point test results appears in appendix 1. Of the futures markets, only the rough rice futures market showed significant nonrandomness in a significant number of contracts.[7]

Kansas City wheat had three contracts which rejected randomness, an event with a probability of only 0.15, assuming independence.[8] The null hypothesis cannot

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[6]The Commodity Futures Trading Commission provided the data used in this study. Limit price moves were included in the series. This may tend to bias the results of the turning-point test toward rejecting the null hypothesis of randomness. If limit moves are omitted from the data, however, limit moves due to inefficiencies in the markets will not be included in the test. This might bias the test results toward failure to reject efficiency when it should be rejected. Most, but not all, studies of daily price changes in stock and futures markets have included limit moves.

[7]Rice price movements were also examined in an earlier study (*15*). Because there were many days with no trading, the closing price in that market need not reflect activity in the market. When opening prices on days in which trades occurred were examined, the randomness hypothesis could not be rejected. The changes in closing price need not have reflected changes in supply and demand. Farmers basing their production plans on the closing price might be misled by the movements in that price. The other markets examined here had much greater volume over the period of analysis. Because of the fewness of opening prices and the nonrandomness in closing price, the author omitted the rough rice market from subsequent analysis.

[8]The contracts examined for each commodity contain observations that overlap in time. For example, the January and March 1981 soybean contracts will both contain observations taken in December 1980. If events in December outside the market influence the January price, they may also influence expectations about the March contract price. If efficiency is falsely rejected for the January contract due to events outside that contract market, the probability that efficiency will be rejected for the March contract will be higher than independence suggests. Alternatively, if outside events prevent market inefficiency from showing up in the January contract, they are likely to have a similar effect on the March contract. Because the data are economic and not experimental, controlling for events outside the markets is difficult. Thus, the probabilities under the null hypothesis given by the binomial distribution most likely have some downward bias.

be rejected, but that does not necessarily mean that the market is efficient. Commodities with associated probabilities that are low, but greater than 0.05, should perhaps be placed into a category of questionable efficiency.

Difference-Sign Test Results

The difference-sign test was applied to the futures data to test for the existence of a risk premium. Table 2 summarizes the test outcomes. A significantly positive t-value for a futures contract shows that there were significantly more upward moves in price (positive changes) than downward moves over the life of the contract and indicates a trend in the price changes. Significant trends were found in a significant number of contracts for the rough rice market and for the cattle and hogs futures markets. In each of these markets the trend was downward. (See appendix table 2 for the t-values for each contract). This may be due to the downward trend in cash commodity prices over much of the period analyzed. If that trend were unanticipated by futures traders, it would show up in the futures price series. A downward trend in futures prices may also be caused by the shorts extracting a risk premium from the longs. If the majority of hedging was long hedging,

Table 1—Turning-point test

Contract	Contracts tested	Significant values[1]	Probability of that number or more[2]
	------Number------		
Corn	27	1	0.75
Cotton	28	2	.41
Kansas City wheat	27	3	.15
Live cattle	34	2	.51
Live hogs	37	2	.56
Orange juice	33	0	1.00
Rough rice	13	3	.02
Soybeans	38	1	.86

[1]This is the number of t-values on individual contracts rejecting the null hypothesis of nonrandomness at a 95-percent confidence level or greater.

[2]Probability of having x or more significant values in N independent tests where the probability of significance is 0.05 for each test, N = number of tests (contracts), x = number of significant values observed, and i = an index. These probabilities were calculated using the binomial distribution:

$$P = \sum_{i=x}^{N} \binom{N}{i} .05^{i} .95^{N-i} = 1 - \sum_{i=0}^{x-1} \binom{N}{i} .05^{i} .95^{N-i}$$

Small values for P imply that price changes are not serially independent unless an unlikely event occurred.

The different contracts examined for each commodity contain observations drawn from overlapping time periods. For example, the January and March 1981 soybean contracts will both contain observations taken in December 1980. If events in December outside the market influence the January price, they may also influence expectations about the March contract price. If efficiency is falsely rejected for the January contract due to events outside that contract market, the probability that efficiency will be rejected for the March contract will be higher than independence suggests. Alternatively, if outside events prevent market inefficiency from showing up in the January contract, they are likely to have a similar effect on the March contract. Because the data are economic and not experimental, it is difficult to control for events outside the markets. Thus, the probabilities under the null hypothesis given by the binomial distribution most likely have some downward bias.

Table 2—Difference-sign test

Contract	Contracts tested	Significant values[1]	Probability of that number or more[2]
	------Number------		
Corn	27	1	0.75
Cotton	28	1	.76
Kansas City wheat	27	2	.39
Live cattle	34	5	.02
Live hogs	37	6	.01
Orange juice	33	2	.50
Rough rice	13	3	.02
Soybeans	38	1	.86

[1]This is the number of t-values on individual contracts rejecting the null hypothesis of nonrandomness at a 95-percent confidence level or greater.

[2]Probability of having x or more significant values in N independent tests where the probability of significance is 0.05 for each test, N = number of tests (contracts), x = number of significant values observed, and i = an index. These probabilities were calculated using the binomial distribution:

$$P = \sum_{i=x}^{N} \binom{N}{i} .05^{i} .95^{N-i} = 1 - \sum_{i=0}^{x-1} \binom{N}{i} .05^{i} .95^{N-i}$$

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then short speculators might require a premium, represented by a higher futures market price than the price actually expected at contract maturity, in order to enter the market and bear the price risk of the hedgers. In the livestock and rough rice markets though, short hedging was larger than long hedging in the period examined. The other markets showed significant trend in only a very few contracts. This result suggests that there was no risk premium to be earned in those markets nor was there a series of unanticipated events causing a trend in the price changes.

Distributional Test Results

The results of the nonparametric tests of randomness do not reject the market efficiency hypothesis at a 95-percent confidence level for any markets but that in rough rice. The livestock markets show some trend, which may indicate some risk premium. This would not itself reject the efficiency hypothesis. Thus, the general assumption in economic models that daily changes in futures prices are serially independent appears justified for the larger markets. Economists often make the further assumption that prices follow a strict random walk. That is, economists commonly assume that the distribution of percentage price changes is stationary, and that the distribution is normal with constant mean and variance.

The validity of this normality assumption can easily be tested with the X^2 goodness-of-fit test. Each of the seven markets which showed randomness in the percentage changes in price as measured by the turning point test was examined with this test (table 3). In every one of the futures markets the test statistics rejected normality. This result supports those of several previous studies of futures markets (*1, 11, 12, 27*). Several authors have suggested possible alternatives to the normal distribution. Fama and Roll found that the distribution of stock price changes could be described by members of the family of stable distributions (*11*). That is, the sample distributions had thicker tails than the normal distributions. Mann and Heifner found that many of the futures contracts they tested had distributions with a characteristic exponent of less than 2 (*27*). That is, they had more observations in the tails of the sample distributions than predicted by the normal distribution.

Table 4 summarizes the estimated parameters of symmetric stable distributions fitted to the distributions of price change. The calculated parameters for each contract appear in appendix table 4. In most contracts for each commodity, the estimate of α, the characteristic exponent, was less than 2. For many contracts the estimate was less than 1.5. The calculated parameters suggest that the distributions of percentage futures price changes are characterized by infinite variance distributions.

A mixture of distributions may also give results that look as though the distribution of price changes is characterized by infinite variance. The percentage changes in futures prices should have increasing variance over the life of the contract, according to Samuelson (*38, 39*). Anderson found that increasing var-

Table 3—X^2 Goodness-of-fit test

Contract	Contracts tested	Significant X^2 values[1]	Probability of that number or more[2]
	-------*Number*-------		
Corn	27	11	<0.01
Cotton	28	17	< .01
Kansas City wheat	27	22	< .01
Live cattle	34	15	< .01
Live hogs	37	26	< .01
Orange juice	33	27	< .01
Soybeans	38	15	< .01

[1]This is the number of t-values on individual contracts rejecting the null hypothesis of nonrandomness at a 95-percent confidence level or greater.

