Parametric Modeling and Simulation of Joint Price-Production Distributions under Non-Normality, Autocorrelation and Heteroscedasticity: A Tool for Assessing Risk in Agriculture

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ABSTRACT

This study presents a way to parametrically model and simulate multivariate distributions under potential non-normality, autocorrelation and heteroscedasticity and illustrates its application to agricultural risk analysis. Specifically, the joint probability distribution (pdf) for West Texas irrigated cotton, corn, sorghum, and wheat production and prices is estimated and applied to evaluate the changes in the risk and returns of agricultural production in the region resulting from observed and predicted price and production trends. The estimated pdf allows for time trends on the mean and the variance and varying degrees of autocorrelation and non-normality (kurtosis and right- or left-skewness) in each of the price and production variables. It also allows for any possible price-price, production-production, or price-production correlation.

Key Words: *agricultural risk analysis, autocorrelation, heteroscedasticity, multivariate non-normal simulation, West Texas agriculture.*

Expected profitability and risk are two fundamental criteria for the evaluation of agricultural policies, trends, and technologies. These are generally assessed using safety-first criteria (Roy) or stochastic dominance analysis (Meyer) if the relevant cumulative distribution functions (cdf's) are not normal. In both cases, precise estimates of the cdf's are required. Therefore, reliable simulation techniques are important for conducting a rigorous agricultural risk analysis.

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Yields have been found to be heteroscedastic over time and space, and non-normally distributed with a tendency towards left-skewness because of widespread pest attack or weather phenomena occasionally causing abnormally low production. They have also been shown to be correlated among each other, even at aggregate levels, presumably because of unfavorable weather conditions affecting several crops simultaneously (Ramirez, Moss and Boggess).

Commodity prices, on the other hand, tend to be autocorrelated through time. Yield non-normality could have implications for the probability distribution of prices during a given time period through the market equilibrium established by supply and demand interac-

tions. Yield left-skewness (i.e. sporadic unusually low yields) could cause price right-skewness (i.e., sporadic unusually high prices), documented by Ramirez and Sosa and Ramirez and Somarriba, which implies that price upswings are relatively more extreme than the downswings. Also, because of the market linkages, macroeconomic cycles, or agricultural policies, real commodity prices could be correlated among each other.

All of these phenomena are critical in agricultural risk analysis. Therefore, parametric modeling methods that can be used for a truly realistic simulation of future price, yield, and production outcomes have been of interest to agricultural economists for many years. In the 1970s Clements, Mapp and Eidman and Richardson and Condra developed a procedure to model and simulate multivariate, correlated random processes under the assumption of normality. In 1974, Anderson stressed the importance of modeling correlation, non-normality (skewness and kurtosis) and changing variances around time/space trends/locations, because these are important characteristics of many stochastic processes in simulation models.

In the mid 1980s, Gallagher advanced a univariate procedure to model and simulate random variables using the Gamma distribution. Because of the importance of skewness and changing variance of soybean yields over time, mainly caused by increased variation in weather conditions, he attempts to model these two characteristics of the corresponding probability distribution. Gallagher recognizes, however, that the Gamma function assumes fixed relations between the mean, the variance and the level of skewness. In addition, these moments depend on the values taken by two parameters only. Thus, in order to model and simulate a changing variance, the corresponding mean and level of skewness also will have to be allowed to vary over time/space according to arbitrary formulae. Taylor was the first to tackle the problem of multivariate non-normal simulation. In his work, published in 1990, he uses a cubic polynomial approximation of a cumulative distribution function

instead of assuming a specific multivariate density for empirical analysis.

