AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

University of California, Davis
Department of Agricultural Economics

Working papers are circulated by the author without formal review. They should not be quoted without author's permission. All inquiries should be addressed to the author, Department of Agricultural Economics, University of California, Davis, California 95616-8512 USA.

FORESIGHT AND SELF-ENFORCEMENT IN ALLOCATIONS OF COMMON PROPERTY

by
Garth John Holloway

Working Paper No. 95-04

Foresight and Self-Enforcement in Allocations of Common Property

Abstract

A conventional model of common-property allocation predicts rent dissipation as potentially large numbers of agents exploit a productive resource, the stock of which is fixed and access to it free. Research has ensued along two themes. One focuses on the design and implementation of regulatory policies that improve allocative efficiency in the conventional model. The other focuses on comparing the inefficiencies associated with alternative behavioral regimes. In this second literature, interest has developed in the extent to which a form of voluntary restraint may be present in the market allocation. The preferred alternative to the conventional model is Nash equilibrium. In the static, one-shot game in which agents have complete information about rivals' behavior, this model predicts inefficiencies, which grow without bound as numbers of agents increase. This paper considers the allocative consequences of an alternative framework, which is Nash equilibrium in a two-stage game. In the initial static phase of the game, agents are uncertain about the effort of rivals. They form conjectural variations and, in a subsequent phase, seek consistency between the conjectures postulated in the initial phase, and the ones observed in the subsequent comparative-static stage. The following results are obtained: A consistent-conjectures equilibrium exists in the allocation of common property. It is the monopolistic-conjectures equilibrium. Agents acts identically, applying effort up to the cartel equilibrium level. The market allocation is efficient. One conclusion is that the predictions derived from the conventional model appear very sensitive to the type of behavioral mode adopted by the firms. Therefore, the conduct of agents exploiting common property needs to be tested. *Key words: foresight, self-enforcement, common property. Journal of Economic Literature Classifications: D23, D43, D61.*

Foresight and Self-Enforcement in Allocations of Common Property

The tragedy of the commons develops in this way. Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. Such an arrangement may work reasonably satisfactorily for centuries because tribal wars, poaching, and disease keep the numbers of both man and beast well below the carrying capacity of the land. Finally, however, comes the day of reckoning, that is, the day when the long desired goal of social stability becomes a reality. At this point, the inherent logic of the commons remorselessly generates tragedy.

As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, more or less consciously, he asks, "What is the utility to me of adding one more animal to my herd?" This utility has one negative and one positive component.

1. The positive component is a function of the increment of one animal. Since the herdsman receives all the proceeds from the sale of the additional animal, the positive utility is nearly +1.
2. The negative component is a function of the additional overgrazing created by one more animal. Since, however, the effects of overgrazing are shared by all the herdsmen, the negative utility for any particular decision making herdsman is only a fraction of -1.

Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another, and another ... But this is the conclusion reached by each and every rational herdsman sharing a commons. Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit—in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom in a commons brings ruin to all.

— *Garrett Hardin, The Tragedy of The Commons, Science 162(1968):1243-48.*

The "tragedy of the commons" results from myopic over-exploitation of a productive resource, the endowment of which is fixed, and access to it free. Its unfortunate implications rest on three tenets. The first is unimpeded access, which leads to diffusion of property rights among those exploiting the resource. The second is costless use, which results in allocations of variable resources in excess of those implied otherwise. The third is lack of foresight and undisciplined behavior. The purpose of this paper is to draw into question the unfailing pessimism surrounding the commons by investigating closely the third tenet in this economic "tragedy."

The common-property problem has spawned a vast and growing literature which, broadly speaking, can be divided into three strands. One attempts resolution of the problem through the allocation of property rights (e.g., Provencher and Burt); another focuses on the optimal form of regulation of access (e.g., Huffaker *et al.*); a third, more recent body of work, investigates the allocative inefficiencies stemming from alternative modes of conduct among the agents who exploit the common resource (e.g., Ostrom *et al.*). Despite allusions to the problem in discursive settings (e.g., Bromley, Libby), there has been scant attention devoted to the formal analysis of this topic in the *American Journal of Agricultural Economics*. The present contribution attempts to fill this void.

A comparison of conduct lends itself to the application of the tools of modern game theory. Despite an enormous growth in the theoretical literature in this area, there have been relatively few applications of the theory to practical problems in resource management. There can be little doubt that the reason for this lies in the mathematical barriers that the theory presents most readers. Perhaps because of this, most attention in the literature has focused on the Nash solution concept, in simple, one-shot equilibria in which agents possess complete information about each other's behavior. The attendant implications are generally well-known: The market allocation can be made efficient by endowing a single agent with sole property rights. Absent such regulation, inefficiency increases as the number of agents exploiting the resource is increased.

That information about the behavior of rivals is usually incomplete, is a well-accepted precept. It is also well-accepted that this type of uncertainty may have important implications for the market allocation, in a variety of settings. One of these is the allocation of common property. One view is that this type of uncertainty can generate a form of aversion in decision-making which closely approximates "self-regulation." The view that exploiters of a common property may wish to exercise voluntary restraint is not novel. In 1985, the *American Agricultural Economics Association* welcomed a symposium entitled "Restraint Under Open Access: Are Voluntary Incentives

Sufficient Or Is Coercion Required ?" Several contributions in that forum (Braden, Field, Runge, Johnson) explored the game theoretic underpinnings of the voluntary-restraints argument. Although models were proposed, there was no explicit formulation of common property allocations. The present work provides one.

In this respect, several comments by Braden are especially pertinent. He identifies two types of uncertainties that influence the exertion of voluntary restraint. The first is uncertainty about the strategic interaction of opponents, the second is uncertainty about the technological possibilities of production. It is the first of these two forms upon which I focus in this paper. As Braden notes:

> "Individuals who are interdependent, as in open access situations, may be presumed to act based on their expectations of others' actions. If those actions cannot be perfectly predicted and assured, each participant must plan according to some probability distribution over the actions available to others. It has been argued that uncertainty may facilitate voluntary agreements among parties for whom non-cooperative behavior is clearly rational and whose interactions are repeated." (p. 356)

In deference to the potentially complex analysis of probability distributions defined over agents' actions, an alternative framework exists. It has been used rather extensively to model strategic interaction in empirical investigations of conduct in the food industries. It is the conjectural-variations model of oligopoly, which was originally proposed by Bowley. Despite its many limitations, this model will provide a useful framework for motivating the central proposition of this paper. This is that the extent of inefficiency in common-property allocations is extremely sensitive to choice of behavioral mode and that, as a consequence, the conduct of the agents who exploit common property needs to be tested.

The paper makes four contributions. The first is to demonstrate that the voluntary-restraints argument can be motivated tractably and lucidly through a simple application of conjectural variations. This formulation proves advantageous because it uncovers the salient features of the common-property problem without the need to resort to sophisticated mathematical arguments. This claim is substantiated from a comprehensive treatment of conjectural variations applied to model the allocation of common property, which is the second contribution of the paper. The third is to provide a formal link between the application in this paper, and its foundations in the static theory of oligopoly. The fourth and final contribution is to elucidate further key issues in the voluntary-restraints debate and,

where possible, resolve them in the context of a model of consistent-conjectures equilibrium. In doing so, I demonstrate that several unsubstantiated claims about self regulation are, in fact, correct, although for reasons different from those argued in the literature.

The paper is structured as follows. The next section presents a simple model. It contains two important features of common-property allocations. The first is that the common-property is essential for production. The second is the existence of an externality. Following Haveman, the externality is modeled as a form of "congestion," in which increased intensity of variable resource use reduces the productivity of other agents' effort. Subsequently we compare the allocations obtained from three regimes, respectively, the efficient allocation chosen by an egalitarian social planner, the allocation achieved in Nash equilibrium, and the many allocations based on arbitrary specifications of conjectural variations. Consistent conjectures are located, and the consistent-conjectures allocation is compared to the previous ones. Subsequently, existence is considered and the practical significance of the results are discussed.