[2]Probability of having x or more significant values in N independent tests where the probability of significance is 0.05 for each test, N = number of tests (contracts), x = number of significant values observed, and i = an index. These probabilities were calculated using the binomial distribution:

$$P = \sum_{i=x}^{N} \binom{N}{i} .05^i .95^{N-i} = 1 - \sum_{i=0}^{x-1} \binom{N}{i} .05^i .95^{N-i}$$

Small values for P imply that price changes are not serially independent unless an unlikely event occurred.

The different contracts examined for each commodity contain observations drawn from overlapping time periods. For example, the January and March 1981 soybean contracts will both contain observations taken in December 1980. If events in December outside the market influence the January price, they may also influence expectations about the March contract price. If efficiency is falsely rejected for the January contract due to events outside that contract market, the probability that efficiency will be rejected for the March contract will be higher than independence suggests. Alternatively, if outside events prevent market inefficiency from showing up in the January contract, they are likely to have a similar effect on the March contract. Because the data are economic and not experimental, it is difficult to control for events outside the markets. Thus, the probabilities under the null hypothesis given by the binomial distribution most likely have some downward bias.

iance and seasonality in variance were common in these distributions (*1*). Fama and Roll noted that their low estimates of α might be caused by nonconstant variance (*12*). They suggested calculating sums of daily changes in the logarithm of price and calculating the characteristic exponents of the resulting distributions. If the values of α increased toward 2, the distribution was characterized by changing variance.

Conover gives a simple nonparametric test for inequality of variances; his test was applied to the futures prices in this study (*6*). The variance of the price changes from a 2-month period in the winter, January and February, was tested against that of a 2-month period in the summer, in most cases July and August. Most crop futures markets receive much greater supply information in the summer months than in the winter ones. The orange juice market, by contrast, has greater supply variability in the winter months. If there is changing variance, these two periods should show a great contrast in the crop futures. The livestock futures markets should show less seasonality, because livestock are produced and marketed year-round.

Table 5 shows the results of the nonparametric variance test. In each market a contract maturing in the fall and one in the spring were chosen. In the fall contracts, one would expect January and February to have lower variance because of Samuelson's maturity effect. In the spring contracts, January and February would have greater variance than the months from the previous summer if the maturity effect were the sole factor affecting the variance of percentage price changes. If there were no significantly greater variance in the summer months than in the winter months, the value of T_1 would be randomly distributed about 0. There may also be seasonality effects that are caused by demand shifts, but the contracts of this analysis were chosen only for possible seasonality in supply. January-February and July-August were chosen because those periods were likely to show a seasonality effect if one existed. Two-month periods were chosen to provide a sufficient number of observations to estimate the variance accurately and perform the inequality of variance test. Appendix table 5 displays the standard deviations of the 2-month segments for each of the seven commodities.

Nine of the 11 corn futures contracts showed significantly greater variance in the summer months, July-August, than the winter months, January-February (four of the five December contracts and five of six May contracts). Clearly the seasonal effect dominated the corn futures market recently. In no case was the variance of price changes greater in the winter months than in the summer months. The maturity effect is harder to detect in this market. The T_1 values of the spring (May) contracts are generally lower than those in the fall (December) contracts, suggesting that a maturity effect exists.

For cotton and soybeans the seasonality is less pronounced. Five of the 11 soybean contracts showed July-August variability to be significantly greater than the variability in January-February. Though the T_1 values in five of the six spring soybean contracts had negative signs, only one was significant at a 95-percent confidence level. The significant negative T_1 value in four of the five fall contracts is influenced by the maturity effect as well as the seasonality effect. The cotton market gave similar results. In all five fall contracts summer variance was greater than winter variance. But of the spring contracts, spring 1980 had significantly greater variance in the winter than in the summer. One of the spring contracts had significantly greater summer variance than winter variance. These results suggest that in this market seasonality is less important as a cause of variability than in the corn and

Table 4—Estimates of characteristic exponent

Commodity	Contracts tested	Number of contracts in which the characteristic exponent is—				
		$\alpha = 2$[1]	$1.75 \le \alpha < 2$	$1.5 \le \alpha < 1.75$	$1.0 \le \alpha < 1.5$	$\alpha < 1.0$
				Number		
Corn	27	1	12	11	3	0
Cotton	28	0	1	23	3	0
Kansas City wheat	27	0	2	17	8	0
Live cattle	34	6	9	15	4	0
Live hogs	37	0	2	28	7	0
Orange juice	33	0	2	9	22	0
Soybeans	38	1	4	30	3	0

[1]The characteristic exponent, α, of the family of symmetric stable distributions can range from 0 to 2. The only value associated with a finite variance distribution (the normal distribution) is $\alpha = 2$.

soybean markets. Time to maturity may play a more important role in the differing variance between time periods, than in the case of corn and soybeans.

For Kansas City wheat, January and February price changes were tested against May and June ones. Contrasting seasons are harder to pick in wheat, because wheat is grown over much of the year. Four of the eleven contracts showed significantly greater May-June variance than January-February variance, though one contract showed the opposite result. Ten of the 11 contracts had a negative sign though, suggesting that seasonality is of some importance in the wheat futures market.

Orange juice should produce the opposite sign for T_1-values. The months of greatest potential supply variability are the winter months when frost damage can significantly reduce the size of the Florida orange crop. Eight of the eleven orange juice contracts tested had significantly greater winter month variance than in the summer months. All contracts had greater winter variability (the T_1-values were all positive). A maturity effect is harder to oetect, as was the case with corn.

In the livestock markets seasonality should play a smaller role. Deviations from expected supply are less likely to be concentrated in one season than is the case with crops. In the cattle futures market there seems to be a slight tendency for greater summer variability in the fall contract and winter variability in the spring contract. In the hog futures market two of the five fall contracts show significantly greater variation in the summer than in the winter, and all five of the contracts maturing in the fall show greater variation in the summer months. Of the spring hog contracts, three of the five have greater variation in the winter months (near maturity) than in the summer months (far from maturity). Two of the three values are significant. There appears to be a stronger maturity effect in hogs than the other markets, but no seasonal effect.

The results of the seasonality of variance test show that seasonality is an important determinant of price variability in some markets. In others, Samuelson's maturity effect also leads to variances that differ significantly from one time period to another in a given futures contract.

Testing the 2-Month Segments for Normality

We have found that the distribution of percentage futures price changes is not normally distributed. The nonparametric tests of variance suggest reasonality and

Table 5—Seasonality of variance: Calculated values of the T_1 statistic for inequality of variance[1]

Contract maturing	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Soybeans
				T_1-value			
Fall 1979	−5.33*	−2.71*	−5.16*	−4.41*	−4.88*	0.56	−2.84*
Fall 1980	−4.08*	−2.33*	−1.14	− .10	− .03	2.04*	−4.06*
Fall 1981	−2.74*	−3.0[illegible]	2.10*	−1.90	− .42	3.50*	− .30
Fall 1982	−1.77	−4.33*	−1.71	2.08*	−1.82	4.65*	−2.28*
Fall 1983	−5.21*	−4.08*	− .97	− .78	−4.35*	6.43*	−4.84*
Spring 1979	−4.49*	1.52	−3.33*	−1.37	− .59	1.24	.32
Spring 1980	−3.42*	3.35*	−3.36*	− .95	− .48	1.40	−1.88
Spring 1981	−2.90*	−1.96*	− .15	− .01	2.52*	6.15*	−1.81
Spring 1982	−3.75*	−1.06	− .44	2.56*	2.35*	2.00*	−3.02*
Spring 1983	− .55	−1.81	−2.53*	.70	1.04	5.10*	−1.61
Spring 1984	−4.30*	−1.35	−1.92	NA	NA	4.95*	−1.67

*Shows values significant at a 95-percent or greater confidence level.
NA = Data not available at time of study.
[1]The value of T_1 statistic can be compared with the normal distribution for more than 10 observations in either of the January-February or July-August segments. Each 2-month group tested contained about 40 observations and the total number of observations for each test was between 80 and 85.