In 1997, Ramirez developed and applied a multivariate model of non-normal, heteroscedastic, time-trending yields. More recently, Ramirez and Somarriba addressed the complementary issue of jointly modeling and simulating sets of autocorrelated, non-normal, time-trending prices that are correlated to each other. This study illustrates how these two procedures can be unified to conduct comprehensive agricultural risk analysis. Specifically, the joint probability distribution (pdf) for West Texas[1] irrigated cotton, corn, sorghum, and wheat production and real (1998) prices is estimated and used to evaluate the changes in the risk and returns of agricultural production resulting from observed and predicted price and production trends[2]. Time series (1909–1998 U.S.) price (USDA/NASS) and (1969–1998 West Texas) production (TASS) data are used to estimate the pdf, which allows for different linear time trends on the mean and the variance and varying degrees of autocorrelation and non-normality (kurtosis and right- or left-skewness) in each of the real price and production variables. It also allows for any possible price-price, production-production, or price-production correlation.

The multivariate pdf used in this study includes different parameters or parametric functions to separately model each of those distributional characteristics. This implies flexibility, in the sense that it can theoretically account for any possible underlying mean, variance, autocorrelation, and covariance structure and a wide variety of kurtosis-skewness combinations. This also means that sta-

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[1] For the purposes of this study, West Texas is defined as the three northern-most crop reporting districts of Texas. These districts consist of the Northern High Plains district (District 1-N) located in the Northwestern section of the Texas panhandle, the Southern High Plains district (District 1-S) located just south of District 1-N, and the Northern Low Plains district (District 2-N) which is adjacent and east of Districts 1-N and 2-N.

[2] These four crops are selected for the analysis because they currently represent the majority of the value of production from irrigated field crops in West Texas (TASS).

tistical tests can be conducted to identify the appropriate distributional attributes that need to be considered in the analysis.

The Multivariate PDF Model

This study combines the procedures of Ramirez and Ramirez and Somarriba, which apply a modified inverse hyperbolic sine (IHS) transformation to a potentially non-normal dependent variable Y to transform it to normality. In these models, $E[Y] = X\beta$, indicating that the mean of the Y distribution is a linear function of a matrix of independent variables (X) and a vector of slope coefficients (β). The variance of Y is determined by a separate parameter or parametric function (σ), while the third and fourth central moments of Y do not depend on σ or $X\beta$. This means that if σ is allowed to change across observations, the variance of the Y distribution changes, but its skewness and kurtosis remain constant. In other words, the parameters β and σ independently control the mean and variance of Y, respectively.

Skewness and/or kurtosis can also be made variable across observations through two other model parameters, μ and Θ, respectively. In both Ramirez, and Ramirez and Somarriba models, if $\Theta > 0$ and μ approaches 0, the Y distribution becomes symmetric, but it remains kurtotic. Higher values of Θ cause increased kurtosis. If $\Theta > 0$ and $\mu > 0$, Y has a kurtotic and right-skewed distribution, while $\mu < 0$ results in a kurtotic and left-skewed distribution. This allows for the specifying and testing of a null hypothesis of symmetric non-normality versus an alternative hypothesis of non-symmetric non-normality (H_0: $\mu = 0$ vs. H_a: $\mu \neq 0$).

Ramirez and Somarriba also present a kurtotic but symmetric version of the formerly described model, in which the third central moment of the Y distribution is zero but its fourth central moment is still determined by Θ. They show that the normal-pdf model is a special case of this later model specification, which in turn is nested to the full skewed and kurtotic pdf model described initially. This implies that the null hypothesis of normality vs. the alter-

native of symmetric non-normality can be tested using an asymptotic t-test (H_0: $\Theta = 0$ vs. H_a: $\Theta > 0$) or a likelihood ratio test of the normal-pdf model vs. the kurtotic but symmetric version of the model. A likelihood ratio test of the full, skewed and kurtotic model vs. the normal-pdf model can be used to test a more general null hypothesis of non-normality (symmetric or asymmetric) vs. an alternative hypothesis of pdf normality.

