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PROVIDE PROJECT

The Provincial Decision-making Enabling Project

Technical Paper 2003: 3

The PROVIDE Project Standard Computable General Equilibrium Model

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PROVIDE PROJECT

The Provincial Decision-making Enabling Project

Overview

The Provincial Decision-Making Enabling (PROVIDE) Project aims to facilitate policy design by supplying policymakers with provincial and national level quantitative policy information. The project entails the development of a series of databases (in the format of Social Accounting Matrices) for use in Computable General Equilibrium models.

The National and Provincial Departments of Agriculture are the stakeholders and funders of the PROVIDE Project. The research team is located at Elsenburg in the Western Cape.

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For the original project proposal and a more detailed description of the project, please visit www.elsenburg.com/provide

The PROVIDE Project Standard Computable General Equilibrium Model[1]

Abstract

The paper describes the Standard Computable General Equilibrium (CGE) model developed for the PROVIDE Project. The model allows for a generalised treatment of trade relationships by incorporating provisions for non-traded exports and imports, and competitive and non-competitive imports, and allows the relaxation of the small country assumption for exported commodities. The model encompasses multiple product activities by differentiating between commodities by the activities that produce them, using a range of production technologies that can be selected by the user. The model is designed for calibration using data compiled as a Social Accounting Matrix (SAM).

The model is designed so that it can be readily adapted by the user to incorporate different and/or additional behavioural assumptions.

If you use this model please acknowledge the source.

1 The author of this paper was Scott McDonald, who gratefully acknowledges the comments and corrections supplied by Cecilia Punt and Lindsay Chant.

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1. Introduction

This document provides a description of a computable general equilibrium (CGE) model. This model is characterised by several distinctive features. First, the model allows for a generalised treatment of trade relationships by incorporating provisions for non-traded exports and imports, i.e., commodities that are neither imported nor exported, competitive imports, i.e., commodities that are imported and domestically produced, non-competitive imports, i.e., commodities that are imported but not domestically produced, commodities that are exported and consumed domestically and commodities that are exported but not consumed domestically. Second, the model allows the relaxation of the small country assumption for exported commodities that do not face perfectly elastic demand on the world market. Third, the model allows for (simple) modeling of multiple product activities through an assumption of fixed proportions of commodity outputs by activities with commodities differentiated by the activities that produce them. Hence the numbers of commodity and activity accounts are not necessarily the same. Fourth, (value added) production technologies can be specified as either Cobb-Douglas or Constant Elasticity of Substitution (CES). And fifth, household consumption expenditure can be represented by either Cobb-Douglas utility functions or Stone-Geary utility functions.

The model is designed for calibration using a reduced form of a Social Accounting Matrix (SAM) that broadly conforms to the UN System of National Accounts (SNA). Table 1 contains a macro SAM in which the active sub matrices are identified by X and the inactive sub matrices are identified by 0. In general the model will run for any SAM that does not contain information in the inactive sub matrices and conforms to the rules of a SAM. In some cases a SAM might contain payments from and to both transacting parties, in which case recording the transactions as net payments between the parties will render the SAM consistent with the structure laid out in Table 1.

The most notable differences between this SAM and one consistent with the SNA are:

1) The SAM is assumed to contain only a single ‘stage’ of income distribution. However, fixed proportions are used in the functional distribution of income within the model and therefore a reduced form of an SNA SAM using apportionment (see Pyatt, 1989) will not violate the model’s behavioural assumptions.
2) The trade and transport margins, referred to collectively as marketing margins, are subsumed into the values of commodities supplied to the economy.
3) A series of tax accounts are identified (see below for details), each of which relates to specific tax instruments. Thereafter a consolidated government account

is used to bring together the different forms of tax revenue and to record government expenditures. These adjustments do not change the information content of the SAM, but they do simplify the modeling process. However, they do have the consequence of creating a series of reserved names that are required for the operation of the model.[2]

Table 1 Macro SAM for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital Accounts	RoW
Commodities	0	X	0	X	X	X	X	X
Activities	X	0	0	0	0	0	0	0
Factors	0	X	0	0	0	0	0	X
Households	0	0	X	0	X	X	0	X
Enterprises	0	0	X	0	0	X	0	X
Government	X	X	X	X	X	0	0	X
Capital Accounts	0	0	0	X	X	X	0	X
RoW	X	0	X	0	0	0	0	0
Total	X	X	X	X	X	X	X	X

The model contains a section of code, immediately after the data have been read in, that resolves a number of common 'problems' encountered with SAM database by transforming the SAM so that it is consistent with the model structure. Specifically, all transactions between an account with itself are eliminated by setting the appropriate cells in the SAM equal to zero. Second, all transfers from domestic institutions to the Rest of the World and between the Rest of the World and domestic institutions are treated net as transfers to the Rest of the World and domestic institutions, by transposing and changing the sign of the payments to the Rest of the World. And third, all transfers between domestic institutions and the government are treated as net and as payments from government to the respective institution. Since these adjustments change the account totals, which are used in calibration, the account totals are recalculated within the model.

In addition to the SAM, which records transactions in value terms, two additional databases are used by the model. The first records the 'quantities' of primary inputs used by each activity. If such quantity data are not available then the entries in the factor use matrix

2 These and other reserved names are specified below as part of the description of the model.

are the same as those in the corresponding sub matrix of the SAM. The second series of additional data are the elasticities of substitution for imports and exports relative to domestic commodities, the elasticities of substitution for the CES production functions, the income elasticities of demand for the linear expenditure system and the Frisch (marginal utility of income) parameters for each household.

All the data are recorded in a GDX (GAMS data exchange) file. The code for converting the data from separate worksheets in an MS Excel workbook into a GDX file is included at the end of the data entry section.

2. The Computable General Equilibrium Model

The model is a member of the class of single country computable general equilibrium (CGE) models that are descendants of the approach to CGE modeling described by Dervis *et al*., (1982). More specifically, the implementation of this model, using the GAMS (General Algebraic Modeling System) software, is a direct descendant and development of models devised in the late 1980s and early 1990s, particularly those models reported by Robinson *et* al., (1990), Kilkenny (1991) and Devarajan *et al*., (1994). The model is a SAM based CGE model, wherein the SAM serves to identify the agents in the economy and provides the database with which the model is calibrated. Since the model is SAM based it contains the important assumption of the law of one price, i.e., prices are common across the rows of the SAM.[3] The SAM also serves an important organisational role since the groups of agents identified by the SAM structure are also used to define sub-matrices of the SAM for which behavioural relationships need to be defined. As such the modeling approach has been influenced by Pyatt's 'SAM Approach to Modeling' (Pyatt, 1989).

The description of the model proceeds in five stages. The first stage is the identification of the behavioural relationships; these are defined by reference to the sub matrices of the SAM within which the associated transactions are recorded. The second stage is definitional, and involves the identification of the components of the transactions recorded in the SAM, while giving more substance to the behavioural relationships, especially with those governing inter-institutional transactions, and in the process defining the notation. The third stage uses a pair of figures to explain the nature of the price and quantity systems for commodity and activity accounts that are embodied within the model. In the fourth stage an algebraic statement of the model is provided; the model's equations are summarised in a table that also provides (generic) counts of the model's equations and variables. A full listing of the parameters and

3 The one apparent exception to this is for exports. However the model implicitly creates a separate set of export commodity accounts and thereby preserves the 'law of one price', hence the SAM representation in the text is actually a somewhat condensed version of the SAM used in the model.

variables contained within the model are located in Appendix 1.[4] Finally in the fifth stage there is a discussion of the default and optional closure rules available within the model.

2.1. Behavioural Relationships

While the accounts of the SAM determine the agents that can be included within the model, and the transactions recorded in the SAM identify the transactions that took place, the model is defined by the behavioural relationships. The behavioural relationships in this model are a mix of non-linear and linear relationships that govern how the model's agents will respond to exogenously determined changes in the model's parameters and/or variables. Table 2 summarises these behavioural relationships by reference to the sub matrices of the SAM.

Households are assumed to choose the bundles of commodities they consume so as to maximise utility where the utility function is either Cobb-Douglas or Stone-Geary. For a developing country a Stone-Geary function may be generally preferable since it allows for subsistence consumption expenditures, which is an arguably realistic assumption when there are substantial numbers of very poor consumers. The households choose their consumption bundles from a set of 'composite' commodities that are aggregates of domestically produced and imported commodities. These 'composite' commodities are formed as Constant Elasticity of Substitution (CES) aggregates that embody the presumption that domestically produced and imported commodities are imperfect substitutes. The optimal ratios of imported and domestic commodities are determined by the relative prices of the imported and domestic commodities. This is the so-called Armington assumption (Armington, 1969), which allows for product differentiation via the assumption of imperfect substitution (see Devarajan *et al.*, 1994). The assumption has the advantage of rendering the model practical by avoiding the extreme specialisation and price fluctuations associated with other trade assumptions, e.g., the Salter/Swan or Australian model. In this model the country is assumed to be a price taker for all imported commodities.

Domestic production uses a two-stage production process. In the first stage aggregate intermediate and aggregate primary inputs are combined using Leontief technology. Hence intermediate input demands are in fixed proportions relative to the output of each activity, and the residual prices per unit of output after paying for intermediate inputs, the so-called value added price, are the amounts available for the payment of primary inputs. Primary inputs are combined to form aggregate value added using either Cobb-Douglas or CES technologies, with the optimal ratios of primary inputs being determined by relative factor prices. The activities are defined as multi-product activities with the assumption that the proportionate

4 The model includes specifications for transactions that were zero in the SAM. This is an important component of the model. It permits the implementation of policy experiments with exogenously imposed changes that impact upon transactions that were zero in the base period.

combinations of commodity outputs produced by each activity/industry remain constant; hence for any given vector of commodities demanded there is a unique vector of activity outputs that must be produced. The vector of commodities demanded is determined by the domestic demand for domestically produced commodities and export demand for domestically produced commodities. Using the assumption of imperfect transformation between domestic demand and export demand, in the form of a Constant Elasticity of Transformation (CET) function, the optimal distribution of domestically produced commodities between the domestic and export markets is determined by the relative prices on the alternative markets. The model can be specified as a small country, i.e., price taker, on all export markets, or selected export commodities can be deemed to face downward sloping export demand functions, i.e., a large country assumption.

Table 2 Behavioural Relationships for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital	RoW	Total	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Utility Functions (CD or Stone-Geary)	Fixed in Real Terms	Fixed in Real Terms and Export Taxes	Fixed Shares of Savings	Commodity Exports	Commodity Demand	Consumer Commodity Price Prices for Exports
Activities	Domestic Production	0	0	0	0	0	0	0	Constant Elasticity of Substitution Production Functions	
Factors	0	Factor Demands (CD or CES)	0	0	0	0	0	Factor Income from RoW	Factor Income	
Households	0	0	Fixed Shares of Factor Income	Fixed (Nominal) Transfers	Fixed (Nominal) Transfers	Fixed (Nominal) Transfers	0	Remittances	Household Income	
Enterprises	0	0	Fixed Shares of Factor Income	0	0	Fixed (Nominal) Transfers	0	Transfers	Enterprise Income	
Government	Tariff Revenue	Indirect Taxes on Activities	Fixed Shares of Factor Income	Direct Taxes on Household Income	Direct Taxes on Enterprise Income	0	0	Transfers	Government Income	
Capital	0	0	Depreciation	Household Savings	Enterprise Savings	Government Savings (Residual)	0	Current Account 'Deficit'	Total Savings	
Rest of World	Commodity Imports	0	Fixed Shares of Factor Income	0	0	0	0	0	Total 'Expenditure' Abroad	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Enterprise Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad		
	Producer Commodity Prices Domestic and World Prices for Imports	Value Added Prices								

The other behavioural relationships in the model are generally linear. A few features do however justify mention. First, all the tax rates are declared as parameters with associated scaling factors that are declared as variables. If a fiscal policy constraint is imposed then one or more of the sets of tax rates can be allowed to vary equiproportionately to define a new vector of tax rates that is consistent with the fiscal constraint. Relative tax rates can be adjusted by resetting the tax rate parameters. Similar scaling factors are available for a number of key parameters, e.g., household savings rates and inter-institutional transfers. Second, technology changes can be introduced through changes in the activity specific efficiency parameters. Third, the proportions of current expenditure on commodities defined to constitute subsistence consumption can be varied. Fourth, although a substantial proportion of the sub matrices relating to transfers, especially with the rest of the world, contain zero entries, the model allows changes in such transfers, e.g., aid transfers to the government from the rest of the world are defined equal to zero in the database but they can be made positive, or even negative, for model simulations. And fifth, the model is set up with a range of flexible closure rules. While the base model has a standard neoclassical model closure, e.g., full employment, savings driven investment and a floating exchange rate, these closure conditions can all be readily altered.