Agents, Technologies, and Markets

There are N individuals indexed { i i=1..N }, which we call "firms." Each selects a quantity, { x_i i=1..N }, of a variable input. For ease of reference throughout the paper it may be useful to think of the variable factor as a combination of several inputs required for production. I shall refer to this amalgamation, simply, as "effort." Effort has an opportunity cost. The real cost per-unit of effort is σ_w. The notation "subscripted sigma" denotes a variable of interest to the firm, the value of which it perceives to be given. Hence, we abstract from possible imperfections in the factor market.

Individual and total effort are related through the aggregation condition:

$$(1) \qquad x = \sum_{i=1}^{N} x_i.$$

Outputs, denoted { y_i i=1..N }, are obtained from applying units of the variable factor to another resource, which is essential for production and is available in fixed and finite supply. The fixed resource is common property and access to it is free. Because the endowment is fixed, increasing applications of the variable factor lead to "congestion."

There is thus an externality in production in which individual returns depend on both the "private" and the "public" levels of effort. Specifically,

(2) $$y_i \leq f_i(x_i , x), \qquad i=1..N,$$

characterize technology. About the functions $\{ f_i(\cdot)\ i=1..N \}$, we make some plausible assumptions, namely that for all $x_i \geq 0$, $x \geq 0$ and $i=1..N$:

ASSUMPTION 1 (Monotonicity): $\partial f_i(\ )/\partial x_i > 0,\ \partial f_i(\ )/\partial x < 0.$

ASSUMPTION 2 (Concavity): $\partial^2 f_i(\ )/\partial x_i^2 < 0.$

ASSUMPTION 3 (Convexity): $\partial^2 f_i(\ )/\partial x^2 > 0.$

ASSUMPTION 4 (Essentiality): $f_i(0 , x) = 0.$

ASSUMPTION 5 (Feasibility): $f_i(x_i , 0) = \varnothing.$

ASSUMPTION 6 (Congestion): $\lim_{x \to \infty} f_i(x_i , x) = 0.$

These assumptions imply the usual restrictions on returns to the variable factor, namely that they diminish with increased intensity of use, and that the factor is essential for production. With respect to aggregate employment, several additional restrictions are implied. Precisely, production is infeasible unless congestion is present and, when it is, output declines asymptotically with respect to total employment.

To motivate further the externality in (2), consider a specific example. Some tractable forms satisfying Assumptions 1-6 are the family of Cobb-Douglas technologies:

(3) $$f_i(x_i , x) \equiv \left(\frac{x_i}{x} \right)^{\alpha_i}, \qquad 0 < \alpha_i < 1, \qquad i=1..N.$$

These examples highlight some important aspects of the problem. Note that the functions $\{ f_i(\cdot)\ i=1..N \}$ are homogeneous of degree zero. In this case, individual output remains unaffected when all firms intensify effort by the

same proportion. We could think of such a situation as one in which congestion is "pure." In this case, in order for individual returns to increase, factor growth must be disproportionately greater in the firm than it is in the rest of the community. Hence, firms strive to "outdo" the competition, increasing effort above the level one expects in the absence of congestion. Inevitably, this will lead to over-exploitation of the common property—unless one of two situations emerges: Either the firms agree to collude, or they exercise moral restraint, the rationale for which is not apparent, at present.

As usual, in a model in which agents are linked by an aggregate of their own actions, intuition is aided by focusing on symmetric situations. In this case technology becomes $\{ f_i(\cdot) \equiv 1/N^{\alpha_i} \; i=1..N \}$ and, thus, depends solely on the number of agents who exploit the resource. Individual output, although always positive, is a declining function of the number of firms. A limiting case, which is approached asymptotically—but is ruled out strictly by Assumptions 2 and 3—is the situation in which each firm's output equals its share in total effort. That is, when production is $\{ f_i(\cdot) = 1/N \; i=1..N \}$. In this case, output declines monotonically and convexly as numbers increase. The elasticity of firm supply with respect to firm numbers is minus unity. The elasticity of aggregate supply, on the other hand, will be zero. To motivate this, note that aggregate output, y, and the individual components, $\{ y_i \; i=1..N \}$, are related

$$(4) \qquad y = \sum_{i=1}^{N} y_i .$$

In the symmetric situation with production $\{ f_i(\cdot) = 1/N \; i=1..N \}$, aggregate output is $y = N \; y_i = 1$, which is constant. It follows that the elasticity of supply with respect to new entry is zero. In such a situation we say that congestion is "complete" because increases in the number of firms fail to affect output.

A description of the environment is incomplete without a link between price and output. We will assume the particular one

$$(5) \qquad \sigma_p = P(y) ,$$

where $P(\cdot)$ declines monotonically in y, and it is made explicit that agents assume that the price of their product is given. Hence, we also ignore the possibilities for strategic interaction in the product market. This may, at first,

seem incongruent with a basic supposition, namely that firms recognize possibilities for strategic interaction. The price-taking assumption, however, is easily rationalized. The output from the common property may be small in relation to total supply. This, in turn, may be justified by noting that, whereas outputs are usually transportable to central markets, common properties are not. Nevertheless, the results of my analysis will be sensitive to this assumption, but then, this simply serves to reinforce the central proposition of the paper.

Abstracting from strategic concerns in the market is important for an additional reason. I wish to highlight the fact that the conventional model of oligopoly, and the price-taking model proposed here are essentially the same. In other words, the model of static, homogeneous-product oligopoly is, itself a model of an externality where the externality arises due to congestion. This interpretation can be motivated with reference, once again, to the examples in (3).

Symmetric specifications of (3) are indicative of situations in which firm and aggregate output move in opposite directions in the face of new entry. In such situations, new entrants "steal business" from incumbents. That is, the combined output of incumbents contracts, but by a lesser proportion than the increment attributed new entrants. Consequently, aggregate output expands. The existence of this phenomena has received considerable attention in the literature on oligopoly (Seade, Mankiw and Whinston). However, its purpose here is to motivate a different issue. This is that the static, homogeneous-product model shares an essential similarity with the common-property model proposed in this paper. In the latter case, there is an externality in production: aggregate effort affects the production of each of the firms. In oligopoly there is an externality in marketing: aggregate output affects the price received by each of the firms. This link between oligopoly and the study of common property is important. The conjectural-variations model employed below, is a time-honored setting within which to generalize conduct, and the framework is equally applicable to both situations.

Thus, allocations of common property generate an externality arising from the effects of congestion. This externality will lead to inefficiency in the market allocation. When production is homogeneous of zero degree, we say that congestion is "pure," and when new entry leaves aggregate output unaffected, we say that congestion is "complete." These features of the problem are suitably illustrated by the examples in (3). Although they are not used directly in subsequent analysis, these specifications help motivate additional concepts and we will refer to them occasionally throughout the remainder of the paper.

Conduct

Facing product price σ_p and factor price σ_w, firms choose effort $\{ x_i \; i=1..N \}$. When making these decisions, they form *conjectural variations*. These are expressions of the form $\{ x_j = \chi_{ij}(x_i) \; j=1..N \}$, which depict perceptions about the relationship between own effort and those of others' within the common property. Each agent solves:

$$(6) \qquad \text{Problem 1:} \quad \begin{cases} \underset{y_i, x_i}{\max:} & \pi_i \equiv \sigma_p \, y_i - \sigma_w \, x_i \\ \text{subject to:} & \\ & y_i \leq f_i(x_i , x) \qquad i=1..N. \\ & x = \sum_{j=1}^{N} x_j \\ & x_j = \chi_{ij}(x_i) \qquad j=1..N \end{cases}$$

To each conjectural variation corresponds a *conjectural elasticity*, or, simply, a *conjecture*, depicting perceptions about rivals' rates of response. For notational simplicity I include $\{ \chi_{ii}(x_i) \equiv x_i \; i=1..N \}$ depicting each agent's perception of itself. Let $\{ \hat{\theta}_{ij} \equiv (\partial \chi_{ij}(\cdot)/\partial x_i)(x_i/\chi_{ij}(\cdot)) \; i,j=1..N \}$ denote *i's* perception about *j's* response, when agent *i* adjusts its contribution to public effort. If agents observe effort levels and know how to count, equation (1) implies a set of correspondences $\{ x = \chi_i(x_i) \equiv \sum_j \chi_{ij}(x_i) \; i=1..N \}$, relating own effort to the community aggregate, and another set $\{ \hat{\theta}_i \equiv \sum_j (x_j/x) \, \hat{\theta}_{ij} \; i=1..N \}$, relating the inter-agent conjectures, to the set of aggregate ones $\{ \hat{\theta}_i \equiv (\partial \chi_i(\cdot)/\partial x_i)(x_i/\chi_i(\cdot)) \; i=1..N \}$. By assuming that agents know how to count, we avoid some substantial algebra. However, the correspondences above should be kept in mind as we proceed.

