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Finite Mixture Estimation of Size Economies and Cost Frontiers in the Face of Multiple Production Technologies

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ABSTRACT

Finite mixture estimation (FME) is compared to estimated generalized least squares (EGLS) in the estimation of economies of size and production cost frontiers for Alabama dairy farms. FME provides several unique insights into the economic forces behind recent changes in Alabama's dairy industry. FME provides estimation of a stochastic average cost frontier with known statistical properties, which it was not otherwise possible to obtain using available stochastic frontier estimation packages.

Key Words: dairy, economies of size, finite mixture estimation, stochastic cost frontier.

Economies-of-size studies have long been used to analyze managerial decisions relative to firm size (Boehlje; Chavas and Klemme; Garcia and Sonka; Hallam 1993a, b; Harrington; Heady; Hildreth; Miller; Moschini 1990). Recently, stochastic cost frontier estimation has gained acceptance in estimating minimum production costs and evaluating relative efficiencies (Aigner, Lovell, and Schmidt; van den Broeck et al.; Forsund, Lovell, and Schmidt; Greene 1980a, b, 1990; Olson, Schmidt, and Waldman; Schmidt and Lovell 1979, 1980; Stevenson; Waldman). In these and similar applications, researchers routinely assume that data represent profit-maximizing or cost-minimizing behavior by firms producing in accord with a uniform, optimal technology—an assumption that is not always justified.

Beard, Caudill, and Gropper elaborate the consequences of specification error when observations taken from more than one distribution are treated as if drawn from a single distribution:

> The simultaneous existence of multiple technologies of production can pose a serious problem for traditional cost function estimation. If a sample includes observations on firms using different technologies, pooling the data in a single regression procedure is likely to produce misleading results. Since the single-technology restriction is a specification error in these cases, the estimated cost function obtained may not be quantitatively (e.g., in coefficient magnitudes), nor qualitatively (e.g., in the implied presence or absence of scale effects) similar to any true underlying cost relationships (p. 655).

In this study, finite mixture estimation (FME) is used, as recommended by Beard, Caudill, and Gropper, not only to detect whether the observations in a sample might have been drawn from more than one distribution, but also as a tool to enhance analyses of the data.

Variations in production technologies or firm efficiencies can stem from many sources: varying rates of technological adaptation, ex-

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perimentation and innovation, differing endowments, or primary enterprise goals other than profit maximization to name a few. If it is known a priori the variations that exist within a sample and which observations correspond to each variation, then regressions can be specified to include and evaluate this important information. The problem, as addressed here, arises when approaching a data set for which such a priori information is not available.

Previous studies provide evidence that a diversity of production methods and efficiencies coexist within the dairy industry. Using national dairy data, Weersink and Tauer report finding large variations, state by state, in rates of technological adaptation as well as in the presence or absence of economies of size. In another study, Tauer examined data from 49 New York dairy producers and found that fewer than half could be classified as approaching profit maximization, while just over half came within 10% of cost minimization.

First-hand experience with Alabama dairy producers leaves no doubt that widely differing approaches to production and levels of efficiency coexist within the industry, from low-input grazing dairies to high-input confined dairies. Better producers from widely differing points along this production spectrum report profitable operations. Product price has been administered through the Federal Milk Market Order system since 1982. For much of this time, there has been a gradual decrease in the real price paid to farmers, as the Milk Market Administrator has responded to regulations requiring that milk prices be brought gradually into line with free market supply and demand levels. Expansion of output by new or existing dairies was not constrained.

During most of the time covered by the data used in this research, the industry was characterized by a coincidence of trends in which some firms expanded, suggesting a possibility of economies of size, while other firms opted to exit the industry. The most recent data, however, indicate that expansion of output among Alabama's remaining dairies has slowed or possibly even reversed, while the volume of producers ceasing production has accelerated, at least in the short run (Smith, Taylor, and Moss). Firm-level data from Alabama's dairy industry are used in this study to provide insight into forces behind these observed trends. FME is employed in estimating economies of size and average cost frontiers for an industry in which various methods of production and levels of efficiency coexist, although for most of these variations available data do not permit the attribution of observations to identifiable firm-level differences in production methods or efficiencies.