The fall contracts were July for Kansas City wheat, November for orange juice and soybeans, and December for corn, cotton, live cattle, and live hogs. The spring contracts were March for Kansas City wheat, April for live cattle and live hogs, and May for corn, cotton, orange juice, and soybeans.

The months tested were January and February versus July and August for all contracts except Kansas City wheat. The months tested for Kansas City wheat were January and February versus May and June.

a maturity effect in the variance of changes in the logarithm of price. Because so many statistical tests and procedures rely on the normality assumption, it is important to know if the distribution of price changes is normal within a specific season and at a given time to maturity. Tables 6 and 7 summarize the results of the turning point test, the difference-sign test, and the X^2 goodness-of-fit test for the 2-month periods for each futures market. Appendix tables 6-12 contain the individual test results. In none of the markets could the null hypothesis of randomness in price changes be rejected at a 95-percent confidence level. The results of the X^2 goodness-of-fit test sharply contrast with those displayed in table 3. In five of the seven markets, the null hypothesis of normality could not be rejected at a 95-percent confidence level. In the two markets which rejected normality, a lower percentage of contracts rejected normality. The results of the X^2 goodness-of-fit test suggest that the common assumption that percentage price changes are normally distributed is reasonable over relatively short periods of time.

A member of the family of symmetric stable distributions was fit to each contract. Table 8 summarizes the estimates of the characteristic exponent, α. The number of estimates having $\alpha = 2$ or $1.75 \leq \alpha < 2$ was much greater with the 2-month samples than when whole contracts were tested (table 4). The hypothesis of

Table 6—Tests of randomness for 2-month segments[1]

Commodity	Contracts	Turning-point test: Significant values[2]	Turning-point test: Probability of that number or more[3]	Difference-sign test: Significant values[2]	Difference-sign test: Probability of that number or more[3]
	----------Number----------			Number	
Corn	22	0	1.00	2	0.30
Cotton	22	3	.09	3	.09
Kansas City wheat	22	1	.68	2	.30
Live cattle	20	2	.26	2	.26
Live hogs	20	1	.64	2	.26
Orange juice	22	1	.68	0	1.00
Soybeans	22	1	.68	3	.09

[1]Because the 2-month segments do not overlap in time, the effect of outside events does not affect adjoining contracts. The assumption of independence between contracts is not violated with the 2-month segments.
[2]Number of values significant at a 95-percent confidence level.
[3]The probability is given by calculating the probability of a given number of rejections in a collection of samples using the binomial formula.

Table 7—X^2 goodness-of-fit test for 2-month segments [1]

Commodity	Contracts	$X^2 \leq 14.07$ [2]	$X^2 > 14.07$	Probability of that number or more [3]
	------------------------Number------------------------			
Corn	22	16	6	<0.01
Cotton	22	20	2	.30
Kansas City wheat	22	18	4	.02
Live cattle	20	19	1	.66
Live hogs	20	19	1	.66
Orange juice	22	21	1	.68
Soybeans	22	22	0	1.00

[1]The segments are July-August and January-February except for wheat, where the segments are January-February and April-May.
[2]The value of X^2 with 7 degrees of freedom at the 95-percent confidence level is 14.07.
[3]Because the 2-month segments do not overlap in time, the effect of outside events does not effect adjoining contracts. The assumption of independence between contracts is not violated with the 2-month segments. The probability is given by calculating the probability of a given number of rejections in a collection of samples using the binomial formula.

Table 8—Estimates of characteristic exponent for 2-month groups

Commodity	Contracts tested	Number of contracts in which the characteristic exponent is—				
		$\alpha = 2$[1]	$1.75 \leq \alpha < 2$	$1.5 \leq \alpha < 1.75$	$1.0 \leq \alpha < 1.5$	$\alpha < 1.0$
		Number				
Corn	22	9	5	4	4	0
Cotton	22	5	6	8	3	0
Kansas City wheat	22	3	3	8	8	0
Live cattle	20	10	2	5	3	0
Live hogs	20	6	2	8	4	0
Orange juice	22	4	4	5	9	0
Soybeans	22	9	6	4	3	0

[1]The characteristic exponent, α, of the family of symmetric stable distributions can range from 0 to 2. The only value associated with a finite variance distribution (the normal distribution) is $\alpha = 2$.

normality isn't as strongly supported by these data as by the results of the X^2 goodness-of-fit test, yet there are many more characteristic exponents equaling 2 than when whole contracts were considered. These tests suggest that normality in the percentage price changes of futures contracts is often a reasonable assumption when the periods analyzed occur during the same season at the same distance to maturity. Thus if one were to correct a time series model for seasonality and time to maturity effects, the result should better fit the underlying assumptions of the model.

Conclusions

In several agricultural futures markets, logarithmic price changes do not follow the normal distribution. That is, the common assumption that price changes are drawn from a distribution with 0 mean and finite and constant variance is not valid in these markets. In most markets, the author found no strong evidence of serial dependence or trend. The X^2 goodness-of-fit test, however, rejected normality in every market tested. The variance of the percentage price changes varied with the season and the time to maturity.

The author also examined the markets over shorter periods of time, within the summer and winter seasons. Again, nonparametric test of randomness confirmed the serial independence of daily changes in futures prices over 2-month intervals during a given season and a given time to maturity, implying that futures market prices are efficient. A X^2 normality test suggested that the 2-month segments within a specific season generally followed the normal distribution. Thus, the thick tails observed for the life-of-contract price distributions may result from combining mixtures of normal distributions with different variances.

Agricultural option pricing models should allow for changing variance over the life of the option contract. Also, those constructing or using economic models using futures prices should consider the nature of the distribution of daily price changes and note the effect of season and time to maturity on variance.

The author makes the following recommendations for analyzing futures market contracts:

- Persons using option pricing models of the Black-Scholes variety should consider including changing variance in their formulas. The assumption of constant variance means that the models will underprice options in high variance seasons and overprice options in times with low variance of price changes.

- Users of Autoregressive Integrated Moving Average (ARIMA) models and spectral analysis models need to correct for the nonstationary variance of percentage price changes before they can profitably apply those models to futures market prices. Researchers using models that assume stationary covariance must adjust the futures market data to eliminate nonstationary variance of percentage price changes before fitting those models.

- Those using econometric forecast and forecast accuracy models need to consider seasonality of variance. The forecast and actual outcome prices may each be drawn from distributions with changing variance if they are taken at different times of the year. Some of the outcome prices may come from a low variance distribution while others may come from a high variance distribution.

