An Applied Procedure for Estimating and Simulating Multivariate Empirical (MVE) Probability Distributions In Farm-Level Risk Assessment and Policy Analysis

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Simulation as an analytical tool continues to gain popularity in industry, government, and academics. For agricultural economists, the popularity is driven by an increased interest in risk management tools and decision aids on the part of farmers, agribusinesses, and policy makers. Much of the recent interest in risk analysis in agriculture comes from changes in the farm program that ushered in an era of increased uncertainty. With increased planting flexibility and an abundance of insurance and marketing alternatives farmers face the daunting task of sorting out many options in managing the increased risk they face. Like farmers, decision makers throughout the food and fiber industry are seeking ways to understand and manage the increasingly uncertain environment in which they operate. The unique abilities of simulation as a tool in evaluating and presenting risky alternatives together with an expected increase in commodity price risk, as projected by Ray, *et al.*, will likely accelerate the interest in simulation for years to come.

Increased interest in risk management tools for assessing alternative farm management strategies led to the creation of the Texas Risk Management Education Program (TRMEP) by the Texas Agricultural Extension Service. The risk management specialists with TRMEP help

farmers evaluate long-term strategic management alternatives by using a client's personal farm data and the farm-level simulation model, Farm Assistance (Klose and Outlaw). The use of farm-level simulation techniques has been essential to the application of the model and the success of the program. Producer interest in the program and demand for the service is growing at an increasing rate as the program enters its third year.

Agribusiness professionals are demanding more emphasis on risk-management tools in their advanced education programs at Purdue. Programs such as the Strategic Agri-Marketing program are incorporating risk analysis into the curriculum for analyzing cases dealing with various aspects of marketing. The use of risk analysis gives managers a better feel for the impacts of alternative marketing strategies and illustrates the inherent uncertainties surrounding an intensely competitive environment. Evaluations from participants in the five-day program have been very positive towards the use of simulation in teaching the concepts of strategic marketing.

Interest in farm-level policy analyses by the House and Senate Agricultural Committees continues to increase as evidenced by the growing number of farm-level policy analyses conducted by the Agricultural and Food Policy Center (AFPC). Policy makers use AFPC's farm-level simulation results to evaluate the merits of various legislative alternatives. At the request of the Agricultural Committees,

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AFPC is presently analyzing several safety net policy options for agriculture.

The widespread availability of microcomputers and the increasing computational power of spreadsheets has permitted applied researchers to develop simulation models using spreadsheets rather than specialized simulation languages. Gray (1998), Richardson and Nixon (1999a and 1999b), and Richardson (1999) have demonstrated that Microsoft Excel is capable of simulating very complex farm level and agribusiness decision models. With the aid of spreadsheet add-ins such as @Risk and Crystal Ball, analysts can develop a simulation model, generate random numbers, and statistically analyze the results without having to learn a specialized programming language. These advances will undoubtedly promote the adoption and use of simulation in risk analysis for academics and professionals.

The current nature of the agricultural industry and the increased interest in simulation call for a review of the techniques available for simulating firm-level models. The basic equations and identities required to simulate a farm or agribusiness are outlined elsewhere (Richardson and Nixon (1986), and Gray (1998)) so this paper will focus on simulating stochastic variables in firm-level models. Specifically, the purpose of the paper is to describe and demonstrate the procedures developed by researchers in the AFPC to simulate stochastic prices and yields in large-scale firm-level simulation models used in policy and strategic planning analyses. The procedure is a semi-parametric Monte Carlo simulation technique, which incorporates intra- and inter-temporal correlation and allows the researcher to control the heteroscedasticity of the random variables over time.

Review of Literature

Numerous books are available on the topic of simulation; however, most are not written for agricultural economists and they do not relate to problems faced by agriculture firm-level simulation modelers (e.g., Law and Kelton, Savage, and Winston). Techniques presented in the majority of the simulation books can be applied to many of the business aspects of a farm, ranch, or agribusiness, but they generally ignore the unique aspects of agricultural firms. Some of the special problems facing firm-level simulation modelers are:

- non-normally distributed random yields and prices,
- intra-temporal correlation of production across enterprises and fields,
- intra- and inter-temporal correlation of output prices,
- heteroscedasticity of random variables over time due to policy changes,
- numerous enterprises that are affected by weather and carried out over a lengthy growing season,
- government policies that affect the shape of the price distributions, and
- strategic risks associated with technology adoption, competitor responses, and contract negotiations.

The focus of this paper is on describing and demonstrating an applied simulation approach for dealing with the first four problems in the list. A portion of the literature in the area of farm-level simulation is reviewed before describing the procedure for generating appropriately correlated random numbers in firm simulation models. The relevant phrase is "appropriately correlated" and it means that whatever procedure is used to simulate random variables must ensure that the historical relationship between all variables is maintained in the simulated variables. This concept can be extended to include coefficient of variation stationarity which means that the relative variability for the random variables must not be changed by the simulation process.

Agrawal and Heady (1972) provided a cursory treatment of simulation in their operations research book but no details were provided on how to construct a firm-level simulation model. Anderson, Dillan and Hardaker (1977) suggested simulation as a tool for analyzing risky decisions but provided no detail for addressing the unique modeling problems listed above. Richardson and Nixon (1986) described the types of equations and

identities used to construct the Farm Level Income and Policy Simulation Model (FLIP-SIM), but provided a minimum amount of detail on how the random variables were simulated. More recently Hardaker, Huirne, and Anderson (1997) have suggested that simulation can be used as a possible tool for helping farmers cope with risk, but they did not provide details on how to build a farm-level simulation model or how to simulate the random variables facing farmers.