Finite Mixture Foundations and Methods

Beard, Caudill, and Gropper cite early work on finite mixture distributions dating back to the late 19th century. Finite mixture theory extends to mixtures drawn from two or more distributions, of which the component marginal densities need not be all of the same parametric family. Early economic applications of finite mixture techniques, also known as switching regressions, date from the 1970s (Quandt; Quandt and Ramsey). The two-distribution mixture model used in this application is taken from work by Beard, Caudill, and Gropper, who followed the approach recommended by Hartley. This approach premises a finite mixture of two univariate normal PDFs: $\Phi_1 \sim N(\mu_1, \sigma_1^2)$ and $\Phi_2 \sim N(\mu_2, \sigma_2^2)$. Average production costs, c_i, are assumed to have been generated by either of two functions, g_1 or g_2, and to be distributed according to Φ_1 or Φ_2, respectively. The probability that an average cost observation, c_i, results from the stochastic process $\{c_i = g_1(y_i; b_1) + e_1\}$ is denoted λ (mixing weight), and consequently the probability that c_i results from a second stochastic process $\{c_i = g_2(y_i; b_2) + e_2\}$ then becomes $1 - \lambda$; $0 \leq \lambda \leq 1$. The y_i are output observations, the b_i are vectors of parameters, and the error terms e_1 and e_2 are assumed to be distributed normally with zero means and variances of σ_1^2 and σ_2^2, respectively. Thus the likelihood function,

$$L = \prod_i [\lambda\Phi_1(c_i - g_1(y_i; \beta_1)) + (1 - \lambda)\Phi_2(c_i - g_2(y_i; \beta_2))],$$

is maximized over β_1, β_2, σ_1^2, σ_2^2, and λ, using the EM algorithm in the manner suggested by Hartley. The EM algorithm is a technique for likelihood estimation with incomplete data (Dempster, Laird, and Rubin). In this case, the missing data would indicate which observations were drawn from either of two distributions. The EM algorithm circumvents a singularity problem that might otherwise be encountered using maximum-likelihood estimation (MLE). Hartley states that limited Monte Carlo experiments have shown point estimates from this approach to be very close to true parameter values for moderate sample sizes of approximately 100 observations (p. 740). As λ approaches either 0 or 1, the mixture model collapses to a single average cost function. Also note that when multiple roots exist, the algorithm may converge on multiple solutions. Hartley advises, "presumably the root which maximizes L is the consistent one," and recommends experimenting with starting values. A slight variation of this assumption is used in this analysis when FME results in two alternative solutions with likelihood ratios that are very close to each other.

Given that convergence occurs, the EM algorithm provides a likelihood value, an estimate of the mixing weight, and estimates for two sets of parameters along with their familiar measures of reliability (variances and t-ratios). Use of this FME algorithm calls for experimentation and judgment on the part of the researcher. As noted by Hartley, the algorithm requires specification of starting parameter values, and a researcher should experiment with a broad range of starting values. Some data will produce solutions that are stable over a broad range of starting values, while other data will result in less robust solutions which are sensitive to variations in starting values. The algorithm either may fail to converge or may converge on alternative solutions as starting values are varied.

In those instances where it is necessary to evaluate alternative solutions, the researcher must evaluate solutions in light of the likelihood values, the estimated mixing weights, t-ratios on key parameters, and the robustness of alternative solutions. As previously cited from Hartley, the solution that maximizes the likelihood value is presumed to be the consistent one. Estimated mixing weights which project one group to contain a very small percentage of the observations are suspect, as they are likely to result from a few outliers. Similarly, low t-ratios on key parameters suggest that the estimated model is incorrectly specified for that group of observations. Finally, solutions that are not robust over a reasonable range of starting values should be considered less likely than alternatives that are more robust. Less robust solutions may represent spikes caused by data anomalies and, in our experience, often are associated with extremely lopsided mixing weights. Graphing the estimated functions over reasonable values of the independent variables also can be helpful in evaluating alternatives. In this usage, reasonable values for independent variables are values that fall within the range of values found in the observations.