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Appendix table 1—Summary of turning-point test (full contract)[1]

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				t-values				
1979:								
Jan.	—	—	—	—	—	−1.30	—	−1.38
Feb.	—	—	—	1.50	0	—	—	—
Mar.	−1.14	1.67	−2.43*	—	—	−1.81	—	−.42
Apr.	—	—	—	1.43	−.78	—	—	—
May	−1.34	−1.01	−.59	—	—	−1.42	—	−.96
June	—	—	—	1.28	.43	—	—	—
July	−1.48	−.85	−3.12*	—	−.04	−.38	—	−.41
Aug.	—	—	—	2.04	.37	—	—	.57
Sept.	.56	—	−1.79	—	—	−1.49	—	.51
Oct.	—	.33	—	.69	1.26	—	—	—
Nov.	—	—	—	—	—	−1.12	—	.28
Dec.	1.12	.38	−.68	.35	.18	—	—	—
1980:								
Jan.	—	—	—	−.94	—	−.08	—	0
Feb.	—	—	—	.52	.09	—	—	—
Mar.	1.53	1.16	.36	—	—	.29	—	2.30*
Apr.	—	—	—	−.17	.43	—	—	—
May	2.04*	1.06	−.05	—	—	−.29	—	1.29
June	—	—	—	.76	2.34*	—	—	—
July	1.25	1.13	.15	—	.14	−.91	—	1.29
Aug.	—	—	—	1.39	1.30	—	—	1.14
Sept.	1.53	—	−.22	—	—	−.21	—	.81
Oct.	—	.30	—	.57	−.18	—	—	—
Nov.	—	—	—	—	—	.59	—	.74
Dec.	1.16	−1.14	.32	.74	1.00	—	—	—
1981:								
Jan.	—	—	—	1.13	—	−1.93	—	.98
Feb.	—	—	—	.59	.09	—	—	—
Mar.	.85	−2.58*	−.27	—	—	−.42	—	.41
Apr.	—	—	—	1.71	.83	—	—	—
May	−.19	−1.75	.70	—	—	−.67	−2.73*	−.74
June	—	—	—	1.12	−.09	—	—	—
July	−.28	−.51	1.78	—	−.50	−1.72	−1.83	.37
Aug.	—	—	—	.97	−.23	—	—	−.72
Sept.	.70	—	1.28	—	—	−.55	.68	.87
Oct.	—	−1.15	—	1.13	−1.12	—	—	—
Nov.	—	—	—	—	—	.81	−1.79	−.96
Dec.	−.94	−.04	−.25	−.54	−.66	—	—	—
1982:								
Jan.	—	—	—	—	—	.63	−1.44	−.51
Feb.	—	—	—	−2.19*	−1.62	—	—	—
Mar.	−1.21	−.50	−1.07	—	—	−.08	−1.82	−1.06
Apr.	—	—	—	−.56	.70	—	—	—
May	0	−.70	−1.61	—	—	.62	−3.10*	−.92
June	—	—	—	−.51	−.49	—	—	—
July	−1.11	.12	−.10	—	−.85	−.12	−.28	−.97
Aug.	—	—	—	−1.44	−.86	—	—	−.19
Sept.	−1.25	—	−.57	—	—	.29	.18	.38
Oct.	—	1.40	—	−.14	−1.40	—	—	—
Nov.	—	—	—	—	—	.25	−1.41	−.73
Dec.	−.04	2.22*	−.44	−.09	−1.62	—	—	—

See footnotes at end of table.

—Continued

Appendix table 1—Summary of turning-point test (full contract)[1]—Continued

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				t-values				
1983:								
Jan.	—	—	—	—	—	−0.21	−1.39	0.04
Feb.	—	—	—	−1.31	−1.39	—	—	—
Mar.	−0.14	1.24	−2.27*	—	—	−1.16	−1.58	.09
Apr.	—	—	—	−.38	−.62	—	—	—
May	−.88	.62	−.87	—	—	−1.12	−2.28*	1.00
June	—	—	—	−.29	.13	—	—	—
July	−.69	.62	1.19	—	−.67	.92	—	.04
Aug.	—	—	—	.52	.18	—	—	.18
Sept.	−1.52	—	1.06	—	—	.96	—	−.83
Oct.	—	−.78	—	.29	1.25	—	—	—
Nov.	—	—	—	—	—	1.86	—	−.09
Dec.	−.30	.86	.26	.82	.61	—	—	—
1984:								
Jan.	—	—	—	—	—	.33	—	−.65
Feb.	—	—	—	.50	−.52	—	—	—
Mar.	−.18	.99	−.56	—	—	.12	—	−.32
Apr.	—	—	—	.90	2.21*	—	—	—
May	−1.96	1.36	−.57	—	—	−1.77	—	−.58

* = The t-value is significant at a 95-percent or greater confidence level.
— = No contracts traded for that contract maturity month.
[1]The t-values shown were calculated for the null hypothesis that the number of turning points in the series is the same as that expected for a random series.

Appendix table 2—Summary of difference-sign test (full contract)[1]

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				t-values				
1979:								
Jan.	—	—	—	—	—	−1.25	—	−2.35*
Feb.	—	—	—	−1.67	−.30	—	—	—
Mar.	−1.60	−1.90	−1.33	—	—	−.46	—	−.20
Apr.	—	—	—	.10	−1.20	—	—	—
May	0	−1.67	−.11	—	—	.46	—	.31
June	—	—	—	−.48	−1.06	—	—	—
July	.20	−.84	−1.77	—	−.60	−.09	—	−.51
Aug.	—	—	—	−2.12*	−1.43	—	—	−.42
Sept.	.31	—	−.12	—	—	.46	—	0
Oct.	—	−.45	—	−2.17*	−.10	—	—	—
Nov.	—	—	—	—	—	2.10*	—	−.71
Dec.	−.59	−.19	−.25	−2.02*	−1.35	—	—	—
1980:								
Jan.	—	—	—	−.10	—	1.10	—	−.51
Feb.	—	—	—	0	−.20	—	—	—
Mar.	−1.96	−.73	−.92	—	—	−.55	—	.61
Apr.	—	—	—	0	1.83	—	—	—
May	.41	−1.30	−.78	—	—	−1.37	—	.51
June	—	—	—	−1.30	−2.55*	—	—	—
July	−.10	−1.16	.56	—	−2.80*	−.36	—	1.43
Aug.	—	—	—	−1.80	−1.53	—	—	.21
Sept.	.51	—	.73	—	—	.82	—	.64
Oct.	—	−.66	—	−.63	−.50	—	—	—
Nov.	—	—	—	—	—	.46	—	−.10
Dec.	−.10	−1.39	2.83*	−.99	−.38	—	—	—
1981:								
Jan.	—	—	—	−1.18	—	−1.49	—	.61
Feb.	—	—	—	−.98	1.44	—	—	—
Mar.	0	−3.31*	−.47	—	—	−.65	—	−.92
Apr.	—	—	—	.11	1.92	—	—	—
May	1.02	−1.54	.82	—	—	−1.67	−.50	−.20
June	—	—	—	−1.23	.29	—	—	—
July	2.14*	−.85	.67	—	.10	−1.76	−2.12*	−.10
Aug.	—	—	—	−2.91*	−2.55*	—	—	−.64
Sept.	.92	—	−.65	—	—	−1.01	−.83	−.43
Oct.	—	−1.13	—	−.76	−1.08	—	—	—
Nov.	—	—	—	—	—	−1.00	−1.16	.92
Dec.	.59	−.64	.87	−1.50	−.69	—	—	—
1982:								
Jan.	—	—	—	—	—	−1.02	−1.55	1.43
Feb.	—	—	—	−.73	−.50	—	—	—
Mar.	−.78	1.46	.94	—	—	−.18	−2.30*	−.82
Apr.	—	—	—	−.41	.87	—	—	—
May	.92	−.18	0	—	—	−.27	−1.21	.41
June	—	—	—	−2.55*	−.29	—	—	—
July	−.92	1.28	1.79	—	−1.30	1.83	−.43	.20
Aug.	—	—	—	−.64	−1.56	—	—	−.85
Sept.	.10	—	−.63	—	—	−1.00	−1.66	−.21
Oct.	—	−.18	—	−1.15	−2.40*	—	—	—
Nov.	—	—	—	—	—	−.18	−1.44	.47
Dec.	.49	.09	−.32	−1.56	−2.69*	—	—	—

See footnotes at end of table.

