Comparing Measures of Productivity growth for Australian Broadacre Agriculture[1]

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Abstract:

Rarely have comparisons of the methodologies to measure Australian agricultural Total Factor Productivity (tfp) been presented with empirical examples. Mullen and Cox (1995) provided an insight into the difference between the translog cost, Tornqvist-Theil, scale adjusted Tornqvist-Theil and non parametric methodologies in measuring productivity for farms with greater than 200 sheep from 1953 to 1988. This paper looks at Australian broadacre agriculture for the period 1977-78 to 1993-94 using a wide range of methodologies including the Fisher, Tornqvist-Theil, Laspeyres and Paasche indices and the Chavas and Cox and Malmquist non parametric approaches. An indication of the upper and lower bounds of productivity growth for Australian broadacre agriculture will be provided by the analysis

[1] Many thanks to Mark Eigenraam, Shawna Grosskopf and Paul Cashin. Any mistakes that remain are solely those of the authors

1.0 Introduction.

Broadacre agriculture has been an important source of economic growth in Australia. An important component of government policy in agriculture has been to foster economic growth by investment in research and extension programs. In an attempt to monitor the performance of agriculture with respect to other industries and the agricultural sectors of other countries there have been a series of studies of productivity growth.

Total Factor productivity estimates for Australian agriculture have traditionally been measured by using the Tornqvist-Theil index procedure. Two well known examples are Lawrence and McKay (1980) and Males et al (1990). The former analysed productivity for farms in the Sheep industry using ABARE survey data from 1952-53 to 1976-77. They found productivity growth to average 2.9% per annum. The latter analysed productivity for all broadacre farms from 1977-78 to 1988-89 and found an annual 2.2% rate of productivity growth.

Some studies in both Australia an d other countries have attempted the next step of relating productivity growth to investments in research and extension. In an Australian context Mullen and Cox (1995) analysed the effect of agricultural research and extension expenditures on Australian broadacre productivity growth. The internal rate of return to agricultural research was found to lie between 15 and 40%. A recent US study is that of Alston *et al* (1994) which found the internal rate of return to Californian public investment in agricultural research for 1949-85 to be approximately 20%.

There is continuing interest in the measurement of productivity. This reflects, *inter alia*, a renewed interest in the causes of economic growth and some concerns with restrictive assumptions about technology implied by the Tornqvist-Theil (TT) approach. For example, Caves *et al*. (1982a) have shown the TT index to be superlative and exact for constant returns to scale, translog transformation functions with constant second order coefficients (across time and/or across firms). To the extent that these maintained hypotheses are not supported by the data, the TT tfp index is likewise potentially biased.

Recently, interest in the measurement of productivity growth has taken two directions. First, alternative index number procedures have been re-examined. Diewert recommends the Fisher index because it satisfies what Fisher (1923) called the factor reversal test (see below). However, the Fisher index will implicitly impose a constant returns to scale quadratic functional form. The second approach has been to develop nonparametric measures, these allow the investigator to get away from some of the restrictions on the nature of technology imposed by traditional productivity indices and econometric models.

Some work has already been done by Mullen and Cox (1995) in examining the differences between the alternative methodologies. They used ABARE data from broadacre farms with greater than 200 sheep and the data period extended from 1953 to 1988. The use of standard and a scale adjusted Tornqvist-Theil index resulted in average rates of productivity growth of 2.3 and 2.2% respectively. The Chavas and Cox (1992) non parametric measure of productivity was used and found to result in a measure of 1.8% productivity growth. A translog cost function was also used and found to result in a growth rate of 1.6%, however the cost function seemed ill behaved

This paper continues the work of Mullen and Cox in examining alternative ways of measuring productivity growth. Both index number and nonparametric approaches are examined. The index number procedures compared include the TT, Fisher Laspeyres and Paasche indices. Two nonparametric procedures will be examined. The Chavas and Cox measure uses prices as well as quantities and provides a dual upper bound measure. The Malmquist approach is a primal measure which utilises quantity data only. A problem exists if there are substantial differences between the index number and nonparametric methodologies. In such a case, there is no method to discriminate between them on empirical grounds. This is because all are nonparametric measures in the sense that the statistical significance of differences cannot be measured[2]

Tentative estimates of upper and lower bounds for broadacre agricultural productivity will be derived. This data set differs from to that used in Mullen and Cox (1995). It covers a more recent time period and it is for broadacre farms in general rather than broadacre farms with more than 200 sheep

This paper makes no attempt to analyse the factors contributing to productivity growth, rather, it compares the two main approaches by which productivity growth is estimated. The assumptions underlying all methods are clearly stated and advantages and disadvantages outlined. Section 2.1 talks about the index number approach to productivity measurement. Sections 2.2 and 2.3 deal with the nonparametric techniques of Chavas and Cox and Malmquist (as implemented by Fare *et* al) respectively. The results are given in section 3.0 with a conclusion following (section 4.0)

This paper uses ABARE surveys data for Australian broadacre agriculture. The time period is from 1977-78 to 1993-94 with twelve outputs and twenty seven inputs. Appendix A provides a more complete description of the data.

A brief explanation on the definition of productivity growth is given in Appendix B.

2.0 Methodology

The following explanation of index numbers draws heavily from Alston et al (1995) whilst the explanation of non parametric measures is taken largely from Chavas and Cox (1994).

2.1 Index Number Approaches

The index number approach is the most simple approach to measuring productivity and consequently the most popular. Fisher (1923) laid the groundwork for testing index numbers more than seventy years ago. More recently, Di[illegible]rt (1992) has further developed Fisher's work to lay down a framework by which index numbers can be judged.

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[2] Strictly, an index number approach is nonparametric. Throughout the rest of the paper the distinction between index number and nonparametric methodologies is used for expository purposes and is consistent with the way the methodologies are referenced in the literature.

Fisher explained index numbers in terms of price relatives. For this paper, quantity relatives are used for expository purposes but the principles are exactly the same. A quantity relative is defined as the ratio of quantity (of a component, say wool) at time i over the quantity at time i-1. Therefore, it will show the percentage change between the two periods. A quantity index may be thought of as the method by which all relevant quantity relatives are averaged. The alternative weighting patterns used imply particular representations of technology or functional forms.

The Laspeyres and Paasche indices assume that the underlying production function is linear. In other words, using the Laspeyres or Paasche indices assumes that all factors are perfect substitutes (Christensen, 1975). Such a restrictive representation of technology is likely to be incorrect. For this reason, most recent work on productivity measurement has focused on using index numbers that are superlative, viz, index numbers that are exact for a flexible functional form. A function is termed flexible if it provides a second order approximation to an arbitrary twice differentiable homogenous production function.

The TT and Fisher indices are exact for the translog and quadratic production functions respectively. Both of these functional forms are flexible so that both these indices are superlative.

Notation

Consider a (N x 1) input vector $X = (x_1, x_2, \ldots, x_N)' \geq 0$ used in the production of a (M x 1) output vector $Y = (y_1, y_2, \ldots, y_M)' \geq 0$. Let the underlying technology be represented by the production possibility set T, where $(Y, -X) \in T$. The set T is assumed to be non-empty, closed, convex and negative monotonic.