Eidman (1971) edited a bulletin on farm-level simulation that included a description of the Hutton and Hinman simulation model and various random number generation schemes. Eidman's bulletin became the basic reference material for farm level modelers during the 70s. The General Farm Simulation Model developed by Hutton and Hinman (1971) addressed many of the problems faced by farm level simulators today but did not address the problems of correlating random yields and prices and dealing with heteroscedasticity. Law and Kelton demonstrate that ignoring the correlation of random variables biases the variance for output variables as follows: a model overestimates variance if a negative correlation between enterprises is ignored, and vise versa.

Clements, Mapp, and Eidman (1971) proposed using correlated random yields and prices for firm-level simulation models. However, the procedure described by Clements, Mapp, and Eidman for correlating two or more random variables only works if the variables are normally distributed, not the case for yields and prices for most agricultural firms. Richardson and Condra (1978 and 1981) reported a procedure for simulating intra-temporally correlated random prices and yields that are not normally distributed. Working independently, King (1979) reported a similar procedure for correlating multivariate non-normal distributions. King's procedure was included in an insurance evaluation program by King, Black, Benson and Pavkov (1988).[1] Taylor

(1990) presented his own procedure for simulating correlated random variables that are not normally distributed.

A procedure for simulating inter-temporally correlated random variables was described by Van Tassel, Richardson, and Conner and demonstrated for simulating monthly meteorological data from non-normal distributions. Their procedure relied on mathematical manipulation of the random deviates to correlate variables from one year to the next and therefore was difficult to expand beyond two or three years for problems involving a large number of random variables.

Simulating Multivariate Non-Normally Distributed Random Variables

Assume we are faced with the analysis of a farm that has four enterprises: corn, soybean, wheat, and sorghum. This means the model will have to simulate eight variables: four yields and four prices. The farm in question only has ten years of yield history (Table 1). Therefore, we have an eight-variable probability distribution that must be parameterized with only ten observations. To make the problem realistic, assume the model is to be simulated for three years, thus requiring the parameters for a multivariate distribution with 24 random variables.

With the limitation of only having ten observations the use of standardized probability distributions can be ruled out because there are too few observations to prove the data fit a particular distribution. The distribution we recommend in this situation is the empirical distribution defined by the ten available observations.[2] Assuming the data are distributed empirically avoids enforcing a specific distribution on the variables and does not limit the ability of the model to deal with correlation and heteroscedasticity.

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[1] Fackler (1991) reported that the procedure described by King was similar to Li and Hammand's procedure reported in 1975.

[2] Law and Kelton provide an overview of the F(x) function for an empirical distribution and the inverse transform method of simulating from the F(x) for an empirical distribution.

Table 1. Historical Yields and Prices for a Representative Farm

| | Yields | | | | National Prices | | | |
Years	Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum
	bu.	bu.	bu.	cwt.	$/bu.	$/bu.	$/bu.	$/cwt.
1	100	29.0	48.0	45.0	2.540	7.42	3.72	2.27
2	155	38.0	46.0	61.0	2.360	5.69	3.72	2.10
3	165	40.0	48.0	55.0	2.280	5.74	2.61	2.12
4	112	33.0	54.0	75.0	2.370	5.58	3.00	2.25
5	80	28.0	65.0	5.0	2.070	5.56	3.24	1.89
6	109	40.0	52.0	37.0	2.500	6.40	3.26	2.31
7	145	45.0	50.0	25.0	2.260	5.45	3.45	2.13
8	90	26.0	48.0	12.0	3.050	6.76	4.37	2.91
9	117	47.0	72.0	60.0	2.710	7.35	4.30	2.24
10	114	46.0	50.0	59.0	2.600	6.50	3.45	2.34
Summary Statistics								
Mean	118.700	37.200	53.300	43.400	2.474	6.245	3.512	2.256
Std Dev	26.435	7.386	8.050	21.919	0.261	0.712	0.516	0.251
Coef Var	0.223	0.199	0.151	0.505	0.105	0.114	0.147	0.111
Minimum	80.000	26.000	46.000	5.000	2.070	5.450	2.610	1.890
Maximum	165.000	47.000	72.000	75.000	3.050	7.420	4.370	2.910

Parameter Estimation for a MVE Probability Distribution

The first step in estimating the parameters for a multivariate empirical (MVE) distribution is to separate the random and non-random components for each of the stochastic variables. There are two ways to remove the random component of a stochastic variable: (a) use regression (or time series) analysis to identify the systematic variability, or (b) use the mean when there is no systematic variability. Yield is often a function of trend so an ordinary least squares (OLS) regression on trend may identify the deterministic component of a random yield variable. When an OLS regression fails to indicate a statistically significant non-random component, then use the simple mean ($\bar{X}$) of the data as defined in equations 1.1 or 1.2 and shown in column 3 of Table 2.[3]

(1)   Non-Random Component of the Historical Values

(1.1) $\hat{X}_{it} = \hat{a} + \hat{b} \cdot \text{Trend}_t + \hat{c} \cdot Z_t$

or

(1.2) $\hat{X}_{it} = \bar{X}_i$ for each random variable X_i and each year t.

The second step for estimating parameters for a MVE distribution is to calculate the random component of each stochastic variable. The random component is simply the residual ($\hat{e}$) from the predicted or non-random component of the variable (Column 4 of Table 2). It is this random component of the variable that will be simulated, not the whole variable.

(2)   Random Component

(2.1) $\hat{e}_{it} = X_{it} - \hat{X}_{it}$ for each random variable X_i and each year t.

The third step is to convert the residuals in equation 2.1 ($\hat{e}_{it}$) to relative deviates about their respective deterministic components. Dividing the $\hat{e}_{it}$ values by their corresponding

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[3] Stochastic prices in a farm-level model present a unique problem. The farm receives local prices that are a function of national prices and a wedge or basis. Due to the effect of farm policy on prices, the model must simulate the national prices and then use the wedge to convert stochastic national prices to stochastic local prices. This is particularly important when simulating the effects of policy changes on farms in different re-

gions because all of the farms must be impacted by the same prices.