—Continued

Appendix table 2—Summary of difference-sign test (full contract)[1]—Continued

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				t-values				
1983:								
Jan.	—	—	—	—	—	0.55	0.28	0.10
Feb.	—	—	—	–1.25	–2.98*	—	—	—
Mar.	–0.50	0.36	–1.26	—	—	– .18	–2.00*	– .19
Apr.	—	—	—	–1.36	– .58	—	—	—
May	–1.22	.55	– .22	—	—	.64	1.00	.38
June	—	—	—	– .96	– .87	—	—	—
July	– .82	1.10	.34	—	– .49	.09	—	– .10
Aug.	—	—	—	– .94	0	—	—	.78
Sept.	– .41	—	– .48	—	—	–2.01*	—	– .20
Oct.	—	.81	—	– .21	.10	—	—	—
Nov.	—	—	—	—	—	–1.64	—	–1.10
Dec.	1.13	1.55	1.84	– .12	–1.06	—	—	—
1984:								
Jan.	—	—	—	—	—	– .36	—	– .41
Feb.	—	—	—	– .56	.38	—	—	—
Mar.	– .29	.09	.89	—	—	–1.83	—	–1.40
Apr.	—	—	—	– .56	.87	—	—	—
May	1.32	1.00	2.61*	—	—	– .36	—	– .69

* = The t-value is significant at a 95-percent or greater confidence level.
— = No contracts traded for that contract maturity month.
[1]The t-values shown were calculated for the null hypothesis that the number of positive changes equals the number of negative changes.

Appendix table 3—Summary of X^2 goodness-of-fit test

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				X^2 values				
1979:								
Jan.	—	—	—	—	—	13.72	—	9.52
Feb.	—	—	—	9.70	16.94*	—	—	—
Mar.	18.84*	8.23	14.95*	—	—	14.02	—	10.07
Apr.	—	—	—	8.26	17.42*	—	—	—
May	13.04	19.17*	26.47*	—	—	12.33	—	7.30
June	—	—	—	6.74	17.98*	—	—	—
July	14.27*	11.99	46.44*	—	13.49	6.25	—	6.63
Aug.	—	—	—	14.08*	15.29*	—	—	9.61
Sept.	25.74*	—	47.41*	—	—	9.57	—	10.47
Oct.	—	6.27	—	13.66	22.97*	—	—	—
Nov.	—	—	—	—	—	9.00	—	6.70
Dec.	22.94*	26.29*	21.67*	13.27	21.12*	—	—	—
1980:								
Jan.	—	—	—	14.58*	—	27.52*	—	18.38*
Feb.	—	—	—	13.00	31.80*	—	—	—
Mar.	34.54*	26.38*	26.12*	—	—	36.67*	—	21.42*
Apr.	—	—	—	25.71*	31.29*	—	—	—
May	15.49*	27.55*	37.55*	—	—	40.54*	—	27.34*
June	—	—	—	14.66*	15.98*	—	—	—
July	18.07*	19.82*	11.08	—	7.20	38.79*	—	8.40
Aug.	—	—	—	11.26	4.09	—	—	18.13*
Sept.	9.71	—	21.84*	—	—	38.94*	—	31.40*
Oct.	—	48.33*	—	4.33	14.65*	—	—	—
Nov.	—	—	—	—	—	55.83*	—	29.27*
Dec.	11.46	36.07*	20.99*	4.62	11.30	—	—	—
1981:								
Jan.	—	—	—	23.91*	—	135.52*	—	14.01
Feb.	—	—	—	7.01	6.12	—	—	—
Mar.	5.43	11.18	25.83*	—	—	95.24*	—	12.34
Apr.	—	—	—	2.81	20.29*	—	—	—
May	7.45	6.06	27.96*	—	—	73.77*	99.62*	10.41
June	—	—	—	9.57	11.83	—	—	—
July	11.47	6.60	13.75	—	9.09	48.01*	111.14*	14.30*
Aug.	—	11.51	—	11.17	18.79*	—	—	9.70
Sept.	2.38	—	28.69*	—	—	15.14*	36.74*	16.12*
Oct.	—	12.34	—	5.29	31.29*	—	—	—
Nov.	—	—	—	—	—	35.51*	12.68*	8.93
Dec.	3.71	20.19*	30.22*	19.95*	14.71*	—	—	—
1982:								
Jan.	—	—	—	—	—	64.14*	36.26*	18.57*
Feb.	—	—	—	20.03*	11.40	—	—	—
Mar.	7.15	30.76*	6.20	—	—	36.96*	61.10*	16.39*
Apr.	—	—	—	15.26*	7.10	—	—	—
May	14.88*	19.23*	16.91*	—	—	27.98*	67.08*	9.42
June	—	—	—	7.31	45.39*	—	—	—
July	5.40	26.64*	5.43	—	15.75*	33.08*	160.34*	15.14*
Aug.	—	—	—	18.19*	11.37	—	—	14.36*
Sept.	11.29	—	27.24*	—	—	23.41*	51.65*	7.55
Oct.	—	25.80*	—	20.78*	20.07*	—	—	—
Nov.	—	—	—	—	—	38.37*	113.88*	9.53
Dec.	2.23	15.03*	16.56*	21.55*	26.96*	—	—	—

See footnotes at end of table.

—Continued

Appendix table 3—Summary of X^2 goodness-of-fit test—Continued

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Rough rice	Soybeans
				X^2 values				
1983:								
Jan.	—	—	—	—	—	57.93*	133.74*	6.49
Feb.	—	—	—	20.33*	17.39*	—	—	—
Mar.	4.98	18.36*	26.73*	—	—	56.23*	117.30*	6.31
Apr.	—	—	—	13.02	32.23*	—	—	—
May	4.37	22.00*	19.62*	—	—	55.95*	105.30*	6.84
June	—	—	—	6.46	13.64	—	—	—
July	4.31	28.89*	21.98*	—	14.51*	40.94*	—	12.53
Aug.	—	—	—	18.31*	19.07*	—	—	11.93
Sept.	10.98	—	53.42*	—	—	54.11*	—	19.68*
Oct.	—	25.31*	—	7.34	43.68*	—	—	—
Nov.	—	—	—	—	—	91.72*	—	17.71*
Dec.	14.49*	11.51	36.41*	19.08*	30.31*	—	—	—
1984:								
Jan.	—	—	—	—	—	127.23*	—	15.63*
Feb.	—	—	—	16.32*	40.97*	—	—	—
Mar.	16.41*	9.51	32.15*	—	—	115.59*	—	13.67
Apr.	—	—	—	11.75	25.39*	—	—	—
May	21.92*	12.69	—	—	—	93.57*		10.93

* = The t-value is significant at a 95-percent or greater confidence level.
— = No contracts traded for that contract maturity month.

Appendix table 4—Summary of characteristic exponent estimates (full contract)[1]