Assuming that production occurs efficiently, the constrained optimization in Problem 1 can be reduced to:

$$(7) \qquad \text{Problem 2:} \quad \underset{x_i}{\max:} \quad \pi_i(x_i \mid \sigma_p , \sigma_w) \equiv \sigma_p \, f_i(x_i , \chi_i(x_i)) - \sigma_w \, x_i, \qquad i=1..N.$$

The corresponding first-order condition is:

$$(8) \qquad \sigma_p \, \partial f_i(\cdot)/\partial x_i + \sigma_p \, \partial f_i(\cdot)/\partial x \times \partial \chi_i(\cdot)/\partial x_i - \sigma_w \equiv \phi_i(x_i \mid \sigma_p , \sigma_w) = 0, \qquad i=1..N.$$

We will reserve discussion of the second-order conditions until later.

The appearance of the conjectures $\{ \partial\chi_i(\cdot)/\partial x_i \; i=1..N \}$ reflect a departure from the usual Nash rule, which equates marginal value product to increments $\{ \sigma_p \partial f_i(\cdot)/\partial x \; i=1..N \}$ above the real cost of the factor. If an interior solution exists under myopic behavior, one will exist when conjectures are zero, but the converse is not necessarily the case. We will assume that the relative slopes in (8) support interior solutions under a variety of conjectures. Applying some standard manipulations, we may rewrite these first-order conditions:

$$(9) \qquad MRS(x_i, x) = \frac{x_i/x}{\theta_i}, \qquad i=1..N,$$

where $MRS(x_i, x) \equiv -\partial\pi_i(\cdot)/\partial x \div \partial\pi_i(\cdot)/\partial x_i \equiv -\sigma_p \partial f_i(\cdot)/\partial x \div \sigma_p \partial f_i(\cdot)/\partial x_i - \sigma_w$ denotes the marginal rate of substitution in profits between private and public effort. An additional regularity assumption can be motivated with reference to the corresponding Nash equilibrium. In this case we can depict profit contours in the x-x_i plane. These contours represen combinations of effort that keep profits constant at a given level. From the arguments above, firms must increase own effort when the aggregate increases in order to keep profits constant. Consequently, the profit contours slope upward. But then, marginal rates of substitution must be positive, and we conclude:

ASSUMPTION 7 (Regularity): $MRS(x_i, x) \geq 0, \qquad i=1..N.$

Figure 1a further motivates the concept. The shapes of the profit contours indicate that, as congestion increases, firms must exert increasingly greater increments of effort in order to compensate. The specifications in (3) illustrate this case, and draw us toward an additional assumption. This is that, with respect to community effort, marginal rates of substitution slope upward . Accordingly, we have:

ASSUMPTION 8 (Irregularity): $\partial MRS(x_i, x)/\partial x \geq 0, \qquad i=1..N.$

We use the term "irregularity" to draw out a distinction between the presentation here and another which it parallels, namely the appropriate provision of a public good. In the public-goods literature, Assumption 7 is also invoked, but in the context of declining marginal rates of substitution. In the present case, rates of substitution increase with respect to the level of the public variable, as the two examples in figure 1b suggest.

(Insert figure 1 about here.)

Although this discussion is useful, we should note that the representation in figures 1a and 1b are not strictly possible when firms have conjectural variations. Instead of the variable x in Problem 2, the firm has a conjecture $\chi_i(x_i)$. It follows that it is not possible to depict the "trade-off" between variables in the x-x_i plane. From the point of view of the firm, the units of measurement for the first axis degenerate. Nevertheless, the Nash interpretation of Problem 2 provides a useful benchmark from which to make comparisons, and we will focus attention in the remainder of the paper on the cases circumscribed by the Nash Assumptions, 7 and 8. An important implications evolves from Assumption 7. The range of our search for consistent conjectures is now considerably reduced. For continuity, we present it as:

ASSUMPTION 9 (Non-Negativity) $\hat{\theta}_i \geq 0 \quad i=1..N.$

In this context, a Nash equilibrium can be simulated by applying the so-called Nash conjectures $\{ \hat{\theta}_i = x_i/x \;\; i=1..N \}$. Many other outcomes are, however, feasible. This, of course—if nothing else—is the redeeming feature of conjectural variations, but it also leads to an 'embarrassment of riches.' Which of the possible solutions to this problem will prevail in equilibrium ? In other words, which values for the conjectures $\{ \hat{\theta}_i \; i=1..N \}$ and the shares in total effort $\{ x_i/x \;\; i=1..N \}$ are consistent with equilibrium ? To answer this question we will use the concept of *consistent conjectures*. Given a set of responses $\{ \theta_i \; i=1..N \}$, about which the agents form beliefs $\{ \hat{\theta}_i \;\; i=1..N \}$, we will use the conditions $\{ \hat{\theta}_i = \theta_i \;\; i=1..N \}$ to identify equilibrium. We will prove it to be unique.

The Allocative Inefficiency of Alternative Regimes

Before turning to examine consistency, we can characterize efficiency in the context of equations (9), and compare the set of Pareto-efficient allocations to the ones generated in Nash and conjectural-variations equilibrium. A set of efficient allocations is a set of output levels $\{ y_i \; i=1..N \}$ and y, and a set of effort levels $\{ x_i \; i=1..N \}$ and x, that maximize net returns to the common property. There are two components. One is the surplus generated in consumption of the output; the other is the surplus generated from producing and marketing the product. Hence, the objective is maximization of consumers' surplus plus profits, subject to the aggregation conditions, and the constraints on the technological possibilities available to the firms. Formally, the set of efficient allocations solves:

(10) Problem 3:

$$\begin{cases} \max: & \Omega(\,) = \int_{\omega_1} P(s)\,ds - \sigma_p\, y \\ & \qquad + \sum_{i=1}^{N} \sigma_p\, y_i - \sum_{i=1}^{N} \sigma_w\, x_i \\ \text{subject to:} & \\ & y = \sum_{i=1}^{N} y_i \\ & x = \sum_{i=1}^{N} x_i \\ & f_i(x_i, x) \geq y_i \qquad i = 1..N \end{cases}$$

After some rearrangement, the Lagrangean can be written with the use of a set of multipliers $\{\lambda_i\ i=1..N\}$ on the production possibilities. This Lagrangean is:

$$(11) \qquad L(y_1..y_N, x_1..x_N; \lambda_1..\lambda_N) \equiv \int_{\omega_1} P(s)\,ds - \sum_{i=1}^{N} \sigma_w\, x_i + \sum_{i=1}^{N} \lambda_i \left(f_i\left(x_i, \sum_{i=1}^{N} x_i\right) - y_i \right).$$