Ramirez and Somarriba's non-normal time-series (autocorrelation) results can be combined with Ramirez's non-normal heteroscedastic specification to obtain a more general model consisting of a set of marginal pdf's representing variables that could be non-normally distributed, heteroscedastic and/or autocorrelated, and correlated among each other. Consider the jth ($n \times 1$) normally but not independently and identically distributed (iid) mean-standardized dependent variable vector ($U_j = Y_j - X_j\beta_j$). Let $\Phi_j = \sigma_j^2 \iota_j$ be the covariance matrix of this vector. Let P_j be an $n \times n$ matrix such that $P_j'P_j = \iota_j^{-1}$, $Y_j^* = P_jY_j$ (an $n \times 1$ vector), and $X_j^* = P_jX_j$ (an $n \times k$ matrix), where Y_j and X_j are the vector and matrix of original dependent and independent variables. Because of the choice of P_j, the transformed mean-standardized vector $P_j(Y_j - X_j\beta_j) = (P_jY_j - P_jX_j\beta_j) = (Y_j^* - X_j^*B_j)$ is iid.

A variety of autocorrelated and/or heteroscedastic pdf-models can be specified depending on the choice of P_j and/or σ_j^2 (Judge et al., pp. 283–291 and 419–445). Heteroscedasticity is modeled by making σ_j^2 systematically change across observations while autocorrelation is accounted for by finding a transforming matrix P_j such that $P_j'P_j = \iota_j^{-1}$. The likelihood function corresponding to a marginal, univariate pdf model that can accommodate non-normality (kurtosis and/or right- or left-skewness) and autocorrelation and/or heteroscedasticity is given by (Ramirez and Somarriba):

$$(1) \quad NNLL_j = -0.5 \times \ln|\iota_j| + \sum_{i=1}^{n} \ln(G_{ji})$$

$$- 0.5 \sum_{i=1}^{n} H_{ji}^2;$$

where

$$G_{ji} = F(\Theta_j, \mu_j)/\{sign(\mu_j)\sigma_j(1 + R_{ji}^2)^{1/2}\};$$

$$H_{ji} = \Theta_j^{-1}ln\{R_{ji} + (1 + R_{ji}^2)^{1/2}\} - \mu_j;$$

$$R_{ji} = (F(\Theta_j, \mu_j)\Theta_j/sign(\mu_j)\sigma_j)$$

$$\times (Y_{ji}^* - X_{ji}^*\beta_j + sign(\mu_j)\sigma_j/\Theta_j);$$

$$F(\Theta_j, \mu_j)_- = \{e(0.5\Theta_j^2)\}\{e(\Theta_j\mu_j) - e(-\Theta_j\mu_j)\}$$

$$\div 2;$$

and $i = 1, \ldots, n$ refers to the observations. When the Y_j and, therefore, the mean-standardized dependent variable vector $(Y_j - X_j\beta_j)$ is believed to be autocorrelated, a first transformation $P_jY_j = Y_j^*$ and $P_jX_j = X_j^*$ is used to obtain a non-autocorrelated vector $(Y_j^* - X_j^*\beta_j)$. This is then converted into a normal random variable vector through Ramirez and Somarriba's transformation. If Y_j is only heteroscedastic, P_j is an $n \times n$ identity matrix. The likelihood function for the multivariate (joint) non-normal, heteroscedastic/autocorrelated pdf model is given by:

(2)   MNNLL

$$= \left\{-(n/2) \times ln|\Sigma| - 0.5 \times \sum_{j=1}^{m} [ln(|\iota_j|)]\right.$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} [ln(G_{ji})] - 0.5$$

$$\left. \times \sum_{i=1}^{n}\sum_{j=1}^{m} [\{H_i^*(\Sigma^{-1})\}^* . H_i]\right\};$$

where Σ is an $m \times m$ positive semi-definite matrix with unit diagonal elements and non-diagonal elements σ_{jk}, which account for the covariance between the m variables of interest; G_{ji} is as defined in equation (1) if Y_j (and thus Y_j^*) is not normally distributed or $G_{ji} = \sigma_j^{-1}$ if Y_j is normally distributed; and H_i is a $1 \times m$ row vector with elements H_{ji} $(j = 1, \ldots, m)$ also defined in equation (1) if Y_j is not normally distributed and $H_{ji} = (Y_{ji}^* - X_{ji}^*\beta_j)/\sigma_j$ if Y_j is normally distributed. The operator * indicates a matrix multiplication and * indicates an element-by-element matrix multiplication. The concentrated multivariate log-likelihood function (equation (2)) links m marginal pdf's [equation (1)] through the covariance matrix Σ.