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Soybeans
				Value of characteristic exponent			
1979:							
Jan.	—	—	—	—	—	1.74	1.73
Feb.	—	—	—	1.56	1.60	—	—
Mar.	1.63	1.66	1.55	—	—	1.82	1.70
Apr.	—	—	—	1.70	1.62	—	—
May	1.72	1.70	1.53	—	—	1.72	1.85
June	—	—	—	1.93	1.51	—	—
July	1.73	1.87	1.30	—	1.65	1.75	1.73
Aug.	—	—	—	2.00	1.58	—	1.65
Sept.	1.39	—	1.23	—	—	1.73	1.63
Oct.	—	1.72	—	2.00	1.46	—	—
Nov.	—	—	—	—	—	1.73	1.67
Dec.	1.43	1.61	1.38	1.84	1.49	—	—
1980:							
Jan.	—	—	—	1.98	—	1.42	1.58
Feb.	—	—	—	2.00	1.42	—	—
Mar.	1.48	1.54	1.48	—	—	1.42	1.65
Apr.	—	—	—	2.00	1.51	—	—
May	1.64	1.43	1.44	—	—	1.50	1.64
June	—	—	—	2.00	1.56	—	—
July	1.51	1.63	1.67	—	1.69	1.50	1.60
Aug.	—	—	—	2.00	1.69	—	1.55
Sept.	1.75	—	1.71	—	—	1.52	1.54
Oct.	—	1.42	—	1.91	1.55	—	—
Nov.	—	—	—	—	—	1.58	1.51
Dec.	1.76	1.33	1.72	1.75	1.69	—	—
1981:							
Jan.	—	—	—	1.49	—	1.47	1.51
Feb.	—	—	—	1.70	1.64	—	—
Mar.	1.78	1.64	1.53	—	—	1.23	1.56
Apr.	—	—	—	1.92	1.58	—	—
May	1.81	1.74	1.47	—	—	1.23	1.67
June	—	—	—	1.67	1.74	—	—
July	1.66	1.69	1.72	—	1.76	1.33	1.59
Aug.	—	—	—	1.59	1.79	—	1.61
Sept.	1.84	—	1.67	—	—	1.55	1.68
Oct.	—	1.54	—	1.77	1.63	—	—
Nov.	—	—	—	—	—	1.45	1.64
Dec.	1.91	1.59	1.53	1.62	1.67	—	—
1982:							
Jan.	—	—	—	—	—	1.35	1.62
Feb.	—	—	—	1.88	1.68	—	—
Mar.	1.98	1.59	1.61	—	—	1.38	1.62
Apr.	—	—	—	1.60	1.67	—	—
May	1.90	1.55	1.53	—	—	1.49	1.68
June	—	—	—	1.70	1.59	—	—
July	1.76	1.65	1.67	—	1.49	1.45	1.65
Aug.	—	—	—	1.57	1.72	—	1.80
Sept.	1.71	—	1.66	—	—	1.53	1.75
Oct.	—	1.55	—	1.48	1.62	—	—
Nov.	—	—	—	—	—	1.43	1.70
Dec.	2.00	1.69	1.77	1.42	1.53	—	—

See footnotes at end of table.

—Continued

Appendix table 4—Summary of characteristic exponent estimates (full contract)[1]—Continued

Contract maturity month	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Soybeans
	Value of characteristic exponent						
1983:							
Jan.	—	—	—	—	—	1.44	1.74
Feb.	—	—	—	1.43	1.60	—	—
Mar.	1.98	1.61	1.79	—	—	1.43	1.84
Apr.	—	—	—	1.53	1.50	—	—
May	1.87	1.57	1.65	—	—	1.40	2.00
June	—	—	—	1.67	1.59	—	—
July	1.88	1.64	1.69	—	1.65	1.46	1.90
Aug.	—	—	—	1.61	1.55	—	1.58
Sept.	1.68	—	1.48	—	—	1.37	1.39
Oct.	—	1.52	—	1.89	1.36	—	—
Nov.	—	—	—	—	—	1.35	1.46
Dec.	1.66	1.51	1.48	1.56	1.39	—	—
1984:							
Jan.	—	—	—	—	—	1.41	1.47
Feb.	—	—	—	1.52	1.38	—	—
Mar.	1.55	1.66	1.62	—	—	1.21	1.55
Apr.	—	—	—	1.60	1.52	—	—
May	1.54	1.64	1.64	—	—	1.26	1.61

— = No contracts traded for that contract maturity month.
[1]The characteristic exponent, α, of the family of symmetric stable distributions can range from 0 to 2. The only value associated with a finite variance distribution (the normal distribution) is $\alpha = 2$.

Appendix table 5—Seasonality of variance: Standard deviations of changes in log of price for 2-month segments

Contract	Segment	Corn	Cotton	Kansas City wheat	Live cattle	Live hogs	Orange juice	Soybeans
				Standard deviations				
Fall 1979	Jan.-Feb.	0.0052	0.0051	0.0078	0.0089	0.0078	0.0155	0.0093
	July-Aug.	.0185	.0083	.0055	.0178	.0217	.0124	.0168
Fall 1980	Jan.-Feb.	.0088	.0137	.0074	.0111	.0115	.0118	.0121
	July-Aug.	.0151	.0169	.0207	.0110	.0166	.0086	.0220
Fall 1981	Jan.-Feb.	.0100	.0060	.0087	.0076	.0127	.0332	.0136
	July-Aug.	.0144	.0206	.0125	.0099	.0141	.0165	.0149
Fall 1982	Jan.-Feb.	.0089	.0045	.0061	.0103	.0116	.0224	.0079
	July-Aug.	.0126	.0106	.0150	.0074	.0142	.0069	.0113
Fall 1983	Jan.-Feb.	.0081	.0054	.0054	.0067	.0059	.0138	.0091
	July-Aug.	.0197	.0121	.0128	.0096	.0151	.0023	.0245
Spring 1979	Jan.-Feb.	.0049	.0105	.0076	.0101	.0117	.0178	.0146
	July-Aug.	.0125	.0078	.0054	.0132	.0136	.0136	.0114
Spring 1980	Jan.-Feb.	.0106	.0147	.0082	.0123	.0130	.0146	.0134
	July-Aug.	.0175	.0082	.0115	.0136	.0154	.0096	.0161
Spring 1981	Jan.-Feb.	.0100	.0120	.0047	.0092	.0168	.0310	.0169
	July-Aug.	.0142	.0155	.0128	.0098	.0110	.0077	.0215
Spring 1982	Jan.-Feb.	.0065	.0067	.0064	.0111	.0160	.0232	.0077
	July-Aug.	.0131	.0084	.0194	.0080	.0118	.0144	.0143
Spring 1983	Jan.-Feb.	.0103	.0079	.0043	.0072	.0111	.0157	.0093
	July-Aug.	.0116	.0086	.0107	.0071	.0095	.0056	.0112
Spring 1984	Jan.-Feb.	.0095	.0085	.0063	NA	NA	.0129	.0170
	July-Aug.	.0191	.0105	.0118	NA	NA	.0028	.0220

NA = Data not available at time of study.

Appendix table 6—Corn contracts: Statistics on 2-month segments

Statistic	December contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	0.0017	−0.0051	0.0004	−0.0005	0.0014	−0.0061	0.0022	−0.0068	0.0014	0.0086
Scale c[1]	.0053	.0151	.0040	.0120	.0078	.0117	.0052	.0087	.0062	.0191
Characteristic exponent α[1]	2.0000	2.0000	1.2400	2.0000	1.9800	2.0000	1.4900	1.6900	1.8600	2.0000
X^2[2]	15.6200	3.2800	6.3800	6.5700	7.9200	11.8100	5.6700	2.2900	2.2800	9.0000
Studentized range[3]	4.1700	3.8900	5.7300	4.0000	4.5200	3.9900	4.5200	4.8900	4.7400	4.4400
Turning-point test t-value[4]	−.9000	1.4600	1.4100	−.4900	.6400	.1200	−.5300	.2500	1.0200	−.6200
Difference-sign test t-value[4]	−1.1500	−.5800	.5800	−2.0200	−1.7300	.5800	−.2900	.2900	.0000	.0000

Statistic	May contract, segment ending											
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982	Feb. 1984	Aug. 1983
Location δ[1]	0.0009	−0.0023	−0.0007	−0.0044	−0.0007	−0.0009	0.0001	−0.0047	0.0041	−0.0055	−0.0011	0.0075
Scale c[1]	.0039	.0092	.0068	.0160	.0073	.0118	.0043	.0092	.0069	.0081	.0028	.0168
Characteristic exponent α[1]	2.0000	1.8310	1.4130	2.0000	1.6640	2.0000	1.7140	1.7810	1.6470	1.7790	1.0210	2.0000
X^2[2]	2.5000	7.0500	15.1000	14.4400	9.4600	8.0240	7.7890	14.6700	8.4360	8.9520	18.6900	14.8500
Studentized range[3]	3.8400	5.1400	5.4700	3.8500	4.5800	3.8910	3.9290	4.2390	4.7040	4.4650	5.9600	4.6800
Turning-point test t-value[4]	−1.5200	−1.3700	1.4100	1.8300	−.5100	.5000	−1.0400	−.1200	.2600	.2500	−.9000	−.6200
Difference-sign test t-value[4]	1.4400	−2.3100	0	.5800	−.5800	−1.7300	.8700	.2900	−.5800	.2900	−1.1500	.5300