2.2. Transaction Relationships

The transactions relationships are laid out in Table 3, which is in two parts. The prices of domestically consumed (composite) commodities are defined as PQD_c, and they are the same irrespective of which agent purchases the commodity. The quantities of commodities demanded domestically are divided between intermediate demand, $QINTD_c$, and final demand, with final demand further subdivided between demands by households, QCD_c, enterprises, $QENTD_c$, government, QGD_c, investment, $QINVD_c$, and stock changes, $dstocconst_c$. The value of total domestic demand, at purchaser prices, is therefore $PQD_c * QQ_c$. Consequently the decision to represent export demand, QE_c, as an entry in the commodity row is slightly misleading, since the domestic prices of exported commodities, $PE_c = PWE_c * ER$, do not accord with the law of one price. The representation is a space saving device that removes the need to include separate rows and columns for domestic and exported commodities. The price wedges between domestic and exported commodities are represented by export duties, te_c, that are entered into the commodity columns. Commodity supplies come from domestic producers who receive the common prices, PXC_c, for outputs irrespective of which activity produces the commodity, with the total domestic production of commodities being denoted as QXC_c. Commodity imports, QM_c, are valued carriage insurance and freight (cif) paid, such that the domestic price of imports, PM_c, is defined as the world price, PWM_c, times the exchange rate, ER, plus an *ad valorem* adjustment for import duties, tm_c. All domestically consumed commodities are subject to a sales tax, ts_c.

Domestic production activities receive average prices for their output, PX_a, that are determined by the commodity composition of their outputs. Since activities produce multiple outputs their outputs can be represented as an index, QX_a, formed from the commodity composition of their outputs. In addition to intermediate inputs, activities also purchase primary inputs, $FD_{f,a}$, for which they pay average prices, WF_f. To create greater flexibility the model allows the price of each factor to vary according to the activity that employs the factor. Finally each activity pays production taxes, the rates, tx_a, for which are proportionate to the value of activity outputs.

The model allows for the domestic use of both domestic and foreign owned factors of production, and for payments by foreign activities for the use of domestically owned factors. Factor incomes therefore accrue from payments by domestic activities and foreign activities, $factwor_f$, where payments by foreign activities are assumed exogenously determined and are denominated in foreign currencies. After allowing for depreciation, $deprec_f$, and the payment of factor taxes, tf_f, the residual factor incomes, $YFDIST_f$, are divided between domestic institutions (households, enterprises and government) and the rest of the world in fixed proportions.

Households receive incomes from factor rentals and/or sales, inter household transfers, $hohoconst_{h,h}$, transfers from enterprises, $hoentconst_h$, and government, $hogovconst_h$, and remittances from the rest of the world, $howor_h$, where remittances are defined in terms of the foreign currency. Household expenditures consist of payments of direct/income taxes, ty_h, after which savings are deducted, where the savings rates, $caphosh_h$, are fixed exogenously in the base model. The residual household income is then divided between inter household transfers and consumption expenditures, with the pattern of consumption expenditures determined by the household utility functions.

Table 3 Transactions Relationships for the Standard Model

	Commodities	**Activities**	**Factors**	**Households**
Commodities	0	$\left(PQD_c * QINTD_c\right)$	0	$\left(PQD_c * QCD_c\right)$
Activities	$\left(PXC_c * QXC_c\right)$ $\left(PX_a * QX_a\right)$	0	0	0
Factors	0	$\left(WF_f * FD_{f,a}\right)$	0	0
Households	0	0	$\sum_f hovash_{h,f} * YFDISP_f$	$\left(\sum_{hh} hohoconst_{hh,h}\right)$
Enterprises	0	0	$\left(\sum_f entvash_f * YFDISP_f\right)$	0
Government	$\left(tm_c * PWM_c * QM_c * ER\right)$ $\left(te_c * PWE_c * QE_c * ER\right)$ $\left(ts_c * PQS_c * QQ_c\right)$	$\left(tx_a * PX_a * QX_a\right)$	$\left(\sum_f govvash_f * YFDISP_f\right)$ $\left(tf_f * YFDISP_f\right)$	$\left(ty_h * YH_h\right)$
Capital	0	0	$\sum_f deprec_f$	$\left(caphosh_h * YH_h\right)$
Rest of World	$\left(PWM_c * QM_c * ER\right)$	0	$\left(\sum_f worvash_f * YFDISP_f\right)$	0
Total	$\left(PQD_c * QQ_c\right)$	$\left(PX_a * QX_a\right)$	YF_f	YH_h

Table 3b Transactions Relationships for the Standard Model

	Enterprises	Government	Capital	RoW	Total
Commodities	$\left(PQD_c * QENTD_c\right)$	$\left(PQD_c * QGD_c\right)$	$\left(PQD_c * QINVD_c\right)$ $\left(PQD_c * dstocconst_c\right)$	$\left(PWE_c * QE_c * ER\right)$	$\left(PQD_c * QQ_c\right)$
Activities	0	0	0	0	$\left(PX_a * QX_a\right)$
Factors	0	0	0	$\left(factwor_f * ER\right)$	YF_f
Households	$hoentconst_h$	$\left(hogovconst_h * HGADJ\right)$	0	$\left(howor_h * ER\right)$	YH_h
Enterprises	0	$\left(entgovconst * EGADJ\right)$	0	$\left(entwor * ER\right)$	$EENT$
Government	$\left(TYEADJ * tye * YE\right)$	0	0	$\left(govwor * ER\right)$	EG
Capital	$\left(YE - EENT\right)$	$\left(YG - EG\right)$	0	$\left(CAPWOR * ER\right)$	$TOTSAV$
Rest of World	0	0	0	0	Total ‘Expenditure’ Abroad
Total	YE	YG	$INVEST$	Total ‘Income’ from Abroad	

The enterprise account receives income from factor sales, primarily in the form of retained profits,[5] transfers from government, *entgovconst*, and foreign currency denominated transfers from the rest of the world, *entwor*. Expenditures then consist of the payment of direct/income taxes, *tye*, consumption, which is assumed fixed in real terms,[6] and savings, which are defined as a residual, i.e., the difference between income, *YE*, and committed expenditure, *EENT*. There is an analogous treatment of government savings, i.e., the internal balance, which is defined as the difference (residual) between government income, *YG*, and committed government expenditure, *EG*. In the absence of a clearly definable set of behavioural relationships for the determination of government consumption expenditure, the quantities of commodities consumed by the government are fixed in real terms, and hence government consumption expenditure will vary with commodity prices.[7] Transfers by the government to other domestic institutions are fixed in nominal terms, although there is a facility to allow them to vary, e.g., with consumer prices. On the other hand government incomes can vary widely. Incomes accrue from the various tax instruments (import and export duties, sales, production and factor taxes, and direct taxes), that can all vary due to changes in the values of production, trade and consumption. The government also receives foreign currency denominated transfers from the rest of the world, *govwor*, e.g., aid transfers.

Domestic investment demand consists of fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. The comparative static nature of the model and the absence of a capital composition matrix underpin the assumption that the commodity composition of fixed capital formation is fixed, while a lack of information means that stock changes are assumed invariant. However the value of fixed capital formation will vary with commodity prices while the volume of fixed capital formation can vary both as a consequence of the volume of savings changing or changes in exogenously determined parameters. In the base version of the model domestic savings are made up of savings by households, enterprises, the government (internal balance) and foreign savings, i.e., the balance on the capital account or external balance, *CAPWOR*. The various closure rules available within the model allow for different assumptions about the determination of domestic savings, e.g., flexible versus fixed savings rates for households, and value of ‘foreign’ savings, e.g., a flexible or fixed exchange rate.

5 Hence the model contains the implicit presumption that the proportions of profits retained by incorporated enterprises are constant.

6 Hence consumption expenditure is defined as the fixed volume of consumption, $QENTD_c$, times the variable prices. It requires only a simple adjustment to the closure rules to fix consumption expenditures. Without a utility function, or equivalent, for enterprises it is not possible to define the quantities consumed as the result of an optimisation problem.

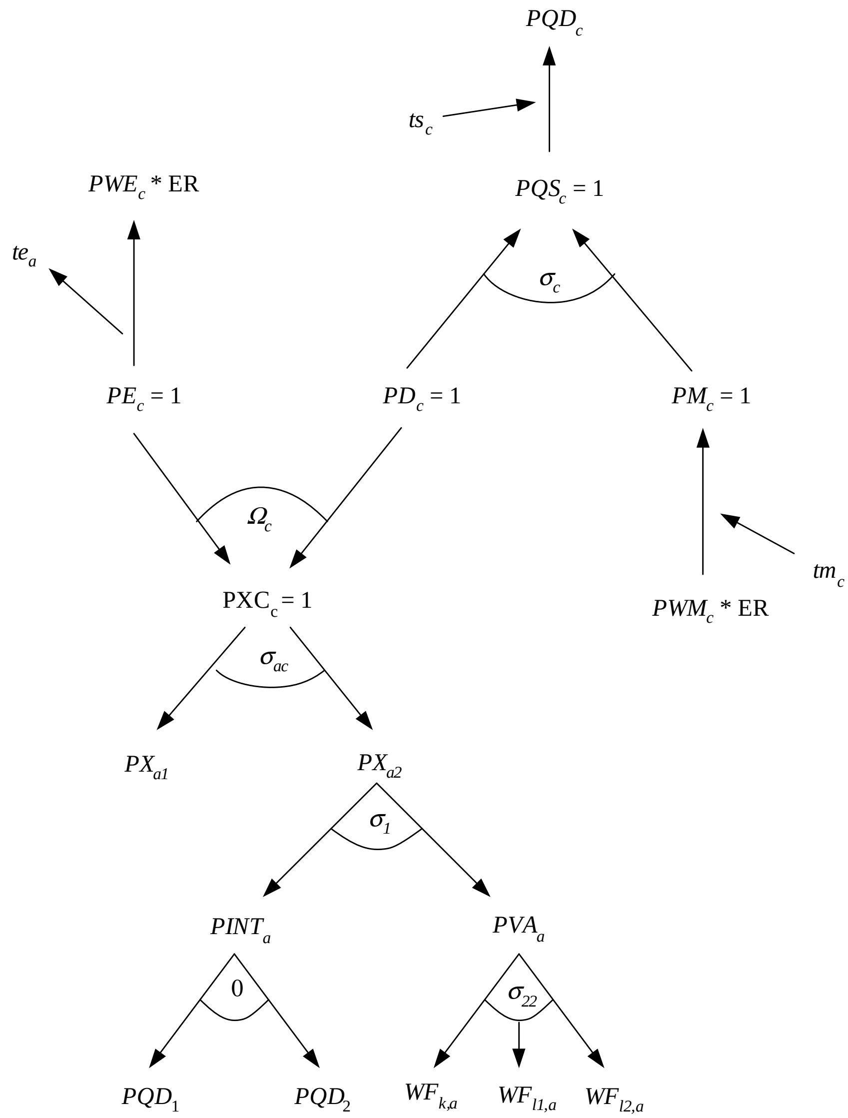
7 The closure rules allow for the fixing of government consumption expenditure rather than real consumption.

Incomes to the rest of the world account, i.e., expenditures by the domestic economy in the rest of the world, consist of the values of imported commodities and factor services. On the other hand expenditures by the rest of the world account, i.e., incomes to the domestic economy from the rest of the world, consist of the values of exported commodities and NET transfers by institutional accounts. All these transactions are subject to transformation by the exchange rate. In the base model the balance on the capital account is fixed at some target value, denominated in foreign currency terms, e.g., at a level deemed equal and opposite to a sustainable deficit on the current account, and the exchange rate is variable. This assumption can be reversed, where appropriate, in the model closure.

Figures 1 and 2 provide further detail on the interrelationships between the prices and quantities. The supply prices of the composite commodities (PQS_c) are defined as the weighted averages of the domestically produced commodities that are consumed domestically (PD_c) and the domestic prices of imported commodities (PM_c), which are defined as the products of the world prices of commodities (PWM_c) and the exchange rate (ER) uplifted by *ad valorem* import duties (tm_c). These weights are updated in the model through first order conditions for optima. The average prices exclude sales taxes, and hence must be uplifted by (*ad valorem*) sales taxes (ts_c) to reflect the composite consumer price (PQD_c). The producer prices of commodities (PXC_c) are similarly defined as the weighted averages of the prices received for domestically produced commodities sold on domestic and export (PE_c) markets. These weights are updated in the model through first order conditions for optima. The prices received on the export market are defined as the products of the world price of exports (PWE_c) and the exchange rate (ER) less any exports duties due, which are defined by *ad valorem* export duty rates (te_c).

