Journal of Agricultural and Applied Economics, 30,2(December 1998):267–276

Fractional Cointegration and the False Rejection of the Law of One Price in International Commodity Markets

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ABSTRACT

This study examines the Law of One Price (LOP) in international commodity markets using fractional cointegration analysis. For proper evaluation of the LOP, fractional cointegration analysis seems to be appropriate because of its flexibility in capturing a wider range of mean reversion behavior than standard cointegration analysis. Out of nine pairs of price series examined, fractional cointegration supports the existence of the LOP in eight cases, as compared to three cases using standard cointegration procedures. Overall, these results suggest that there is a long-run tendency for the LOP to hold for commodity prices.

Key Words: fractional cointegration, international commodity markets, Law of One Price.

The Law of One Price (LOP) is the notion that commodity prices in spatially separated markets, adjusted for exchange rates and transportation costs, should be equal. This equality is established and maintained by the profit-seeking actions of international commodity arbitragers (Goodwin 1990a). The assumption that the LOP holds is an important component of most international trade models because it allows the use of a single representative price. On the other hand, deviations from the LOP can explain the short-run volatility of exchange rates and "overshooting effects" (Ardeni).

The empirical validity of the LOP in international commodity markets has received a great deal of attention among researchers. Many studies (Officer; Carter and Hamilton; Zanias; Jung and Doroodian; Buongiorno and Uusivuori) have failed to support the LOP hypothesis. The frequent empirical rejection of the LOP is troubling because it is difficult to believe that rational traders are incapable of finding profitable arbitrage opportunities or that markets function so imperfectly that deviations in prices for the same goods can persist for long periods of time. It is possible that the LOP has been falsely rejected in these studies either because important factors such as transportation costs, price expectations, or market power were not taken into account, or because the nature of the methods employed was insufficiently flexible to capture the true relationships among the price series examined.

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This is Journal Paper No. J-16676 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa, and Project No. 3310, supported by Hatch Act and State of Iowa funds.

The purpose of this analysis is to explore the second of these explanations by using an alternative method—fractional cointegration—to test for the LOP in commodity markets.

Before introducing this method, it is instructive to review some of the studies of the LOP which have focused on various factors that may condition the behavior of prices in important commodity markets. Goodwin (1992) used multivariate cointegration tests (the Johansen approach) to examine the LOP in international wheat markets. He rejected the LOP before accounting for transportation costs, but the LOP was not rejected after accounting for transportation costs. Similarly, Baffes examined the LOP in international commodity markets for seven commodities in four countries with explicit consideration of transactions costs. In most cases, he concluded that the LOP cannot be rejected as the maintained hypothesis.

Goodwin (1990a, b) tested the LOP in international commodity markets by incorporating price expectations rather than utilizing contemporaneous prices. He argued that it takes time to move goods from one market to another. Thus, one should not expect the LOP to hold for contemporaneous prices unless arbitragers have perfect foresight or unless prices are constant. Goodwin and Schroeder (1991a) empirically evaluated spatial price linkages in regional cattle markets using cointegration tests. Even after incorporating such market characteristics as distance between markets, industry concentration ratios, market volumes, and market types, they rejected integration among several markets.

Sexton, Kling, and Carman examined market integration in U.S. celery markets using a switching regression model to test which of three possible characterizations—efficient arbitrage, shortage, and glut—best represented the spatial integration of these markets. Their results did not support the notion that these markets demonstrate efficient arbitrage because there were significant deviations from the LOP in both the California and Florida celery markets.

While the results of these studies are mixed, it is clear that even when the corrections for transportation costs and other factors are made, the empirical evidence does not always support the LOP. One reason for the weak empirical support for the LOP may be due to the fact that standard cointegration methods are too restrictive. Standard cointegration methods test a discrete hypothesis that the order of integration of the equilibrium errors is either zero or one. If the order of integration of the equilibrium errors in two price series is found to be zero, then there exists a long-run relationship between these prices and the LOP is confirmed. If the order of integration is one, the LOP is not supported. This discrete hypothesis testing limits the ability of cointegration to correctly verify long-run relationships.