The set $I = \{1, 2, \ldots, I\}$ contains I observations on (Y, X) in a given industry. Assume that the output-input vectors $(Y_i, -X_i) \in I$ and the corresponding price vectors (P_i, W_i) are observed for each observation $i \in I$. Here, $Y_i > 0$ and $P_i \geq 0$ are (Mx1) vectors of output quantities and prices, and $X_i \geq 0$ and $W_i \geq 0$ are (Mx1) vectors of input quantities and prices.

The Divisia Index

The Laspeyres, Paasche, Fisher and TT indices are discrete approximations to the continuous Divisia index (D) which can be written as:

$$D_i = D_b \exp \int_b^i ((W'_s \Delta X_s) / (W'_s X_s)) ds \qquad (2.1.01)$$

where:

D_b = the index in the base period b
D_i = The index value in period i
ΔX_s = the change in input quantities

The Divisia index has the property of being *invariant*, therefore, if production changes involve movement along an isoquant (rather than shifts of an isoquant) from time period i-1 to i, then the index value remains unchanged. Approximating the continuous Divisia index with discrete alternatives will mean that some information will be lost.

Each of the following indices is chained rather than fixed base. A chain based index uses a series of rolling weights whereas a fixed base index uses the same weights over the entire sample period.

The following formulae are for (input) quantities but the price indices are analogous.

Laspeyres

The Laspeyres index, denoted La is

$$La_i = La_{i-1}(W'_{i-1}X_i / W'_{i-1}X_{i-1}) \qquad (2.1.02)$$

Paasche

The Paasche index approximation is denoted Pa and:

$$Pa_i = Pa_{i-1}(W'_iX_i / W'_iX_{i-1}) \qquad (2.1.03)$$

Note that the Paasche and Laspeyres indices shown are chain indices, viz, the weighting factors (W'_i and W'_{i-1}) will change from periods i-1 to i. Alston et al (1995) gave the indices in this way and in his seminal text, Fisher (1923) placed no emphasis on whether these indices should be chain or fixed base but concentrates on the formula giving the value at any point in time. The implicit Laspeyres quantity index (the value ratio divided by the price index) is the same as the Paasche direct quantity index and vice versa.

Fisher

Fisher (1923) advocated the use of what he termed the *ideal* index number formula which subsequently came to be known as the Fisher index. Fisher attempted to test a myriad of index numbers using, *inter alia*, two great reversal tests; the time reversal and the factor reversal test. The *ideal* formula passed both of these tests which is why Fisher named it thus.

Of the time reversal test, Fisher's (pg 64) words were, "the formula for calculating an index number should be such that it will give the same ratio between one point of comparison and the other point, *no matter which of the two is taken as the base*" (italics in original). For example, if an index number stated that quantities doubled from time period 1 to 2 using time period 1 as a base, then that index should also say that quantities halved from time periods 2 to 1, using time period 2 as a base. Put differently, the time reversal test requires that the index value calculated using period 1 as a base, multiplied by the index value using time period two as a base should equal unity.

The factor reversal test is somewhat more complicated than the time reversal test. The best way to illustrate the factor reversal test is by an example. Fisher used an example utilising bacon and rubber (pg 73). Table (2.1.01) follows the accompanying text. "Suppose the price of bacon is twice as high in 1918 as in 1913 while the price of rubber is exactly the same in

1918 as in 1913; and suppose that the *quantity* of bacon sold in 1918 is half as much as the quantity sold in 1913 while the quantity of rubber is the same in both years. Evidently the *value* of bacon sold in 1918 is the same as the value of that used in 1913 (since half the quantity of bacon is sold at twice the price) and likewise the value of the rubber remains unchanged since both its price and quantity remain unchanged). Consequently, the total value of both together remains unchanged also. A good index number of these prices multiplied by the corresponding index number of these quantities ought, therefore, to give (in this case) 100 per cent" (italics in original).

Table 2.1.01 Factor Reversal Test eg.			
		1913	1918
Bacon	Price	100	200
	Quantity	100	50
	Value	*10 000*	*10 000*
Rubber	Price	100	100
	Quantity	100	100
	Value	*10 000*	*10 000*
	Total Value	20 000	20 000

The Fisher index is denoted F

$$F_t = (La_t)^{1/2}(Pa_t)^{1/2} \quad (2.1.04)$$

So the Fisher index is simply the geometric mean of the Laspeyres and Paasche indices. As such it always lies between the two.

TT index and the Allen and Diewert test

The TT index is the most commonly used index in the literature on productivity measurement. It satisfies the time reversal test but not the factor reversal test. In other words, the price index value multiplied by the quantity index value at any point in time, will not necessarily equal the value ratio.

To ensure that price by quantity equals value the direct price (quantity) index is calculated and the quantity (price) index derived implicitly by dividing value by price (quantity). This is then called the implicit price (quantity) index.

The problem is that the choice between an implicit or direct quantity index for productivity calculations is not obvious. To overcome this for superlative indices, Allen and Diewert (1982) argued that if there is less variation in price ratios than quantity ratios then the direct price index and corresponding implicit quantity index should be used.

Let P_{kI} define the price of the kth output at time Γ. The procedure to determine if there is less variation in price ratios then quantity ratios is firstly, to regress $\log(P_{k\Gamma}/P_{kI})$, where k = 1,...,M, on a constant which yields $SSR(P_\Gamma,P_1)$ as the sum of squared residuals. This procedure is repeated for quantities and analogously results in $SSR(X_\Gamma/X_1)$. If (say)

$SSR(P_T,P_1) > SSR(X_T/X_1)$ then prices vary more than quantities and so the implicit price index should be used.

Keeping the above in mind, the TT index is defined as:

$$TT_i = TT_{i-1} \prod_{k=1}^{N} (X_{k,i}/X_{k,i-1})^{S_{ki}}$$

where

$$S_{ki} = 1/2(s_{k,i} + s_{k,i-1})$$

and

$$s_{k,i} = X_{k,i}W_{k,i}/(\sum_{k=1}^{N} X_{k,i}W_{k,i}) \qquad (2.1.05)$$

which is commodity k's share of the total value of inputs in period i.

In words, the change in quantities between i-1 and i for each (kth) commodity is weighted by an average share (between the two periods) of each (kth) commodity.

Diewert (1992) argues strongly in favour of the Fisher productivity index over alternatives such as the TT. This conclusion is drawn after analysing indices using a "test" approach and an "economic" approach. The test approach involves logical or accounting tests which index numbers should satisfy. The economic approach analyses which indices conform to basic theorems about economic agents' behaviour. Diewert finds that the Fisher index satisfies all 20 tests in the former and the appropriate theorems in the latter.

Caveats for the index number approach

There are problems in aggregating inputs and outputs by indexing. If the quality of a given unit of measurement is changing over time then results may be misleading. For example, if labour is measured in weeks worked and the skill of labour is improving over time, then one week worked in 1978 may not be as effective as one week worked in 1993. *Ceteris Paribus*, this would mean that a labour week in 1993 should be scaled up to suit the greater skill of the labour exertion. The same type of argument applies equally to outputs.