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 7—Cotton contracts: Statistics on 2-month segments

Statistic	December contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	0.0013	0.0018	0.0034	0.0027	−0.0024	−0.0043	−0.0004	−0.0042	−0.0007	0.0022
Scale c[1]	.0031	.0062	.0096	.0173	.0039	.0080	.0035	.0076	.0038	.0078
Characteristic exponent α[1]	1.4900	1.7400	1.6800	2.0000	1.5900	2.0000	1.9000	1.7900	1.8800	1.6200
X^2[2]	7.5000	4.6700	12.0000	26.0000	7.4100	8.0000	15.6800	8.5100	6.3800	6.0700
Studentized range[3]	4.8710	5.2300	4.4000	3.0200	4.7200	4.5700	4.7200	4.5300	4.0300	3.9900
Turning-point test t-value[4]	− .3800	1.7200	− .1500	0	− .900	− .1200	.5200	3.1200	− .9000	1.2500
Difference-sign test t-value[4]	2.900	− .2900	− .3500	−2.0200	−1.5500	.2900	− .8700	− .5800	.5800	1.1500

Statistic	May contract, segment ending											
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982	Feb. 1984	Aug. 1983
Location δ[1]	−0.0001	0.0020	0.0050	0.0021	−0.0030	0.0018	−0.0013	−0.0041	−0.0004	−0.0038	−0.0016	0.0020
Scale c[1]	.0065	.0045	.0100	.0057	.0097	.0144	.0051	.0066	.0048	.0073	.0059	.0071
Characteristic exponent α[1]	1.5000	1.4200	1.8600	1.6800	2.0000	2.0000	1.7800	1.8500	1.4500	2.0000	1.6700	1.7500
X^2[2]	5.8700	13.8800	3.2700	2.6600	2.7900	11.0000	3.5800	12.9000	4.3300	5.5000	9.5000	13.0000
Studentized range[3]	5.0600	5.0600	3.9500	4.3100	3.7800	3.2800	4.2300	4.8100	5.4900	4.7800	5.0100	4.0700
Turning-point test t-value[4]	− .9000	.1200	.0000	.8700	− .9000	−1.6600	.5200	−1.0000	−1.6600	2.6500	− .3800	2.2700
Difference-sign test t-value[4]	−2.3100	0	0	.5800	1.1500	−2.8900	.8700	−1.1500	0	− .2900	− .2900	1.4400

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 8—Kansas City wheat contracts: Statistics on 2-month segments

Statistic	July contract, segment ending									
	Feb. 1979	June 1979	Feb. 1980	June 1980	Feb. 1981	June 1981	Feb. 1982	June 1982	Feb. 1983	June 1983
Location δ[1]	– 0.0002	0.0073	0.0002	– 0.0022	– 0.0031	– 0.0008	– 0.0010	– 0.0017	0.0016	– 0.0008
Scale c[1]	.0038	.0137	.0071	.0094	.0080	.0072	.0037	.0080	.0024	.0037
Characteristic exponent α[1]	1.8100	1.6600	1.5100	1.5300	1.5800	2.0000	1.3500	2.0000	1.1200	1.6800
X^2 [2]	4.5000	8.4800	12.5400	12.9000	8.9500	9.4900	5.6800	7.5400	25.3600	2.7600
Studentized range[3]	4.3400	4.6400	6.1000	5.3900	4.5500	3.9300	4.5000	3.9900	5.8800	4.5300
Turning-point test t-value[4]	0	– .8600	1.0200	2.0000	.6400	.8700	– 2.7600	.1200	1.3000	1.2500
Difference-sign test t-value[4]	1.4400	– .8700	.5800	– 1.7300	1.1500	1.7300	1.7300	.5800	– .8700	1.7300

Statistic	March contract, segment ending											
	Feb. 1979	June 1978	Feb. 1980	June 1979	Feb. 1981	June 1980	Feb. 1982	June 1981	Feb. 1983	June 1982	Feb. 1984	June 1983
Location δ[1]	– 0.0001	– 0.0003	– 0.0013	0.0074	– 0.0040	– 0.0018	– 0.0012	– 0.0007	0.0016	– 0.0024	– 0.0020	– 0.0025
Scale c[1]	.0036	.0066	.0068	.0153	.0061	.0067	.0034	.0050	.0016	.0046	.0027	.0052
Characteristic exponent α[1]	1.6900	1.5200	1.3800	1.8600	1.4000	1.4400	1.3100	1.4600	1.0700	2.0000	1.5000	1.9700
X^2 [2]	3.5000	5.6000	13.0500	6.1000	8.9500	15.3400	8.3200	10.3000	19.2100	8.0200	8.0000	19.9000
Studentized range[3]	4.7300	4.3400	6.7400	4.7000	5.1900	6.0000	4.4300	5.0700	4.6400	3.6300	4.6200	5.1800
Turning-point test t-value[4]	.7600	– .3700	.6400	– .8600	– 1.2800	1.2500	– 1.4300	.3900	0	– 1.8900	– 0.3800	– 0.8600
Difference-sign test t-value[4]	– .2900	1.1500	.5800	– 3.1800	0	– 1.1500	.2900	.5800	– 2.3100	.8700	.8700	1.4400

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 9—Live cattle contracts: Statistics on 2-month segments

Statistic	December contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	0.0012	−0.0001	0.0011	0.0007	−0.0011	0.0010	−0.0004	0.0009	0.0013	0.0002
Scale c[1]	.0054	.0162	.0080	0097	.0058	.0079	.0079	.0038	.0050	.0044
Characteristic exponent α[1]	1.5300	2.0000	2.0000	2.0000	1.9600	2.0000	2.0000	1.4000	1.7400	1.2200
X^2[2]	11.000	8.3950	9.000	2.7600	6.9000	12.2900	9.8900	5.14000	4.1600	5.6200
Studentized range[3]	5.0700	3.9900	3.5400	3.6200	4.6400	3.8100	4.5400	5.3700	4.4400	4.9100
Turning-point test t-value[4]	.7600	− .7300	1.2500	− .4900	−2.4300	−1.2300	.5200	− .8600	.4300	2.0900
Difference-sign test t-value[4]	.8700	1.1500	1.1500	1.4400	−1.7300	− .2900	.2900	−2.0200	1.4100	.2900

Statistic	June contract, segment ending									
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982
Location δ[1]	0.0009	−0.0011	0.0015	−0.0004	−0.0010	0.0001	0.0006	−0.0008	0.0008	0.0004
Scale c[1]	.0082	.0089	.0099	.0096	.0079	.0080	.0099	.0054	.0052	.0042
Characteristic exponent α[1]	1.9500	1.7000	2.0000	2.0000	2.0000	2.0000	2.0000	1.5900	1.6700	1.4000
X^2[2]	3.0000	5.5900	9.0000	14.6700	5.8700	8.0000	9.8900	5.1000	12.5400	7.5400
Studentized range[3]	4.2300	4.0200	3.4000	3.3200	4.1900	3.5900	3.4600	4.4500	5.1500	5.6700
Turning-point test t-value[4]	.7600	2.0000	.8700	− .4900	− .1300	1.1400	− .2600	− .6200	1.0200	−1.3700
Difference-sign test t-value[4]	2.0200	0	1.1500	−1.4400	−1.7300	1.4400	− .2900	−2.3100	0	1.7300