It can be shown that fractionally integrated equilibrium errors are also mean reverting, although they may exhibit significant persistence in the short run. Thus, the long-run behavior of prices in commodity markets actually may be related, although this relationship cannot be found through standard cointegration tests. If this is the case, the false rejection of the LOP may be due to the inability of standard analytical techniques to discover the fractionally integrated nature of long-run price relationships.

In this study, we use a fractional cointegration approach, developed by Granger, and Granger and Joyeux, to test the LOP. This approach combines the concept of cointegration introduced by Engle and Granger and fractional differencing introduced by Hosking. Both cointegration and fractional cointegration test for long-run relationships between economic variables or the mean-reverting behavior of equilibrium errors with few restrictions on the short-run dynamics, but they differ in the manner the hypotheses are tested. In addition, fractionally cointegrated variables show more significant short-run persistence to shock than fully cointegrated variables. Fractional cointegration analysis allows the equilibrium errors to follow a fractionally cointegrated process, such that the order of integration is a fraction between zero and one. Thus, by avoiding the discrete hypothesis of unit-roots/no-unit-roots in equilibrium, this method permits analysis of a wider range of mean-reversion behavior than standard cointegration analysis. This gain in flexibility in testing subtle mean-reverting dynamics is shown to be vital in the proper evaluation of the LOP.

The remainder of this article is organized as follows. The next section describes the LOP and relates it to both standard cointegration and fractional cointegration tests. This is followed by a description of fractional cointegration tests. Next, data and estimation procedures are discussed. The final sections of the article include economic interpretation of the results and a brief conclusion.

The Law of One Price

A generalized version of the LOP for a single homogeneous commodity can be expressed as:

$$(1) \qquad P_t^1 = \alpha(P_t^2 r_t)^{\beta} T^{\gamma},$$

where P_t^1 and P_t^2 are the domestic and foreign prices in their respective currencies, r_t is the exchange rate for foreign currency in terms of domestic currency, and T_t are the transfer costs. The constant term α includes factors that are not taken into account in other variables such as costs, and trade impediments (Zanias). Typically, the LOP is supported if β and γ are not significantly different from one.

Due to problems in obtaining explicit information on transfer costs, most studies have assumed them to be constant or a constant proportion of nominal product prices over the study period. This assumption enables them to remove T as a variable in equation (1), since the analysis is conducted in a regression framework. In such cases, the influences of transfer costs on commodity prices are reflected in the constant term, which can thus assume any value.

Following Goodwin (1990a), this study assumes that transfer costs are a constant proportion of nominal commodity prices. After removing T and taking logarithms of both sides, equation (1) can be rewritten as:

$$(2) \qquad \ln(P_t^1) = \alpha + \beta \ln(P_t^2) + \epsilon_t,$$

where P_t^2 is the foreign price for the commodity expressed in the domestic currency, and ϵ_t is the error term. As pointed out by Goodwin, Grennes, and Wohlgenant, prices may vary in a nonsynchronous manner within a band created by transportation costs and, in that case, any value of β could be consistent with the LOP. But the presence of nonstationarity in variables makes the hypothesis tests regarding the values of α and β estimated from the conventional model unreliable (Stock). To overcome this problem, cointegration tests have been utilized.

Cointegration and Fractional Integration

Let y_t^1 and y_t^2 be represented by a vector X_t. If elements of X_t are integrated of order d, denoted by I(d), then the linear combination ($z_t = \eta X_t$) also will be integrated of the same order. If a vector η exists such that z_t is I($d - b$) with $b > 0$, then y_t^1 and y_t^2 are said to be cointegrated of order (d, b). The typical case considered in empirical work is one in which $b = d = 1$, i.e., the components of X_t are I(1) and the equilibrium error z_t is I(0). The procedure developed by Engle and Granger, which has been widely used for testing cointegration, involves regressing y_t^1 on y_t^2 (or y_t^2 on y_t^1) and then testing to determine if the residual is integrated of order zero using a unit root test. The elements of X_t are cointegrated if the equilibrium error (z_t) is I(0). It is generally tested by the augmented Dickey-Fuller (ADF) method. The ADF test is based on the following regression:

$$(3) \qquad \Delta z_t = \alpha_0 + \beta z_{t-1} + \sum_{i=1}^{m} \delta_i \Delta z_{t-i} + \nu_t,$$

where z is the equilibrium error, Δ is the first-difference operator, and ν_t is the stationary error term. The null hypothesis of no cointegration is rejected if the estimated β is significantly negative.