Data

Two sets of cost data are used in this study. The Dairy Herd Improvement Association (DHIA) collects data from a majority of the dairies operating in Alabama (Bertrand et al.). This provides a large data base, with information relative to changes in size, production per cow, and similar variables. The panels drawn from these data span the years 1980–94, and include 1,157 observations from Holstein dairies, 187 from Jersey dairies, and 73 from dairies using other breeds of cattle. The limitation with this sample is that DHIA does not collect data for inputs to production other than feed and livestock or for outputs other than milk.

The Alabama Farm Analysis Association (AFAA) provides a smaller panel, 110 observations spanning the years 1984–92. These records include detailed observations on all inputs to and outputs from production. As pointed out by Garcia and Sonka, a farm record keeping system which collects standardized data on a regular basis can overcome many of the data pitfalls otherwise encountered. The Alabama farm records system is designed similarly to the Illinois system as described by Casler. In this system, commodities produced on-farm are entered into farm inventories at values comparable to prevailing market prices, which facilitates evaluation of dairy farm performance separately from the performance of other farm enterprises. This approach is supported by the finding of Moschini (1988) that, for Ontario dairy farms, a hypothesis of nonjointness with other farm outputs could not be rejected. Unlike DHIA files, however, this record keeping system does not record information on the breeds of the dairy cattle being used.

All of the above data sets were chosen to predate the introduction of bovine somatotropin (bST) for widespread use by the dairy industry. It is assumed that adoption of bST by dairy producers will have resulted in significant structural shifts in both production and efficiency.

All dollar amounts have been inflated to 1994 equivalents. Feed costs were inflated by the feed price index published in the *Agricultural Outlook* [U.S. Department of Agriculture (USDA)]. Other costs were inflated using the Implicit Price Deflator Index of Gross Domestic Product (Council of Economic Advisors). For a more complete description of the assumptions and compilation methods used in these data sets, see Smith.

Model Specification and Estimation Procedures

The literature on agricultural economies of size mentions that "sagging L-shaped" average cost curves are typically found in studies of agricultural enterprises (Ahearn, Whittaker, and El-Osta; Hallam 1993a, b; Heady). Functional forms approximating this and more complex shapes were evaluated both conceptually and by testing them on small subsets of data. A form employing the reciprocal of total production as the principal independent variable was selected. The reciprocal form approximates the sagging L-shape, and has an additional intuitive appeal for this type of study. A positive sign on the estimated parameter for the reciprocal of output indicates average costs which decrease as output increases. In this form the dependent variable, average cost of production, is asymptotic to a value. Given the correct sign on the estimated parameter, this value of the asymptote can be viewed as a minimum possible average cost of production.

Dependent variable observations from the two data sources are different. AFAA data contain observations on average total costs per cwt of milk produced, while DHIA data contain only average feed costs per cwt of milk. Feed costs normalized with the USDA Index of Feed Prices were not found to exhibit statistically significant time trends at the customary 5% level. Thus two slightly different functional forms are estimated:

$$(1) \quad C_t = \alpha + \beta/Q_t + \gamma T_t + \epsilon_t$$

and

$$(2) \quad F_t = \alpha + \beta/Q_t + \epsilon_t,$$

where C in equation (1) is the average total cost of the milk produced, F in equation (2) is average feed cost, Q is the total amount of milk produced expressed in hundredweights, and T is time trend consisting of the final two digits of the year from which the records were taken, 80–94.