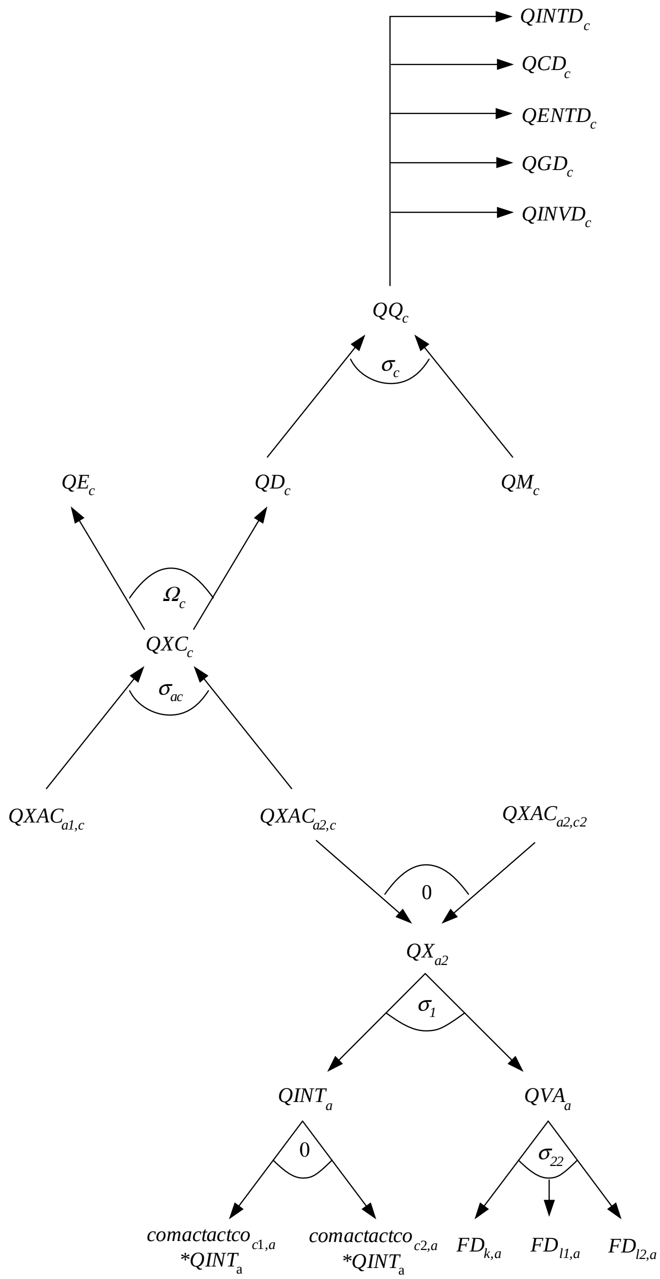
The average price per unit of output received by an activity (PX_a) is defined as the weighted average of the domestic producer prices, where the weights are constant. After paying indirect/production/output taxes (tx_a), this is divided between payments to aggregate value added (PVA_a), i.e., the amount available to pay primary inputs, and aggregate intermediate inputs ($PINT_a$). Total payments for intermediate inputs per unit of aggregate intermediate input are defined as the weighted sums of the prices of the inputs (PQD_c).

Figure 1 Price Relationships for a Standard Model



Total demands for the composite commodities, QQ_c, consist of demands for intermediate inputs, $QINTD_c$, consumption by households, QCD_c, enterprises, $QENTD_c$, and government, QGD_c, gross fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. Supplies from domestic producers, QDD_c, plus imports, QM_c, meet these demands; equilibrium conditions ensure that the total supplies and demands for all composite commodities equate. Commodities are delivered to both the domestic and export, QE_c, markets subject to equilibrium conditions that require all domestic commodity production, QXC_c, to be either domestically consumed or exported.

Figure 2 Quantity Relationships for a Standard Model



The presence of multiple product activities means that domestically produced commodities can come from multiple activities, i.e., the total production of a commodity is defined as the sum of the amount of that commodity produced by each activity. Hence the domestic production of a commodity (*QXC*) is a CES aggregate of the quantities of that commodity produced by a number of different activities (*QXAC*), which are produced by each activity in activity specific fixed proportions, i.e., the output of *QXAC* is a Leontief (fixed proportions aggregate of the output of each activity (*QX*).

Production relationships by activities are defined by a series of nested Constant Elasticity of Substitution (CES) production functions. The nesting structure is illustrated in Figure 3, where, for illustration purposes only, two intermediate inputs and three primary inputs ($FD_{k,a}$, $FD_{l1,a}$ and $FD_{l2,a}$) are identified. Activity output is a CES aggregate of the quantities of aggregate intermediate inputs (*QINT*) and value added (*QVA*), while aggregate intermediate inputs are a Leontief aggregate of the (individual) intermediate inputs and aggregate value added is a CES aggregate of the quantities of primary inputs demanded by each activity (*FD*). The allocation of the finite supplies of factors (*FS*) between competing activities depends upon relative factor prices via first order conditions for optima. The base model contains the assumption of full employment, but this can be relaxed.

3. Algebraic Statement of the Model

The model uses a series of sets, each of which is required to be declared and have members assigned. For the majority of the sets the declaration and assignment takes place simultaneously in a single block of code.[8] However, the assignment for a number of the sets, specifically those used to control the modeling of trade relationships is carried out dynamically by reference to the data used to calibrate the model. The following are the basic sets for this model

$$c = \{\text{commodities}\}$$
$$a = \{\text{activities}\}$$
$$f = \{\text{factors}\}$$
$$h = \{\text{households}\}$$
$$g = \{\text{government}\}$$

and for each set there is an alias declared that has the same membership as the corresponding basic set. The notation used involves the addition of a '*p*' suffix to the set label, e.g., the alias for *c* is *cp*.

However, for practical purposes, mainly associated with reading in the data, these basic sets are declared and assigned as subsets of a global set, *sac*,

$$sac = \{c, a, f, h, g, ent, kap, dstoc, row, total\}.$$

All the dynamic sets relate to the modeling of the commodity accounts and therefore are subsets of the set *c*. The subsets are

8 For practical purposes it is often easiest if this block of code is contained in a separate file that is then called up from within the *.gms file. This is how the process is implemented in the worked example.

$$
\begin{aligned}
ce(c) &= \{\text{export commodities}\} \\
cen(c) &= \{\text{non-export commodities}\} \\
ced(c) &= \{\text{export commodities with export demand functions}\} \\
cedn(c) &= \{\text{export commodities without export demand functions}\} \\
cm(c) &= \{\text{imported commodities}\} \\
cmn(c) &= \{\text{non-imported commodities}\} \\
cx(c) &= \{\text{commodities produced domestically}\} \\
cxn(c) &= \{\text{commodities NOT produced domestically AND imported}\} \\
cd(c) &= \{\text{commodities produced AND demanded domestically}\} \\
cdn(c) &= \{\text{commodities NOT produced AND demanded domestically}\}
\end{aligned}
$$

and members are assigned using the data used for calibration.

Finally a set is declared and assigned for a macro SAM that is used to check model calibration. This set and its members are

$$ss = \{commdty, activity, valuad, hholds, entp, govtn, kapital, world, totals\}.$$

Reserved Names

The model also uses a number of names that are reserved, in addition to those specified in the set statements detailed above. The majority of these reserved names are components of the government set; they are reserved to ease the modeling of tax instruments. The required members of the government set, with their descriptions, are

$$g = \left\{\begin{array}{cc} \text{DIRTAX} & \text{Direct Taxes} \\ \text{SALTAX} & \text{Sales Taxes} \\ \text{IMPTAX} & \text{Import Taxes} \\ \text{EXPTAX} & \text{Export Taxes} \\ \text{INDTAX} & \text{Indirect Taxes} \\ \text{FACTTAX} & \text{Factor Taxes} \\ \text{GOVT} & \text{Government} \end{array}\right\}.$$

The other reserved names are for the factor account and for the capital accounts. For simplicity the factor account relating to residual payments to factors has the reserved name of *GOS* (gross operating surplus); in many SAMs this account would include payments to the factors of production land and physical capital, payments labeled mixed income and payments for entrepreneurial services. Where the factor accounts are fully articulated *GOS* would refer to payments to the residual factor, typically physical capital and entrepreneurial services.

The capital account includes provision for two expenditure accounts relating to investment. All expenditures on stock changes are registered in the account *dstoc*, while all investment expenditures are registered to the account *kap*. All incomes to the capital account accrue to the *kap* account and stock changes are funded by an expenditure levied on the *kap* account to the *dstoc* account.

Conventions

The equations for the model are set out in nine 'blocks'; which group the equations under the following headings 'prices', 'production', 'trade', 'income', 'expenditure', 'taxes', 'market clearing', 'GDP' and 'model closure'. This grouping of equations is intended to ease the reading of the model rather than being a requirement of the model.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modeling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- The standard prefixes for variable names are: *P* for price variables, *Q* for quantity variables, *E* for expenditure variables, *Y* for income variables, and *V* for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.
- A series of variables are declared that allow for the equiproportionate adjustment of groups of parameters. These variables are named using the convention ***ADJ*, where ** is the parameter series they adjust.
- All parameters are in lower case, except those used to initialise variables.
- Names for parameters are derived using account abbreviations with the row account first and the column account second, e.g., actcom** is a parameter referring to the activity:commodity (supply or make) sub-matrix;
- Parameter names have a two or five character suffix which distinguishes their definition, e.g., ***sh* is a share parameter, ***av* is an average and **const is a constant parameter;
- The names for all parameters and variables are kept short.

3.1. Price Block Equations

The price block consists of ten price equations; four of which refer to the treatment of trade. The domestic price of competitive imports (P1) is the product of the world price of imports

(*PWM*), the exchange rate (*ER*) and one plus the import tariff rate (tm_c) multiplied by the tariff rate adjustment variable (*TMADJ*).

$$PM_c = PWM_c * ER * \left(1 + \left(TMADJ * tm_c\right)\right) \quad \forall cm \tag{P1}$$

These equations are only implemented for members of the set *c* that are imported, i.e., for members of the subset *cm*. The domestic price of exports (P2) is defined as the product of the world price of exports (*PWE*), the exchange rate (*ER*) and one minus the export tax rate multiplied by an export tax adjustment variable (*TEADJ*)

$$PE_c = PWE_c * ER * \left(1 - \left(TEADJ * te_c\right)\right) \qquad \forall ce \tag{P2}$$

and are only implemented for members of the set *c* that are exported, i.e., for members of the subset *ce*. The world price of imports and exports are declared as variables to allow relaxation of the small country assumption, and are then fixed as appropriate in the model closure block.

Domestic agents consume composite consumption commodities (*QQ*) that are aggregates of domestically produced and imported commodities. The prices of these composite commodities (*PQD*) are defined as the supply prices of the composite commodities plus an *ad valorem* sales tax (*ts*), which can be scaled (using *TSADJ*), i.e.,

$$PQD_c = PQS_c * \left(1 + \left(TSADJ * ts_c\right)\right) \tag{P3}$$

and the supply prices are defined as the volume share weighted sums of expenditure on domestically produced (*QD*) and imported (*QM*) commodities. These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QQ*) below.[9]

$$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \quad \forall cd \textbf{ OR } cm \tag{P4}$$

This equation is implemented for all commodities that are imported (*cm*) and for all commodities that are produced and consumed domestically (*cd*). Similarly, domestically produced commodities (*QXC*) are supplied to either or both the domestic and foreign markets (exported). The supply prices of domestically produced commodities (*PXC*) are defined as the volume share weighted sums of expenditure on domestically produced and exported (*QE*) commodities. These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QXC*) below[10] (P5).

$$PXC_c = \frac{PD_c * QD_c + \left(PE_c * QE_c\right)\$ce_c}{QXC_c} \qquad \forall cx \tag{P5}$$

9 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

10 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

This equation is implemented for all commodities that are produced domestically (*cx*), with a control to only include terms for exported commodities when there are exports (*ce*).

The supply prices of domestically produced commodities are determined by purchaser prices of those commodities on the domestic and international markets. Adopting the assumption that domestic activities produce commodities in fixed proportions (*actcomactsh*), the proportions provide a mapping (P6) between the supply prices of commodities and the (weighted) average activity prices (*PX*).[11]

$$PX_a = \sum_c actcomactsh_{a,c} * PXC_c \ . \tag{P6}$$

In this model a two-stage production process is adopted, with the top level as a CES function. The value of activity output can therefore be expressed as the volume share weighted sums of the expenditures on inputs after allowing for the production taxes, which are the product of tax rates (*tx*) and a tax rate-scaling factor (*TXADJ*), i.e.,

$$PX_a * \left(1 - \left(TXADJ * tx_a\right)\right) * QX_a = \left(PV_a * QVA_a\right) + \left(PINT_a * QINT_a\right). \tag{P7}$$

But the aggregate price of intermediates (*PINT*) is not defined. This is defined as the intermediate input-output coefficient weighted sum of the prices of intermediate inputs, i.e.,

$$PINT_a = \sum_c comactactco_{c,a} * PQD_c \tag{P8}$$

where $comactactco_{c,a}$ are the intermediate input-output coefficients.

The price block is completed by two price indices that can be used for price normalisation. Equation (P8) is for the consumer price index (*CPI*), which is defined as a weighted sum of composite commodity prices (*PQD*) in the current period, where the weights are the shares of each commodity in total demand (*comtotsh*)

$$CPI = \sum_c comtotsh_c * PQD_c \ . \tag{P9}$$

The domestic producer price index (*PPI*) is defined (P9) by reference to the supply prices for domestically produced commodities (*PD*) with weights defined as shares of the value of domestic output for the domestic market (*vddtotsh*)

$$PPI = \sum_c vddtotsh_c * PD_c \ . \tag{P10}$$

11 In the special case of each activity producing only one commodity **and** each commodity only being produced by a single activity, which is the case in the reduced form model reported in Dervis *et al.*, (1982), then the aggregation weights *actcomactsh* correspond to an identity matrix.