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 10—Live hog contracts: Statistics on 2-month segments

Statistic	December contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	0.0030	0.0024	0.0006	0.0016	0.0016	0.0013	−0.0007	0.0015	−0.0007	0.0053
Scale c[1]	.0051	.0158	.0079	.0136	.0102	.0086	.0069	.0129	.0034	.0108
Characteristic exponent α[1]	1.5900	1.8400	1.6300	2.0000	2.0000	1.5400	1.4800	2.0000	1.4000	1.8800
X^2 [2]	2.5000	7.4700	4.6100	8.4800	11.0000	8.4800	6.2100	13.7100	14.5900	5.6200
Studentized range[3]	4.4400	3.9100	5.2300	4.0500	4.1300	4.1700	4.8800	3.5700	6.1000	4.4900
Turning-point test t-value[4]	.7600	−1.1000	.1200	.6200	−.5100	−.4900	.5200	−1.6000	1.4100	.6200
Difference-sign test t-value[4]	.2900	−2.3100	−.5800	−.2900	1.7300	−.8700	−1.4400	−1.4400	−1.7300	1.4400

Statistic	June contract, segment ending									
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982
Location δ[1]	0.0038	0.0013	−0.0016	0.0009	0.0016	0.0018	−0.0006	−0.0016	−0.0020	−0.0015
Scale c[1]	.0081	.0077	.0096	.0089	.0137	.0087	.0131	.0069	.0074	.0059
Characteristic exponent α[1]	1.6900	1.5700	1.6800	1.4800	2.0000	2.0000	2.0000	1.4800	1.7400	1.5500
X^2 [2]	3.0000	5.1000	8.5100	3.7100	7.4100	11.9300	4.6300	3.6300	4.3300	5.1000
Studentized range[3]	4.3400	3.7300	4.3700	4.2300	3.4800	4.1500	3.7400	4.4800	4.0600	4.1600
Turning-point test t-value[4]	2.6500	1.2500	1.6200	.2500	1.0200	−.2500	1.3000	.5000	1.4100	.5000
Difference-sign test t-value[4]	−.8700	.5800	−1.1500	−2.6000	.5800	.5800	−.2900	.5800	−1.1500	−1.7300

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.
[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.
[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.
[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 11—Orange juice contracts: Statistics on 2-month segments

Statistic	November contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	−0.0026	0.0017	0.0031	0.0009	0.0127	−0.0005	−0.0066	0.0009	−0.0060	0.0005
Scale c[1]	.0076	.0088	.0086	.0051	.0240	.0117	.0132	.0045	.0101	.0016
Characteristic exponent α[1]	1.3300	2.0000	1.8300	1.4700	1.7800	1.8800	1.4700	1.6300	1.8000	1.4400
X^2[2]	9.5000	6.0900	13.5600	3.0500	12.0300	3.2400	5.6800	3.1500	3.5800	3.2400
Studentized range[3]	5.2600	3.6600	4.3500	5.3800	4.9800	4.4800	4.6900	4.5300	4.7900	5.0000
Turning-point test t-value[4]	.3800	− .1200	1.4100	.5200	.6400	.2500	− 1.0400	.5000	2.0700	0.9900
Difference-sign test t-value[4]	− 1.4400	.2900	1.7300	− .2900	− .5800	− .2900	.2900	0	− .2900	− .8700

Statistic	May contract, segment ending											
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982	Feb. 1984	Aug. 1983
Location δ[1]	−0.0025	−0.0001	−0.0046	0.0023	0.0125	0.0001	−0.0079	0.0015	−0.0065	−0.0002	−0.0001	0.0009
Scale c[1]	.0099	.0089	.0084	.0075	.0262	.0045	.0135	.0109	.0103	.0039	.0062	.0013
Characteristic exponent α[1]	1.4900	1.6000	1.4300	2.0000	2.0000	1.4800	1.4700	2.0000	1.6300	1.7200	1.3300	1.6000
X^2[2]	14.0000	4.5000	13.5000	8.0200	3.8200	3.8200	7.2600	7.5400	3.8200	10.0000	10.5000	15.3400
Studentized range[3]	4.4300	4.2300	4.5600	3.8200	3.8300	5.1600	4.6400	4.2600	4.6300	5.0200	4.6000	5.0900
Turning-point test t-value[4]	.3800	.3800	.2600	− .2500	1.4100	− .9000	− 1.0400	.5000	.2600	.3800	1.1400	− .2500
Difference-sign test t-value[4]	− .2900	.8700	0	1.1500	− .5800	− .5800	.2900	− .5800	0	− .8700	.2900	− 1.1500

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

Appendix table 12—Soybean contracts: Statistics on 2-month segments

Statistic	November contract, segment ending									
	Feb. 1979	Aug. 1979	Feb. 1980	Aug. 1980	Feb. 1981	Aug. 1981	Feb. 1982	Aug. 1982	Feb. 1983	Aug. 1983
Location δ[1]	0.0010	−0.0028	−0.0010	−0.0035	−0.0007	−0.0042	0.0002	−0.0049	0.0019	0.0137
Scale c[1]	.0078	.0116	.0069	.0200	.0092	.0090	.0065	.0096	.0055	.0202
Characteristic exponent α[1]	2.0000	1.7600	1.4000	2.0000	1.6100	1.6000	2.0000	2.0000	1.5000	2.0000
X^2[2]	3.0000	.9500	8.9500	7.0000	9.4600	4.1900	5.6700	6.1000	5.8700	8.5100
Studentized range[3]	4.0300	4.4600	6.1000	3.4400	4.2600	4.8300	4.0700	4.1900	4.6000	4.1200
Turning-point test t-value[4]	.3800	1.4600	1.4100	0	−.9060	.6200	−2.1300	−.1200	1.3000	−1.3700
Difference-sign test t-value[4]	1.4400	−.5800	−.5800	−.5800	−.5800	.2900	−2.0200	.2900	1.4000	0

Statistic	May contract, segment ending											
	Feb. 1979	Aug. 1978	Feb. 1980	Aug. 1979	Feb. 1981	Aug. 1980	Feb. 1982	Aug. 1981	Feb. 1983	Aug. 1982	Feb. 1984	Aug. 1983
Location δ[1]	0.0019	0.0032	−0.0015	−0.0021	−0.0025	−0.0030	−0.0012	−0.0036	0.0005	−0.0049	−0.0041	0.0122
Scale c[1]	.0083	.0078	.0082	.0114	.0133	.0202	.0057	.0103	.0065	.0098	.0113	.0183
Characteristic exponent α[1]	1.4800	1.8000	1.4600	1.7500	2.0000	2.0000	1.9400	1.7900	1.7700	2.0000	1.6300	2.0000
X^2[2]	4.5000	4.2100	10.4900	4.6700	11.0000	3.2800	10.4200	3.2400	2.7900	6.1000	5.8700	10.4600
Studentized range[3]	5.1000	4.2100	5.7200	4.2000	4.4700	3.4400	4.0300	4.9600	4.3500	4.0800	4.4600	4.1100
Turning-point test t-value[4]	.7600	.6200	1.4100	1.4600	.2600	0	−.6500	.6200	1.7900	−.8600	−.9000	−.6200
Difference-sign test t-value[4]	−1.4400	−.8700	0	0	0	−1.1600	−2.0200	1.4400	0	.8700	0	−2.3100

[1]The location δ, scale c, and characteristic exponent α are the parameters estimated for members of the family of symmetric stable distributions.

[2]X^2 is the value of X^2 statistic in the goodness-of-fit test. Values greater than 14.07 reject the null hypothesis of normality at the 95-percent confidence level.

[3]The Studentized range is a measure of the dispersion of the data. It is the range of the data divided by its standard deviation.

[4]The turning-point and difference-sign test t-values are nonparametric tests used to examine randomness and trend in the data.

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