Table 2. Steps for Estimating the Parameters for an Empirical Distribution

Observa- tion	Random Vari- able X_{it}	Deterministic Component $\hat{X}_t$	Stochastic Components $\hat{e}_t$	Relative Variability D_{it}	Sorted Deviates S_{it}	Probability of Occurrence $P(S_{it})$
Pmin					−0.1370	0.00
1	48.0	53.3	−5.30	−0.0994	−0.1370	0.05
2	46.0	53.3	−7.30	−0.1369	−0.0994	0.15
3	48.0	53.3	−5.30	−0.0994	−0.0994	0.25
4	54.0	53.3	0.70	0.0131	−0.0994	0.35
5	65.0	53.3	11.70	0.2195	−0.0619	0.45
6	52.0	53.3	−1.30	−0.0243	−0.0619	0.55
7	50.0	53.3	−3.30	−0.0619	−0.0244	0.65
8	48.0	53.3	−5.30	−0.0994	0.0131	0.75
9	72.0	53.3	18.70	0.3508	0.2195	0.85
10	50.0	53.3	−3.30	−0.0619	0.3508	0.95
Pmax					0.3508	1.00

predicted values in the same period results in fractions that express the relative variability of each observation as a fraction of the predicted values (Column 5 in Table 2).

(3)   Relative Variability of Each Observation (Deviates)

(3.1) $D_{it} = \hat{e}_{it}/\hat{X}_{it}$ for each of the 10 years t and for each random variable X_i.

The fourth step is to sort the relative deviates in equation 3.1 and to create pseudo-minimums and pseudo-maximums for each random variable. The relative deviates, D_{it}, are simply sorted from the minimum deviate to the maximum to define the points on the empirical distribution for each random variable X_i (Column 6 in Table 2). In a standard empirical distribution the probability of simulating the minimum or maximum of the data is equal to zero (Law and Kelton). In reality these points were each observed in history with a 10-percent probability, for a variable with ten years of data. The problem can be corrected by adding two pseudo observations. Pseudo-minimum and pseudo-maximum values are calculated and added to the data resulting in a 12-point empirical probability distribution. The pseudo-minimum and maximums are defined to be very close to the observed minimum and maximum and cause the simulated distribution to return the extreme values

with approximately the same frequency they were observed in the past.

(4)   Sorted Deviates and Pmin and Pmax

(4.1) $S_{it} = \text{Sorted}[D_{it}$ from min to max]

for all years t and each random variable X_i

(4.2) $Pmin_i = \text{Minimum } S_{it} \cdot 1.000001$

(4.3) $Pmax_i = \text{Maximum } S_{it} \cdot 1.000001$

The fifth step is to assign probabilities to each of the sorted deviates in equations 4.1–4.3. The probabilities for the end points (Pmin and Pmax) are defined to be 0.0 and 1.0 to ensure that the process conforms to the requirements for a probability distribution (Column 7 in Table 2). Each of the ten observed deviates had an equal chance of being observed (1/T) in history so in the simulation process that assumption must be maintained.[4] The intervals created by the addition of the Pmin and Pmax deviates are assigned one half of the probability assigned to the other intervals. Based on this empirical formulation, outcomes approximating the minimum are real-

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[4] However, the flexibility of this procedure allows for assigning any probability between 0 and 1 to the sorted deviates. Thus, elicitation processes can be incorporated to reflect management's/experts' opinions about the distributions for each variable.

Table 3. Parameters for a Sample MVE Probability Distribution

Observation	Yields				National Prices				Probability of Occurrence
	Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum	
Sorted Fractional Deviates (Dit) and Probability of Occurrence (P(Sit))									
Pmin	-0.3260	-0.3011	-0.1370	-0.8848	-0.1633	-0.1273	-0.2568	-0.1622	0.00
1	-0.3260	-0.3011	-0.1370	-0.8848	-0.1633	-0.1273	-0.2568	-0.1622	0.05
2	-0.2418	-0.2473	-0.0994	-0.7235	-0.0865	-0.1097	-0.1458	-0.0691	0.15
3	-0.1575	-0.2204	-0.0994	-0.4240	-0.0784	-0.1065	-0.0774	-0.0603	0.25
4	-0.0817	-0.1129	-0.0994	-0.1475	-0.0461	-0.0889	-0.0718	-0.0559	0.35
5	-0.0564	0.0215	-0.0619	0.0369	-0.0420	-0.0809	-0.0177	-0.0071	0.45
6	-0.0396	0.0753	-0.0619	0.2673	0.0105	0.0248	-0.0177	-0.0027	0.55
7	-0.0143	0.0753	-0.0244	0.3594	0.0267	0.0408	0.0592	0.0062	0.65
8	0.2216	0.2097	0.0131	0.3825	0.0509	0.0825	0.0592	0.0239	0.75
9	0.3058	0.2366	0.2195	0.4055	0.0954	0.1769	0.2244	0.0372	0.85
10	0.3901	0.2634	0.3508	0.7281	0.2328	0.1882	0.2443	0.2899	0.95
Pmax	0.3901	0.2634	0.3508	0.7281	0.2328	0.1882	0.2443	0.2899	1.00
Intra-Temporal Correlation Matrix									
Corn	1	0.5826	-0.3793	0.4829	-0.3111	-0.3977	-0.3591	-0.3162	
Soybean	0	1	0.1621	0.4597	-0.0856	-0.0164	-0.0796	-0.2588	
Wheat	0	0	1	-0.0744	-0.0797	0.2014	0.2297	-0.3065	
Sorghum	0	0	0	1	-0.0062	0.0726	-0.2146	-0.1445	
Corn	0	0	0	0	1	0.7274	0.7489	0.9252	
Soybean	0	0	0	0	0	1	0.6584	0.4899	
Wheat	0	0	0	0	0	0	1	0.5656	
Sorghum	0	0	0	0	0	0	0	1	

Table 3. Continued

	Yields				National Prices				Probability of Occurrence
Observation	Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum	

Inter-Temporal Correlation Coefficients

	Corn
Corn	0.0536
Soybean	−0.1810
Wheat	−0.1187
Sorghum	0.0132
Corn	0.1531
Soybean	0.1426
Wheat	0.4231
Sorghum	−0.1577

	Yields				Prices			
Years	Corn bu.	Soybean bu.	Wheat bu.	Sorghum cwt.	Corn $/bu.	Soybean $/bu.	Wheat $/bu.	Sorghum $/cwt.