3.2. Production Block Equations

With CES technology the output by an activity (*QX*) is determined by the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and intermediates used (*QINT*), where δ_a^x is the share parameter, $rhoc_a^x$ is the substitution parameter and ad_a^x is the efficiency parameter

$$QX_a = ad_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + \left(1-\delta_a^x\right) QINT_a^{-rhoc_a^x} \right)^{-\frac{1}{rhoc_a^x}}. \tag{X1a}$$

The associated the first order conditions defining the optimum ratios of value added to intermediate inputs can be expressed in terms of the relative prices of value added (*PVA*) and intermediate inputs (*PINT*) as

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PV_a} * \frac{\delta_a^x}{\left(1-\delta_a^x\right)} \right]^{\frac{1}{\left(1+rhoc_a^x\right)}}. \tag{X1b}$$

The production function for QVA is a multi-factor CES function, i.e.,

$$QVA_a = ad_a^{va} * \left[\sum_f \delta_{f,a}^{va} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}} \tag{X2a}$$

where δ_a^{va} is the share parameter, $rhoc_a^{va}$ is the substitution parameter and ad_a^{va} is the efficiency parameter. The associated first order conditions for profit maximisation determine the wage rate of factors (*WF*), where the ratio of factor payments to factor *f* from activity *a* (*WFDIST*) are included to allow for non-homogenous factors, and is derived directly from the first order condition for profit maximisation as equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity,[12] i.e.,

$$\begin{aligned} WF_f * WFDIST_{f,a} &= PV_a * ad_a^{va} * \left[\sum_f \delta_{f,a}^{va} * FD_{f,a}^{-\rho_a^{va}} \right]^{-\left(\frac{1+\rho_a^{va}}{\rho_a^{va}}\right)} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)} \\ &= PVA_a * QVA_a * ad_a^{va} * \left[\sum_f \delta_{f,a}^{va} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)} \end{aligned}. \tag{X2b}$$

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12 The formulation in (X2b) implies that both the activity outputs (*QX*) and factor demands are solved simultaneously through the profit maximisation process. However this formulation would not work if there was production rationing, i.e., activity outputs (*QX*) were fixed, but there was still cost minimisation. For such a model X2b could be written, by simple substitution, as

$$WF_f .wfdist_{f,a} = PV_a .QX_a \left[\sum_f \delta_{f,a}^x .FD_{f,a}^{-\rho_a^x} \right]^{-1} .\delta_{f,a}^x FD_{f,a}^{\left(-\rho_a^x-1\right)}.$$

This formulation also works as an alternative to (X2b). Thanks are due to Sherman Robinson for the explanation as to the theoretic and practical distinction between these alternative, but mathematically identical, formulations.

The assumption of a two-stage production nest with Constant Elasticity of Substitution between aggregate intermediate input demand and aggregate value added and Leontief technology on intermediate inputs means that intermediate commodity demand (*QINTD*) is defined as the product of the fixed (Leontief) input coefficients of demand for commodity *c* by activity *a* (*comactco*), multiplied by the quantity of activity output (*QX*)

$$QINTD_c = \sum_a comactactco_{c,a} * QINT_a \,. \tag{X3}$$

Equation (X4) aggregates the commodity outputs by each activity (*QXAC*) to form the composite supplies of each commodity (*QXC*). It is assumed that the activity specific commodities are differentiated and therefore imperfect substitutes, hence the use of a CES aggregator function with $adxc_c$ as the shift parameter, $\delta^{xc}_{a,c}$ as the share parameter and ρ^{xc}_c as the elasticity parameter.

$$QXC_c = adxc_c * \left[\sum_a \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c}\right]^{-1/\rho^{xc}_c} \tag{X4}$$

The matching first order condition for the optimal combination of commodity outputs is therefore given by

$$\begin{aligned} PXAC_{a,c} &= PXC_c * adxc_c * \left[\sum_a \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c}\right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)} \\ &= PXC_c * QXC_c * \left[\sum_a \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c}\right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)} \end{aligned} \,. \tag{X5}$$

Finally the output to commodity supplies, where the ‘weights’ (*actcomcomsh*) identify the amount of each commodity produced per unit of output of each activity

$$QXAC_{a,c} = actcomcomsh_{a,c} * QX_a \,. \tag{X6}$$

This equation not only captures the patterns of secondary production it also provides the market closure conditions for equality between the supply and demand of domestic output.

3.3. Trade Block Equations

Trade relationships are modeled using the Armington assumption of imperfect substitutability between domestic and foreign commodities. The set of nine equations provides a general structure that accommodates most eventualities found with single country CGE models. In particular these equations allow for traded and non-traded commodities while simultaneously accommodating commodities that are produced or not produced domestically and are consumed or not consumed domestically and allowing a relaxation of the small country assumption of price taking for exports.

The output transformation functions (T1), and the associated first-order conditions (T2), establish the optimum allocation of domestic commodity output (*QXC*) between domestic demand (*QD*) and exports (*QE*), by way of Constant Elasticity of Transformation (CET) functions, with commodity specific share parameters (γ), elasticity parameters (*rhot*) and shift/efficiency parameters (*at*), i.e.,

$$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1-\gamma_c) * QD_c^{rhot_c} \right)^{\frac{1}{rhot_c}} \quad \forall ce \textbf{ AND } cd \qquad \text{(T1)}$$

with the first order conditions defining the optimum ratios of exports to domestic demand in relation to the relative prices of exported (*PE*) and domestically supplied (*PD*) commodities, i.e.,

$$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1-\gamma_c)}{\gamma_c} \right]^{\frac{1}{(rhot_c - 1)}} \qquad \forall ce \textbf{ AND } cd \,. \qquad \text{(T2)}$$

But T1 is only defined for commodities that are both produced and demanded domestically (*cd*) **and** exported (*ce*). Thus, although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced **and** demanded domestically but **not** exported, and those cases where commodities are produced domestically **and** exported but **not** demanded domestically.

If commodities are produced domestically but **not** exported, then domestic demand for domestically produced commodities (*QD*) is, by definition (T3), equal to domestic commodity production (*QXC*), i.e.,

$$QXC_c = QD_c \qquad \forall cen \textbf{ AND } cd \qquad \text{(T3)}$$

where the sets *cen* (commodities not exported) and *cd* (commodities produced and demanded domestically) control implementation. On the other hand if commodities are produced domestically but **not** demanded by the domestic output, then domestic commodity production (*QXC*) is, by definition (T4), equal to commodity exports (*QE*), i.e.,

$$QXC_c = QE_c \qquad \forall ce \textbf{ AND } cdn \qquad \text{(T4)}$$

where the sets *ce* (commodities exported) and *cdn* (commodities produced but not demanded domestically) control implementation.

The domestic supply equations are modeled using Constant Elasticity of Substitution (CES) functions and associated first order conditions to determine the optimum combination of supplies from domestic and foreign (import) producers. The domestic supplies of the composite commodities (*QQ*) are defined as CES aggregates (T5) of domestic production supplied to the domestic market (*QD*) and imports (*QM*), where aggregation is controlled by the share parameters (δ), the elasticity of substitution parameters (*rhoc*) and the shift/efficiency parameters (*ac*), i.e.,

$$QQ_c = ac_c\left(\delta_c QM_c^{-rhoc_c} + (1-\delta_c)QD_c^{-rhoc_c}\right)^{-\frac{1}{rhoc_c}} \qquad \forall cm \textbf{ AND } cx \tag{T5}$$

with the first order conditions defining the optimum ratios of imports to domestic demand in relation to the relative prices of imported (*PM*) and domestically supplied (*PDD*) commodities, i.e.,

$$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)}\right]^{\frac{1}{(1+rhoc_c)}} \qquad \forall cm \textbf{ AND } cx\,. \tag{T6}$$

But T5 is only defined for commodities that are both produced domestically (*cx*) **and** imported (*cm*). Although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced but **not** imported, and those cases where commodities are **not** produced domestically **and** are imported.

If commodities are produced domestically but **not** imported, then domestic supply of domestically produced commodities (*QD*) is, by definition (T7), equal to domestic commodity demand (*QQ*), i.e.,

$$QQ_c = QD_c \qquad \forall cmn \textbf{ AND } cx \tag{T7}$$

where the sets *cmn* (commodities not imported) and *cx* (commodities produced domestically) control implementation. On the other hand if commodities are **not** produced domestically but are demanded on the domestic market, then commodity supply (*QQ*) is, by definition (T8), equal to commodity imports (*QM*), i.e.,

$$QQ_c = QM_c \qquad \forall cm \textbf{ AND } cxn \tag{T8}$$

where the sets *cm* (commodities imported) and *cxn* (commodities not produced domestically) control implementation.

The equations T1 to T8 are sufficient for a general model of trade relationships when combined with the small country assumption of price taking on all import and export markets. However, it may be appropriate to relax this assumption in some instances, most typically in cases where a country is a major supplier of a commodity to the world market, in which case it may be reasonable to expect that as exports of that commodity increase so the export price (*PE*) of that commodity might be expected to decline, i.e., the country faces a downward sloping export demand curve. The inclusion of export demand equations (T9) accommodates this feature

$$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c} \right)^{-eta_c} \qquad \forall ced \qquad \text{(T9)}$$

for which the export demands are defined by constant elasticity export demand functions, with constants (*econ*), elasticities of demand (*eta*) and prices for substitutes on the world market (*pwse*).

3.4. Income Block Equations

There are seven equations in the income block; in each case the equations are defined from the perspective of income received by institutional accounts.

There are two sources of income for factors. First there are payment to factor accounts for services supplied to activities, i.e., domestic value added, and second there are payments to domestic factors that are used overseas, the value of these are assumed fixed in terms of the foreign currency. Factor incomes (*YF*) are therefore defined as the sum of all income to the factors across all activities (Y1)

$$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right). \qquad \text{(Y1)}$$

Before distributing factor incomes to the institutions that supply factor services allowance is made for depreciation (*deprec*) and factor taxes (*tf*) so that factor income for distribution (*YFDISP*) is defined (Y2) as

$$YFDISP_f = \left(YF_f - deprec_f \right) * \left(1 - \left(TFADJ * tf_f \right) \right). \qquad \text{(Y2)}$$

Households receive income from a variety of sources (Y3). Factor incomes are distributed to households as fixed proportions (*hovash*) of the distributed factor income for all factors owned by the household, plus interhousehold transfers (*HOHO*), payments from enterprises (*hoentconst*), transfers from government (*hogovconst*) that are adjustable using a scaling factor (*HGADJ*) and transfer from the rest of the world (*howor*) converted into domestic currency units, i.e.,

$$\begin{aligned} YH_h = & \left(\sum_f hovash_{h,f} * YFDISP_f \right) + \left(\sum_{hp} HOHO_{h,hp} \right) \\ & + hoentconst_h + \left(hogovconst_h * HGADJ \right) + \left(howor_h * ER \right) \end{aligned}. \qquad \text{(Y3)}$$

Similarly, income to enterprises (Y4) comes from the share of distributed factor incomes accruing to enterprises (*entvash*) and transfers from government (*entgovconst*) that are adjustable using a scaling factor (*EGADJ*) and the rest of the world (*entwor*) converted in the domestic currency units, i.e.,

$$YE = \left(\sum_f entvash_f * YFDISP_f \right) + \left(entgovconst * EGADJ \right) + \left(entwor * ER \right) \quad (Y4)$$

The economy also employs foreign owned factors whose services must be recompensed. It is assumed that these services receive fixed proportions of the factor incomes available for distribution, i.e.,

$$YFWOR_f = worvash_f * YFDISP_f . \quad (Y5)$$

The sources of income to the government account (Y6) are more complex. Income accrues from 6/7 tax instruments; tariff revenues (*MTAX*), export duties (*ETAX*), sales taxes (*STAX*), production taxes (*ITAX*), factor taxes (*FTAX*) and direct taxes (*DTAX*), which are defined in the tax equation block below. In addition the government can receive income as a share (*govvash*) of distributed factor incomes and transfers from abroad (*govwor*) converted in the domestic currency units, i.e.,

$$YG = MTAX + ETAX + STAX + ITAX + FTAX + DTAX + \left(\sum_f govvash_f * YFDIST_f \right) + \left(govwor * ER \right) \quad (Y6)$$

It would be relatively easy to subsume the tax revenue equations into the equation for government income, but they are kept separate to facilitate model testing and the implementation of fiscal policy experiments. Ultimately however the choice is a matter of personal preference.

The final equation details the sources of income to the capital account. Total savings in the economy are defined (Y7) as fixed shares (*caphosh*) of households' after tax income, where direct taxes (*ty*) have first call on household income, plus the allowance for depreciation, enterprise savings (*CAPENT*) , the government budget deficit/surplus (*CAPGOV*) and the current account 'deficit' (*CAPWOR*), i.e.,

$$TOTSAV = \sum_h \left(\left(YH_h * \left(1 - \left(TYADJ * ty_h \right) \right) \right) * \left(SADJ * caphsh_h \right) \right) + \left(\sum_f deprec_f \right) + CAPENT + CAPGOV + \left(CAPWOR * ER \right) \quad (Y7)$$

the last three terms of Y7 are defined below by equations in the market clearing block. The scaling factor on households' savings (*SADJ*) is included to allow for a specification where household savings rates can vary and the income tax scaling factor (*TYADJ*) is included to allow for flexible income tax rates.