The corresponding Kuhn-Tucker conditions are:

$$\begin{aligned} &\partial L(\cdot)/\partial y_i \equiv \sigma_p - \lambda_i \leq 0, && y_i\, \partial L(\cdot)/\partial y_i = 0, \quad i=1..N. \\ (12) \quad &\partial L(\cdot)/\partial x_i \equiv -\sigma_w + \lambda_i \left(\partial f_i(\,)/\partial x_i + \partial f_i(\,)/\partial x_i \right) \leq 0, && x_i\, \partial L(\cdot)/\partial x_i = 0, \quad i=1..N. \\ &\partial L(\cdot)/\partial \lambda_i \equiv f_i(\cdot) - y_i \geq 0, && \lambda_i\, \partial L(\cdot)/\partial \lambda_i = 0, \quad i=1..N. \end{aligned}$$

Confining attention to interior solutions to (12), we can solve for the set of efficient allocations of the common property. After some algebra and rearrangement, we find that this set of allocations is characterized by the condition that the sum of marginal rates of substitution in profits across the N agents equals the value one. In other words, the condition

$$(13) \qquad \sum_{i=1}^{N} MRS(x_i, x) = 1,$$

characterizes the set of efficient allocations. Through this condition, we wish to compare sets of alternative allocations generated under different modes of conduct. In this venture, equations (9) will prove extremely useful. Summing over the N agents, we obtain:

$$(14) \qquad \sum_{i=1}^{N} \mathrm{MRS}(\, x_i \,,\, x \,) \;=\; \sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} \quad .$$

The expression on the right side will reappear at a subsequent juncture. We will refer to this term as the "gamma function" $\Gamma(\,\cdot\,) \in [\,0\,,\,\infty\,)$. It follows that $\Gamma(\,\cdot\,)$ provides a measure of the extent of inefficiency in the conjectural-variations allocation. In the case of myopic behavior, the values $\{\, \hat{\theta}_i = 0 \;\; i=1..N \,\}$ can be viewed as the limits of a set of strictly positive conjectures. As the belief of any single firm approaches zero, the extent of inefficiency in the market allocation approaches its maximum, which is infinity. Other modes of conduct will yield alternative measures of inefficiency and, without further specification of the values of the conjectures, little more can be said. Three particular situations are, however, noteworthy. The first is the value obtained in Nash equilibrium; the second is the value observed when there is concordance among the firms; and the third is the value derived from a simple transformation of the Nash conjectures. Below, we consider each case in turn.

Nash equilibrium, recall, is synthesized by the conjectures $\{\, \hat{\theta}_i = x_i/x \;\; i=1..N \,\}$. In this case, each of the agents conjecture in relation to their relative contributions in total effort. When these shares are inserted into the right-hand side of (14), we obtain a condition representing the market allocation in Nash equilibrium. It is:

$$(15) \qquad \sum_{i=1}^{N} \mathrm{MRS}(\, x_i \,,\, x \,) \;=\; N.$$

Comparing (15) and (13), the extent of departure from an efficient allocation is seen to increase monotonically as the number of agents increases. Maximum efficiency—although minimum equity—is attained if the right to the common property is granted to a single individual. This conceptual solution to the common-property problem is well known (e.g., Weitzman). Figure 1c illustrates optimal and Nash-equilibrium solutions.

As an alternative to Nash equilibrium, suppose that the agents have homogeneous beliefs. That is, suppose there exists concordance of the form $\{\, \hat{\theta}_i = \hat{\theta} \;\; i=1..N \,\}$. How such a situation comes to be realized, we leave unanswered for the moment. Inserting these conjectures into the right side of (14), we compute

$$(16) \qquad \sum_{i=1}^{N} \mathrm{MRS}(\, x_i \,,\, x \,) \;=\; \hat{\theta}^{\,-1} \,,$$

where we have used the fact that the shares sum to one. In this case, the extent of inefficiency in the market allocation depends on the extent to which $\hat{\theta}$ departs from the value one. Values below one lead to values for $\Gamma(\cdot)$ that exceed one, to over-allocations of effort, and thus to over-exploitation, values that exceed one lead to values for $\Gamma(\cdot)$ below one, and to under-allocations. Figure 1d illustrates a situation in which effort is over-allocated.

Now, consider a simple transformation of Nash equilibrium, in which the agents conjecture values that are N times the so-called Nash conjectures. In such a situation $\{ \hat{\theta}_i = Nx_i/x \;\; i=1..N \}$ and, consequently, the right side of (14) equals one. It follows that the set of conjectures N times the static Nash conjectures locates an efficient solution. Once again, we leave unanswered the forces that might bring the agents to this conclusion, but we return to it subsequently.

This discussion highlights the importance of three features that determine efficiency in the market allocation. The first is the distribution of relative effort $\{ x_i/x \;\; i=1..N \}$. The second is the distribution of beliefs $\{ \hat{\theta}_i \;\; i=1..N \}$. The third is the number of agents who exploit the common property. The extent of departure from an efficient allocation depends on the magnitude of each agent's conjecture in relation to its share in total effort, and the function $\Gamma(\cdot) \in [\, 0 , \infty \,)$ can be used as a convenient index within which to nest the various outcomes and compare their relative efficiency. The condition $\Gamma(\cdot) = 1$ identifies efficient allocations.

The Specific Nature of The Common-Property Game

Before turning to characterize consistency, a few comments about the equilibrium in (14) are in order. The first is that it is static. It is static in both the Nash situation and in the general, conjectural-variations setting. In both cases it is assumed that equations (8) define a strict local maximum on the interior of each agent's choice set. In Nash equilibrium, Assumptions 1-6 are sufficient to guarantee this, but they are insufficient, in general, under conjectural variations. We will assume, momentarily, that they are satisfied and, subsequent to deriving consistent conjectures, ascertain the further restrictions that are required for them to hold.

An Independence Axiom

Equations (8) contain an important implication for the remaining analysis. The expressions $\{ \phi_i(x_i \mid \sigma_p , \sigma_w) = 0 \;\; i=1..N \}$ are independent of rivals' levels of effort. The intuition is straight-forward: Since the firm has a conjecture

about each of its rivals, it believes others' allocations to depend on its own. Such a dependence cannot prevail among all agents, and in every equilibrium, except in one particular circumstance, namely the consistent-conjectures equilibrium. These observations were made long ago (Stackelberg, Leontief), but they went unnoticed until quite recently (Pace and Gilley). Their significant implication lies in uncovering a conceptual error perpetrated in previous attempts to derive consistent conjectures. We turn now to examine this issue.

Antecedents

The consistent-conjectures problem received a good deal of attention in the oligopoly literature in the 1980's. However, both a general characterization of it, and a general solution to it, proved elusive. The proposed methodology is to compute the response between any pair of firms, *ceteris paribus*, by applying the implicit function theorem to one of the firm's first-order conditions. Unfortunately, this procedure is flawed. The reason is similar to the one outlined in the derivation of the graphics. It stems from the independence axiom and, specifically, the definitions of the conjectural variations $\{ \chi_{ij}(x_i) \; j=1..N \; j\neq i \}$. Since agent i has *conjectures* about the effort of each of its rivals, the levels $\{ x_j \; j=1..N \; j\neq i \}$, no longer appear in agent i's objective function. Consequently, these levels are absent in the corresponding first-order conditions. It follows that the traditional methodology cannot be applied because it requires us to perturb the effort of at least two agents, when the level of effort of only one of them appears. Any approach that perturbs the effort of a rival in the first-order condition of another firm is inconsistent with the conjectural variations paradigm. An alternative approach is required.

The Equilibrium Concept

The method used below is more akin to traditional equilibrium analysis: Equations (1) and (8) are assumed to determine an equilibrium in the $N+1$ endogenous variables $\{ x^* \; \{ x_i^* \; i=1..N \} \}$, given the prices σ_p and σ_w. When these prices change, we compute the corresponding changes in effort which occur, and proceed to find values for the conjectures that equate the observed adjustments with the ones conjectured in the initial equilibrium. We say that a set of conjectures $\{ \dot{0}_i \; i=1..N \}$ that meet this criteria are consistent.