If the natural resource base is being depleted by the process of agricultural production, and this is not taken into account, then productivity will be overstated. This is essentially the same as placing a zero value on the service flow of an input.

Due to fluctuations which occur from year to year the average annual growth rate is calculated by regressing the natural log of the index against a time trend and using the coefficient as the growth rate. This eliminates sensitivity to end point values (Males et al, 1990).

2.2 Non Parametric Approaches

Both primal and dual approaches to representing technology nonparametrically have been developed. Caves *et al* (1982b) succinctly summarise the relationship between distance functions, as developed by Shephard (1970), and productivity indices. Afriat (1972), Banker and Maindiratta (1988), and Chavas and Cox (1992) have shown that these distance functions can be readily computed with standard nonparametric techniques. Banker and Maindiratta (1988) have shown that the primal and dual approaches give nonparametric bounds to the underlying production technology

Shephard (1970, p 64-78) defines the input distance function as:

$$D_T(Y, X) = \sup\{\delta : (Y, -X/\delta) \in T\} \qquad (2.2.01)$$

where δ is a scalar

The input distance function yields the input requirement set $IR_T(Y) = \{X: D_T(Y, X) \geq 1\}$ as well as the frontier isoquant of a production set $IS_T(Y) = \{X: D_T(Y, X) = 1)\}$ (Shephard, p. 67) Hence, the input distance function completely characterises the technology T and measures the proportional (or radial) reduction in all inputs X that would bring the firm to the frontier isoquant $IS_T(Y)$

Figure (2.2.01) shows the a graphical representation of the input distance function for the case of two inputs, x1 and x2. Point "a" is an inefficient point lying inside the input requirement set. The same output that is produced at point "a" could be produced at point "b", which is connected to "a" via a ray from the origin. Point "b" lies on the isoquant. The distance function in this case measures 0a/0b. So graphically, δ can now be interpreted as the scaling factor which would bring "a" to the frontier (so that it lay on top of point "b").

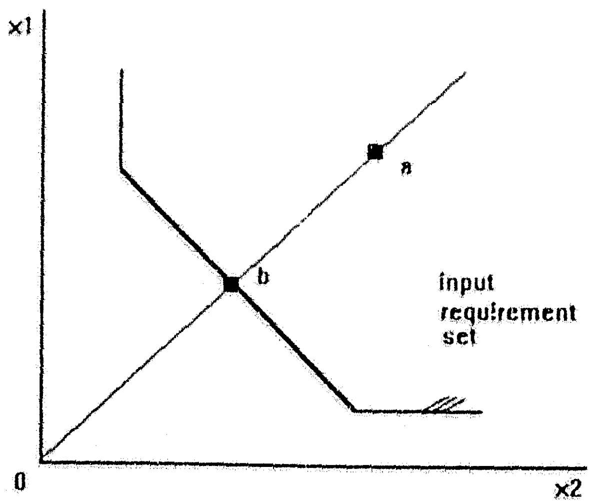

Figure 2.2.01

The Input Distance Function

The input distance function has been of great interest in efficiency analysis. It is the reciprocal of the Farrell (1957) measure of technical efficiency, where $1/D_T(Y, X) = 1$ corresponds to technical efficiency while $1/D_T(Y, X) < 1$ identifies technical inefficiency. Similarly, $[1 - 1/D_T(Y, X)]$ can be interpreted as the proportional reduction in production cost that can be achieved by moving to the frontier isoquant.

Similarly, the output distance function is defined by Shephard (p. 206-212) as:

$$F_T(Y, X) = \inf\{\delta: (Y/\delta, -X) \in T\} \qquad (2.2.02)$$

The output distance function yields the production correspondence $PC_T(X) = \{Y: F_T(Y, X) \leq 1\}$ and the frontier correspondence $FC_T(X) = \{Y: F_T(Y, X) = 1\}$ (Shephard, p. 209). It follows that $F_T(Y, X)$ in (2.2.02) defines the substitution alternatives among the outputs Y, given inputs X. Hence, as with the input distance function, the output distance function provides a complete characterisation of the underlying technology where $1/F_T(Y, X)$ measures the proportional rescaling of all outputs, Y, that would bring the firm to the frontier production correspondence $FC_T(X)$. Then, $[1/F_T(Y, X) - 1]$ can be interpreted as the proportional increase in revenue that can be achieved by moving to the frontier correspondence.

Figure (2.2.02) shows a production possibilities frontier for two outputs, y1 and y2. Point "a" lies inside the frontier and could be classed as technically inefficient. The output distance function measures the distance 0a/0b and δ is now interpreted as the scaling factor that would project point "a" to the frontier.

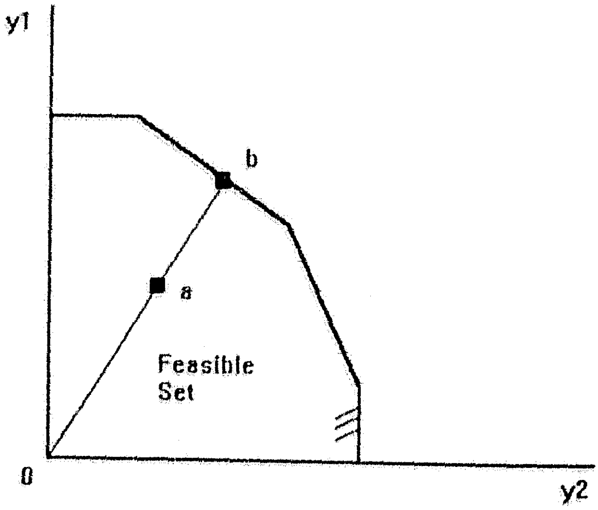

Figure 2.2.02

The Output Distance Function

Assuming that each observation is technically efficient, Caves *et al.* (1982b) propose the input based productivity index , IP, as:

$$IP = 1/D_T(Y, X) \qquad (2.2.03)$$

which measures the radial inflation factor for all inputs such that the inflated inputs (IP.X) = $X/D_T(Y, X)$ lie on the frontier isoquant $IR_T(Y)$ generated by technology T (Caves et al., p. 1407). In this context, a firm choosing (Y, X) has a higher (lower) productivity than the reference technology T if IP > 1 (< 1). Caves et al. (1982b) also propose the output based productivity index, OP, as:

$$OP = F_T(Y,X) \qquad (2.2.04)$$

which measures the radial deflation factor for all outputs by which the deflated outputs (Y/OP) = $Y/F_T(Y, X)$ lie on the frontier correspondence $FC_T(X)$ generated by technology T (Caves et al., p. 1402). Thus, a firm choosing (Y, X) has a higher (lower) productivity than the reference technology T if OP > 1 (< 1).