3.5. Expenditure Block Equations

The expenditure block consists of eleven equations that concentrate upon expenditure upon commodities and total expenditures by institutions. Inter household transfer (*HOHO*) are defined as a fixed proportion of household income (*YH*) after payment of direct taxes and savings, i.e.,

$$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1-\left(TYADJ * ty_h\right)\right)\right) * \left(1-\left(SADJ * caphsh_h\right)\right). \tag{E1}$$

Household consumption expenditure (*HEXP*) is defined as household after tax income less savings and transfers to other households (E2),

$$\begin{aligned} HEXP_h = {} & \left(YH_h * \left(1-\left(TYADJ * ty_h\right)\right)\right) * \left(1-\left(SADJ * caphsh_h\right)\right) \\ & -\left(\sum_{hp} HOHO_{hp,h}\right). \end{aligned} \tag{E2}$$

Households are then assumed to maximise utility subject to a Cobb-Douglas (CD) utility function or a Stone-Geary utility function. If the utility function is CD, then expenditures are allocated in fixed proportions to each consumption commodity (*comhoav*) such that the volumes of each commodity consumed are given by

$$QCD_c = \frac{\sum_h \left(comhoav_{c,h} * HEXP_h\right)}{PQD_c} \tag{E3a}$$

which ensures that all disposable income is exhausted. But if the utility function is Stone-Geary then household consumption demand consists of two components, ‘subsistence’ demand (*qcdconst*) and ‘discretionary’ demand, and the equation must therefore capture both elements. This can be written as

$$QCD_c = \frac{\left(\sum_h \left(PQ_c * qcdconst_{c,h} + \sum_h beta_{c,h} * \left(HEXP_h - \sum_c \left(PQ_c * qcdconst_{c,h}\right)\right)\right)\right)}{PQD_c}. \tag{E3b}$$

where discretionary demand is defined as the marginal budget shares (*beta*) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand.

The consumption of commodities by enterprises (*QENTD*) are defined (E4) in terms of fixed volumes (*comentconst*), which can be varied via the volume adjuster (*QENTDADJ*),

$$QENTD_c = comentconst_c * QENTDADJ \ . \tag{E4}$$

Associated with any given volume of enterprise final demand there is a level of expenditure defined by

$$VENTD = \left(\sum_c QENTD_c * PQD_c \right). \tag{E5}$$

If *QENTDADJ* is made flexible, then *comentconst* ensures that the quantities of commodities demanded are varied in fixed proportions; clearly this specification of demand is not a consequence of a defined set of behavioural relationships, as was the case for households, which reflects the difficulties inherent to defining utility functions for non-household institutions. If *VENTD* is fixed then the volume of consumption by enterprises (*QENTD*) must be allowed to vary, via the variable *QENTDADJ*. Then total enterprise expenditure (*EENT*) is defined (E6) as the sum of expenditure by enterprises on consumption demand at current prices, plus transfers to households (*hoentconst*) plus corporation tax, where *tye* is the direct/income tax rate and *TYEADJ* the tax rate scaling factor

$$\begin{aligned} EENT = & \left(\sum_c QENTD_c * PQD_c \right) + \left(\sum_h hoentconst_h \right). \\ & + \left(TYEADJ * tye * YE \right) \end{aligned} \tag{E6}$$

The demand for commodities by the government for consumption (*QGD*) is also defined (E7) in terms of fixed proportions (*comgovconst*) that can be varied with a scaling adjuster (*QGDADJ*)

$$QGD_c = comgovconst_c * QGDADJ \ . \tag{E7}$$

Associated with any given volume of government final demand there is a level of expenditure defined by

$$VGD = \left(\sum_c QGD_c * PQD_c \right). \tag{E8}$$

Hence, total government expenditure (*EG*) can be defined (E9) as equal to the sum of expenditure by government on consumption demand at current prices, plus transfers to households (*hogovconst*) that can be adjusted using a scaling factor (*HGADJ*) and transfers to enterprises (*entgovconst*) that can also be adjusted by a scaling factor (*EGADJ*)

$$\begin{aligned} EG = & \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ \right) \\ & + \left(entgovconst * EGADJ \right) \end{aligned} \tag{E9}$$

As with enterprises there are difficulties inherent to defining utility functions for a government. Changing *QGDADJ*, either exogenously or endogenously, by allowing it to be a variable in the closure conditions, provides a means of changing the behavioural assumption with respect to the 'volume' of commodity demand by the government. If the value of government final demand (*VGD*) is fixed then government expenditure is fixed and hence the volume of consumption by government (*QGD*) must be allowed to vary, via the *QGDADJ* variable. If it is deemed appropriate to modify the patterns of commodity demand by the government then the components of *comgovconst* must be changed.

The same structure of relationships is adopted for investment demand (E10). The volumes of commodities purchased for investment are determined by the volumes in the base period (*invconst*) and can be varied using the adjuster (*IADJ*)

$$QINVD_c = \left(IADJ * invconst_c\right). \tag{E10}$$

Then value of investment expenditure (*INVEST*) is equal (E11) to the sum of investment demand valued at current prices plus the current priced value of stock changes (*dstocconst*) that are defined as being fixed in volume terms at the levels in the base period

$$INVEST = \sum_c \left(PQD_c * \left(QINVD_c + dstocconst_c\right)\right). \tag{E11}$$

If *IADJ* is made variable then the volumes of investment demand by commodity will adjust equiproportionately, in the ratios set by *invconst*, such as to satisfy the closure rule defined for the capital account. Changes to the patterns of investment demand require changes in the ratios of investment demand set by *invconst*.

3.6. Tax Block Equations

Although it is not necessary to keep the tax revenue equations separate from other equations, e.g., they can be embedded into the equation for government income (*YG*), it does aid clarity and assist with implementing fiscal policy simulations. For this model there are six tax revenue equations, for each one of which there is a tax rate adjuster, labeled ***ADJ* where ** is the tax rate label, that allows for the equiproportionate changing of the respective tax rates. If the patterns of taxes are to be changed then the tax rates must be altered directly. In all cases the tax rates can be negative indicating a 'transfer' from the government.

There are three tax instruments that are dependent upon expenditure on commodities, with each expressed as an *ad valorem* tax rate. Tariff revenue (*MTAX*) is defined (F1) as the sum of the product of tariff rates (*tm*) and the value of expenditure on imports at world prices, i.e.,

$$MTAX = \sum_c \left(TMADJ * tm_c * PWM_c * ER * QM_c\right). \tag{F1}$$

The revenue from export duties (*ETAX*) is defined (F2) as the sum of the product of export duty rates (*te*) and the value of expenditure on exports at world prices, i.e.,

$$ETAX = \sum_{c} \left(TEADJ * te_c * PWE_c * ER * QE_c \right). \tag{F2}$$

Finally the revenue from sales taxes (*STAX*) is defined (F2) as the sum of the product of sales tax rates (*ts*) and the value of domestic expenditure on commodities, i.e.,

$$\begin{aligned} STAX &= \sum_{c} \begin{pmatrix} TSADJ * ts_c * PQS_c * \\ \left(QINTD_c + QCD_c + QENTD_c + QGD_c + QINVD_c + dstocconst_c \right) \end{pmatrix}. \\ &= \sum_{c} \left(TSADJ * ts_c * PQS_c * QQ_c \right) \end{aligned} \tag{F3}$$

There is a single tax on production (*ITAX*). As with other taxes this is defined (F4) as the sum of the product of indirect tax rates (*tx*) and the value of output by each activity evaluated in terms of the activity prices (*PX*), i.e.,

$$ITAX = \sum_{a} \left(TXADJ * tx_a * PX_a * QX_a \right). \tag{F4}$$

These are the tax instruments most likely to yield negative revenues through the existence of production subsidies.

The tax on factors (FTAX) is defined (F5) as the product of factor tax rates (tf) and factor incomes for all factors,

$$FTAX = \sum_{f} \left(TFADJ * tf_f * \left(YF_f - deprec_f \right) \right). \tag{F5}$$

Finally, the revenue from direct taxes (DTAX) is defined (F6) as the sum of the product of household income tax rates (*ty*) and household incomes plus the product of the direct tax rate for enterprises (*tye*) and enterprise income,

$$DTAX = \sum_{h} \left(TYADJ * ty_h * YH_h \right) + \left(TYEADJ * tye * YE \right). \tag{F6}$$

3.7. Market Clearing Block Equations

The market clearing equations ensure the simultaneous clearing of all markets. In this model there are six relevant markets: factor and commodity markets and enterprise, government, capital and rest of world accounts. Market clearing with respect to activities has effectively been achieved by (X4), wherein the supply and demand for domestically produced commodities was enforced, while the demand system and the specification of expenditure relationships ensures that the household markets are cleared.

Adopting an initial assumption of full employment, which the model closure rules will demonstrate can be easily relaxed, amounts to requiring that the factor market is cleared by equating factor demands and factor supplies (M1) for all factors

$$FS_f = \sum_a FD_{f,a} \,. \tag{M1}$$

Market clearing for the composite commodity markets requires that the supplies of the composite commodity (*QQ*) are equal to total of domestic demands for composite commodities, which consists of intermediate demand (*QINTD*), household (*QCD*), enterprise (*QENTD*) and government (*QGD*) and investment (*QINVD*) final demands and stock changes (*dstocconst*) (M2)

$$QQ_c = QINTD_c + QCD_c + QENTD_c + QGD_c + QINVD_c + dstocconst_c \,. \tag{M2}$$

Since the markets for domestically produced commodities are also cleared (X4) this ensures a full clearing of all commodity markets.

The total value of domestic final demand (*VFDOMD*) is defined as the sum of the expenditures on final demands by households and domestic institutions (enterprises, government and investment), i.e.,

$$VFDOMD = \sum_c PQD_c * \left(QCD_c + QENTD_c + QGD_c + QINVD_c + dstocconst_c \right) \qquad \text{(M3).}$$

It is also useful to express the value of final demand by each non-household domestic institution as a proportion of the total value of domestic final demand; this allows the implementation of what has been called a 'balanced macroeconomic closure'.[13] Hence the share of the value of final demand by enterprises can be defined as a proportion of total final domestic demand, i.e.,

$$VENTDSH = {}^{VENTD}\!\big/_{VFDOMD} \tag{M4}$$

and similarly for government's value share of final demand

$$VGDSH = {}^{VGD}\!\big/_{VFDOMD} \tag{M5}$$

and for investment's value share of final demand

$$INVESTSH = {}^{INVEST}\!\big/_{VFDOMD} \,. \tag{M6}$$

If the share variables (*VENTDSH*, *VGDSH* and *INVESTSH*) are fixed then the quantity adjustment variables on the associated volumes of final demand by domestic non-household institutions (*QENTDADJ*, *QGDADJ* and *IADJ* or *SADJ*) must be free to vary. On the other hand if the volume adjusters are fixed the associated share variables must be free so as to allow the value of final demand by 'each' institution to vary.

13 The adoption of such a closure rule for this class of model has been advocated by Sherman Robinson and is a feature, albeit implemented slightly differently, of the IFPRI standard model.

Making savings a residual for each account clears the three institutional accounts that are not cleared elsewhere – enterprises, government and rest of the world. Thus the enterprise account clears (M7) by defining enterprise savings (*CAPENT*) as the difference between enterprise income and other expenditures, i.e., a residual

$$CAPENT = YE - EENT\ . \qquad \text{(M7)}$$

The government account clears by defining government savings (*CAPGOV*) as the difference between government income and other expenditures, i.e., a residual

$$CAPGOV = YG - EG\ . \qquad \text{(M8)}$$

And the rest of world account clears (M9) by defining the balance on the capital account (*CAPWOR*) as the difference between expenditure on imports, of commodities and factor services, and total income from the rest of the world, which includes export revenues and payments for factor services, transfers from the rest of the world to the household, enterprise and government accounts, i.e., it is a residual

$$\begin{aligned} CAPWOR = &\left(\sum_c pwm_c * QM_c\right) + \left(\sum_f YFWOR_f\right) \\ &- \left(\sum_c pwe_c * QE_c\right) - \left(\sum_f factwor_f\right) \ . \qquad \text{(M9)} \\ &- \left(\sum_h howor_h\right) - entwor - govwor \end{aligned}$$

The final account to be cleared is the capital account. Total savings (*TOTSAV*), see Y6 above, is defined within the model and hence there has been an implicit presumption in the description that the total value of investment (*INVEST*) is driven by the volume of savings. This is the market clearing condition imposed by (M10)

$$TOTSAV = INVEST + WALRAS\ . \qquad \text{(M10)}$$

But this market clearing condition includes another term, *WALRAS*, which is a slack variable that returns a zero value when the model is fully closed and all markets are cleared, and hence its inclusion provides a quick check on model specification.