Projected Means for Simulation Period

Years	Corn bu.	Soybean bu.	Wheat bu.	Sorghum cwt.	Corn $/bu.	Soybean $/bu.	Wheat $/bu.	Sorghum $/cwt.
2000	118.7	37.2	53.3	43.4	1.960	4.520	2.910	3.232
2001	121.1	37.9	54.4	44.3	2.000	4.710	2.990	3.375
2002	123.5	38.7	55.5	45.2	2.060	4.870	3.090	3.482

Assumed Expansion Factors

Years	Corn bu.	Soybean bu.	Wheat bu.	Sorghum cwt.	Corn $/bu.	Soybean $/bu.	Wheat $/bu.	Sorghum $/cwt.
2000	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2001	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2002	1.0	1.0	1.0	1.0	1.4	1.4	1.4	1.4

ized about 10 percent of the time, and the same for the maximum. Equation 5 illustrates the assigning of probabilities for each of the deviates.

(5)   Probabilities of Occurrence for the Deviates

(5.1) $P(Pmin_i) = 0.0$

(5.2) $P(S_{i1}) = (1/T) \cdot 0.5$

(5.3) $P(S_{i2}) = (1/T) + P(S_{i1})$

(5.4) $P(S_{i3}) = (1/T) + P(S_{i2})$

$\vdots$

(5.11) $P(S_{i10}) = (1/T) + P(S_{i9})$

(5.12) $P(Pmax_i) = 1.0$

The sixth step for estimating the parameters for a MVE distribution is to calculate the M × M intra-temporal correlation matrix for the M random variables (Table 3).[5] The intra-temporal correlation matrix is calculated using the unsorted, random components ($\hat{e}_{it}$) from equation 2.1 and is demonstrated for a 2 × 2.

(6)   Intra-Temporal Correlation Matrix for X_i
        to X_j

$$\rho_{ij} = \begin{bmatrix} \rho_{(\hat{e}_{it},\hat{e}_{it})} & \rho_{\hat{e}_{it},\hat{e}_{jt}} \\ 0 & \rho_{(\hat{e}_{jt},\hat{e}_{jt})} \end{bmatrix}$$

The seventh step is to calculate the inter-temporal correlation coefficients for the random variables. The inter-temporal correlation coefficients are calculated using the unsorted residuals ($\hat{e}_{it}$) from equation 2.1 lagged one year, or the correlation of $\hat{e}_{it}$ to $\hat{e}_{it-1}$ (Table 3). The inter-temporal correlation coefficients are used to create a separate matrix for each random variable. The inter-temporal correlation matrices are 3 × 3 for a three-year simulation problem. A zero in the upper-right-most cell of the inter-temporal matrix assumes no second-order autocorrelation of the variables, a reasonable assumption given only ten observations.

(7)   Inter-Temporal Correlation Matrix for
        Variable X_{it}'s Correlation to X_{it-1}

$$\rho_{i(t,t-1)} = \begin{bmatrix} 1 & \rho_{(\hat{e}_{it},\hat{e}_{it-1})} & 0 \\ & 1 & \rho_{(\hat{e}_{jt},\hat{e}_{jt-1})} \\ & & 1 \end{bmatrix}$$

The seventh step completes the parameter estimation for a MVE distribution. The parameters used for simulation are summarized in equation 8.

(8) $[\hat{X}_{ik},\ S_{it},\ Pmin_i,\ Pmax_i,\ P(S_{it}),\ \rho_{ij(M \times M)}$ and

        $\rho_{i(t,t-1)(K \times K)}]$ for random variables X_i,

i = 1, 2, 3, . . . , M,

historical years t = 1, 2, 3, . . . , T,

and simulated years k = 1, 2, 3, . . . , K.

The completed MVE probability distribution can be simulated in Excel using @Risk or in any other computer language that generates independent standard normal deviates (i.e., values drawn independently from a normal distribution with a mean of 0.0 and a standard deviation of 1.0). The steps to simulate the MVE are provided next to demonstrate how the parameters are used to simulate a MVE probability distribution.

Prior to simulation, the square root of the intra-temporal (ρ_{ij}) correlation matrix and each of the inter-temporal (ρ_{it-1}) correlation matrices must be calculated. The square root procedure for factoring a covariance matrix, described by Clements, Mapp and Eidman, is used to factor the correlation matrices (one intra-temporal and one inter-temporal) and is named MSQRT.[6]

(9)   Factored Correlation Matrices

(9.1) $R_{ij(M \times M)} = MSQRT(\rho_{ij(M \times M)})$

(9.2) $R_{i(t,t-1)(K \times K)} = MSQRT(\rho_{i(t,t-1)(K \times K)})$

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[5] When using the data to estimate the correlation coefficients, Fackler (1991, p. 1093) agrees that one should estimate the rank correlation coefficient directly and then calculate the appropriate random values.

[6] An Excel program to factor a correlation matrix by the square root method is available from the authors.

Simulation of a MVE Probability Distribution

The first step for simulating a MVE distribution is to generate a sample of independent standard normal deviates (ISND). The number of ISNDs generated must equal the number of random variables; in the case of this example 24 ISNDs are needed for eight variables and three years. The best solution to the problem of generating ISNDs is to use @Risk to generate the ISNDs and to select the Latin Hypercube option. By using @Risk to generate the ISNDs, one can take advantage of @Risk's ability to manage the iterations and calculate statistics for the model's output variables, while controlling the process to simulate stochastic variables. During the simulation process @Risk fills the ISND vector each iteration with a new sample of random standard normal deviates and Excel calculates the equations for correlating the deviates.[7]

(10)   Vector of ISNDs

$$\text{ISND}_{i(24 \times 1)} = Risknormal(0, 1)$$

generate 24 ISNDs by repeating the @Risk formula in 24 cells.