An issue that we leave unanswered momentarily, is the process by which the firms come to be consistent. This is another aspect of the theory that is controversial—but it is also controversial in the alternative Nash equilibrium.

The controversy relates to philosophical issues and will not be pursued further at this point. One immediate justification for considering consistent conjectures is that, without such a restriction, we can say little more about common property allocations than we have done so above. Since we know already a good deal about the allocation in Nash equilibrium, it seems reasonable, moreover natural, to extend knowledge in this direction.

Genesis of The Fixed Point Problem

A focus on consistent conjectures, requires that we examine comparative statics. Consequently, the equilibrium is no longer static. There are now two phases to the game. In the first stage, firms form conjectures and exert effort conditional on the responses they expect from their rivals. This part of equilibrium is encompassed by equations (1) and (8). An important point to recognize is that, given the information available to them, each agent exerts effort in an optimal manner. There is, therefore, no tendency for adjustment until some force, exogenous to equilibrium, displaces the variables in (8). When adjustments occur, agents observe their rivals' responses and compare these to the ones they conjectured in the initial equilibrium. When the conjectures and adjustments conform, we say that conjectures are *consistent*. However, the equilibrium concept possesses an additional subtlety. The responses computed around the equilibrium will depend on the values of all parameters within the model, and the conjectures $\{\hat{\theta}_j \mid j=1, N\}$ comprise a subset of these. It follows that a solution to a set of fixed-point equations is implied.

Preliminary Caveats

Before characterizing a consistent-conjectures equilibrium -- which requires some mathematical digression--it should be understood that neither the assumption of Nash behavior, nor that of consistent conjectures is made under the illusion that these modes of conduct correctly model actual behavior. They are but two of, possibly, many solution concepts that one could employ. Their importance lies in extending the investigation beyond the myopic case in which agents equate marginal-value-product to the real cost of effort. Without the imposition of consistency, or some similar criteria, the allocations in (14) are ad hoc and arbitrary.

There are three relevant questions within this context. The first is the theoretical robustness of the equilibrium concept; the second is the extent to which the theoretical development aids new insight; and the third is the ease and tractability with which these insights are obtained. The consistent-conjectures equilibrium located below performs

quite well under the latter two criteria, but not the first. Whether, under this failure, it is a suitable characterization of real behavior, or an approximation thereof, remains open to question.

Locating Consistent Conjectures

The arguments that follow are an abridged version of derivations contained in a related work (Anon., 1995), which is available upon request. Its focus is a set of philosophical issues implicit in consistent-conjectures economies. One of these is that the computations necessary to locate the equilibrium are within the intellectual capacities of the agents. Hence, they will be brought inextricably to our conclusions. To derive them, we initiate a search, which we predicate as follows.

Preliminary Concepts

The first step requires formulation of several definitions. The first pertains to comparative-static perturbations of the static equilibrium. A technical detail that will prove useful is to express derivatives in proportional-change terms. This approach allows not only the qualitative effects of certain adjustments to be ascertained, but also their relative magnitudes. Henceforth, a tilde above a variable will denote a change in its value relative to that in initial equilibrium. That is, for some variable, v, let $\tilde{v} \equiv \Delta v/v$ denote its proportional change. Applying this calculus in equations (1) and (8), the adjustments $\{ \tilde{x}, \{ \tilde{x}_i \; i=1..N \} \}$ emanating from changes in prices $\{ \tilde{\sigma}_p \; , \tilde{\sigma}_w \}$ are:

$$\tilde{x} = \sum_{i=1}^{N} \frac{x_i}{x} \tilde{x}_i \, , \tag{17}$$

$$\upsilon_i \tilde{x}_i + \mu_i \tilde{\sigma}_p + \eta_i \tilde{\sigma}_w = 0, \qquad i=1..N, \tag{18}$$

where $\upsilon_i \equiv x_i \partial^2 \pi_i(\cdot)/\partial x_i^2 \equiv \upsilon_i(\hat{\theta}_i)$, $\mu_i \equiv \sigma_p \partial^2 \pi_i(\cdot)/\partial x_i \partial \sigma_p \equiv \mu_i(\hat{\theta}_i)$ and $\eta_i \equiv \sigma_w \partial^2 \pi_i(\cdot)/\partial x_i \partial \sigma_w \equiv \eta_i(\hat{\theta}_i)$ depend, implicitly, on the conjectures. The fixed-point problem arises from the fact that the responses depend on structural parameters which, in turn, depend on conjectures. With these concepts at hand, we can state the following:

DEFINITION (Consistent-Conjectures Equilibria): A consistent-conjectures equilibrium is a set of efforts $\{ x^* \mid x_i^* \; i=1..N \}$ that satisfy (1) and (8), a set of adjustments $\{ \tilde{x} \mid \tilde{x}_i \; i=1..N \}$ that satisfy (17) and (18), and a set of conjectures $\{ \hat{\theta}_i \; i=1..N \}$ that solve the fixed-point problem

$$\hat{\theta}_i = \frac{\tilde{x}(\hat{\theta}_1 \ldots \hat{\theta}_N)}{\tilde{x}_i(\hat{\theta}_i)} \qquad i=1..N$$

There are two points to note about these concepts. The first is that they are not readily amenable to further manipulation. The second is that they identify conditions that are sufficient for equilibrium. The strategy adopted below is to identify a set of conditions that are only necessary, but which are more amenable to manipulation, and restrict the range of the search. Since these conditions are only necessary ones, they admit a potentially larger set of conjectures than the ones we ultimately seek. Therefore, I term these conditions *admissibility criteria* and the conjectures that meet them *admissible conjectures*.

Admissibility

There are three notions of admissibility. One, termed *firm-wise admissibility*, pertains to the firms making correct conjectures about aggregate responses. Another, termed *pair-wise admissibility*, pertains to the firms making correct conjectures about rivals' responses. The third, termed *group-wise admissibility*, pertains to the first notion and, the addition criterion that the conjectured adjustments conform to the aggregation condition in (17). Interest in each of these concepts lies in the extent to which they narrow the search for consistent conjectures. We will discover that they narrow the search quite substantially.

Search Strategy

We begin by first applying the group-wise admissibility criterion and, subsequently, consider admissibility between pairs of firms. Pair-wise admissibility will derive the most general form of the fixed-point equations that are consistent with equilibrium. This is a reduced-form in which the observed adjustments $\{ \theta_i \; i=1..N \}$ are expressed as functions of the conjectured adjustments $\{ \hat{\theta}_i \; i=1..N \}$ and the set of shares $\{ x_i/x \; i=1..N \}$. The key result is to show that every pair-wise admissible equilibrium in which group-wise admissibility prevails is a consistent-conjectures

equilibrium. Existence is considered subsequently. Some intermediate results will be necessary, but most of these can be traced to the admissibility criteria.

Group-Wise Admissibility

A single restriction evolves from manipulating the firm-wise admissibility criterion, combining it with equation (17), and applying some traditional methodology to a linear system. Firm-wise admissibility requires that the conjectured adjustments equal the actual ones, that is $\{ \hat{\theta}_i = \theta_i \;\; i=1..N \}$. The observed adjustments are defined $\{ \theta_i \equiv \tilde{x} / \tilde{x}_i \;\; i=1..N \}$. Hence, conjectures are firm-wise admissible when they satisfy $\{ \hat{\theta}_i = \tilde{x} / \tilde{x}_i \;\; i=1..N \}$. Rewriting these conditions we have N equations

$$\tilde{x} = \hat{\theta}_i \, \tilde{x}_i , \qquad i=1..N. \tag{19}$$

Combining these equations with the condition in (17) we obtain the linear system

$$\Sigma \, \tilde{x} = 0, \tag{20}$$

where $\tilde{x} \equiv (\tilde{x}, \tilde{x}_1 .. \tilde{x}_N)^T$, 0 denotes the N+1 null vector, and Σ is defined

$$\Sigma \equiv \begin{pmatrix} 1 & -x_1/x & -x_2/x & \cdots & -x_N/x \\ 1 & -\hat{\theta}_1 & & & \\ 1 & & -\hat{\theta}_2 & & \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & & & & -\hat{\theta}_N \end{pmatrix}. \tag{21}$$

Since the system in (20) is homogeneous, the matrix Σ must be singular in order to force a non-trivial solution $\tilde{x} \neq 0$. Therefore, we restrict attention to configurations that satisfy the restriction $|\Sigma| = 0$. This can be written explicitly:

$$\prod_{i=1}^{N} -\hat{\theta}_i + \sum_{i=1}^{N} x_i/x \prod_{j \neq i}^{N} -\hat{\theta}_j = 0. \tag{22}$$

We say that a set of conjectures $\{ \hat{\theta}_i \;\; i=1..N \}$ that satisfy this condition are group-wise admissible.