Table 4 Equation and Variable Counts for the Standard Model

Name	Equation	Number of Equations	Variable	Number of Variables
	PRICE BLOCK			
			PDD_c	c
$PMDEF_c$	$PM_c = PWM_c * ER * \left(1 + \left(TMADJ * tm_c\right)\right) \quad \forall cm$	cm	PM_c	cm
$PEDEF_c$	$PE_c = PWE_c * ER * \left(1 - \left(TEADJ * te_c\right)\right) \quad \forall ce$	ce	PE_c	ce
$PQDDEF_c$	$PQD_c = PQS_c * \left(1 + \left(TSADJ * ts_c\right)\right)$	c	PQD_c	c
$PQSDEF_c$	$PQS_c = \dfrac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \quad \forall cd \textbf{ OR } cm$	c	PQS_c	c
$PXCDEF_c$	$PXC_c = \dfrac{PD_c * QD_c + \left(PE_c * QE_c\right)\$ce_c}{QXC_c} \quad \forall cx$	cx	PXC_c	cx
$PXDEF_a$	$PX_a = \sum_c actcomactsh_{a,c} * PXC_c$	a	PX_a	a
$PVDEF_a$	$PX_a * \left(1 - \left(TXADJ * tx_a\right)\right) * QX_a = \left(PV_a * QVA_a\right) + \left(PINT_a * QINT_a\right)$	a	PV_a	a
$PINTDEF_a$	$PINT_a = \sum_c comactactco_{c,a} * PQD_c$	a	$PINT_a$	a
$CPIDEF$	$CPI = \sum_c comtotsh_c * PQD_c$	1	CPI	1
$PPIDEF$	$PPI = \sum_c vddtotsh_c * PD_c$	1	PPI	1

Name	Equation	Number of Equations	Variable	Number of Variables
	PRODUCTION BLOCK			
$QXPRODFN_a$	$QX_a = ad_a^x\left(\delta_a^x QVA_a^{-rhoc_a^x} + \left(1-\delta_a^x\right)QINT_a^{-rhoc_a^x}\right)^{-\frac{1}{rhoc_a^x}}$	a	QX_a	a
$QXFOC_a$	$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PV_a} * \frac{\delta_a^x}{\left(1-\delta_a^x\right)}\right]^{\frac{1}{\left(1+rhoc_a^x\right)}}$	a	$QINT_a$	a
$QVAPRODFN_a$	$QVA_a = ad_a^{va} * \left[\sum_f \delta_{f,a}^{va} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1/\rho_a^{va}}$	a	QVA_a	a
$QVAFOC_{f,a}$	$WF_f * WFDIST_{f,a} = PVA_a * QVA_a * ad_a^{va} * \left[\sum_f \delta_{f,a}^{va} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)}$	$(f*a)$	$FD_{f,a}$	$(f*a)$
$QINTEQ_c$	$QINTD_c = \sum_a comactactco_{c,a} * QINT_a$	c	$QINTD_c$	c
$COMOUT_c$	$QXC_c = adxc_c * \left[\sum_a \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}}\right]^{-1/\rho_c^{xc}}$	c	QXC_c	c
$COMOUTFOC_{a,c}$	$PXAC_{a,c} = PXC_c * adxc_c * \left[\sum_a \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}}\right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)} * \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)}$	$(a*c)$	$PXAC_{a,c}$	$(a*c)$
$ACTIVOUT_{a,c}$	$QXAC_{a,c} = actcomcomsh_{a,c} * QX_a$	$(a*c)$	$QXAC_{a,c}$	$(a*c)$

Name	Equation	Number of Equations	Variable	Number of Variables
	TRADE BLOCK			
CET_c	$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1-\gamma_c) * QD_c^{rhot_c} \right)^{\frac{1}{rhot_c}} \quad \forall ce \textbf{ AND } cd$	c	QDD_c	c
$ESUPPLY_a$	$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1-\gamma_c)}{\gamma_c} \right]^{\frac{1}{(rhot_c - 1)}} \quad \forall ce \textbf{ AND } cd$	c	QE_c	c
$EDEMAND_c$	$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c} \right)^{-eta_c} \quad \forall ced$			
$CET2_c$	$QXC_c = QD_c \quad \forall cen \textbf{ AND } cd$			
$CET3_c$	$QXC_c = QE_c \quad \forall ce \textbf{ AND } cdn$			
$ARMINGTON_c$	$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + (1-\delta_c) QD_c^{-rhoc_c} \right)^{-\frac{1}{rhoc_c}} \quad \forall cm \textbf{ AND } cx$	c	QQ_c	c
$COSTMIN_c$	$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)} \right]^{\frac{1}{(1+rhoc_c)}} \quad \forall cm \textbf{ AND } cx$	c	QM_c	c
$ARMINGTON2_c$	$QQ_c = QD_c \quad \forall cmn \textbf{ AND } cx$			
$ARMINGTON3_c$	$QQ_c = QM_c \quad \forall cm \textbf{ AND } cxn$			

Name	Equation	Number of Equations	Variable	Number of Variables
	INCOME BLOCK			
$YFEQ_f$	$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right)$	f	YF_f	f
$YFDISPEQ_f$	$YFDISP_f = \left(YF_f - deprec_f \right) * \left(1 - \left(TFADJ * tf_f \right) \right)$	f	$YFDIST_f$	f
$YHEQ_h$	$YH_h = \left(\sum_f hovash_{h,f} * YFDISP_f \right) + \left(\sum_{hp} HOHO_{h,hp} \right)$ $+ hoentconst_h + \left(hogovconst_h * HGADJ \right) + \left(howor_h * ER \right)$	h	YH_h	h
$YEEQ$	$YE = \left(\sum_f entvash_f * YFDISP_f \right)$ $+ \left(entgovconst * EGADJ \right) + \left(entwor * ER \right)$	1	YE	1
$YGEQ$	$YG = MTAX + ETAX + STAX + ITAX + FTAX + DTAX$ $+ \left(\sum_f govvash_f * YFDISP_f \right) + \left(govwor * ER \right)$	1	YG	1
$YFWOREQ_f$	$YFWOR_f = worvash_f * YFDISP_f$	f	$YFWOR_f$	f
$TOTSAVEQ$	$TOTSAV = \sum_h \left(\left(YH_h * \left(1 - \left(TYADJ * ty_h \right) \right) \right) * \left(SADJ * caphsh_h \right) \right)$ $+ \left(\sum_f deprec_f \right) + CAPENT + CAPGOV + \left(CAPWOR * ER \right)$	1	$TOTSAV$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	EXPENDITURE BLOCK			
$HOHOEQ_{h,hp}$	$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1-\left(TYADJ * ty_h\right)\right)\right) * \left(1-\left(SADJ * caphsh_h\right)\right)$	$h*hp$	$HOHO_{h,hp}$	$h*hp$
$HEXPEQ_h$	$HEXP_h = \left(YH_h * \left(1-\left(TYADJ * ty_h\right)\right)\right) * \left(1-\left(SADJ * caphsh_h\right)\right) - \left(\sum_{hp} HOHO_{hp,h}\right)$	h	$HEXP_h$	h
$QCDEQ_c$	$QCD_c = \frac{\sum_h \left(comhoav_{c,h} * HEXP_h\right)}{PQD_c}$ **OR** $QCD_c = \frac{\left(\sum_h \left(PQD_c * qcdconst_{c,h} + \sum_h beta_{c,h} * \left(HEXP_h - \sum_c \left(PQD_c * qcdconst_{c,h}\right)\right)\right)\right)}{PQD_c}$	c	QCD_c	c
$QENTDEQ_c$	$QENTD_c = comentconst_c * QENTDADJ$	c	$QENTD_c$	c
$VENTDEQ$	$VENTD = \left(\sum_c QENTD_c * PQD_c\right)$	1	$VENTD$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	EXPENDITURE BLOCK			
EENTEQ	$EENT = \left(\sum_c QENTD_c * PQD_c \right) + \left(\sum_h hoentconst_h \right) + \left(TYEADJ * tye * YE \right)$	1	*EENT*	1
$QGDEQ_c$	$QGD_c = comgovconst_c * QGDADJ$	*c*	QGD_c	*c*
VGDEQ	$VGD = \left(\sum_c QGD_c * PQD_c \right)$	1	*VQGD*	1
EGEQ	$EG = \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ \right) + \left(entgovconst * EGADJ \right)$	1	*EG*	1
$QINVDEQ_c$	$QINVD_c = \left(IADJ * invconst_c \right)$	*c*	$QINVD_c$	*c*
INVEST	$INVEST = \sum_c \left(PQD_c * \left(QINVD_c + dstocconst_c \right) \right)$	1	*INVEST*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX BLOCK			
MTAXEQ	$MTAX = \sum_c \left(TMADJ * tm_c * PWM_c * ER * QM_c \right)$	1	*MTAX*	1
ETAXEQ	$ETAX = \sum_c \left(TEADJ * te_c * PWE_c * ER * QE_c \right)$	1	*ETAX*	1
STAXEQ	$STAX = \sum_c \left(\begin{array}{l} TSADJ * ts_c * PQS_c * \\ \left(\begin{array}{l} QINTD_c + QCD_c + QENTD_c \\ +QGD_c + QINVD_c + dstocconst_c \end{array} \right) \end{array} \right)$ $= \sum_c \left(TSADJ * ts_c * PQS_c * QQ_c \right)$	1	*STAX*	1
ITAXEQ	$ITAX = \sum_a \left(TXADJ * tx_a * PX_a * QX_a \right)$	1	*ITAX*	1
FTAXEQ	$FTAX = \sum_f \left(TFADJ * tf_f * \left(YF_f - deprec_f \right) \right)$	1	*FTAX*	1
DTAXEQ	$DTAX = \sum_h \left(TYADJ * ty_h * YH_h \right) + \left(TYEADJ * tye * YE \right)$	1	*DTAX*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MARKET CLEARING BLOCK			
$FMEQUIL_f$	$FS_f = \sum_a FD_{f,a}$	f	FS_f	f
$QEQUIL_c$	$QQ_c = QINTD_c + QCD_c + QENTD_c + QGD_c + QINVD_c + dstocconst_c$	c		
$VFDOMDEQ$	$VFDOMD = \sum_c PQD_c * \left(\begin{array}{l} QCD_c + QENTD_c + QGD_c + QINVD_c \\ + dstocconst_c \end{array} \right)$	1	$VFDOMD$	1
$VENTDSHEQ$	$VENTDSH = VENTD / VFDOMD$	1	$VENTDSH$	1
$VGDSHEQ$	$VGDSH = VGD / VFDOMD$	1	$VGDSH$	1
$INVESTSHEQ$	$INVESTSH = INVEST / VFDOMD$	1	$INVESTSH$	1
$CAPENT$	$CAPENT = YE - EENT$	1	$CAPENT$	1
$CAPGOVEQ$	$CAPGOV = YG - EG$	1	$CAPGOV$	1
$CAEQUIL$	$CAPWOR = \left(\sum_c pwm_c * QM_c \right) + \left(\sum_f YFWOR_f \right) - \left(\sum_c pwe_c * QE_c \right) - \left(\sum_f factwor_f \right) - \left(\sum_h howor_h \right) - entwor - govwor$	1	$CAPWOR$	1
$WALRASEQ$	$TOTSAV = INVEST + WALRAS$	1	$WALRAS$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MODEL CLOSURE			
			$\overline{ER}$ or $\overline{CAPWOR}$	1
			$\overline{PWM_c}$ and $\overline{PWE_c}$ or $\overline{PWE_{cedn}}$	$2c$
			$\overline{SADJ}$ or $\overline{IADJ}$ or $\overline{INVEST}$ or $\overline{INVESTSH}$	1
			$\overline{QENTDADJ}$ or $\overline{VENTD}$ or $\overline{VENTDSH}$	1
			At least **one** of $\overline{TMADJ}$, $\overline{TEADJ}$, $\overline{TSADJ}$, $\overline{TXADJ}$, $\overline{TFADJ}$, $\overline{TYADJ}$, $\overline{TYEADJ}$ and $\overline{CAPGOV}$	7
			at least **two** of $\overline{QGDADJ}$, $\overline{HGADJ}$, $\overline{EGADJ}$, $\overline{VGD}$ and $\overline{VGDSH}$	3
			$\overline{FS_f}$ and $\overline{WFDIST_{f,a}}$	$(f*(a+1))$
			$\overline{CPI}$ or $\overline{PPI}$	1

4. Model Closure Conditions or Rules

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of the main options available with this formulation of the model by reference to the accounts to which the rules refer.

4.1. Foreign Exchange Account Closure

The closure of the rest of the world account can be achieved by fixing either the exchange rate variable (C1a) or the balance on the current account (C1b). Fixing the exchange rate is appropriate for countries with a fixed exchange rate regime whilst fixing the current account balance is appropriate for countries that face restrictions on the value of the current account balance, e.g., countries following structural adjustment programmes.

$$ER = \overline{ER} \qquad \text{(C1a)}$$

or

$$CAPWOR = \overline{CAPWOR}\,. \qquad \text{(C1b)}$$

It is a common practice to fix a variable at its initial level by using the associated parameter, i.e., ***0, but it is possible to fix the variable to any appropriate value.

The model is formulated with the world prices for traded commodities declared as variables, i.e., PWM_c and PWE_c. If a strong small country assumption is adopted, i.e., the country is assumed to be a price taker on all world commodity markets, then all world prices will be fixed. When calibrating the model the world prices will be fixed at their initial levels, i.e.,

$$PWE_c = \overline{PWE_c}$$
$$PWM_c = \overline{PWM_c} \qquad \text{(C1c)}$$

but this does not mean they cannot be changed as parts of experiments.

However, the model allows a relaxation of the strong small country assumption, such that the country may face a downward sloping demand curve for one or more of its export commodities. Hence the world prices of some commodities are determined by the interaction of demand and supply on the world market, i.e., they are variables. This is achieved by limiting the range of world export prices that are fixed to those for which there are no export demand function, i.e.,

$$PWE_{cedn} = \overline{PWE_{cedn}} \qquad \text{(C1d)}$$

and canceling the first part of (C1c).

4.2. Capital Account Closure

To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. This is achieved by fixing either the saving rates for households or the volumes of commodity investment. This involves fixing either the savings rates adjuster (C2a) or the investment volume adjuster (C2b), i.e.,

$$SADJ = \overline{SADJ} \qquad \text{(C2a)}$$

or

$$IADJ = \overline{IADJ} \qquad \text{(C2b)}$$

Note that fixing the investment volume adjuster (C2b) means that the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*). Note also that the adjustment in such cases takes place through equiproportionate changes in the savings rates of households despite the fact that there are other sources of savings. The magnitudes of these other savings sources can also be changed through the closure rules (see below).