Cointegration also can be tested by Johansen's maximum-likelihood procedure using an error correction model.[1] The main advantage of Johansen's approach is that it resolves a limitation of the ADF tests, i.e., the simultaneity biases caused by the use of more than one endogenous variable at the same time. In addition, Engle and Granger's technique is limited to bivariate cointegration, whereas Johansen's maximum-likelihood approach can be extended to multiple variables.

Both the Engle-Granger and Johansen procedures test whether the equilibrium error is I(0) or I(1). If the equilibrium error is found to be I(0), then the null hypothesis of no cointegration is rejected. In that case, y_t^1 and y_t^2 are found to be cointegrated (i.e., z_t is a mean-reverting process) and any shock to the system will die out, which means the LOP between the two series holds. Thus, the mean-reversion behavior of the equilibrium error is of primary interest in testing for long-run equilibrium relationships among economic variables.

The equilibrium error could be mean reverting without being exactly I(0). A fractionally integrated error term also will display mean-reverting behavior (Granger and Joyeux; Hosking). The advantage of fractional cointegration relative to standard cointegration methods is that it is able to discern long-run price behavior despite substantial short-run deviations from equilibrium. As Cheung and Lai argue, a method that can distinguish between high and low frequencies and detect long-run relationships in noisy data is needed for proper analysis of the LOP.[2] Fractional cointegration appears to be such a method.

A fractionally integrated process z_t can be represented as follows:

$$(4) \qquad C(L)(1 - L)^d z_t = D(L)\nu_t,$$

where L is the lag operator, and $C(L)$ and $D(L)$ are polynomials of the lag operator, i.e., $C(L) = 1 - C_1L - \ldots - C_pL^p$, and $D(L) = 1 + D_1L + \ldots + D_qL^q$. The fractional differencing operator, $(1 - L)^d$, is defined as $(1 - L) = \Sigma^\infty \Gamma(k - d)L^k/\Gamma(k + 1)\Gamma(-d)$, where Γ is the gamma function. The error term (ν_t) is i.i.d. $(0, \sigma^2)$. Equation (4) is referred to as the autoregressive fractionally integrated moving average (ARFIMA) model of order (p, q, d), and is similar to the standard autoregressive moving average (ARIMA) model where d is restricted to integers. In the ARFIMA model, d can take any real value between zero and one. According to Hosking, for d values between 0 and 0.5, the autocorrelation of z_t shows a hyperbolic decay at a rate proportional to k^{2d-1}, as compared to a faster geometric decay in a standard ARIMA process where $d = 1$. The distinction between $d = 1$ and $d < 1$ is crucial in terms of the mean-reversion property of z_t and the cointegration property of y_t^1 and y_t^2. For $d < 1$, the effect of any shock will die out slowly, whereas for $d = 1$, it will remain forever (Cheung and Lai). As with Engle and Granger's technique, the fractional cointegration approach is limited to two variables. In order to extend fractional cointegration to more than two variables, it would be necessary to estimate an error correction model. Cheung and Lai report that efficient estimation of an error correction model in a fractional cointegration framework does not appear to be straightforward.

Testing for Fractional Cointegration

Engle and Granger's technique can be easily extended to test if the residual is I(d), where

[1] Johansen, and Johansen and Juselius cointegration tests involve a maximum-likelihood estimation procedure that provides estimates of cointegrating vectors for a given number of variables. It is based on the following error correction representation:

$$\Delta X_t = \sum_{j=1}^{k} \alpha_j \Delta X_{t-j} + \theta(r)X_{t-1} + \epsilon_t,$$

where X_t is a 2×1 vector of I(1) processes. The rank of $\theta(r)$ equals the number of cointegrating vectors, which is tested by maximum eigenvalue and trace statistics. Critical values for these statistics are found in Johansen and Juselius.