Under constant returns to scale, the input and output distance functions are reciprocal to each other (Shephard, p. 207-208). Hence the input based and output based productivity measures in (2.2.03) and (2.2.04), respectively, will be identical under constant returns to scale (Caves *et al.*, p. 1408). Therefore, empirical evidence that these measures are different indicates the existence of variable versus constant returns to scale.

The above productivity indices can help evaluate the rate of technical change in an industry. Their use typically depends on the nature of the data available. With cross-section data, firm level information is available only for a given time period. In such a situation, productivity indices are basically undistinguishable from radial technical efficiency indices. Yet, if the industry is affected by technical progress, these indices can reflect different adoption rates of new technology across firms as the production possibility set expands. With time series data, the productivity indices allow a measurement of the rate of shift of frontier technology over time. If firm level data are available both across firms and over time (eg. the case of panel data), then it becomes possible to distinguish between efficiency and productivity: the cross section information across firms provides a basis for estimating technical efficiency indices within each period, and the time series information allows the estimation of productivity indices across periods.

In order to make productivity indices empirically tractable, we need to obtain some representation of the reference technology T. In this section, we explore how nonparametric methods can be used for that purpose. We will limit our discussion to the analysis of productivity based on time series data.

Assuming that the set I involves time series data, this information can be used to measure productivity over time. In a sequential analysis of productivity for time i, the reference technology T is evaluated based on the set I of observations made up to time i. Alternatively, in an inter temporal analysis, the reference set T is evaluated based on the observations for all time periods.

Consider the maintained hypothesis of profit maximisation:

$$\mathrm{Max}_{Y,X}\,[P_i'Y_i - W_i'X_i : (Y_i, -X_i) \in T] \qquad (2.2.05)$$

for each $i \in I$. Let $Y_i^* = Y_i(P_i, W_i)$ and $X_i^* = X_i(P_i, W_i)$ denote the profit maximising output supply and input demand functions corresponding to (2.2.05) for firm $i \in I$. Then, by definition of the maximisation problem in (2.2.05), profit maximising behaviour must satisfy the following set of inequalities:

$$(P_i'Y_i^* - W_i'X_i^*) - (P_i'Y_j^* - W_i'X_j^*) \geq 0 \qquad (2.2.06)$$

for all $i \in I$ and all $j \in I$. Expression (2.2.06) corresponds to Varian's Weak Axiom of Profit Maximisation: it is a necessary as well as a sufficient condition for profit maximisation given the I observations on production behaviour (Varian (1984), p. 584). There are many production possibility sets that satisfy (2.2.06) over a finite number of observations (Varian(1984), p. 591). Banker and Maindiratta ((1988), p. 1321) propose constructing two families of production possibility sets, S and L, that provide a lower and upper bound on the reference technology T where the set S is given by:

$$S \equiv [(Y, -X) : Y \leq \sum_{i \in I} \lambda_i Y_i, X \geq \sum_{i \in I} \lambda_i X_i, \sum_{i \in I} \lambda_i = 1,$$

$$Y \geq 0, X \geq 0, \lambda_i \geq 0]. \qquad (2.2.07)$$

where λ is a scalar.

The set L is given by

$$L \equiv [(Y, -X) : P_i'Y - W_i'X \leq P_i'Y_i - W_i'X_i, i \in I,$$

$$(2.208)$$

$$Y \geq 0, X \geq 0]$$

Each set S or L is convex, closed, negative monotonic and admissible, and corresponds to a general variable-return-to-scale technology. Banker and Maindiratta ((1988), p. 1321) have shown that, for any admissible production possibility set T, then $S \subseteq T \subseteq L$.

In other words, S in (2.2.07) is the <u>tightest lower bound</u> while L in (2.2.08) is the <u>tightest upper bound</u> for any set T which is consistent with profit maximisation. This establishes a basis for investigating the nonparametric bounds of the production possibility set that can be generated by a finite number of observations on production behaviour.

Note that the inner bound S in (2.2.07) requires only quantity information on Y and X. Hence, it corresponds to a <u>primal</u> approach to production analysis and focuses on the nonparametric estimation of the production frontier (see Afriat (1972), Fare, Grosskopf and Lovell (1985)). This approach has also been called Data Envelopment Analysis (DEA) in the management science literature (eg., Banker et al. (1984)). Note, in contrast, the outer bound L in (12) requires information on both quantities (Y, -X) and prices (P, W). Hence, it corresponds to a <u>dual</u> nonparametric approach to production analysis, as developed by Afriat (1972), Hanoch and Rothschild (1972), Varian (1984), Chavas and Cox (1988), or Cox and Chavas (1990).

Given (2.2.07) and (2.2.08) as a representation of the bounds on the reference technology T, the evaluations of the distance functions (2.2.01) and (2.2.02) are straightforward. Obtaining the input distance function $D_S(Y, X)$ in (2.2.01) associated with the lower bound S involves solving the linear programming problem

$$1/D_S(Y_j, X_j) = \min_{\delta,\lambda}\{\delta : Y_j \le \sum_{i \in I}\lambda_i Y_i,\ X_j\delta \ge \sum_{i \in I}\lambda_i X_i, \quad \sum_{i \in I}\lambda_i = 1,\ \lambda_i \ge 0\}, \tag{2.2.09}$$

for all j ∈ I. The input distance function $D_L(Y, X)$ in (2.2.01) associated with the upper bound L can be obtained from the solution of the linear programming problem:

$$1/D_L(Y_j, X_j) = \min_{\delta}\{\delta : P_i'Y_j - W_i'X_j\delta \le P_i'Y_i - W_i'X_i,\ i \in I\}, \tag{2.2.10}$$

for all j ∈ I.

Similarly, the output distance function $F_S(Y, X)$ in (2.2.02) associated with the lower bound S is given by the solution of the linear programming problem:

$$1/F_S(Y_j, X_j) = \max_{\delta,\lambda}\{\delta : Y_j\delta \le \sum_{i \in I}\lambda_i Y_i,\ X_j \ge \sum_{i \in I}\lambda_i X_i, \quad \sum_{i \in I}\lambda_i = 1,\ \lambda_i \ge 0\}, \tag{2.2.11}$$

for all j ∈ I. And the output distance function $F_L(Y, X)$ in (2.2.02) associated with the upper bound L is obtained in a similar manner from solving:

$$1/F_L(Y_j, X_j) = \max_{\delta}\{\delta : P_i'Y_j\delta - W_i'X_j \le P_i'Y_i - W_i'X_i,\ i \in I\}, \tag{2.2.12}$$

for all j ∈ I.

This primal-dual nonparametric approach to productivity measurement was applied by Chavas and Cox (1994) to U.S. agriculture. Chavas and Cox (1994, p13) pointed out that the choice between the dual and primal approaches is partly influenced by the nature of the data available. They concluded that in situations where there was significant variations in prices, as was the case for Australian broadacre agriculture from 1953 to 1988, the dual approach, which generates an upper bound representation of technology, may be more informative than the primal approach.