When working with three time-series data sets of different lengths, as it is the case in this study, a weighted form of the concentrated log-likelihood function (equation (2)) has to be used (Ramirez and Somarriba).

If the jth mean-standardized dependent variable vector $(Y_j - X_j\beta_j)$ is believed to be autocorrelated, P_j and ι_j must be specified to make equation (2) operational. In the case of the first-order autocorrelation process assumed in this application, for example, the element on the first row and column of P_j is $(1 - \rho_j^2)^{1/2}$, where ρ_j is the correlation coefficient between $(Y_{ji} - X_{ji}\beta_j)$ and $(Y_{ji-1} - X_{ji-1}\beta_j)$. The other elements in the principal diagonal of P_j are equal to 1. All elements immediately below the principal diagonal are equal to $-\rho_j$ and the remaining elements of P_j are zero. Also $|\iota_j|$ $= |(P_j'P_j)^{-1}| = 1/(1 - \rho_j^2)$. Judge et al. also derive P_j and $|\iota_j|$ for higher-order autoregressive processes.

Although many types of conditional mean and variance specifications are conceptually feasible in this approach, only the possibilities of quadratic time-trends in the mean and linear time-trends in the standard deviation of the price and production pdf's are considered for the purposes of this illustration, by letting:

(3) $X_j\beta_j = \beta_{j0} + t\beta_{j1} + t^2\beta_{j2}$

$$(t = 1, \ldots, T; j = 1, \ldots, m),$$

(4) $\sigma_j(t) = \sigma_{j0} + t\sigma_{j1}$

$$(t = 1, \ldots, T; j = 1, \ldots, m).$$

The maximum likelihood estimates for the parameters β_{j0}, β_{j1}, β_{j2}, σ_{j0}, σ_{j1}, Θ_j, μ_j and μ_{jk}'s, and future realizations/forecasts of the exogenous variables can be used to jointly simulate the pdf's for the variables under analysis (Ramirez and Somarriba). First, an $N \times M$ matrix M of correlated normal random vectors (V_j) with means μ_j and covariance matrix Σ is generated using standard procedures. Then, N simulated values for the each of the endogenous variables are obtained by letting $V = V_j$ and applying equation (5) to each of the $(N \times 1)$ columns (V_j) of M:

(5) $\mathbf{Y}_j = [\{\text{sign}(\mu_j)\sigma_j/\Theta_j F(\Theta_j, \mu_j)\}$

$\times \sinh(\Theta_j \mathbf{V}_j)] + \mathbf{X}_j \boldsymbol{\beta}_j$

$- \text{sign}(\mu_j)\sigma_j/\Theta_j;$

$F(\Theta_j, \mu_j)_- = \{e(0.5\Theta_j^2)\}\{e(\Theta_j\mu_j) - e(-\Theta_j\mu_j)\}$

$\div 2;$

where $\text{sign}(\mu_j)$ indicates the sign of the parameter estimate for μ_j, $\sinh(\cdot)$ denotes the hyperbolic sine function, and $e(\cdot)$ denotes the standard exponential function. If the objective is to simulate the pdf's into the immediate future, the best predictor for the mean of $\mathbf{Y}_j$ under a standard first-order autocorrelation process assumption $(Y_{jF(T+t)})$ replaces $\mathbf{X}_j\boldsymbol{\beta}_j$ in equation (5) to center the simulated pdf's. This best predictor is given by (Ramirez and Somarriba):