Substantial variations in average characteristics exist between dairy cattle of different breeds. Attributes such as body weight, feed requirements, pounds of milk produced, percentage of butter fat, and even tolerance to summer heat stress are quite different from one breed to another. However, the inclusion of breed as a variable in the estimation of dairy production or cost functions was not found in a review of the literature. For this reason, it was decided to employ the information on cattle breeds which is available in DHIA data as a means to illustrate the potential distortions that can occur when such important differences are ignored and data are lumped into a single regression. As a first step, therefore, economies of size are separately estimated using estimated generalized least squares (EGLS) on DHIA data sorted by breed. EGLS using Harvey's correction was necessary due to heteroskedasticity encountered in the

Table 1. Parameters for Average Feed Cost: EGLS Regressions on DHIA Data

	Dairy Breed		
Parameter Estimates	Holstein	Jersey	Other Breeds
Intercept	5.98	6.69	7.134
Reciprocal term	3,189.5	−2,800.9	−784.8
	(3.85)[a]	(−1.46)	(−6.23)

[a] Numbers in parentheses are *t*-values.

data. The subsequent FME regressions also use data transformed according to Harvey's method.

Next, since it is known a priori that other unidentifiable production and efficiency differences exist even among dairy farms using the same breed of cattle, FME is applied to these same DHIA samples. Results from both regression methods, EGLS and FME, are reported together in order to facilitate comparison of the analytical contributions obtained with each approach. Finally, the Alabama Farm Analysis Association data are analyzed, again using both EGLS and FME.

Estimation Results

Economies of size estimates from the panels of DHIA data which have been separated by breed show contrasting results (table 1). These results demonstrate the potential losses of information which occur when observations are pooled into a single regression without consideration of breed differences. The dairy farms classified as "other breeds" are shown to have been less efficient than their competition in terms of feed conversion, a common ratio relating feed used to the output obtained. In fact, other breed farms evidence statistically significant diseconomies of size in this regard. Similarly, the negative sign on the estimated coefficient for Jersey farms indicates diseconomies of size, but this estimate is not statistically significant at a customary 5% level. Nevertheless, it reveals that farms using Jersey cattle were not obtaining feed conversion economies of size such as those achieved on Holstein dairy farms. Next, this same data set is evaluated again, this time using FME.

FME is a large sample technique. The "other breeds" panel was found to be too small and disparate to produce meaningful results with FME. The Holstein and Jersey panels, however, do provide illuminating estimates (table 2). The FME estimates from both the Holstein and Jersey panels were robust. Both show a larger proportion of the observations to be drawn from dairy farms which are not obtaining significant economies of size in feed costs, while smaller groups of observations from farms of each breed are shown to be obtaining statistically significant economies of size in feed costs. Note also that the magnitude of the parameter estimates indicates that these economies of size are much stronger than those that were estimated for the Holstein dairy farms by EGLS. This is consistent with what would be intuitively expected to result

Table 2. Parameters for Average Feed Cost: FME Regressions on DHIA Data

	Dairy Breed			
Parameter Estimates	Holstein 1	Holstein 2	Jersey 1	Jersey 2
Intercept	6.12	5.33	6.72	6.18
Reciprocal term	1,659.5	10,677.6	−5,693.8	20,017.3
	(1.03)[a]	(2.95)	(−2.08)	(6.01)
Mixing weight (λ)	80.9%	19.1%	84.3%	15.7%
	(6.46)		(14.3)	

[a] Numbers in parentheses are *t*-values.

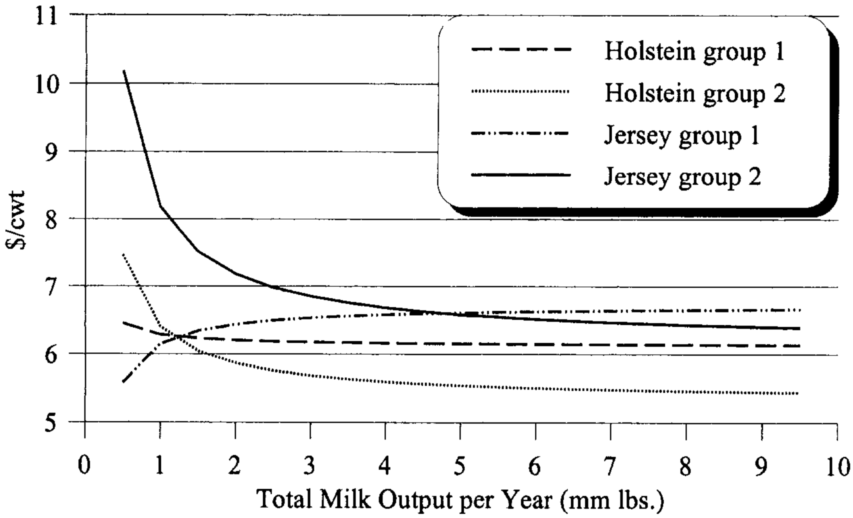

Figure 1. Average feed costs from finite mixture estimation on DHIA data

from removing the observations from farms not obtaining economies of size.