2.3 The Malmquist Approach

There has been strong interest in the Malmquist approach to productivity measurement in recent literature. Fare and Grosskopf *et al* (1992, 1994 and 1995) have utilised the method extensively in recent times. Despite this strong interest in the use of the Malmquist technique overseas, it has not yet been used to measure productivity of broadacre agriculture in Australia. The Malmquist productivity index is a primal nonparametric representation of technology so that it is closely linked to (2.2.11).

The output distance which will be used in implementing the Malmquist productivity index is :

$$F_{Ti}(X_i, Y_i) = \inf\{\delta : (Y_i / \delta, -X_i) \in T_i\}$$

$$= (\sup\{\delta : (\delta Y_i, -X_i) \in T_i\})^{-1}.$$

(2.3.01)

where T_i represents the technology set at time i, this is often called the reference technology in the literature (Fare et al, 1994). The output distance function $F_{T,i+1}(X_{i+1}, Y_{i+1})$ is similarly defined.

Figure (2.3.01) shows how this distance function can be envisaged in the one input, one output case. The line T_i represents the production frontier at time i and T_{i+1} represents the production frontier at time i+1. If production at time i occurs at the point "a" then, as with the point "a" in Figure 2.0.02, this is construed as technical inefficiency. The reason for this is that output could be increased to "b" (moving production to the coordinate (x_i, y_i')) without any increase in inputs. The output distance function, in terms of the y axis, equals the ratio $0y_i/0y_i'$. Note that this value will necessarily be less than or equal to unity.

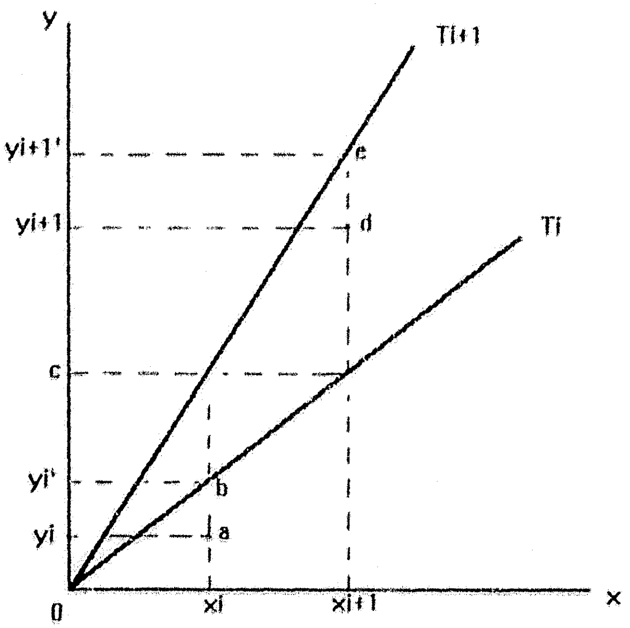

Figure 2.3.01

The Malmquist Output Based Productivity Index

The Malmquist productivity index also requires computation of a distance function when the observation period is different to the reference technology period. For example, when comparing observations at time i+1 to the reference technology at time i. The output distance function in this case is written as:

$$F_{Ti}(X_{i+1}, Y_{i+1}) = \inf\{\delta : (Y_{i+1} / \delta, -X_{i+1}) \in T_i\} \qquad (2.3.02)$$

In Figure (2.3.01) the distance function above is given by the distance $0y_{i+1}/0c$. In other words, it compares the coordinate (x_{i+1}, y_{i+1}) to the reference technology at time i for the given x_{i+1} inputs. This value would be greater than unity since (in this diagram) technical progress has occurred. The distance function $F_{T,i+1}(X_i, Y_i)$ is defined analogously.

The Malmquist productivity index for the reference technology i is defined as:

$$M_i = F_{Ti}(X_{i+1}, Y_{i+1}) / F_{Ti}(X_i, Y_i) \qquad (2.3.03)$$

or using reference technology at time i+1:

$$M_{i+1} = F_{T,i+1}(X_{i+1}, Y_{i+1}) / F_{T,i+1}(X_i, Y_i). \qquad (2.3.04)$$

It is not obvious which of the two above formulae should be used to measure productivity between periods i and i+1. Therefore a geometric mean is used such that:

$$M = (M_i . M_{i+1})^{1/2} \qquad (2.3.05)$$

As with the nonparametric methods discussed in section 2.3, the Malmquist can be used to decompose measured changes in efficiency into technical progress (movements of the frontier) and catch up efficiency (movements towards the frontier) when panel data are available. However, with only one (say) country's data the maintained assumption is that the (sole) country is on the frontier.[3]

In Figure (2.3.01) this means that a sole country, given inputs x_i at time i, would be at the point "b". Similarly, given x_{i+1} at time i+1, the sole country is assumed to be at point "e".. It can be seen then that the value of the distance function (2.3.01) and its t+1 counterpart will be equal to unity. The Malmquist formula for the one country case then simplifies to the solution of two (rather than four) distance functions. The relevant distance functions are (2.3.02) and its analogous counterpart. Therefore, (2.3.05) can then be written as:

$$M = (F_{Ti}(X_{i+1}, Y_{i+1}) / F_{T,i+1}(X_i, Y_i))^{1/2}. \qquad (2.3.06)$$

Once again, the distance functions are solved via linear programming techniques. The linear programming problem corresponding to the output distance function (2.3.02) for the one country case is:

$$(F_{Ti}(X_{i+1}, Y_{i+1}))^{-1} = \max\delta$$

subject to:

[3] In the case where data is available for only (say) one country, then a "window" approach can be used to derive a frontier from observations in other periods (for eg, observation i+1, i+2 and i+3 relative to observation i). However, in agriculture where seasonal fluctuations cause large changes from year to year this does not seem appropriate.

$\delta Y_{i+1} \leq \lambda Y_i$
$\lambda X_i \leq X_{i+1}$
$\lambda \geq 0.$ (2.3.07)

where δ and λ are scalars.

The linear programming problem for $F_{T,i+1}(X_i,Y_i)$ will be identical with the time subscripts (i and i+1) reversed. Each distance function must be solved for every two periods which warrant comparison. In the current context, the seventeen years of data require the solution to be computed sixteen times for each distance function.

This is a constant returns to scale specification so the solution after substituting to (2.3.05), will be equal to the input oriented Malmquist approach.

3.0 Results

Figure 3.0.01 shows the alternative productivity measures (Fisher, TT, Laspeyres and Paasche indices as well as the Dual nonparametric measure) for Australian broadacre agriculture from 1977-78 to 1993-94.

The Laspeyres direct quantity and Paasche direct quantity indices envelop the Fisher and TT indices. The Laspeyres index runs from 100 to 168.63 while the Paasche index runs from 100 to 151.17. The average growth rate of the two indices is 3.02 and 2.29% respectively. Rao (1995) stated that these two indices should provide upper and lower bounds to the rate of productivity growth.

The Fisher direct quantity index (equal to the implicit quantity index because the factor reversal test is satisfied) grows from 100 in 1977-78 to 159.66 in 1993-94. The average growth rate of this index is 2.66%. This is very similar to the TT implicit quantity index which rises from 100 to 160.05 at the approximately same average rate of growth. The direct quantity TT ranges from 100 to 159.55 over the relevant period at an average growth rate of 2.64%. Therefore, with this data set there does not seem to be much difference between the use of an implicit or direct TT index.