(6) $Y_{jF(T+t)} = \mathbf{X}_{jf(T+t)}\boldsymbol{\beta}_{je} + \{\rho_j^t(Y_{j(T)} - \mathbf{X}_{j(T)}\boldsymbol{\beta}_{je})\}$

where $Y_{jF(T+t)}$ is the predicted value for Y_j at time period $T + t$, $\mathbf{X}_{jf(T+t)}$ is the vector of independent variable values/forecasts at $T + t$ for which the prediction/simulation is desired, $Y_{j(T)}$ is the value taken by the dependent variable during the most recent time period (T), $\mathbf{X}_{j(T)}$ is the vector of values taken by the independent variables at time period T, and $\boldsymbol{\beta}_{je}$ is an estimate from a consistent estimator of the vector of the mean-controlling parameters $(\boldsymbol{\beta}_j)$.

The Estimated Production and Price PDF Models

The parameter estimates and related statistics for the joint production-price pdf model are presented in Table 1. The statistical non-significance of the kurtosis and skewness parameters (θ and μ) in the case of cotton production and price indicates that their marginal pdf's can not be shown to be non-normal at the 10 percent certainty level. While West Texas cotton production does not appear to be cyclical, U.S. cotton prices follow temporal cycles ($\rho = 0.4534$). Expected cotton production has linearly increased at a constant average rate of 9.578 million lbs./year from 577.32 million lbs. in 1968 to 864.66 million lbs. in 1998.

The variability of production as measured by its standard deviation, however, has increased at a relatively faster rate of 8.977 million lbs./year from 128.12 million lbs. in 1968 to 397.43 million lbs. in 1998. In real (1998 U.S.$) terms, the mean of the cotton price distribution is estimated to have decreased at a constant rate of 1.8 cents/lb. per decade from 66.8 cents/lb. in 1968 to 61.4 cents/lb. in 1998. The standard deviation of the distribution has diminished at a relatively faster rate of about 1.3 cents/lb. per decade from 10 cents/lb. in 1968 to 6.1 cents/lb. in 1998 (Tables 1 and 2).

In contrast to cotton, the skewness and kurtosis parameters of the corn price and of the production distribution are statistically significant at the 10 percent level, indicating non-normality, but the corn production distribution is considerably more skewed and kurtotic than the corn price distribution (Tables 1 and 2 and Figure 1). Its mean has linearly increased from 32.75 million bushels in 1968 to 147.95 million bushels in 1998, while its degree of variability, as measured by the standard deviation, does not appear to be changing through time. The mean of the corn price distribution has decreased at a constant rate of 17 cents/bu. per decade form 2.89 $/bu. in 1968 to 2.38 $/bu. in 1998. In relative terms, this rate of decline is substantially higher than that of cotton prices. In relation to the mean of the distributions, however, the standard deviation of the corn price distribution has diminished at a faster rate than the standard deviation of the cotton price distribution, declining from 48 cents/bu. in 1968 to 20 cents/bu. in 1998.

The sorghum production and price distributions are also non-normal, exhibiting right-skewness and kurtosis (Tables 1 and 2 and Figure 2). In contrast to cotton and corn, sorghum production in West Texas has decreased substantially, at a non-linear decreasing rate, from an average of 168.06 million bushels in 1968 to 49.65 million bushels in 1998. The model also predicts that sorghum production reached a minimum level in 1990. The variance of the sorghum production distribution does not appear to be changing through time, which implies that production variability has

Table 1. Parameter Estimates and Related Statistics for the Production/Price PDF Model