[2] Many time series with long time spans tend to show dependence between distant observations. These series can be best represented in a frequency domain, bounded by frequencies between zero and π. The goal is to determine how important cycles of different frequencies are in accounting for the behavior of the series. Low-frequency data refers to the value of the periodic function at zero, and high-frequency data to the value at π.

$d < 1$. This involves direct estimation of d, whereas in standard cointegration tests, the distinct hypotheses of I(1) and I(0) are tested using the unit root test. However, studies by Diebold and Rudebusch, and by Sowell showed that standard unit root tests, such as the Dickey-Fuller test, may have weaker power than fractional alternatives.

In this study, a test based on spectral regression, developed by Geweke and Porter-Hudak (GPH), is used to test for fractional cointegration. Cheung and Lai measured the power of the GPH test against a conventional unit root test. Using a simulation approach, they showed that the GPH test performs at least as well as the augmented Dickey-Fuller (ADF) test against the usual unit root alternatives; but against the fractional alternative, the GPH test performs significantly better than the ADF test. They also confirmed that the power of either the GPH or the ADF test rises as the sample size increases. For sample sizes of 200 or fewer, the GPH test has a potential power advantage over the ADF test. Details on the derivation of the GPH test are provided in the appendix.

The choice of the number of low-frequency ordinates, n, used in the GPH regression [appendix equation (A4)] necessarily involves judgment (Cheung and Lai). A value of n that is too large will contaminate the estimate of d due to medium- or high-frequency ordinates, whereas a value that is too small will result in an imprecise estimate due to limited degrees of freedom. GPH used the rule $n = T^{\mu}$, where T is the sample size, and with $\mu = 0.5$, 0.6, and 0.7. They found that the effect of increasing the sample size is small. Their results also suggested that in empirical work, n should be kept small if d appears to be sensitive to the choice of μ. Similarly, Cheung and Lai conducted a Monte Carlo experiment to obtain the size of n for their sample size of 76, and used a range of values of μ for the sample size function, $n = T^{\mu}$. Use of this range of values provided information on the sensitivity of the results to the choice of n. Based on the simulation results, they found better performance for $\mu = 0.55$, 0.575, and 0.6. In another investigation, Cheung used $\mu = 0.5$ (which is commonly used to test for fractional integration), and also reports results for $\mu = 0.45$ and 0.55 to check the sensitivity of the estimates. Overall, it may be inferred that, irrespective of sample size, a value of μ between 0.5 and 0.6 appears to be the ideal choice.

Data and Estimation

The quarterly price series used to test the LOP through fractional cointegration includes five commodities (wheat, wool, sugar, tea, and zinc) and four countries (Australia, Canada, the United Kingdom, and the United States). Some of the price series are unit values and others are market prices.[3] Even though unit values are not ideal, the nonavailability of prices in some instances forced us to use unit values as a proxy for market prices.

The commodities and countries analyzed are similar to those used by Ardeni, with the exception that most of the series used in this study are updated and the nonavailability of data forced us to abandon some price series. The primary reason for using these price series is that it allows us to compare our results with those of Ardeni. The original idea was to collect data for the period 1966:1 through 1993:4, but discontinuities in some of the price series forced us to use a shorter sample period for some commodities. A brief description of each price series, along with its sample range, is provided in table 1. All prices are expressed in U.S. dollars.

Before testing for cointegration, it is necessary to check for unit roots in the individual price series. The order of integration of each price series was determined using both ADF and GPH tests. ADF and GPH unit root test results for each price series are presented in table 2. The ADF test statistics were calculated by using equation (3). The number of lags to include in the equations was determined by using the Akaike information criterion.

The GPH test was conducted for individual

[3] Both market prices and export or import unit values are obtained from *International Financial Statistics* (IMF). Unit values are calculated from reported value and volume data for individual commodities.