The second step for simulating a MVE distribution is to correlate the ISNDs within each year of the simulation period ($k = 1, 2, \ldots,$ K) by multiplying the factored correlation matrix (R_{ij}) and eight of the values in the ISND vector. The matrix multiplication is repeated once for each year (k) to be simulated, using the same R_{ij} matrix each time but a different set of eight ISNDs. The resulting eight values in each of three vectors are intra-temporally correlated standard normal deviates (CSNDs) (see Richardson and Condra (1978)). For large samples (number of iterations) the correlated standard normal deviates in equation 11 exhibit similar intra-temporal correlation to that

observed in the ρ_{ij} correlation matrix in equation 8.

(11)   Correlated Standard Normal Deviates for Simulated Years 1–3

(11.1) $\text{CSND}_{i(8 \times 1)}^{k=1} = R_{ij(8 \times 8)} \cdot \text{ISND}_{i(8 \times 1)}$

for the first eight ISND values,

(11.2) $\text{CSND}_{i(8 \times 1)}^{k=2} = R_{ij(8 \times 8)} \cdot \text{ISND}_{i(8 \times 1)}$

for the second eight ISND values,

(11.3) $\text{CSND}_{i(8 \times 1)}^{k=3} = R_{ij(8 \times 8)} \cdot \text{ISND}_{i(8 \times 1)}$

for the last eight ISND values.

The third step in simulation is to capture the inter-temporal correlation of the random variables. The values in the three 8×1 vectors of CSNDs (equation 11) are used in a second matrix multiplication to add the inter-temporal correlation to each random variable. Equation 12 is repeated for each of the eight variables and does not significantly diminish the intra-temporal relationship established in equation 11.[8] A single-step approach to correlating random variables that combines equations 11 and 12 into one 24×24 correlation matrix would be superior. However, the problem with a single-step approach is that even for small models the (MT $\times$ MT) correlation matrix can be impossible to factor. The two-step correlation process in equations 11 and 12 overcomes that problem and allows for a large number of random variables to be appropriately correlated in a multi-year simulation model.[9]

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[7] While @Risk includes a correlation function, the rank-order correlation procedure used by @Risk presents several difficulties when incorporating inter-temporal correlation and large intra-temporal correlation matrices.

[8] The two-step approach is an improvement over Van Tassel, Richardson, and Conner's mathematical manipulation of deviates one year at a time, because it permits a large number of variables to be correlated over 10 or more years.

[9] The ACSNDs can be used to simulate multivariate normal (MVN) random variables by applying the adjusted correlated deviates as follows: $\bar{X}_{ik} = \bar{X}_{ik} + \hat{\sigma}_i \cdot \text{ACSND}_{ik}$ for each random variable X_i and where $\hat{\sigma}_i$ is the standard deviation for X_i. This procedure for simulating MVN distributions incorporates both inter- and intra-temporal correlation for large scale models with numerous variables and years in the planning ho-

(12)   Adjusted Correlated Standard Normal Deviates for Variable X_i in Simulated Years 1–3

(12.1)
$$\begin{bmatrix} ACSND_i^{k=3} \\ ACSND_i^{k=2} \\ ACSND_i^{k=1} \end{bmatrix} = R_{it-1(3\times3)} \cdot \begin{bmatrix} CSND_i^{k=3} \\ CSND_i^{k=2} \\ CSND_i^{k=1} \end{bmatrix}$$

for each of the i random variables.

The fourth step in simulating a MVE distribution is to transform the ACSNDs from equation 12 to uniform deviates. This step is accomplished using Excel's command = *normsdist*($CSND_i$) for each of the 24 values. Most simulation languages contain a similar error function which can be used to integrate the standard normal distribution from minus infinity to the $ACSND_i$. Because the input for the error function (ACSND) is appropriately correlated, the output is a vector of correlated deviates distributed uniform zero-one.

(13)   Correlated Uniform Deviates

$$CUD_{i(24\times1)} = normsdist(ACSND_{i(24\times1)})$$

The fifth step in simulation is to use the CUD_is to simulate random deviates for the empirical distribution of each variable X_i. Using the CUD_i along with the respective variable's S_i and $P(S_i)$ one simply interpolates among the S_i values to calculate a random deviate for variable X_i. In Excel the interpolation can be accomplished using a table lookup function for each random variable X_i, thus calculating 24 fractional deviates.[10] The interpolation process does not affect the correlation implicit in the CUD_i's so the resulting random deviates are appropriately correlated fractional deviates (or CFD_i).

(14)   Interpolation of an Empirical Distribution for Variable X_i Using the CUD_i

$$CFD_{ik} = \begin{bmatrix} Pmin & 0.0 \\ S_1 & P(S_1) \\ S_2 & P(S_2) \\ S_3 & P(S_3) \\ S_4 & P(S_4) \\ S_5 & P(S_5) \\ S_6 & P(S_6) \\ S_7 & P(S_7) \\ S_8 & P(S_8) \\ S_9 & P(S_9) \\ S_{10} & P(S_{10}) \\ Pmax & 1.0 \end{bmatrix} \leftarrow CUD_{ik}.$$

The sixth step in simulating a MVE distribution is to apply the correlated fractional deviates to their respective projected means and make any needed adjustment for heteroscedasticity. Projected mean yields for years 1–3 can be the historical means or the projected values from the OLS regressions in equation (1). Projected mean prices for years 1–3 can be from the OLS results in equation (1) or from projections by FAPRI or any other macro model that projects national prices. The CFD_i values are fractions of the mean so as the mean changes, the MVE distribution keeps the relative variability or coefficient of variation constant.[11] An expansion factor (E_{ik}) is included in equation 15 to allow for managing of the coefficient of variation over time. If the variable is assumed to have the same relative variability over time the E_{ik} factors are 1.0 for all years t; however if the relative risk is assumed to increase 10 percent per year the E_{ik} factors are 1.1, 1.2, and 1.3, respectively, for the first three years.