Crop	Param.	PROD. Estimate	PRICE Estimate	Crop	Param.	PROD. Estimate	PRICE Estimate
Cotton	θ	N.S.	N.S.	Sorghum	θ	0.6068	0.5472
	σ_0	128.1240	0.1771		σ_0	17.1647	0.8484
	σ_1	8.9774	−0.0013		σ_1	N.S.	−0.0076
	μ	N.S.	N.S.		μ	18.1500	1.1580
	ρ	N.S.	0.4534		ρ	0.3552	0.2733
	β_0	577.3200	0.7744		β_0	168.0550	6.4873
	β_1	9.5778	−0.0018		β_1	−11.6501	−0.0307
	β_2	N.S.	N.S.		β_2	0.25676	N.S.
Corn	θ	0.4271	0.3629	Wheat	θ	0.7080	0.5443
	σ_0	19.8600	0.2984		σ_0	11.4508	0.5221
	σ_1	N.S.	−0.0027		σ_1	N.S.	N.S.
	μ	19.9572	0.8338		μ	−11.0724	N.S.
	ρ	0.6020	0.3242		ρ	0.6624	0.5259
	β_0	32.7474	3.8990		β_0	26.8532	5.4964
	β_1	3.8389	−0.0171		β_1	N.S.	−0.0256
	β_2	N.S.	N.S.		β_2	N.S.	N.S.

Correlation Matrix

	PRODUCTION				PRICE			
PROD.	Cotton	Corn	Sorghum	Wheat	Cotton	Corn	Sorghum	Wheat
Cotton	1.0000	N.S.	−0.3753	N.S.	N.S.	—	—	—
Corn	N.S.	1.0000	0.0977	N.S.	—	N.S.	—	—
Sorghum	−0.3753	0.0977	1.0000	0.3856	—	—	N.S.	—
Wheat	N.S.	N.S.	0.3856	1.0000	—	—	—	N.S.
PRICE								
Cotton	N.S.	—	—	—	1.0000	0.3325	0.1619	0.2569
Corn	—	N.S.	—	—	0.3325	1.0000	0.7537	0.5525
Sorghum	—	—	N.S.	—	0.1619	0.7537	1.0000	0.3791
Wheat	—	—	—	N.S.	0.2569	0.5525	0.3791	1.0000

Note: N.S. indicates not significant at the 10 percent level, according to a single-parameter likelihood ratio test. All other parameters are statistically significant at the 10 percent level, according to single-parameter likelihood ratio tests. These tests are conducted by re-estimating the model setting the corresponding parameter equal to zero and calculating the test statistic $\chi^2_{(1)} = 2*$(maximum value reached by the unrestricted log-likelihood function-maximum value reached by the restricted log-likelihood function), which under the null hypothesis of parameter non-significance is distributed as a chi-square random variable with one degree of freedom (Judge, *et al.*).

increased substantially in relation to the expected production level.

The mean of the sorghum price distribution has decreased at a constant rate of 30 cents/cwt per decade from 4.67 $/cwt in 1968 to 3.76 $/cwt in 1998. In relative (percentage) terms, this rate of decline is similar to that of corn prices. The standard deviation of the sorghum price distribution has also diminished at a relative rate comparable to that of the standard deviation of the corn price distribution

from 72 cents/cwt in 1968 to 32 cents/cwt in 1998.

The wheat production and price distributions are non-normal as well (Table 1). The non-significance of the skewness parameter (μ) in the price distribution, however, indicates that it is kurtotic but symmetric, while the negative parameter estimate for μ in the production distribution implies left-skewness. Figure 3 illustrates the substantial degrees of kurtosis and left-skewness that characterize the esti-

Table 2. Mean, Standard Deviation, Skewness and Kurtosis Coefficients of the Estimated/ Simulated 1968, 1978, 1988 and 1998 Cotton, Corn, Sorghum and Wheat Production and Price Distributions