Table 1. Description of the Price Series (U.S. $)

Variable	Description	Sample Range
PWH_{AUS}	Australian wheat export price, unit value ($/bushel)	1966:1–1993:4
PWH_{US}	U.S. wheat export price, No. 1 hard red winter, Gulf ($/bushel)	1966:1–1993:4
PWH_{CAN}	Canadian wheat export price, hard red spring, unit value ($/bushel)	1966:1–1990:2
PWO_{AUS}	Australian wool export price (¢/kg)	1975:1–1993:4
PWO_{UK}	UK wool import price Australia–New Zealand 50s: UK dominion (¢/kg)	1975:1–1993:4
$PSUG_{AUS}$	Australian sugar import price, unit value (¢/kg)	1975:1–1993:4
$PSUG_{UK}$	Sugar, London daily spot price (¢/lb.)	1975:1–1993:4
$PTEA_{US}$	Tea, mid-month U.S. import price (¢/lb.)	1966:1–1985:4
$PTEA_{UK}$	Tea, London auction price, UK, warehouse CIF (¢/lb.)	1966:1–1985:4
PZN_{US}	Zinc FOB price, New York (¢/lb.)	1966:1–1993:4
PZN_{CAN}	Canadian zinc import price, unit value (¢/lb.)	1966:1–1989:2
PZN_{UK}	Zinc spot price, London metal exchange, CIF, 98% pure (¢/lb.)	1966:1–1989:2

Source: International Financial Statistics (IMF).

price series using appendix equation (A4) to check for fractional integration. The unit root hypothesis can be tested by determining whether the GPH estimate of d is significantly different from one. The sample sizes for the GPH regressions were determined with the formula $n = T^{\mu}$. In this study, we chose $\mu =$ 0.5, 0.55, and 0.575, considering our sample size and the findings of other studies. In estimating equation (A4), the error variance was restricted to its theoretical value of $\pi^2/6$.

Results

Based on the critical values calculated from McKinnon, the ADF test statistics indicate that the unit root hypothesis cannot be rejected, even at the 1% significance level, for all price series. Since lag order determination using statistical tests alone has been criticized, the ADF test was performed using different lag orders. The ADF results were robust to a change in lag orders. Similarly, the GPH test statistics

Table 2. ADF and GPH Unit Root Test Results

	ADF Test Statistics		GPH Test Statistics[a]		
Variable	Levels	First Differences	$\mu = 0.50$	$\mu = 0.55$	$\mu = 0.575$
PWH_{AUS}	−2.02	−4.40**	0.97	0.42	0.38
PWH_{US}	−2.18	−4.84**	1.17	1.13	1.44
PWH_{CAN}	−1.79	−4.04**	0.18	1.00	1.31
PWO_{AUS}	−2.41	−4.23**	2.23	1.33	2.02
PWO_{UK}	−2.48	−3.86**	2.02	1.86	2.55
$PSUG_{AUS}$	−2.29	−4.73**	1.77	1.32	1.74
$PSUG_{UK}$	−2.14	−5.94**	1.47	0.74	1.34
$PTEA_{US}$	−1.17	−3.96**	1.22	2.11	1.91
$PTEA_{UK}$	−2.20	−5.82**	0.61	2.33	1.40
PZN_{US}	−1.57	−4.53**	4.67*	2.76	1.73
PZN_{CAN}	−0.07	−3.91**	0.60	0.13	0.49
PZN_{UK}	−1.69	−5.08**	4.07*	2.93	2.06

Notes: Single and double asterisks (*) denote significance at the 10% and 5% levels, respectively. Critical McKinnon statistics are calculated from McKinnon, and are different for each price series because of sample size.

[a] The GPH test statistics are F-statistics from the spectral regression. For the GPH test, the null hypothesis of $d = 1$ is tested against the alternative $d \neq 1$.