The dual nonparametric Chavas and Cox measure follows the direct quantity Paasche index closely, particularly from the early eighties onwards. The dual measure reaches a value of 151.15 in 1993-94. The average rate of growth is 2.26% which is slightly below the Paasche's 2.29%. The shape of the productivity index is very similar to that of the index number approaches.

The range for productivity growth found here is consistent with the previously cited studies of Lawrence and McKay (2.9% per annum) and Males et al (2.2% per annum). The fact that the dual nonparametric measure lies below the index number approaches is consistent with Mullen and Cox (1995) where the dual measure found an average annual productivity growth of 1.8% relative to the TT's 2.2%. For that dataset, the dual nonparametric measure also lay below the Paasche measure (Mullen, Pers. Comm).

There is no means by which to test whether the differences between measures recorded here are statistically significant differences or not. This means that any arguments in favour of one approach over another will have to be made on *a priori* grounds. Advocates of nonparametric measures use this fact to argue in favour of their less restrictive approach.

Figure 3.0.02 shows variations in the Malmquist index compared to the Chavas and Cox dual nonparametric approach.

The results of the Malmquist index seem somewhat erratic compared to those of the other productivity measures. The Malmquist productivity index runs from 100 to 112 and gives a negative growth rate (-0.77%) over the observation period.

For most observations, the direction in which the Malmquist moves is consistent with the other measures, however, there are some notable exceptions. From 1990-91 to 1991-92 and from 1991-92 to 1992-93 are two cases in point. Here the other productivity measures fall and rise respectively whilst Malmquist does the opposite. The reason for this seems to be the fact that Malmquist is not using prices to "weight" commodities. From 1990-91 to 1991-92 there is a large fall in the quantities of wheat and wool output. These two outputs make up a large proportion of the total value of output (approximately 38%) but the Malmquist cannot take account of this as the other measures do. Consequentially, the Malmquist productivity measure rises. It seems as if the Malmquist is affected by the very large rise in the output quantity of grain sorghum (132%) even though this does not make up a large proportion of the value of output. This is not to say that the Malmquist is a totally unweighted measure, however, not taking account of price information when it is available would seem to be injudicious.

Such an argument confirms the view stated by Chavas and Cox (1994, pg14) : "The dual approach generating the upper bound representation of technology may be more informative than the primal approach, especially in situations where there are significant price variations across observations (as typically found in time series data)" (underlining and brackets in original). Unless price data are not available and are thought not to vary, then the a primal approach may give misleading results.

4.0 Conclusion

In this paper, several alternative measures of productivity growth in Australian broadacre agriculture, for the period 1977-78 to 1993-94, have been reported. According to the Fisher index, productivity grew at an annual rate of 2.7% and suggests that productivity growth has returned to levels discussed by Lawrence and McKay for the period 1952-53 to 1976-77 after falling to about 2.2 as calculated by Males *et al* for the 1977-78 to 1988-89 period. This rate of productivity growth is also higher than that found by Mullen and Cox. The difference may reflect the different observation period but it also reflects the fact that the dataset used by Mullen and Cox contained no cropping specialists. Knopke, Strappazzon and Mullen (1995) found, using the current dataset, rates of productivity growth were much higher for cropping specialists than for other enterprise types. This higher rate of productivity growth suggests that Mullen and Cox may have underestimated the rate of return to investment on research in broadacre agriculture, at least over the more recent observation period. Mullen and Cox noted some evidence that the rate of return from research had increased over the observation period.

The rate of productivity growth estimated by the Chavas and Cox approach averaged 2.3% but no productivity growth was identified by the Malmquist primal approach. This result echoed the experience of Chavas and Cox in measuring productivity in US agriculture. They concluded that the dual measure is preferred because it uses price information in addition to quantity information[4].

A problem this paper has not resolved is that the nonparametric measures of productivity growth lie below the Paasche index. The Paasche index was expected to provide a lower bound estimate of productivity growth. Because none of the measures used here provide goodness of fit statistics, it cannot be ascertained whether the differences between these measures are statistically significant.

On *a priori* grounds an important difference between the two is that index number approaches imply particular functional forms whereas nonparametric measure do not.

With respect to the index number approach, one line of research is to estimate the production technology using the functional form underlying the particular index number measured. In preliminary work Mullen and Cox have estimated a translog cost model of broadacre agriculture over the period 1953 to 1988. Recall that a translog production function is implied by a TT index. They found that the estimated model did not satisfy all the requirements of a well behaved cost function.

This finding prompted an investigation of the Fisher Index. As yet a quadratic cost function has not been estimated but expectations are that, since the Fisher and TT indices track each other so closely, the quadratic model will also fail to satisfy all the conditions of a well behaved cost function.

One response to this issue is to investigate functional forms such as the Fourier that are more flexible than second order approximations. However, there are two reasons why this is unlikely to be fruitful. First, it is unlikely that any sample of data is going to perfectly match a behavioural postulate such as cost minimisation. Second, since there are no confidence intervals for the measured rate of productivity, the extent of bias using the TT and Fisher indices is unknown but may be small.

Another difference between the index number approaches and the Malmquist index as applied above, versus the Chavas and Cox dual nonparametric approach, is that the former have been estimated under the assumption of constant returns to scale which is another source of bias. However, Mullen and Cox estimated a scale adjusted TT measure which tracked the Fisher index very closely suggesting little divergence from constant returns to scale, at least for that dataset and observation period.

Hence the choice between index number and nonparametric methodologies remains unclear. Both approaches impose quite strong restrictions on the nature of technical change. In both cases a form of neutral technical change is implied. Obviously an important issue for future research is to resolve the difference between index number and non parametric approaches.

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[4] It should be noted, however, that in the strictest sense productivity is defined in terms of a physical relationship between inputs and outputs (see Appendix B)

Within index number approaches, the Fisher index is most attractive because it satisfies the factor reversal test. Of the nonparametric approaches, the dual Chavas and Cox measure is the most attractive because it uses both price and quantity information.

The Malmquist approach does not appear to give sensible results, at least for this dataset. While the difference between the Fisher and Chavas and Cox measures appear to be small it would be comforting if the nonparametric measures had fallen within the bounds of the Paasche and Laspeyres indices as expected.