Excluding Myopic Conduct

Since there are N conjectures, but only one constraint, there are, in general, an infinite number of combinations that satisfy the group-wise admissibility criterion. The next step is to rule out competitive behavior. In order for the conjectures $\{ \hat{\theta}_i = 0 \; i=1 \ldots N \}$ to be firm-wise admissible, the conditions $\{ \tilde{x} / \tilde{x}_i = 0 \; i=1 \ldots N \}$ must be met. There are two possible situations in which this may arise. The first is an infinite adjustment on the part of the firm, which we exclude from further consideration. The second is the case where aggregate effort is fixed, but the firm adjusts effort by a finite amount. In this case, the effort of at least one other firm must move in opposite direction. But then there must exist a conjecture that has a negative sign, which implies a contradiction. In particular, Assumption 7 has already confined attention to the case where marginal rates of transformation in profits are non-negative. It follows that, the set of firm-wise admissible conjectures must satisfy $\{ \hat{\theta}_i > 0 \; i=1 \ldots N \}$.

Admissibility, Efficiency, and The Gamma Criterion

Multiplying through in equation (22) with the product $\prod_{i=1}^{N} \hat{\theta}_i \neq 0$, we obtain

$$(23) \qquad \sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} \equiv \Gamma(\cdot) = 1.$$

Comparing equations (13) and (14) with (23), a fundamental results emerges: Every group-wise admissible allocation is an efficient allocation. Since group-wise admissibility is necessary for consistency, it follows that every consistent-conjectures allocation is also efficient.

These two findings summarize the central theme of the paper: When agents have foresight about the behavior of the group, a form of self enforcement prevails in which resources are allocated in a non-exploitive manner. It follows that admissible conjectures provide a formal foundation for the self-restraint argument posed by Braden, which was quoted at the outset of the investigation. The intuition for the result is straight-forward: The common-property problem is, essentially, a property-rights problem in which an externality arises. When agents correctly conjecture about the actions of their rivals, they internalize the externality. They make decisions on behalf of the community as a whole, and form production plans that replicate those of a social planner wherein the externality is internalized. The resultant market allocation is efficient.

One could conclude the investigation at this point and consider the empirical relevance of these results. Before doing so, however, we should consider the specific values for the conjectures $\{ \hat{\theta}_i \ i=1..N \}$ that forge efficiency in the equilibrium allocation, how these specific beliefs may come to be realized, and the generality of the equilibria that can support them. Unfortunately, the equilibrium structures that generate consistent conjectures are quite restrictive. They are, however, the structures that we encounter most frequently in the literature.

A Nash Interpretation of The Solution Concept

In this section we ask the question about the types of values for the conjectures $\{ \hat{\theta}_i \ i=1..N \}$ that satisfy the condition $\Gamma(\cdot) = 1$. In order to tie down the consistent conjectural variations, an additional restriction is deemed necessary. The restriction employed is a form of Nash discipline imposed on the beliefs of the agents. Its justification is its obvious precedent in the one-shot, static game.

To motivate a Nash interpretation of the consistent conjectural variations, rearrange the condition $\Gamma(\cdot) = 1$ to obtain:

$$\hat{\theta}_j = \frac{x_j/x}{1 - \sum_{i \neq j}^{N} \frac{x_j/x}{\hat{\theta}_i}} \tag{24}$$

Suppose that all firms other than the j^{th} conjecture collusively. That is, suppose $\{ \hat{\theta}_i = 1 \ i=1..N \ i \neq j \}$. Satisfaction of equation (24) then requires $\hat{\theta}_j = 1$. Recall that the Gamma criterion is necessary for consistency and interpret this result as follows: Given that the other firms conjecture collusively, firm j—wishing to form a consistent conjecture—must also conjecture collusively. Alternatively, suppose that the other firms each have conjectures that are N times the so-called Nash conjectures. That is, suppose $\{ \hat{\theta}_i = Nx_j/x \ i=1..N \ i \neq j \}$. Satisfaction of (24) then requires $\hat{\theta}_j = Nx_j/x$. In this case, firm j must also form a conjecture that is N times the value of its share in group effort. Thus, the desire for consistency forces a type of Nash discipline on each of the firms. Note that this type of discipline is lacking in the general, conjectural-variations model. It is precisely for this reason that we appeal to consistency.

The interpretation of consistent conjectures as Nash strategies in a two-stage game has gone unnoticed in the literature.

Pair-Wise Admissibility

Until this point, little has been said about the accuracy of inter-firm predictions. We will exploit a restriction embedded in the firm-wise admissibility conditions in order to narrow the search. Specifically, we will use this restriction to identify the degree of heterogeneity that is permissible among the firms.

The inter-firm conjectures are the N^2 values $\{ \hat{\theta}_{ij} \; i,j=1..N \}$ which depict *i's* conjecture about *j's* response. This set is deemed pair-wise admissible when each conjecture equals the corresponding observed response. That is, when the equalities $\{ \hat{\theta}_{ij} = \theta_{ij} \; i,j=1..N \}$ are attained. The observed effects are $\{ \theta_{ij} \equiv \tilde{x}_j / \tilde{x}_i \; i,j=1..N \}$. It follows that conjectures are pair-wise admissible when they satisfy $\{ \hat{\theta}_{ij} = \tilde{x}_j / \tilde{x}_i \; i,j=1..N \}$.

We can avoid working with the inter-firm conjectures and redirect attention toward the aggregate responses with the help of a simple observation. This is that firm-wise admissibility implies $\{ \hat{\theta}_i / \hat{\theta}_j = \tilde{x}_j / \tilde{x}_i \quad i,j=1..N \}$. This is obtained by dividing through, in pairs, the firm-wise admissibility criteria $\{ \hat{\theta}_i = \tilde{x} / \tilde{x}_i \; i=1..N \}$. It follows that the set of equalities

$$\hat{\theta}_1 \tilde{x}_1 = \ldots = \hat{\theta}_N \tilde{x}_N, \tag{25}$$

are implied. Another, more direct approach is to note, simply, that these expressions are implied by the left-hand sides of equations (19). The motivation above, although more complex, serves to draw out the fact that the restriction in (25) permits there to be accurate predictions among each and every pair of firms. It follows that, firms' demand responses must be similar, and of the form

$$\tilde{x}_i = \hat{\theta}_i^{-1} \phi, \qquad i=1..N, \tag{26}$$

where ϕ is a common, non-zero scalar equal to the aggregate adjustment, $\tilde{x}$. Employing these forms in (17), the corresponding movement in aggregate effort is

$$\tilde{x} = \sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} \phi . \tag{27}$$

Finally, normalizing repeatedly on each of the responses in (26), we obtain the set of reduced-forms

$$(28) \qquad \theta_i = \hat{\theta}_i \sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} , \qquad i=1..N.$$

These depict the dependence of observed outcomes on the distribution of beliefs and effort across the firms. We say that an equilibrium is pair-wise admissible if it generates these reduced-forms.