Table 3. ADF Cointegration Test Statistics

Price	ADF Test Statistics
PWH_{US} and PWH_{AUS}	−4.70**
PWH_{US} and PWH_{CAN}	−3.30*
PWH_{CAN} and PWH_{AUS}	−2.02
PWO_{UK} and PWO_{AUS}	−1.92
$PSUG_{UK}$ and $PSUG_{AUS}$	−1.51
$PTEA_{US}$ and $PTEA_{UK}$	−4.08**
PZN_{US} and PZN_{CAN}	−2.39
PZN_{US} and PZN_{UK}	−2.48
PZN_{CAN} and PZN_{UK}	−2.39

Notes: Single and double asterisks (*) denote significance at the 10% and 5% levels, respectively. Critical values are calculated from McKinnon statistics.

failed to reject the I(1) hypothesis, confirming the findings of the ADF tests.

Having confirmed that the price series are integrated of order one, we conducted cointegration tests using both ADF and GPH tests. Testing for cointegration between two series using either ADF or GPH tests involves regressing one series on the other and testing the order of integration of the residuals. ADF test statistics for the residual of each pair of price series are reported in table 3. Out of nine pairs of price series, the hypothesis of no cointegration was not rejected in six cases, even at the 10% significance level. Only in the cases of U.S. and Australian wheat prices (PWH_{US} and PWH_{AUS}), U.S. and Canadian wheat prices (PWH_{US} and PWH_{CAN}), and U.S. and UK tea prices ($PTEA_{US}$ and $PTEA_{UK}$) was the hypothesis of no cointegration rejected either at the 5% or 10% significance levels. The results from reverse cointegration regressions were largely similar. In addition, cointegration was also tested for each pair of price series using Johansen's maximum-likelihood procedure. This yielded results similar to those found with the ADF tests.

In the next step, cointegration was tested using the GPH test for the same pairs of price series. This involved estimating appendix equation (A4) for the residuals obtained from each pair of series. The μ values are similar to the ones used for testing the order of integration of individual price series. As before, the error variance was restricted to its theoretical value ($\pi^2/6$). The estimated d values, along with F-statistics for the null hypotheses of $d = 1$ and $d = 0$, are reported in table 4. In most cases, the results vary little across the different values of μ. This suggests that the results are not sensitive to the choice of μ. The null hypothesis of $d = 1$ was rejected in all but one case, implying the presence of cointegration and possibly fractional cointegration between each of the eight pairs of prices. The only case where the null hypothesis of $d = 1$ was not rejected was that for UK and Australian sugar prices ($PSUG_{UK}$ and $PSUG_{AUS}$). Of the eight cases where the null hypothesis of d

Table 4. GPH Cointegration Test Statistics

	$\mu = 0.50$			$\mu = 0.55$			$\mu = 0.575$		
Price	d	H_0: $d = 1$	H_0: $d = 0$	d	H_0: $d = 1$	H_0: $d = 0$	d	H_0: $d = 1$	H_0: $d = 0$
PWH_{US} and PWH_{AUS}	0.36	3.96*	0.86	0.39	3.62*	1.02	0.27	4.24*	0.48
PWH_{US} and PWH_{CAN}	0.19	3.95*	0.22	0.31	4.33*	0.89	0.16	11.74**	0.41
PWH_{CAN} and PWH_{AUS}	0.49	4.66*	7.25**	0.52	4.11*	8.25**	0.63	3.79*	10.61**
PWO_{UK} and PWO_{AUS}	0.39	4.21*	3.65*	0.37	4.01*	3.56**	0.56	3.54*	4.99**
$PSUG_{UK}$ and $PSUG_{AUS}$	0.75	3.02	26.99**	0.88	0.89	50.47**	0.99	0.01	63.98**
$PTEA_{US}$ and $PTEA_{UK}$	0.43	6.14**	3.48	0.21	9.31**	0.64	0.33	9.25**	2.28
PZN_{US} and PZN_{CAN}	0.67	3.97*	15.95**	0.53	7.39**	9.24**	0.45	4.36*	2.79
PZN_{US} and PZN_{UK}	0.57	5.72**	9.49**	0.62	5.58**	15.17**	0.64	8.29**	25.17**
PZN_{CAN} and PZN_{UK}	0.61	5.66**	13.86**	0.55	11.85**	18.05**	0.43	18.89**	10.83**

Notes: Single and double asterisks (*) denote significance at the 10% and 5% levels, respectively. The GPH test statistics are F-statistics from the spectral regression. The null hypotheses of $d = 1$ and $d = 0$ are tested against the alternatives of $d \neq 1$ and $d \neq 0$, respectively.