Figure 3.0.01 Index Numbers and Dual Nonparametric Productivity

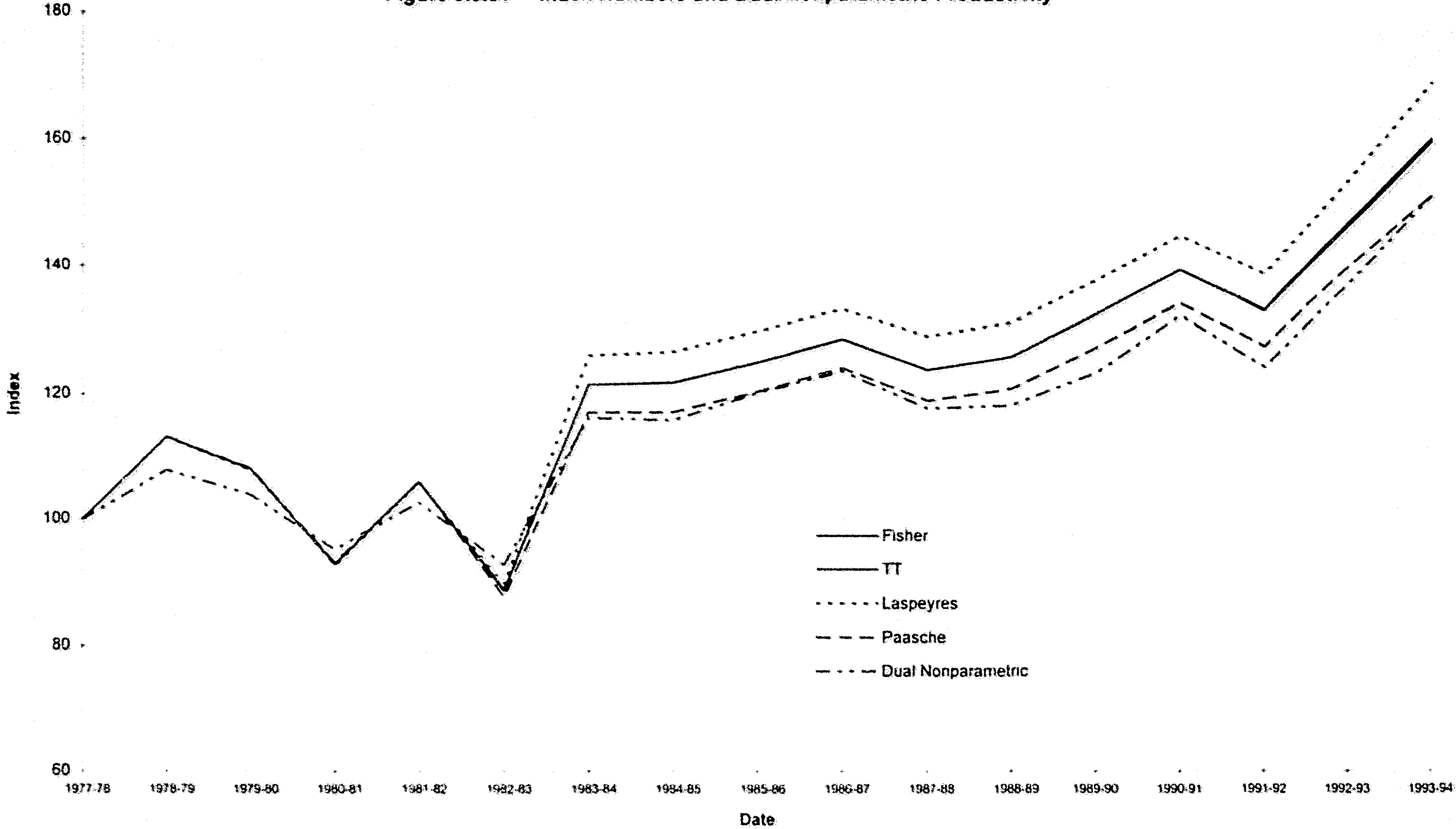
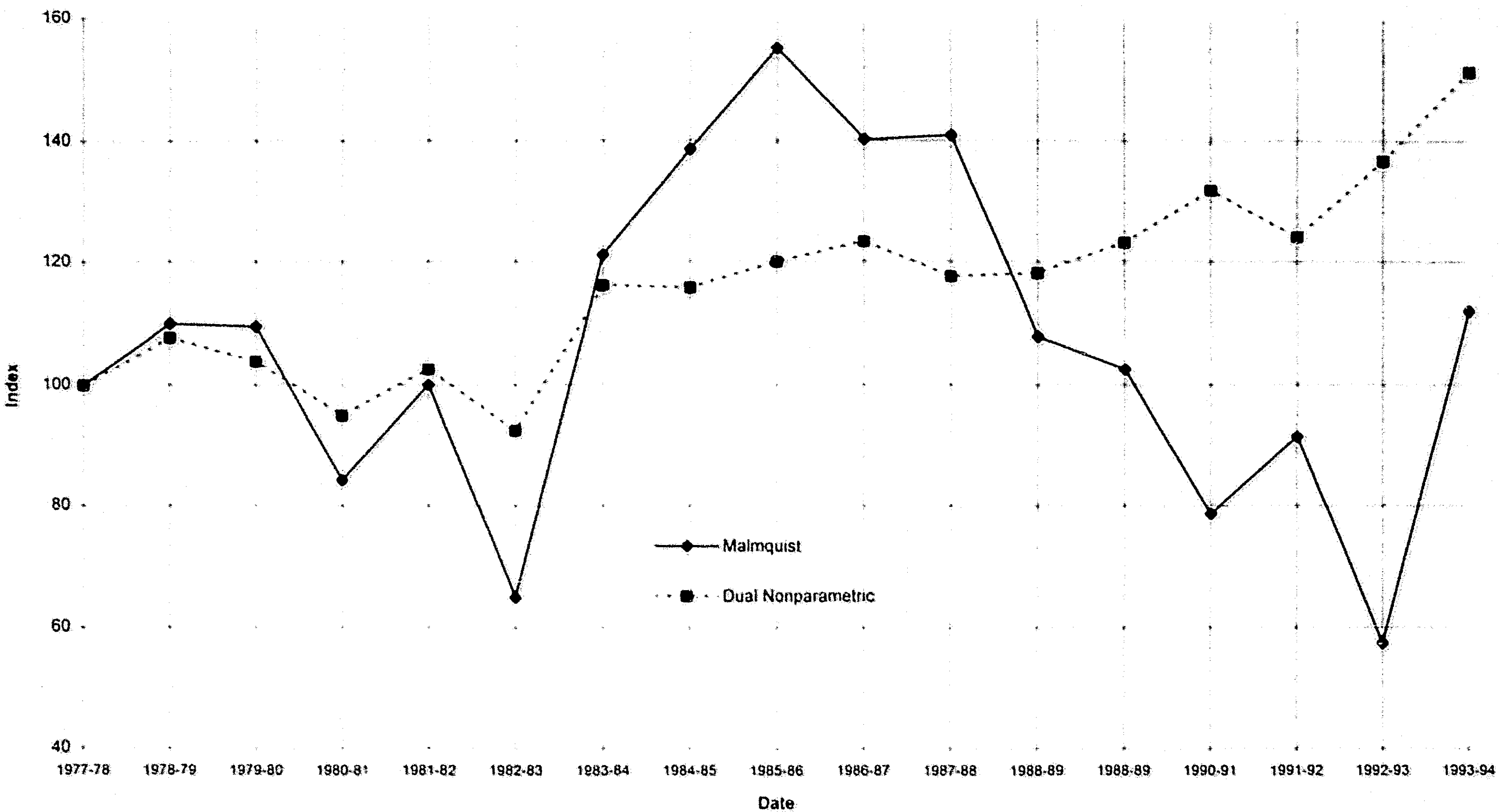


Figure 3.0.02 Malmquist and Dual Nonparametric productivity



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Appendix A

Data Description

The data used are drawn principally from ABARE's annual surveys of broadacre industries (for further information about these surveys, see ABARE 1995a). If quantity variables are not available, these are derived by deflating survey data by the appropriate ABARE prices paid and received indexes (ABARE 1995b). As far as is practicable, the prices used are taken at the farm gate. State indexes are applied to survey means at the state level.