(15)   Simulate Random Values in Year k for Variable X_i

$$\tilde{X}_{ik} = \hat{X}_{ik} \cdot (1 + CFD_{ik} \cdot E_{ik})$$

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rizon. If the model being simulated contains both normal and non-normal distributions, the normal distributions use the above equation and the ACSNDs while the non-normal distributions use equation 15. In this manner the procedure outlined here is capable of appropriately (intra- and inter-temporally) correlating any distribution and any combination of distributions.

[10] Addin software for Excel to simplify the interpolation step is available from the authors.

[11] An explanation of coefficient of variation stationarity for the empirical distribution is provided by Richardson (1999, pp. 104–111). The use of heteroscedasticity adjustments to simulate random variables is explained in the same paper (pp. 140–144).

Table 4. Results of Simulating Yields and Prices for Three Years

	Yields				National Prices			
	Corn	Soy-bean	Wheat	Sor-ghum	Corn	Soybean	Wheat	Sorghum
Year 1								
Mean	118.61	37.17	53.29	43.29	1.96	4.52	2.91	3.23
Std Deviation	25.93	7.29	7.83	21.65	0.20	0.51	0.42	0.34
Coef Var.	0.22	0.20	0.15	0.50	0.10	0.11	0.14	0.11
Minimum	80.00	26.00	46.00	5.00	1.64	3.94	2.16	2.71
Maximum	165.00	47.00	72.00	75.00	2.42	5.37	3.62	4.17
Year 2								
Mean	121.02	37.99	54.30	44.31	2.00	4.71	2.99	3.37
Std Deviation	26.38	7.46	7.96	22.03	0.21	0.53	0.43	0.36
Coef Var.	0.22	0.20	0.15	0.50	0.10	0.11	0.14	0.11
Minimum	81.60	26.52	46.92	5.10	1.67	4.11	2.22	2.83
Maximum	168.30	47.94	73.44	76.50	2.47	5.60	3.72	4.35
Year 3								
Mean	123.55	38.74	55.41	45.14	2.06	4.86	3.09	3.48
Std Deviation	27.20	7.55	8.12	22.60	0.30	0.76	0.62	0.51
Coef Var.	0.22	0.19	0.15	0.50	0.14	0.16	0.20	0.15
Minimum	83.23	27.05	47.86	5.20	1.59	4.00	1.98	2.69
Maximum	171.67	48.90	74.91	78.03	2.73	6.15	4.15	4.90

Excel repeats the process described in simulation steps 1–5 automatically as @Risk simulates each iteration. The resulting random values can be used in the firm-level simulation model to simulate receipts and other variables of interest. The process described here to estimate the parameters and simulate a MVE probability distribution is easily expanded to accommodate models with a large number of random variables. It should be noted that as the correlation matrix gets larger it often becomes difficult to factor.

Random variables generated from the MVE distribution described here have the following properties:

• The variables are intra-temporally correlated the same as the historical period.
• The variables are inter-temporally correlated the same as the historical period.
• The variables have the same means, minimums, and maximums as their parent distributions, if the $\hat{X}_{ik}$ values in equation (15) equal their respective historical means and the E_{ik}s equal one. If the $\bar{X}_{ik}$ in equation 15

is not equal to the historical mean the random variable's average will equal the $\bar{X}_{ik}$ and the minimum will be less than the mean by the same percentage as observed in the historical data.
• The random variables are coefficient of variation (CV) stationary over time if the expansion factors (E_{ik}) are equal to 1.0 for all years.
• When the expansion factors (E_{ik}) are not equal to 1.0 the coefficient of variation in any year t equals the historical coefficient of variation (CV_0) times the expansion factor, or $CV_t = CV_0 \cdot E_{ik}$.
• The standard deviations for the output variables are less likely to be overstated or understated due to ignoring the correlation among enterprises and across years.
• The distributions for the random variables are similar to their parent distributions in terms of shape.

Once the parameters for the MVE are estimated, the distribution can be used to simulate a variety of assumptions about the pre-

Table 5. Correlation Matrix Calculated from Simulation Results for Yields and Prices Over Three Years

Yields for Year 3				Prices for Year 3				Yields for Year 2			
Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum
1.000	0.519	−0.318	0.440	−0.279	−0.342	−0.297	−0.273	**0.045**	0.026	−0.011	0.019
	1.000	0.133	0.403	−0.064	−0.007	−0.058	−0.224	−0.088	**−0.166**	−0.024	−0.077
		1.000	−0.080	−0.061	0.152	0.158	−0.226	0.060	−0.002	**−0.087**	0.013
			1.000	−0.004	0.068	−0.170	−0.130	−0.005	0.000	0.010	**−0.002**
				1.000	0.683	0.684	0.827	−0.054	−0.016	−0.020	−0.010
					1.000	0.576	0.409	−0.054	−0.003	0.007	0.003
						1.000	0.416	−0.151	−0.019	0.088	−0.093
							1.000	0.038	0.026	0.028	0.017
								1.000	0.517	−0.306	0.443
									1.000	0.139	0.415
										1.000	−0.053
											1.000

dicted means without changing the relative variability for the variables. This feature is particularly useful for analyzing technological changes that assume changes in the mean yields. An added feature is that the MVE procedure allows one to experiment with alternative levels of relative variability in the future, due to policy changes and or new varieties which may have more or less risk.