= 1 is rejected, in five cases the null hypothesis of $d = 0$ also is rejected. This means that estimates of d lie between zero and one, suggesting the possibility of fractional cointegration. The GPH test results thus provide a wider and more significant support for the LOP than the ADF test results.

The GPH test results are particularly interesting when compared to the ADF test results in individual cases. As shown in table 3, the ADF tests support cointegration among three pairs of price series—U.S. and Australian wheat prices (PWH_{US} and PWH_{AUS}), U.S. and Canadian wheat prices (PWH_{US} and PWH_{CAN}), and U.S. and UK tea prices ($PTEA_{US}$ and $PTEA_{UK}$)—in which the residuals are integrated of order zero. Comparing the GPH test results, it can be seen in table 4 that the null hypothesis of $d = 0$ cannot be rejected for these three pairs of price series. This suggests that both ADF and GPH tests produce similar results if the estimated value of d is zero. For the remaining six pairs of price series, the ADF tests fail to support cointegration, whereas the GPH tests find evidence of fractional cointegration in all but one pair of price series, i.e., $PSUG_{UK}$ and $PSUG_{AUS}$.

Evidence that the three pairs of prices (PWH_{US} and PWH_{AUS}, PWH_{US} and PWH_{CAN}, and $PTEA_{US}$ and $PTEA_{UK}$) are fully cointegrated implies that any shock to one of these markets is quickly dissipated, and equilibrium is restored quickly as compared to those markets where prices are fractionally cointegrated. It has been shown that the United States is the price leader in international wheat markets which appear to exhibit an imperfectly competitive market structure (Mohanty, Peterson, and Kruse; Goodwin and Schroeder 1991b). If the United States is the price leader, it seems logical to expect that there would be equilibrium relations between U.S. and Australian, and U.S. and Canadian wheat markets. In that case, Australian and Canadian wheat prices follow U.S. prices (and are cointegrated), which means that they tend to follow each other as well but are one step removed (so they are fractionally cointegrated). In tea markets, the U.S. import price of tea and the UK import price of tea are cointegrated. This may be due to the fact that both markets are supplied from the same source and that there are no impediments to trade that might cause prices to diverge.

On the other hand, UK and Australian sugar prices ($PSUG_{UK}$ and $PSUG_{AUS}$) are neither fully nor fractionally cointegrated, suggesting that these two markets are not integrated. This result is not surprising because, as a member of the European Union (EU), the United Kingdom falls under the EU's common sugar policy. This policy operates through a system of production quotas with over-quota production dumped on the world market. This has the effect of isolating EU sugar markets from the world sugar market in which Australia operates. Under these conditions, it would be expected that divergences in prices would persist over time.

Fractional integration of zinc and wool prices suggests that there are long-run equilibrium relationships in these markets. One interpretation is that factors such as trade policies, exchange rates, or transportation costs slow the adjustments of prices in these markets, but eventually the effects of arbitrage bring them into a long-run equilibrium relationship. Another explanation might be that economic fundamentals such as money supply and interest rates are the cause of this slower adjustment.

Another interesting aspect of these results is revealed when comparing them with Ardeni's results obtained using standard cointegration techniques for similar series. Ardeni found cointegration in the same three cases for which we found d values to be zero, suggesting that both the standard cointegration technique and fractional cointegration provide similar results if the series are fully cointegrated (i.e., equilibrium errors are integrated of order zero). But differences arise for the cases in which standard cointegration techniques reject the hypothesis of cointegration. Based on the GPH test, it can be concluded that these series are fractionally cointegrated although full cointegration is rejected. Overall, our results do not contradict Ardeni's findings, but rather, add to them by identifying those cases not fully cointegrated that still have long-run relationships.