Fixing savings, and thus deeming the economy to be savings-driven, can be considered a Neo-Classical approach. Closing the economy by fixing investment however makes the model reflect the Keynesian investment-driven assumption for the operation of an economy.

The model includes a variable for the value of investment (*INVEST*), which can also be used to close the capital account. If *INVEST* is fixed in an investment driven closure, i.e.,

$$INVEST = \overline{INVEST} \qquad \text{(C2c)}$$

then the model will need to adjust the savings rates to maintain equilibrium between the value of savings (*TOTSAV*) and the fixed value of investment. This can only be achieved by changes in the volumes of commodities demanded for investment (*QINVD*) or their prices (*PQD*). But the prices (*PQD*) depend on much more than investment, hence the main adjustment must take place through the volumes of commodities demanded, i.e., *QINVD*, and therefore the volume adjuster (*IADJ*) must be variable, as must the savings rate adjuster (*SADJ*).

Alternatively the share of investment expenditure in the total value of domestic final demand can be fixed, i.e.,

$$INVESTSH = \overline{INVESTSH} \tag{C2d}$$

which means that the total value of investment is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *INVEST* equal to some level.

4.3. <u>Enterprise Account Closure</u>

Fixing the volumes of commodities demand by enterprises, i.e.,

$$QENTDADJ = \overline{QENTDADJ} \tag{C3a}$$

closes the enterprise account (C3). Note that this rule allows the value of commodity expenditures by the enterprise account to vary, which *ceteris paribus* means that the value of savings by enterprises (*CAPENT*) and thus total savings (*TOTSAV*) vary. If the value of this adjuster is changed, but left fixed, this imposes equiproportionate changes on the volumes of commodities demanded.

If *QENTDADJ* is allowed to vary then another variable must be fixed; the most likely alternative is the value of consumption expenditures by enterprises (*VENTD*), i.e.,

$$VENTD = \overline{VENTD}\,. \tag{C3b}$$

This would impose adjustments through equiproportionate changes in the volumes of commodity demand, and would feed through so that enterprise savings (*CAPENT*) reflecting directly the changes in the income of enterprises (*YE*). Alternatively the share of enterprise expenditure in the total value of domestic final demand can be fixed, i.e.,

$$VENTDSH = \overline{VENTDSH} \tag{C3c}$$

which means that the total value of enterprise consumption expenditure is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *VQENTD* equal to some level.

4.4. Government Account Closure

The closure rules for the government account are slightly more tricky because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income and expenditure are 'fixed', government savings must be free to adjust.

Thus in the base specification all the tax rates are fixed by declaring the tax rates as parameters and then fixing all the tax rate scaling factors (C4a – C4f), i.e.,

$$TMADJ = \overline{TMADJ} \tag{C4a}$$

$$TEADJ = \overline{TEADJ} \tag{C4b}$$

$$TSADJ = \overline{TSADJ} \tag{C4c}$$

$$TXADJ = \overline{TXADJ} \tag{C4d}$$

$$TFADJ = \overline{TFADJ} \tag{C4b}$$

$$TYADJ = \overline{TYADJ} \tag{C4b}$$

$$TYEADJ = \overline{TYEADJ} . \tag{C4f}$$

Consequently changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (F1 to F6).

The two other sources of income to the government are controlled by parameters, *govvash* and *govwor*, and therefore are not a source of concern for model closure.[14]

In the base specification government expenditure is controlled by fixing the volumes of commodity demand (*QGD*) through the government demand adjuster (*QGDADJ*), i.e.,

$$QGDADJ = \overline{QGDADJ} \tag{C4g}$$

Alternatively either the value of government consumption expenditure can be fixed, i.e.,

$$VQGD = \overline{VQGD} \tag{C4h}$$

or the share of government expenditure in the total value of domestic final demand can be fixed, i.e.,

$$VGDSH = \overline{VGDSH} . \tag{C4i}$$

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14 The values of income from non-tax sources can of course vary because each component involves a variable.

The scaling factor on the values of transfers to households and enterprises through the household (*HGADJ*) and enterprise (*EGADJ*) adjusters, i.e.,

$$HGADJ = \overline{HGADJ} \tag{C4j}$$

$$EGADJ = \overline{EGADJ} \tag{C4k}$$

also need to be fixed.

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not <u>directly</u> control. Hence the equilibrating condition is that government savings, the internal balance, is not fixed.

If however the model requires government savings to be fixed (C4l), i.e.,

$$CAPGOV = \overline{CAPGOV} \tag{C4l}$$

then either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

In its simplest form this can be achieved by allowing one of the previously fixed adjusters (C4a to C4i) to vary. Thus if the sales tax adjuster (*TSADJ*) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables *TMADJ* and *CAPGOV* would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the sales tax mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates it is necessary to change the relevant tax parameters. Typically such changes would be implemented in policy experiment files rather than within the closure section of the model.

4.5. <u>Numeraire</u>

The model specification allows for a choice of two price normalisation equations, the consumer price index and a producer price index, i.e.,

$$CPI = \overline{CPI} \tag{C5a}$$

or

$$PPI = \overline{PPI}\,. \tag{C5b}$$

A *numeraire* is needed to serve as a base since the model is homogenous of degree zero in prices and hence only defines relative prices.

4.6. Factor Market Closure

The factor market closure rules are more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

4.6.1. Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply of and total demand for factors equate. The total supplies of each factor are determined exogenously and hence

$$FS_f = \overline{FS}_f \tag{C6a}$$

defines the first set of factor market closure conditions. The demands for factor *f* by activity *a* and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages, by allowing for the possibility that the payments to notionally identical factors might vary across activities through the variable that captures the 'sectoral proportions for factor prices'. These proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, and are therefore assumed fixed, i.e.,

$$WFDIST_{f,a} = \overline{WFDIST_{f,a}}\,. \tag{C6b}$$

Finally bounds are placed upon the average factor prices, i.e.,

$$\begin{aligned} &\text{Min } WF_f = 0 \\ &\text{Max } WF_f = +\text{infinity} \end{aligned} \tag{C6c}$$

so that meaningful results are produced.

4.6.2. Factor Immobility and/or Unemployment Closures

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as factors. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations

$$\begin{aligned} FS_{fact} &= \overline{FS_{fact}} \\ WFDIST_{fact,a} &= \overline{WFDIST_{fact,a}} \\ \text{Min } WF_{fact} &= 0 \\ \text{Max } WF_{fact} &= +\text{infinity} \\ FD_{fact,a} &= \overline{FD_{fact,a}} \\ WF_{fact} &= \overline{WF_{fact}} \\ WFDIST_{fact,activ} &= \overline{WFDIST_{fact,activ}} \\ \text{Min } FS_{fact} &= 0 \\ \text{Max } FS_{fact} &= +\text{infinity} \end{aligned} \qquad \text{(C6d)}$$

where *fact* indicates the specific factor and *activ* a specific activity. The block of equations in (C6d) includes all the variables that were declared for the model with reference to factors plus an extra equation for *WFDIST*, i.e., $WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}}$, whose role will be defined below. The choice of which equations are binding and which are not imposed will determine the factor market closure conditions.

As can be seen the first four equations in the block (C6d) are the same as those in the 'Full Factor Mobility and Employment Closure'; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the 'Full Factor Mobility and Employment Closure'. Assume that this set of conditions represents the starting point, i.e., the first four equations are binding and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, i.e., the condition

$$FD_{fact,a} = \overline{FD_{fact,a}} \tag{C6e}$$

must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., the condition

$$WFDIST_{fact,a} = \overline{WFDIST_{fact,a}} \tag{C6f}$$

must now be relaxed.

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition

$$FS_{fact} = \overline{FS_{fact}} \tag{C6g}$$

is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e.,

$$WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}} \tag{C6h}$$

which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.[15]

Start again from the closure conditions for full factor mobility and employments and then assume that there is unemployment of one or more factors in the economy; typically this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing the condition that

$$WF_{fact} = \overline{WF_{fact}} \tag{C6i}$$

and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing

$$FS_{fact} = \overline{FS_{fact}} \tag{C6j}$$

It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions

$$\begin{aligned} &\text{Min } FS_{fact} = 0 \\ &\text{Max } FS_{fact} = +\text{infinity} \end{aligned} \tag{C6k}$$

can be imposed.[16]

15 It can be important to ensure a sensible choice of reference activity. In particular this is important if a factor is not used, or little used, by the chosen activity.

4.6.3. Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

If all factors used by an activity are fixed, this requires imposing the conditions that

$$FD_{f,activ} = \overline{FD_{f,activ}} \tag{C6e}$$

must be imposed, where *activ* refers to the activity of concern. But the returns to these factors in this activities must now be allowed to vary, i.e., the conditions

$$WFDIST_{f,activ} = \overline{WFDIST_{f,activ}} \tag{C6f}$$

must now be relaxed. In this case the condition fixing the total supply of the factor is not redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to mange the model by gathering together factor market conditions that are inspired by activity characteristics after the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supercedes previous conditions.

5. References

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16 If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

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6. Appendix

6.1. Parameter and Variable Lists

The parameter and variable listings are in alphabetic order, and are included for reference purposes. The parameters listed below are those used in the behavioural specifications/equations of the model, in addition to these parameters there are a further set of parameters. This extra set of parameters is used in model calibrated and for deriving results; there is one such parameter for each variable and they are identified by appending a '0' (zero) to the respective variable name.

Table 5 Parameter List

Parameter	Description
ac(c)	Shift parameter for Armington CES function
actcomactsh(a,c)	Share of commodity c in output by activity a
actcomcomsh(a,c)	Share of activity a in output of commodity c
ad(a)	Shift parameter for CES production functions
adxc(c)	Shift parameter for commodity output CES aggregation
alpha(f,a)	Share parameters for Cobb-Douglas production function
alphah(c,h)	Expenditure share by commodity c for household h
at(c)	Shift parameter for Armington CET function
beta(c,h)	Marginal budget shares
caphosh(h)	Shares of household income saved (after taxes)
comactco(c,a)	use matrix coefficients
deltax(f,a)	Share parameters for CES production functions
deltaxc(a,c)	Share parameters for commodity output CES aggregation
dstocconst(c)	Stock change demand volume
comentconst(c)	Enterprise demand volume
comgovconst(c)	Government demand volume
comhoav(c,h)	Household consumption shares
comtotsh(c)	Share of commodity c in total commodity demand
delta(c)	Share parameter for Armington CES function
deprec(f)	depreciation by factor f
econ(c)	constant for export demand equations
entgovconst	Government transfers to enterprises
entvash(f)	Share of income from factor f to enterprises
entwor	Transfers to enterprise from world (constant in foreign currency)
eta(c)	export demand elasticity
factwor(f)	factor payments from RoW (constant in foreign currency)
frisch(h)	Elasticity of the marginal utility of income
gamma(c)	Share parameter for Armington CET function
govvash(f)	Share of income from factor f to government
govwor	transfers to government from world (constant in foreign currency)

Parameter	Description
hexps(h)	Subsistence consumption expenditure
hoentconst(h)	transfers to hhold h from enterprise (nominal)
hogovconst(h)	transfers to hhold h from government (nominal but scalable)
hohoconst(h,hp)	interhousehold transfers
hohosh(h,hp)	Share of h'hold h after tax and saving income transferred to hp
hovash(h,f)	Share of income from factor f to household h
howor(h)	Transfers to household from world (constant in foreign currency)
invconst(c)	Investment demand volume
lesscal(h)	Scaling factor for hexps
predelta(c)	dummy used to estimated delta
pwse(c)	world price of export substitutes
qcdconst(c,h)	Volume of subsistence consumption
rhoc(c)	Elasticity parameter for Armington CES function
rhot(c)	Elasticity parameter for Output Armington CET function
rhox(a)	Elasticity parameter for CES production function
rhoxc(c)	Elasticity parameter for commodity output CES aggregation
te(c)	export subsidy rate by commodity c
tf(f)	Factor tax rate
tm(c)	tariff rates on commodity c
tmr(c)	Real tariff rate
ts(c)	Sales tax rates
tx(a)	Indirect tax rate on activity a
ty(h)	Direct tax rate on household h
tye	Direct tax rate on enterprises
sumelast(h)	Weighted sum of income elasticities
use(c,a)	use matrix transactions
vddtotsh(c)	Share of value of domestic output for the domestic market
worvash(f)	Share of income from factor f to RoW
yhelast(c,h)	(Normalised) household income elasticities

Table 6 Variable List

Variable Name	Variable Description
CAPENT	Enterprise savings
CAPGOV	Government Savings
CAPWOR	Current account balance
CPI	Consumer price index
DTAX	Direct Income tax revenue
EENT	Enterprise expenditure
EG	Expenditure by government
EGADJ	Transfers to enterprises by government Scaling Factor
ER	Exchange rate (domestic per world unit)
ETAX	Export tax revenue
FD(f,a)	Demand for factor f by activity a
FS(f)	Supply of factor f
FTAX	Factor tax revenue
HEXP(h)	Household consumption expenditure
HGADJ	Scaling factor for government transfers to households
HOHO(h,hp)	Inter household transfer
IADJ	Investment scaling factor
INVEST	Total investment expenditure
INVESTSH	Value share of investment in total final domestic demand
ITAX	Indirect tax revenue
MTAX	Import tariff revenue
PD(c)	Consumer price for domestic supply of commodity c
PE(c)	Domestic price of exports by activity a
PM(c)	Domestic price of competitive imports of commodity c
PPI	Producer (domestic) price index
PQD(c)	Purchaser price of composite commodity c
PQS(c)	Supply price of composite commodity c
PV(a)	Value added price for activity a
PWE(c)	World price of exports in dollars
PWM(c)	World price of imports in dollars
PX(a)	Composite price of output by activity a
PXC(c)	Producer price of composite domestic output
PXAC(a,c)	Activity commodity prices
QCD(c)	Household consumption by commodity c
QD(c)	Domestic demand for commodity c
QE(c)	Domestic output exported by commodity c
QENTD(c)	Enterprise consumption by commodity c
QENTDADJ	Enterprise demand volume Scaling Factor
QGD(c)	Government consumption demand by commodity c
QGDADJ	Government consumption demand scaling factor
QINTD(c)	Demand for intermediate inputs by commodity
QINVD(c)	Investment demand by commodity c