The steps for parameter estimation and simulation of MVE distributions are robust and perform efficiently for large-scale agricultural economics simulation models. In addition, the procedure is easily adapted to a variety of programming languages and/or software. The MVE procedure is used by FLIPSIM, Farm Assistance, POLYSYS's crops model, and FAPRI's crops model (Richardson and Nixon 1985; Klose and Outlaw; Ray, *et al.*; and Adams). Gray (1998) was the first to apply the MVE procedure to a large-scale agribusiness simulation model in Excel. Richardson (1999, pp. 184–245) demonstrates the use of the MVE procedure in several agricultural economics oriented simulation models that are programmed in Excel.

Numerical Application of the MVE Distribution

A simple farm-level simulation example is presented in this section. Ten years of actual yields for a farm growing corn, soybeans, sorghum, and wheat are combined with ten years of national prices to develop an MVE yield and price distribution for a farm (Tables 1–3). The farm is simulated for three years using stochastic yields and prices to estimate the distribution of total crop receipts for the farm, assuming 100 acres planted to each crop.

Table 5. Extended

Prices for Year 2				Yields for Year 1				Prices for Year 1			
Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum	Corn	Soybean	Wheat	Sorghum
-0.012	-0.015	-0.018	-0.011	0.003	-0.002	-0.009	0.005	0.008	-0.001	-0.008	0.013
0.002	-0.014	-0.002	0.027	0.001	-0.004	-0.017	0.008	0.009	0.005	-0.007	0.013
-0.010	-0.042	-0.031	0.008	-0.003	0.004	-0.008	0.013	0.006	0.009	0.004	0.004
0.003	-0.004	0.003	0.001	-0.008	-0.021	-0.007	-0.013	0.002	-0.004	-0.007	0.012
0.158	0.112	0.101	0.148	0.018	-0.007	-0.007	-0.001	-0.012	-0.013	-0.022	-0.010
0.102	**0.120**	0.073	0.080	0.010	-0.003	-0.007	0.015	-0.006	-0.007	-0.015	-0.007
0.325	0.277	**0.396**	0.239	-0.002	-0.003	0.001	0.010	-0.008	-0.004	-0.013	-0.008
-0.118	-0.050	-0.067	**-0.119**	0.016	-0.006	0.000	-0.009	-0.005	-0.011	-0.008	-0.003
-0.298	-0.352	-0.308	-0.285	**0.057**	0.031	-0.030	0.025	-0.003	-0.013	-0.008	-0.005
-0.071	-0.002	-0.055	-0.224	-0.091	**-0.155**	-0.032	-0.059	0.025	0.020	0.017	0.050
-0.054	0.165	0.164	-0.233	0.033	-0.009	**-0.077**	0.018	0.009	-0.009	-0.026	0.031
-0.014	0.060	-0.194	-0.133	0.015	0.017	-0.016	**0.013**	0.001	0.003	-0.007	-0.007
1.000	0.677	0.681	0.831	-0.037	-0.005	-0.019	0.010	**0.138**	0.092	0.081	0.125
	1.000	0.574	0.403	-0.042	-0.001	0.015	0.024	0.090	**0.118**	0.067	0.062
		1.000	0.415	-0.143	-0.029	0.079	-0.068	0.308	0.259	**0.395**	0.226
			1.000	0.046	0.039	0.040	0.013	-0.141	-0.082	-0.100	**-0.141**
				1.000	0.537	-0.304	0.440	-0.279	-0.346	-0.332	-0.275
					1.000	0.139	0.419	-0.075	-0.014	-0.076	-0.223
						1.000	-0.066	-0.089	0.140	0.185	-0.252
							1.000	0.009	0.066	-0.202	-0.115
								1.000	0.682	0.715	0.880
									1.000	0.606	0.435
										1.000	0.513
											1.000

The MVE distribution is simulated for three years using historical mean yields and projected national prices for 2000–2002 from the FAPRI November 1999 baseline. For the simulation, it was assumed that the relative variability of yields would be the same in the future as it has been in the past. However, the relative variability of crop prices is assumed to be 40 percent greater in the last year of the historical period. The results of the simulation are summarized in Tables 4 and 5.

A comparison of the simulated and historical distribution statistics can validate the MVE procedure. The simulated means for each crop's yield in year 1 compare very favorably to the historical means as do the other statistics. The simulated mean national prices are very close to the mean forecasts provided by FAPRI. By separating the non-random component from the random component, the MVE has the flexibility to impose the historical variability on any assumed mean. The simulated mean yields in years 2 and 3 reflect the 2-percent per year increase in the assumed mean yields.

The simulated coefficient of variation (CV) is the same as the historical CV for all yields and the first two years of all prices, where the expansion factors were 1.0. Using the percentage deviations as parameter estimates in the MVE forces the CV stationary process, even when the mean changes from year to year. The standard deviation for corn yield increases from 25.93 to 26.83 as the simulated mean rises from 118.61 to 121.02, in year 1 and 2, respectively, thus maintaining a 0.22 CV (Table 4). A process that uses a constant standard deviation would generate a declining CV. The price distributions show the CV stationary process between year 1 and 2. How-

ever, in year 3 the CV increases by 40 percent, reflecting the assumed expansion factors of 1.4 (Table 3). Again, the flexibility of this procedure allows one to control the stochastic process in many dimensions.

The results in Table 4 indicate that the stochastic procedure does a good job of simulating the given means and historical relative variability, and provides flexibility in controlling the relative variability over time. However, a significant contribution of this research centers around the multivariate process. Table 5 reports the simulated 24 × 24 correlation matrix for the random variables. The intra-temporal correlation coefficients, in the triangular areas below the outlined blocks, can be compared directly to the intra-temporal correlation matrix in Table 3. The bold numbers along the diagonal of each outlined box are the simulated first-order inter-temporal correlation coefficients that can be compared to the input inter-temporal correlation coefficients shown in Table 3.