	Production				Price			
1998	Cotton	Corn	Sorghum	Wheat	Cotton	Corn	Sorghum	Wheat
Mean	860.4836	148.1411	49.7985	26.9496	0.6139	2.3790	3.7597	3.2187
Std. Dev.	396.9422	21.1894	19.4255	13.1868	0.0614	0.1995	0.3225	0.6084
Skew.	0.0068	1.5305	2.4158	−3.0926	−0.0084	0.3848	1.4360	−0.0134
Kurt.	0.0141	4.5981	11.3139	18.3661	−0.0233	0.9631	6.4425	1.7303
1988	Cotton	Corn	Sorghum	Wheat	Cotton	Corn	Sorghum	Wheat
Mean	768.7502	109.5518	37.6177	26.7363	0.6328	2.5483	4.0627	3.4797
Std. Dev.	310.2508	20.6890	18.7163	13.1347	0.0742	0.2928	0.4567	0.6141
Skew.	−0.0031	1.3991	2.1936	−2.7203	−0.0467	0.3605	1.1510	−0.0742
Kurt.	0.0555	3.6390	8.4625	13.0405	−0.0042	0.7811	3.4494	1.9972
1978	Cotton	Corn	Sorghum	Wheat	Cotton	Corn	Sorghum	Wheat
Mean	673.0545	71.1480	77.3027	26.7715	0.6496	2.7202	4.3741	3.7334
Std. Dev.	218.6591	20.8419	18.9542	13.2701	0.0875	0.3833	0.5974	0.6114
Skew.	−0.0193	1.4284	2.3898	−3.1962	−0.0016	0.4048	1.2389	0.0035
Kurt.	−0.0325	3.6150	12.4136	21.3548	−0.0054	0.9457	4.4810	2.3961
1968	Cotton	Corn	Sorghum	Wheat	Cotton	Corn	Sorghum	Wheat
Mean	577.7613	32.5976	167.9965	26.9448	0.6685	2.8928	4.6742	3.9923
Std. Dev.	128.5369	20.7576	18.4563	12.8619	0.1000	0.4787	0.7249	0.6096
Skew.	0.0021	1.4354	2.0832	−2.8985	−0.0069	0.4065	1.1221	0.0095
Kurt.	−0.0129	3.7203	7.7587	18.2239	−0.0663	0.9661	3.2100	2.3127

Note: The mean, standard deviations, skewness and kurtosis coefficients were calculated from 20,000 draws simulated from each of the estimated PDF's, using the pre-programmed functions in Microsoft Excel (Office 97).

mated wheat production distribution. In contrast to the other three crops, wheat production in West Texas has remained stable during the last 30 years, averaging 26.85 million bu./year, with a similarly constant standard deviation of 11.45 million bu./year. The mean of the wheat price distribution has decreased at a linear rate of 26 cents/bu. per decade from $3.99/bu. in 1968 to $3.22/bu. in 1998. In relative terms, this rate of decline is surprisingly similar to those of corn and sorghum prices. The standard deviation of the wheat price distribution appears to have remained constant during the last 90 years at 61 cents/bu.

The model also estimates statistically significant, positive bivariate correlation coefficients between all crop prices (Table 1), as strong as 0.754 between corn and sorghum and 0.552 between corn and wheat to as weak as 0.162 between cotton and sorghum prices. A moderate but statistically significant nega-

tive correlation between annual cotton and sorghum production was also found. This is explained by the fact that sorghum is often planted as an alternative crop in areas where cotton has been damaged by unfavorable weather. Moderate to weak positive correlation coefficients between wheat and sorghum (0.386) and between sorghum and corn production (0.098) are predicted as well.