Variable Name	Variable Description
QM(c)	Imports of commodity c
QQ(c)	Supply of composite commodity c
QX(a)	Domestic production by activity a
QXC(c)	Domestic production by commodity c
QXAC(a,c)	Domestic commodity output by each activity
SADJ	Savings rate scaling factor
STAX	Sales tax revenue
TEADJ	Export subsidy Scaling Factor
TFADJ	Factor Tax Scaling Factor
TMADJ	Tarrif rate Scaling Factor
TOTSAV	Total savings
TSADJ	Sales tax rate scaling factor
TXADJ	Indirect Tax Scaling Factor
TYADJ	Income Tax Scaling Factor
TYEADJ	Enterprise income tax Scaling Factor
VFDOMD	Value of final domestic demand
VENTD	Value of enterprise consumption expenditure
VGD	Value of Government consumption expenditure
VGDSH	Value share of Govt consumption in total final domestic demand
VENTDSH	Value share of Ent consumption in total final domestic demand
WALRAS	Slack variable for Walras's Law
WF(f)	Price of factor f
WFDIST(f,a)	Sectoral proportion for factor prices
YE	Enterprise income
YF(f)	Income to factor f
YFDISP(f)	Factor income for distribution after depreciation
YFWOR(f)	Foreign factor income
YG	Government income
YH(h)	Income to household h

6.2. GAMS Code

```
*############### 14. EQUATIONS ######################################
*## PRICE BLOCK

* For some c there are no imports hence only implement for cm(c)

 PMDEF(c)$cm(c)..  PM(c) =E= (PWM(c) *(1 + (TMADJ * tm(c)))) * ER ;

* For some c there are no exports hence only implement for ce(c)

 PEDEF(c)$ce(c)..  PE(c) =E= PWE(c) * ER * (1 - (TEADJ * te(c))) ;

 PQDDEF(c)$(cd(c) OR cm(c))..
                  PQD(c) =E= PQS(c) * (1 + (TSADJ*ts(c))) ;

 PQSDEF(c)$(cd(c) OR cm(c))..
            PQS(c)*QQ(c) =E= (PD(c)*QD(c))+(PM(c)*QM(c)) ;

 PXCDEF(c)$cx(c)..
          PXC(c)*QXC(c) =E= (PD(c)*QD(c)) + (PE(c)*QE(c))$ce(c) ;

 PXDEF(a)..        PX(a) =E= SUM(c,actcomactsh(a,c)*PXC(c)) ;

 PVADEF(a)..  PX(a)*(1-(TXADJ*tx(a)))*QX(a)
                         =E= (PVA(a)*QVA(a)) + (PINT(a)*QINT(a)) ;

 PINTDEF(a)..    PINT(a) =E= SUM(c,comactactco(c,a) * PQD(c)) ;

 CPIDEF..            CPI =E= SUM(c,comtotsh(c)*PQD(c)) ;

 PPIDEF..            PPI =E= SUM(c,vddtotsh(c)*PD(c)) ;

*## PRODUCTION BLOCK

* CES aggregation functions for Level 1 of production nest

 QXPRODFN(a)$aqx(a)..
                   QX(a) =E= adx(a)*(deltax(a)*QVA(a)**(-rhocx(a))
                              + (1-deltax(a))*QINT(a)**(-rhocx(a)))
                              **(-1/rhocx(a)) ;

 QXFOC(a)$aqx(a)..
                  QVA(a) =E= QINT(a)*((PINT(a)/PVA(a))*(deltax(a)/
                                 (1-deltax(a))))**(1/(1+rhocx(a))) ;

* Leontief aggregation functions for Level 1 of production nest

 QINTDEF(a)$aqxn(a)..
                 QINT(a) =E= ioqintqx(a) * QX(a) ;

 QVADEF(a)$aqxn(a)..
                  QVA(a) =E= ioqvaqx(a) * QX(a) ;

* CES aggregation functions for Level 2 of production nest

 QVAPRODFN(a)..   QVA(a) =E= adva(a)*(SUM(f$deltava(f,a),
                              deltava(f,a)*FD(f,a)**(-rhocva(a))))
                              **(-1/rhocva(a)) ;

 QVAFOC(f,a)$deltava(f,a)..
       WF(f)*WFDIST(f,a) =E= PVA(a)*QVA(a)
                             *(SUM(fp$deltava(fp,a),deltava(fp,a)
                             *FD(fp,a)**(-rhocva(a))))**(-1)
                             *deltava(f,a)*FD(f,a)**(-rhocva(a)-1) ;

 QINTDEQ(c)..   QINTD(c) =E= SUM(a,comactactco(c,a)*QINT(a)) ;

 COMOUT(c)$cx(c)..
                  QXC(c) =E= adxc(c)*(SUM(a$deltaxc(a,c),deltaxc(a,c)
                             *QXAC(a,c)**(-rhocxc(c))))**(-1/rhocxc(c)) ;

 COMOUTFOC(a,c)$deltaxc(a,c)..
               PXAC(a,c) =E= PXC(c)*QXC(c)
                             *(SUM(ap$deltaxc(ap,c),deltaxc(ap,c)
                             *QXAC(ap,c)**(-rhocxc(c))))**(-1)
                             *deltaxc(a,c)*QXAC(a,c)**(-rhocxc(c)-1) ;
```

```
 ACTIVOUT(a,c)$actcomactsh(a,c)..
              QXAC(a,c) =E= actcomactsh(a,c) * QX(a) ;

*## TRADE BLOCK

* For some c there are no exports hence only implement for ce(c)

 CET(c)$(cd(c) AND ce(c))..
                 QXC(c) =E= at(c)*(gamma(c)*QE(c)**rhot(c) +
                                (1-gamma(c))*QD(c)**rhot(c))**(1/rhot(c)) ;

 ESUPPLY(c)$(cd(c) AND ce(c))..
                  QE(c) =E= QD(c)*((PE(c)/PD(c))*((1-gamma(c))
                                /gamma(c)))**(1/(rhot(c)-1)) ;

* For c with no exports domestic supply from domestic production is CET2

 CET2(c)$(cd(c) AND cen(c))..
                 QXC(c) =E= QD(c) ;

* For c with no domestic production domestic supply is for export by CET3

 CET3(c)$(cdn(c) AND ce(c))..
                 QXC(c) =E= QE(c) ;

* For some c there are no imports or domestic production
* hence only implement for cd(c) AND cm(c)

 ARMINGTON(c)$(cx(c) AND cm(c))..
                 QQ(c)  =E= ac(c)*(delta(c)*QM(c)**(-rhoc(c)) +
                               (1-delta(c))*QD(c)**(-rhoc(c)))**(-1/rhoc(c)) ;

 COSTMIN(c)$(cx(c) AND cm(c))..
                  QM(c) =E= QD(c)*((PD(c)/PM(c))*(delta(c)/
                                (1-delta(c))))**(1/(1+rhoc(c))) ;

* For c with no imports domestic supply equals domestic production

 ARMINGTON2(c)$(cx(c) AND cmn(c))..
                   QQ(c) =E= QD(c) ;

* For c with no domestic production supply equals imports

 ARMINGTON3(c)$(cxn(c) AND cm(c))..
                   QQ(c) =E= QM(c) ;

*## INCOME BLOCK

 YFEQ(f)..         YF(f) =E= SUM(a,WF(f)*WFDIST(f,a)*FD(f,a))
                             + (factwor(f)*ER) ;

 YFDISPEQ(f).. YFDISP(f) =E= (YF(f) - deprec(f))*(1-(TFADJ*tf(f))) ;

 YHEQ(h)..         YH(h) =E= SUM(f,hovash(h,f)*YFDISP(f))
                             + SUM(hp,HOHO(h,hp))
                             + hoentconst(h)
                             + (HGADJ*hogovconst(h))
                             + (howor(h)*ER) ;

 YEEQ..               YE =E= SUM(f,entvash(f)*YFDISP(f))
                             + (EGADJ*entgovconst)
                             + (entwor*ER) ;

 YGEQ..               YG =E= MTAX + ETAX + STAX + ITAX + FTAX + DTAX
                             + SUM(f,govvash(f)*YFDISP(f))
                             + (govwor*ER) ;

 YFWOREQ(f)..   YFWOR(f) =E= worvash(f)*YFDISP(f) ;

 TOTSAVEQ..       TOTSAV =E= SUM(h,YH(h)*(1-(TYADJ*ty(h)))*(SADJ*caphosh(h)))
                             + SUM(f,deprec(f))
                             + CAPENT
                             + CAPGOV
                             + (CAPWOR*ER) ;

*## EXPENDITURE BLOCK

 HOHOEQ(h,hp)..
             HOHO(h,hp) =E= hohosh(h,hp)
                             *((YH(h)*(1-(TYADJ*ty(h))))
                                   *(1-(SADJ*caphosh(h)))) ;
```

```
 HEXPEQ(h)..     HEXP(h) =E= ((YH(h)*(1-(TYADJ*ty(h))))*(1-(SADJ*caphosh(h))))
                             - SUM(hp,HOHO(hp,h)) ;

* QCDEQ(c).. PQD(c)*QCD(c) =E= SUM(h,comhoav(c,h)*HEXP(h)) ;

 QCDEQ(c)..
          PQD(c)*QCD(c) =E= SUM(h,PQD(c)*qcdconst(c,h))
                            + SUM(h,beta(c,h)
                             *(HEXP(h)-SUM(cp,PQD(cp)*qcdconst(cp,h)))) ;

 QENTDEQ(c)..   QENTD(c) =E= QENTDADJ*comentconst(c) ;

 EENTEQ..           EENT =E= SUM(c,QENTD(c)*PQD(c))
                            + SUM(h,hoentconst(h))
                            + (TYEADJ*tye*YE) ;

 VENTDEQ..       VENTD =E= SUM(c,QENTD(c)*PQD(c)) ;

 QGDEQ(c)..       QGD(c) =E= QGDADJ*comgovconst(c) ;

 EGEQ..               EG =E= SUM(c,QGD(c)*PQD(c))
                            + SUM(h,hogovconst(h)*HGADJ)
                            + (EGADJ*entgovconst) ;

 VGDEQ..           VGD =E= SUM(c,QGD(c)*PQD(c)) ;

 QINVDEQ(c)..   QINVD(c) =E= (IADJ*invconst(c)) ;

 INVESTEQ..       INVEST =E= SUM(c,PQD(c)*(QINVD(c) + dstocconst(c))) ;

*## TAX BLOCK

 MTAXEQ..           MTAX =E= SUM(c,TMADJ*tm(c)*PWM(c)*ER*QM(c)) ;

 ETAXEQ..           ETAX =E= SUM(c,TEADJ*te(c)*PWE(c)*ER*QE(c)) ;

 STAXEQ..           STAX =E= SUM(c,TSADJ*ts(c)*PQS(c)*QQ(c)) ;

 ITAXEQ..           ITAX =E= SUM(a,TXADJ*tx(a)*PX(a)*QX(a)) ;

 FTAXEQ..           FTAX =E= SUM(f,TFADJ*tf(f)*(YF(f)-deprec(f))) ;

 DTAXEQ..           DTAX =E= SUM(h,TYADJ*ty(h)*YH(h))
                            + (TYEADJ*tye*YE) ;

*## MARKET CLEARING BLOCK

 FMEQUIL(f)..      FS(f) =E= SUM(a,FD(f,a)) ;

 QEQUIL(c)..      QQ(c)  =E= QINTD(c) + QCD(c) + QENTD(c) + QGD(c) + QINVD(c)
                              + dstocconst(c) ;

 VFDOMDEQ..       VFDOMD =E= SUM(c, PQD(c) *
                               (QCD(c) + QENTD(c) + QGD(c)
                                + QINVD(c) + dstocconst(c))) ;

 INVESTSHEQ..
       INVESTSH * VFDOMD =E= INVEST ;

 VGDSHEQ..
         VGDSH * VFDOMD =E= VGD ;

 VENTDSHEQ..
       VENTDSH * VFDOMD =E= VENTD ;

 CAPENTEQ..       CAPENT =E= YE - EENT ;

 GOVEQUIL..       CAPGOV =E= YG - EG ;

 CAEQUIL..        CAPWOR =E= SUM(cm,PWM(cm)*QM(cm))
                            + SUM(f,YFWOR(f))
                            - SUM(ce,PWE(ce)*QE(ce))
                            - SUM(h,howor(h))
                            - entwor - govwor
                            - SUM(f,factwor(f)) ;

 WALRASEQ..       TOTSAV =E= INVEST + WALRAS ;
```

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