Conclusions

This study tests the long-run LOP for international commodity prices using a generalized notion of cointegration, called fractional cointegration. The analysis of fractional cointegration allows the equilibrium error to be a fractionally integrated process rather than forcing a choice between I(1) and I(0). For the LOP to hold, the equilibrium error must be mean reverting. Since fractionally integrated equilibrium errors identify a wide range of mean-reversion behavior, it is important to consider this possibility for the proper evaluation of the existence of the LOP.

Fractional cointegration analysis is applied to nine pairs of price series. The empirical results indicate that all but one of these series are fractionally cointegrated even when the hypothesis of cointegration has been rejected. Out of nine cases examined, fractional cointegration supports the existence of the LOP in eight cases, as compared to three cases in the standard cointegration analysis. These findings suggest that there is a long-run tendency for the LOP to hold for these commodity prices. Based on our results, the use of a representative price in trade models may be justified. Even though fractional cointegration permits analysis of a wider range of mean-reversion behavior than standard cointegration, it is limited to two variables. Future research should be aimed at estimating an error correction model in a fractional cointegration framework, so that this approach can be extended to more than two variables.

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Appendix

Fractional integration behavior of a series z_t can be seen from its spectral density $f_z(w)$, which behaves like w^{-2d}, as $w \rightarrow 0$. For $d > 0$, $f_z(w)$ is unbounded at frequency $w = 0$, rather than bounded as for a stationary ARIMA series (Cheung and Lai). Geweke and Porter-Hudak make use of this relationship to develop a procedure to estimate fractional integration behavior. An integrated series—as in text equation (2), where the error term is a stationary linear process with finite spectral density function $f_z(w)$—is bounded away from zero and continuous on the interval $[-\pi, +\pi]$. Assuming normality in the error term, the spectral density function of z_t (where $t = 1, 2, \ldots, T$) is:

$$\text{(A1)}\quad f_z(w) = (\sigma^2/2\pi)4\sin^2(w)^{-d}f_u(w).$$

Taking the logarithm of both sides of equation (A1),

$$\text{(A2)}\quad \mathrm{Log}[f(w)] = \mathrm{Log}[\sigma^2 f_u(0)/2\pi] - d\,\mathrm{Log}[4\sin^2(w/2)] + \mathrm{Log}[f_u(w)/f_u(0)].$$

Adding $I(w_j)$ on both sides of equation (A2) and evaluating at harmonic frequencies, $w_j = 2\pi j/T$ (where $j = 0, 1, 2, \ldots, T-1$), equation (A2) yields:

$$\text{(A3)}\quad \mathrm{Log}[I(w_j)] = \mathrm{Log}[\sigma^2 f_u(0)/2\pi] - d\,\mathrm{Log}[4\sin^2(w_j/2)] + \mathrm{Log}[f_u(w_j)/f_u(0)] + \mathrm{Log}[I(w_j)/f(w_j)],$$

where $I(w_j)$ is the periodogram of the series z at frequency w_j, and is defined as:

$$I(w) = \frac{1}{2\pi T}\left[\sum_{t=1}^{T} e^{itw}(z_t - \bar{z})\right]^2.$$

For low-frequency ordinates w_j at near zero (say $j \leq n \leq T$), the term $\mathrm{Log}[f_u w_j/f_u(0)]$ in equation (A2) becomes negligible compared with the other terms. In that case, it may be estimated using the following simple linear regression equation:

$$\text{(A4)}\quad \mathrm{Log}[I(w_j)] = c - d\,\mathrm{Log}[4\sin^2(w_j/2)] + \eta_t,$$

where c and η_t are equal to $\mathrm{Log}(\sigma^2 f_u(0)/2\pi)$ and $\mathrm{Log}[I(w_j)/f(w_j)]$, respectively, and $j = 1, 2, \ldots, n$ (where $n = T^{\mu} < T$) is an increasing function of T. Geweke and Porter-Hudak used $n = T^{\mu}$ for $0 < \mu < 1$, and showed that least square estimates of d are consistent. The theoretical variance of η_t is known to be equal to $\pi^2/6$ and is often imposed in estimation to raise efficiency (Cheung and Lai).