Consistency

Rewriting the reduced forms in (28) $\{ \theta_i = \hat{\theta}_i \Gamma(\ ) \ i=1..N \}$, an important implication is observed. This is that, when firms have pair-wise admissible demand responses, the criterion $\Gamma(\ ) = 1$ is sufficient to locate consistent conjectures. In other words, every pair-wise admissible equilibrium in which group-wise admissibility prevails is a consistent-conjectures equilibrium.

Previously, we motivated the cartel conjectures $\{ \hat{\theta}_i = 1 \ i=1..N \}$ and the N-firm multiples of the Nash conjectures $\{ \hat{\theta}_i = Nx_i/x \ i=1..N \}$ as candidate consistent conjectures. There are now two issues at hand. The first is to examine to which, if either of these two sets of conjectures that the economy converges; the second is to examine whether the equilibrium, once converged, can support consistent conjectures.

Convergence

The convergence concept pertains to a simple question: Beginning from an arbitrary distribution of shares and conjectures across the firms, is convergence toward consistent conjectures possible ? Moreover, is convergence ever likely ? These matters are too complex to be handled in detail within this paper. However, we can shed light on them with reference to the results of several, simple simulations. These are reported in the background paper (Anon., 1995), and are available upon request.

Experiments are conducted within the context of a game with repeated play and updating. Firms are endowed with pair-wise admissible responses. In an initial period they realize effort from a random distribution and form Nash conjectures $\{ \hat{\theta}_i = x_i/x \ i=1..N \}$. A sequence of random shocks is generated and, through the common parameter ϕ, is assumed to influence effort according to equations (23). The responses in successive periods are computed and the observed responses are compared to the conjectured ones. The conjectures are updated according to some common decision rules, including hindsight, adaptive expectations, and the means of past histories. The results are quite

compelling For industries with few firms—generally, less than twenty—convergence to the conjectures $\{ \hat{\theta}_i = Nx_i/x \;\; i=1..N \}$ occurs frequently and, usually, quite rapidly. With sufficient numbers of positive shocks, the distribution of effort becomes symmetric, in which case the conjectures converge toward the collusive ones $\{ \hat{\theta}_i = 1 \;\; i=1..N \}$. Generally speaking, exceptions occur when there are many firms, when the shocks are large, and—in a few cases—when the initial distribution of output is highly asymmetric.

Existence

The final question is whether the equilibrium can support responses like those in (23). We will discover that it can, but only in a very special circumstance. This is the symmetric equilibrium with identical firms and, hence, the set of collusive conjectures $\{ \hat{\theta}_i = 1 \;\; i=1..N \}$. This conclusion is derived by examining the explicit definitions of the structural parameters, which appear below (18), and comparing the responses that they generate with the ones in (23). The responses in (18) can be rewritten:

$$\tilde{x}_i = - \frac{\mu_i(\hat{\theta}_i)}{\upsilon_i(\hat{\theta}_i)} \tilde{\sigma}_p - \frac{\eta_i(\hat{\theta}_i)}{\upsilon_i(\hat{\theta}_i)} \tilde{\sigma}_w , \qquad i=1..N, \tag{29}$$

where $\upsilon_i(\hat{\theta}_i) \equiv x_i \, \partial^2 f_i(\cdot)/\partial x_i^2 + 2 x \hat{\theta}_i \, \partial^2 f_i(\cdot)/\partial x_i \partial x + \hat{\theta}_i^{\,2} x^2 x_i^{-1} \partial^2 f_i(\cdot)/\partial x^2 + x_i \, \partial f_i(\cdot)/\partial x \times \partial^2 \chi_i(\cdot)/\partial x_i^2$, $\eta_i(\hat{\theta}_i) \equiv \sigma_w$, $\mu_i(\hat{\theta}_i) \equiv -\sigma_w$, and use has been made of an equality in the first-order conditions, (8).

Two important features of the structural forms are observed. The first is that the parameters $\{ \eta_i(\hat{\theta}_i) \; i=1..N \}$ and $\{ \mu_i(\hat{\theta}_i) \; i=1..N \}$ are constant across firms. The second is that they are independent of the conjectures. It follows that, in order for the responses to conform with those in (23), the denominators in the right side of (29) must factor in proportion to the conjectures. In other words, the set of factorizations $\{ \upsilon_i(\hat{\theta}_i) \propto \hat{\theta}_i \;\; i=1..N \}$ is required. From the definition of $\upsilon_i(\hat{\theta}_i)$ below (29), such factorizations can exist only in one very, special circumstance. This is where heterogeneity degenerates. It follows that the identical-firms, symmetric equilibrium with collusive conjectures, $\{ \hat{\theta}_i = 1 \;\; i=1..N \}$, is unique. There is, however, one additional caveat. This is that the second-order conditions are satisfied. In the symmetric, collusive-conjectures equilibrium these conditions are

$$\partial^2 \pi_i(\cdot)/\partial x_i^2 \equiv \partial^2 f_i(\cdot)/\partial x_i^2 + 2N \, \partial^2 f_i(\cdot)/\partial x_i \partial x + N^2 \partial^2 f_i(\cdot)/\partial x^2 < 0, \qquad i=1..N. \tag{30}$$

The first expression on the right side is negative, as is the second. The third, however, is positive. When the number of agents increases, this third term grows exponentially as the second grows only linearly, and it follows that there exists a maximum number of agents permitted in the common property. In general, satisfaction of (30) depends on the specific nature of technology. The important point is this. Even if collusion is a valid prediction, it may be so only for small communities.

The issue is highlighted further with reference to examples (3). Under myopic behavior, a well-defined maximum is ensured by Assumption 2. In Nash equilibrium the second-order conditions are similar to the ones in (30). However, for the example technologies in (3), it can be derived that a symmetric, Nash equilibrium prevails for all common-properties with size $N \geq 1$. In the symmetric, consistent-conjectures equilibrium, the examples in (3) lead to exact equality in (30). The reason stems from the fact that the technologies in (3) are homogeneous of degree zero. Under consistent conjectures, firms internalize group effort, but this just says that returns to scale are constant. It follows that the solutions to (8) are not unique, unless production is non-homogeneous. Uniqueness and completeness of congestion are incompatible with consistent conjectures.

Comments

The equilibrium concept is highly restrictive. Inevitably, it draws into question the practical implications of the exercise. In its defense, however, this very restrictive setting is the one most frequently encountered, in empirical applications of conjectural variations in the *American Journal of Agricultural Economics* and elsewhere. Thus, the scope of the results rests on the predictive performance of the model in casual empirical settings. In this regard, several situations are identified which compare quite well with the predictions derived above. An early case in point is evident from Gordon's synopsis of Canadian Atlantic-coast lobster conservation. About these activities, he notes (pp. 133-34):

> The method of control here is by seasonal closure ... The result has been a steady growth in the number of lobster traps set by each fisherman ... At a fairly conservative estimate, the same quantity of lobsters could be caught with half the present number of traps. In a few places the fishermen have banded together into a local monopoly, preventing entry and controlling their own operations. By this means the amount of fishing gear has been greatly reduced and incomes considerably improved.

More recently, Lueck (pp. 104-07) cites examples of similar types of self-enforcement, in a variety of diverse settings. These include informal jurisdictions over lobster territories in Maine; tribal-conservation of hunting, trapping, and fishing grounds among Cree Indians; and efficient management of common pasture in alpine Switzerland. In the first case, cartel-like behavior evolves from familial and ethnic ties to existing facilities, and by punishment of interlopers guilty of encroachment. In the second case, informal cartels have evolved through group consensus about sanctions and penalties to impose upon infractors. In the third case collusion is governed through inheritance and through informal covenants on the types of uses for which access is permitted, and its duration.

That each of these situations may be representative of a larger and more compelling set of circumstances, remains open to question.

Conclusions

We began this investigation by citing four objectives. The first was to demonstrate that the self-restraint argument in the allocation of common property can be motivated through a model in which agents possess uncertainty about rivals' behavior. A second objective was achieved by presenting a fairly comprehensive treatment of conjectural variations applied to model the allocation of common property. Third, we forged a link between this application and its foundations in the static theory of oligopoly. Finally, we highlighted key issues in the debate about voluntary restraints in the allocation of common property and provided a formal justification for it.