A difficult part of simulating multivariate distributions is accurate generation of the historical correlation. When comparing the simulated intra-temporal correlation to the historical correlation signs, all elements of the matrix are correct except one (the correlation between corn price and soybean price in the first year is 0.009 when it should be −0.006; however, both values are about equal to zero). A closer examination reveals that the order of magnitude for each element of the simulated correlation matrix is similar to historical observations. For instance, the simulated correlation between sorghum price and soybean price of 0.880, 0.831, and 0.827 in years 1, 2, and 3, respectively, is similar in magnitude to the 0.9252 historical correlation coefficient. The same can be said for the simulated correlation between corn and soybean yields of 0.537, 0.517, and 0.519 compared to the historical correlation of 0.5826. In fact, while none of the simulated coefficients matches the historical coefficients exactly, the order of magnitude for all is reasonable.

The procedure used for incorporating inter-temporal correlation also generates acceptable simulated correlation coefficients. With the ex-

ception of the inter-temporal correlation between sorghum yield in years 2 and 3 of −0.002, the signs for all of the first-order auto-correlation coefficients are correct (Table 5). Comparing the simulated coefficients to the input matrix (Table 3) reveals that the order of magnitude for all of the first-order inter-temporal coefficients is acceptable. For example, the simulated first-order inter-temporal correlation coefficient for wheat price was 0.395 and 0.396 between years 1 and 2 and 2 and 3, respectively, which is similar to the input of 0.4231. The inter-temporal coefficients across commodities and higher-than-first-order intra temporal coefficients are spurious and approximately zero in most cases.

The most encouraging result is that this procedure can incorporate a complete correlation matrix into the multivariate simulation for a non-normal distribution with limited historical data. With limited data it is often impossible to estimate a non-singular 24 × 24-input correlation matrix that can be factored. For this reason, among others, using the correlation capabilities of @RISK may not work. However, the two-stage procedure described here avoids the singular matrix problem, incorporates first-order inter-temporal correlation, and produces acceptable intra- and inter-temporal correlation for all of the random variables.

To illustrate the importance of capturing the intra- and inter-temporal correlation effects, a simulation of the joint distribution of revenue for the example was conducted. Assuming the farm plants 100 acres each of corn, soybeans, wheat, and sorghum, the joint distribution of price times yield was simulated 10,000 iterations.[12] This simulation was repeated for four scenarios with the assumptions of no correlation, only intra-temporal correlation, only inter-temporal correlation, and complete correlation.

Statistics summarizing the present value of

[12] Effects of the loan deficiency payments were ignored in this analysis to illustrate the impact of multivariate simulation on the ability to more accurately characterize the joint distribution of total revenue before any risk-management intervention.

Table 6. Sum of Present Value of Total Revenue Assuming Alternative Levels of Correlation Among the Random Variables

	No Correlation	Only Inter-Temporal Correlation	Only Intra-Temporal Correlation	Total Correlation
Mean	199.41	199.43	198.74	198.72
Minimum	135.56	134.88	126.69	126.65
Maximum	264.33	263.01	279.06	274.61
Coefficient of Variation (%)	8.83	8.98	11.01	11.10

the total revenue over the three-year simulation period are summarized in Table 6. The mean for present value of total revenue is only slightly different as one goes from no correlation to total correlation. However, the minimum and maximums for the total revenue distributions increase as more and more correlation is added to the simulation. The results in Table 6 indicate that incorporating full correlation, in this case, increases the variability of the joint distribution for total revenue substantially. This result has serious implications for policy analysis. As U.S. farm policy makers continue to search for risk-reducing policy tools, it is important that the nature of the joint distribution for total revenue be accurately characterized. Incorporating intra- and inter-temporal correlation into a multivariate simulation process can improve the characterization of this joint distribution and more accurately quantify the impact of alternative risk-management policies.

Summary

Demand for simulation as an analytical tool has been increasing rapidly in recent years. Farmers, agribusiness managers, and policy makers are increasingly interested in risk-management tools and policies. The widespread availability of microcomputers and the increasing computational power of spreadsheets has allowed applied researchers to develop simulation models using spreadsheets to meet the increasing demand. The current volatility in the agricultural industry will undoubtedly continue to increase the demand for simulation in the future. The purpose of this paper was to describe and demonstrate an applied procedure to simulate stochastic prices and yields

in large-scale firm-level simulation models used in policy and strategic planning analysis.

Assuming the analysis is faced with an applied simulation problem including limited data on historical prices and yields, the paper describes a simple process for estimating the parameters for a multivariate empirical distribution. The paper then goes on to demonstrate the process for simulating a multivariate empirical distribution with eight random variables simulated across three years. The important contributions of the method include the use of non-normal distributions and an intra-temporal (across commodity) and inter-temporal (across time) correlation matrix to generate correlated stochastic error terms that can be applied to any forecasted mean. The procedure overcomes the problem of a singular correlation matrix that is often encountered when building a multi-year simulation model with a large number of random variables and limited historical data.

An application of the method was conducted using 10 years of actual farm-level historical data for corn, soybeans, wheat, and sorghum. The simulation model was run for 10,000 iterations and the simulated statistics and correlation matrix were compared to the historical input values. Analysis of the simulated statistics showed that the stochastic procedure does a good job of simulating the given means and historical relative variability, and provides flexibility for controlling the stochastic process. Further evaluation of the simulated correlation matrix indicated that the expected signs on the correlation were attained and the order of magnitude for both the intra- and inter-temporal coefficients were consistent.

Finally, an illustration of the impact of in-

cluding multivariate stochastic processes was conducted using the joint distribution of revenues for an example farm. By including both intra- and inter-temporal correlation coefficients, the spread of the joint PDF increased dramatically. This result suggests that including correlation in stochastic simulation models that deal with analysis of risk-management alternatives is critical. The process described in this paper allows applied researchers to address risk-management analysis using simulation when historical data is limited and not normally distributed.

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