The outputs and inputs are specified below.

Inputs

Inputs consist of 27 items which can be split into five major groups, namely, capital, livestock purchases, labour, materials, and services.

Capital

Capital is divided into land, plant and machinery, structures, and livestock. The value variable for land and livestock (beef cattle and sheep) are the opportunity cost of investing funds in those capital items. These are calculated as the average capital value (that is, the average of opening and closing values) multiplied by a real interest rate. The value variables for plant and structures capital are the opportunity costs plus depreciation.

The quantity variable used for land is the area operated. For beef cattle and sheep it is the average of opening and closing numbers. For buildings and plant capital, it is the average value of capital stock deflated by the respective prices paid indexes for each.

Livestock purchases

Livestock purchases are split into beef, sheep and other livestock. Their value variables equal purchases plus negative operating gains

The quantity variables for sheep and beef is derived from the respective value variables (above) and respective prices received indexes for sheep meats and slaughtered beef. For the relatively small category of other livestock, the quantity variable is derived from the value of purchases and a prices received index for livestock products.

Labour

Labour consists of four items: owner-operator and family labour, hired labour, shearing costs, and stores and rations. The value of the owner operator and family labour input is imputed using weeks worked (collected during the survey) and an award wage.

The value of hired labour is wages paid, and the value of shearing and stores and rations are expenditure. The quantity variables for owner operator and family labour and hired labour are weeks worked. Expenditure deflated by a shearing prices paid index is the quantity variable for shearing.

Materials and Services
There are seven items in the materials group: fertiliser, fuel, crop chemicals, livestock materials, seed, fodder, and other materials; and there are seven items in the services group: motor vehicle costs, rates and taxes, miscellaneous livestock costs administrative costs, repairs, contracts, and other services. For each item in both groups the value item is expenditure. The quantity variables are derived by deflating the expenditure on each by the appropriate prices paid index.

Outputs

Outputs consist of eleven items which can be divided into four major groups, namely, crops, livestock sales, wool, and other farm income.

Crops
Crops are split into wheat, barley, oats, grain sorghum, oilseeds and other crops The value variable for wheat is the quantity harvested multiplied by the Australian Wheat Board's average net return for that years pool. For other grains and other crops it is net receipts in that year The quantity variable for each of the grains is the quantity harvested. For the other crops, it is receipts deflated by the prices received index for crops.

Livestock sales
For beef and sheep, the value variable is sales plus positive operating gains. For the minor category of other livestock, the value variable is sales.
The quantity variables for beef and sheep are derived from the respective value variables (above) and the prices received indexes for slaughtered beef and sheep meats. For the category of other livestock, the quantity variable is derived from the value of sales and a prices received index for livestock products.

Wool
The value variable is wool receipts and the quantity is wool shorn (in kilograms).

Other farm income
The value variable is receipts and the quantity is receipts deflated by the farm sector prices received index.

Appendix B

The concept of Productivity

In a static sense, productivity can be measured as the output per unit input. The easiest case is where one input (x) is used to produce one output (y) so that productivity (tfp) over a given period of time is:

$$tfp = y/x \tag{B.01}$$

Usually the investigator is interested in the growth of productivity rather than its level. Rewriting (B.01) in a growth form yields:

$$\dot{tfp} = \dot{y} - \dot{x} \tag{B.02}$$

where :

$\dot{tfp}$= the proportionate rate of growth of tfp
$\dot{y}$= the rate of growth of output y
$\dot{x}$= the rate of growth of input x

all in a given time frame.

In words, (B.02) represents the difference between the growth in outputs and inputs. Also called the Solow residual, $\dot{tfp}$ can be thought of as the growth in output that can not be explained by growth in inputs. Often (B.02) is written in a natural log (ln) form:

$$\dot{tfp} = \ln(y_i/y_{i-1}) - \ln(x_i/x_{i-1}) \tag{B.03}$$

where i and i-1 represent the current and past time periods respectively.

Rarely will the investigator be analysing a case where there is one input and one output. Usually a measure of the productivity taking account of many outputs and all factors - Total Factor Productivity (hence tfp) - is usually calculated. With more than one input and output the component parts must be aggregated to give a single value which can then be substituted into (B.01) to (B.03) above. The index number approach to productivity measurement provides a methodology for this . Essentially, an aggregate (say) output will be calculated as a weighted sum of all component outputs. Different indexing procedures determine the manner in which components are weighted.

Non parametric productivity estimates use linear programming to align the data with axioms of economic theory, for example, profit maximisation. Within the nonparametric approach, a dual or primal linear programming problem may be defined. The primal approach uses information about quantities only whilst the dual approach uses both price and quantity data. In the case where price information is available, it may be argued that the dual approach represents a more complete picture of productivity growth, especially if prices have fluctuated considerably over the time period analysed (Chavas and Cox, 1994).

Figure (B.01) shows a graphical representation of productivity improvement. Initially the isoquant y0 lies on the isocost line PP'. An improvement in productivity may result in more output from the given inputs which would see y1 >y0 being produced without a change in input use. Alternatively, a productivity improvement may be viewed as the same output being produced from less inputs In this case y0' = y0 is produced at relatively lower cost since the tangential isocost line P1P1' lies below PP'

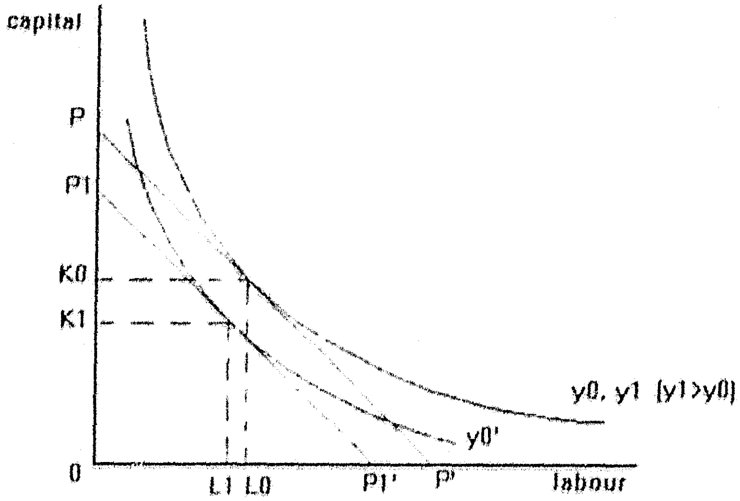

Figure B.01

Productivity Improvement