Figure 1 presents graphs of these estimated functions over the range of the observed output. While these graphs show that Holstein dairy farms have an advantage in average feed costs, note that the smaller, lower cost group of Jersey farms are within a range sufficient to render them competitive in terms of their feed conversion ratios.

Table 3 provides the results from both EGLS and FME regressions on the Alabama Farm Analysis Association data. Recall that these data, unlike the DHIA data, include average total costs per unit of production as opposed to just average feed costs. These FME solutions were less robust than those from the DHIA data, in part because of a smaller sample size. Therefore, two almost equally strong FME solutions are presented (A and B). Note that the likelihood values for these two solutions are quite close (−222.9 and −223.6). The FME algorithm employed in this study is limited to estimating probabilities for only two distributions. As will be subsequently explained in more detail, these alternative solutions are consistent with alternative groupings of three underlying distributions.

Table 3. Parameters for Average Total Cost: EGLS and FME Regressions on Alabama Farm Analysis Association Data

Parameter		FME			
Estimates	EGLS	A1	A2	B1	B2
Intercept	41.73	44.59	12.33	31.4	63.64
Reciprocal term	18,014.0	16,494.3	8,283.9	13,454.4	6,775.4
	(1.93)[a]	(1.81)	(1.55)	(1.50)	(0.63)
Time trend	−0.311	−0.340	−0.0098	−0.204	−0.531
	(−3.86)	(−4.03)	(−0.73)	(−2.32)	(−5.81)
Mixing weight	N/A	92.8%	7.2%	62.9%	37.1%
(λ)		(32.8)		(5.95)	
Likelihood value	N/A	−222.9		−223.6	

Note: FME found two solutions with very similar likelihood values. A1 and A2 are the estimates for the first of these; B1 and B2 are the estimates for the other. (Refer to the text for interpretation of these results.)

[a] Numbers in parentheses are t-values.

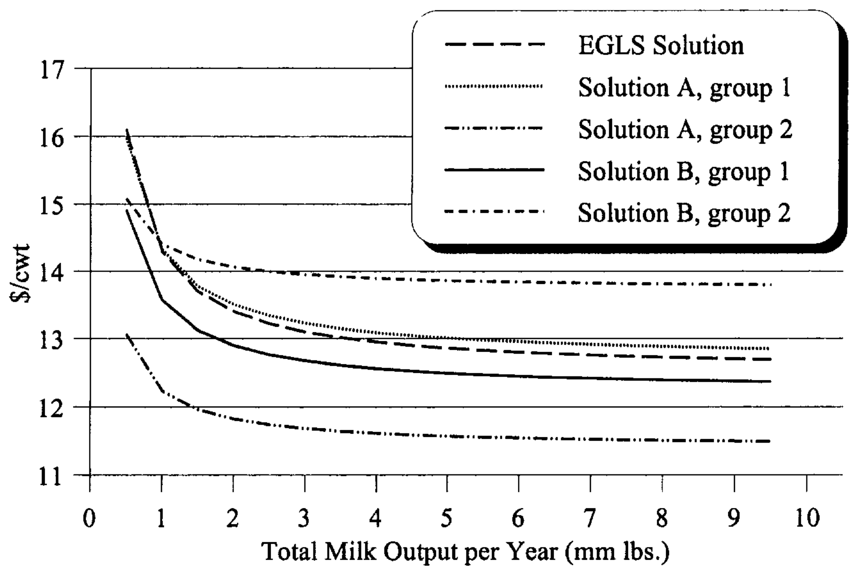

Figure 2. Average total costs from finite mixture estimation on Alabama Farm Analysis Association data