An important aspect of the common-property problem is its production externality. Inefficiencies that stem from this imperfection are exacerbated by myopic behavior, which leads to over-exploitation and rent dissipation. Nash equilibrium in the one-shot, static game predicts over-allocation and attendant inefficiency, which increases with group size. Nash equilibrium in the two-stage game generates efficient allocations: Consistency forces firms to internalize an externality which is inherent in common-property production. Foresight leads to self-enforcement and market allocations that are efficient.

The broader limitations of this study leave plenty of scope for future developments, but they draw into question the practical implications of the theory. The restrictiveness of the equilibrium derived here should guide us toward more robust game-theoretic frameworks. Whether extensive-form or evolutionary games can motivate arguments in a reader-friendly manner remains open to conjecture. Nevertheless, the theory presented here suggests an important

consideration, which has gone unnoticed to date. This is that, when numbers of agents are small, when there is little heterogeneity among them, and when they have the opportunity to observe actions during repeated experiments, the market allocation may be fairly efficient. That self-enforcing situations stem from foresight of this kind, is a conjecture that needs to be tested.

References

Anon. *Conjectural Variations With Fewer Apologies.* Working Paper, March 1995.

Bowley, A. L. *The Mathematical Groundwork of Economics.* Oxford: Oxford University Press, 1924.

Braden, J. B. "Uncertainty and Open Access: Implications from the Repeated Prisoner's Dilemma Game." *American Journal of Agricultural Economics* 67(1985):356-59.

Bromley, D. W. "Arresting Renewable Resource Degradation in the Third World: Discussion." *American Journal of Agricultural Economics* 72(1990):1274-75.

Field, B. C. "The Optimal Commons." *American Journal of Agricultural Economics* 67(1985):364-67.

Gordon, H. S. "The Economic Theory Of A Common Property Resource." *Journal of Political Economy* 62(1954):124-142.

Hardin, G. "The Tragedy of The Commons." *Science* 162(1968):143-48.

Haveman, R. H. "Common Property, Congestion, and Environmental Pollution." *Quarterly Journal of Economics* 87(1973):278-87.

Huffaker, R., J. Wilen and B. D. Gardner. "Multiple Use Benefits On Public Rangelands: An Incentive-Based Fee System." *American Journal of Agricultural Economics* 71(1989):670-678.

Johnson, R. N. "Restraint Under Open Access: Are Voluntary Incentives Sufficient Or Is Coercion Required ? : Discussion." *American Journal of Agricultural Economics* 67(1985):373-76.

Leontief, W. "Stackelberg on Monopolistic Competition." *Journal of Political Economy* 34(1936):554-59.

Libby, L. W. "Conflict On The Commons: Natural Resource Entitlements, The Public Interest, and Agricultural Economics." *American Journal of Agricultural Economics* 76(1994):997-1009.

Lueck, D. "Common Property As An Egalitarian Share Contract." *Journal of Economic Behavior and Organization* 25(1994):93-108.

Mankiw, G. M. and M. D. Whinston. "Free Entry and Social Inefficiency." *Rand Journal of Economics* 17(1986):48-58.

Ostrom, E. R. Gardner and J. Walker. *Rules, Games and Common Pool Resources.* Ann Arbor: University of Michigan, 1994.

Pace, R. K. and O. W. Gilley. "A Note on Consistent Conjectures: The Leontief Legacy." *Southern Economic Journal* 56(1990):788-92.

Provencher, W. and O. Burt. "A Private-Property Rights Regime for The Commons: The Case For Groundwater." *American Journal of Agricultural Economics* 76(1994):875-88.

Runge, C. F. "The Innovation of Rules and the Structure of Incentives in Open Access Resources." *American Journal of Agricultural Economics* 67(1985):368-72.

Seade, J. "On the Effects of Entry." *Econometrica* 48(1980):479-89.

von Stackelberg, H. *Marktform und Gleichgewicht.* Vienna: Julius Springer, 1934.

Weitzman, M. L. "Free Access versus Private Ownership as Alternative Systems for Managing Common Property." *Journal of Economic Theory* 8(1974):225-34.

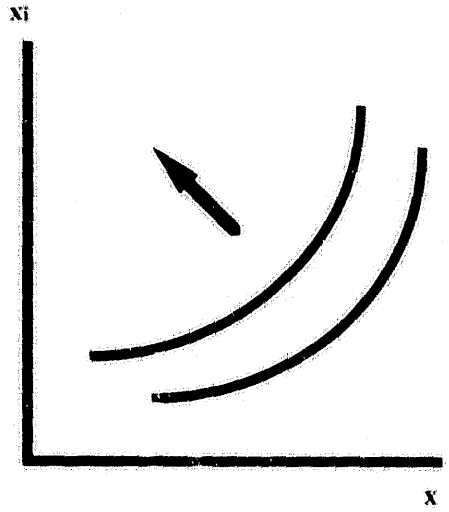

Figure 1a. Isoprofit contours

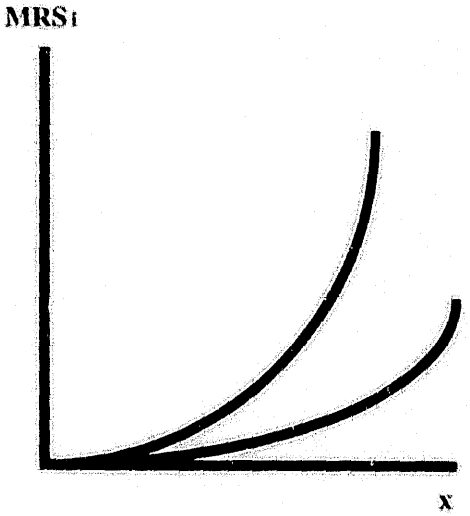

Figure 1b. Marginal rates of substitution

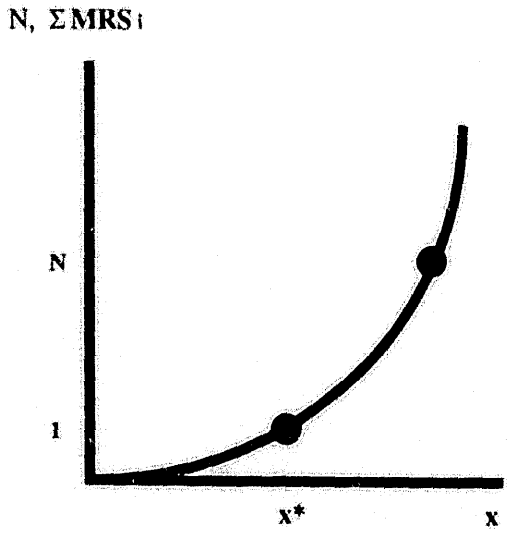

Figure 1c. Nash equilibrium allocations

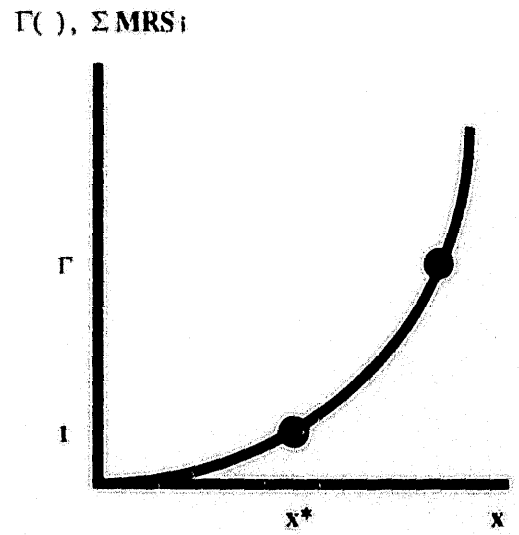

Figure 1d. Conjectural-variations allocations

Figure 1. Profit, production, and allocations of common property