Figure 2 displays graphs of these estimated EGLS and FME functions over the observed range of outputs, with the time trend variable evaluated for 1994. Again, the EGLS results not only serve to demonstrate the additional insight to be gained through using FME, but also provide evidence of the reasonableness of the FME estimates. In both FME solutions, the larger groups of observations produce estimated parameters and functions which are quite close to the EGLS solution, deviating from it in the direction consistent with the nature of the smaller group of observations that was removed from the estimation. Solution A of the FME regressions divides the observations into a large group (92.8% of the observations) and a smaller group (7.2%). The smaller group (A2) contains observations which exhibit the very lowest average costs of production. The FME estimates in solution B divide the observations differently than A. Low- and medium-cost observations are lumped together into a single, larger group, which then isolates a group with higher than average production costs (37.1% of the observations). As in solution A, the parameters estimated for the larger group in solution B are close to those from the EGLS regression and deviate in the direction consistent with removing a group of higher cost observations.

All of the estimated coefficients on the reciprocal terms have the expected sign, which indicates economies of size. They have t-ratios below the accustomed 2.0 demarcation, but four out of five of the t-ratios are sufficiently large to indicate a definite probability that economies of size are being obtained. The t-ratio on the fifth coefficient, that of the high-cost group (B2), is much smaller and shows that these operators have had much less success in obtaining economies of size.

All time trend coefficients except those for the lowest cost group (A2) exhibit decreasing real average costs of production throughout the period of the observations. This is interpreted as an industrywide response to the steady decline in real product prices, as mandated under the federal Milk Market Program. Recall that the lowest cost group (A2) was estimated to contain 7.2% of the observations, drawn from the most efficient farms. The lack of a significant time trend in this group is consistent with the view that these most efficient operations were already at or very near the efficiency frontier. The less efficient dairy farms could respond to decreasing real milk prices by becoming more efficient in ways that the most efficient producers had already exploited. This is important because, as the literature and available software for stochastic frontier estimation point out, when the third moment of the residuals from a preliminary OLS regression turns out to have an incorrect sign, this indicates a data skew that is inconsistent with the assumptions inherent in the distributions for which stochastic frontier algorithms have been developed: exponential, half-normal, and gamma. When this third moment of the residuals has an inappropriate sign, as it does with these data, it has been shown that parameter estimates obtained from the preliminary OLS regression constitute local maxima, and one is left with the options of either using ad hoc estimates having unknown statistical properties or reverting to a nonstochastic, full frontier approach (Waldman, p. 278; Greene 1990, p. 153). It is the contention of this study, in line with suggestions from Caudill offered in a departmental seminar, that FME-estimated function A2 may serve as a reasonable estimate of a stochastic cost frontier function with known statistical properties.

Conclusions

FME has provided a method not only to assess the probability that multiple distributions underlie the data panels, but also has facilitated insight into recent dairy farm industry dynamics. A fuller picture of the role that economies of size played in decisions to either expand production or exit the industry was provided. It was found that only the more efficient operators, those showing lower than average costs of production, had been successful in obtaining economies of size, which explains why some producers exited the industry while others expanded. FME, by estimating the probability that observations are drawn from two distributions, was able to show that existing economies of size, for those producers able to capture them, were much stronger than the economies of size estimated using EGLS and treating all observations as if drawn from a single distribution.

Where economies of size are found, they prevail only up to levels of output approximately equal to the current average size among Alabama dairies. This explains the recently observed slowdown in dairy farm expansion. Furthermore, it was shown through the improved time trend estimates produced with FME that, while size has played an important role in dairy farm efficiency, large improvements in dairy farm efficiency were accomplished in ways not directly related to quantity of output.

Finally, it was shown that the estimated average production cost of the lowest cost group of producers can reasonably be taken as an estimate of a stochastic average cost frontier. Thus, FME was able to provide a stochastic average cost frontier estimate, with known statistical properties. This was not previously available using other stochastic frontier estimation techniques.

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