Risk and Returns of Irrigated Agricultural Production in West Texas

Revenues from agricultural production are critical to the West Texas economy. The probability distribution of the gross revenues generated can be useful to assess the risk and returns of irrigated crop production from the perspective of its contribution to the regional economic activity. Alternatively, the probability distribution of the net revenues generated
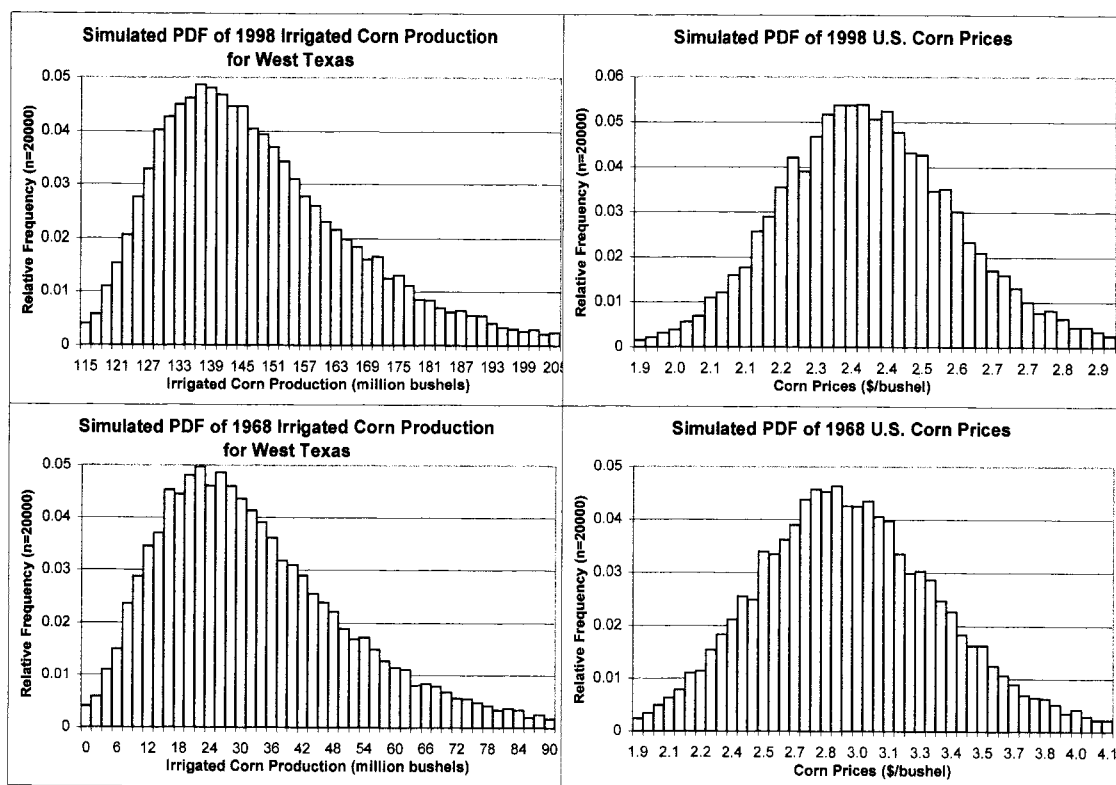
Figure 1.

is more pertinent to evaluate the risk and returns from the producers' perspective. In 1968, the expected gross revenues from irrigated (cotton, corn, sorghum and wheat) crop production were $1028 million with a standard deviation of $170 million. The probability distribution of 1968 gross revenues was moderately right-skewed and kurtotic (Figure 4 and Table 3). This kind of departure from normality could be desirable (*ceteris paribus*) since it implies that a higher proportion of the probability distribution's variability is upward variability.

The previously discussed trends in the mean price and production levels and in their degree of variability have resulted in gross revenue probability distributions with moderately higher expected values, but substantially increased variances in more recent years. The distributions are also becoming more normal. The 1998 distribution, for instance, has a mean of $1072 million and a standard deviation of

$260 million. Its skewness and kurtosis coefficients are reduced to 0.16 and 0.20, respectively (Table 3 and Figure 4), implying that a higher variability is now almost evenly distributed among both tails of the distribution representing both downward and upward variability in nearly equal proportions. As a result, for example, gross revenues under $800 million/year, which was a one-in-13-years event in 1968, has become a one-every-seven-year event in 1998. The probability of a gross revenue occurrence below $950 million/year was lower in 1968 than it is today. Despite its higher mean, the 1998 distribution fails to stochastically dominate the 1968 distribution.

In terms of production, cotton is the main crop in West Texas, followed by corn and wheat. Different gross revenue distribution scenarios for 1998 can be constructed by realizing that the aggregate crop production distributions are proportional to the planted acreage distributions:
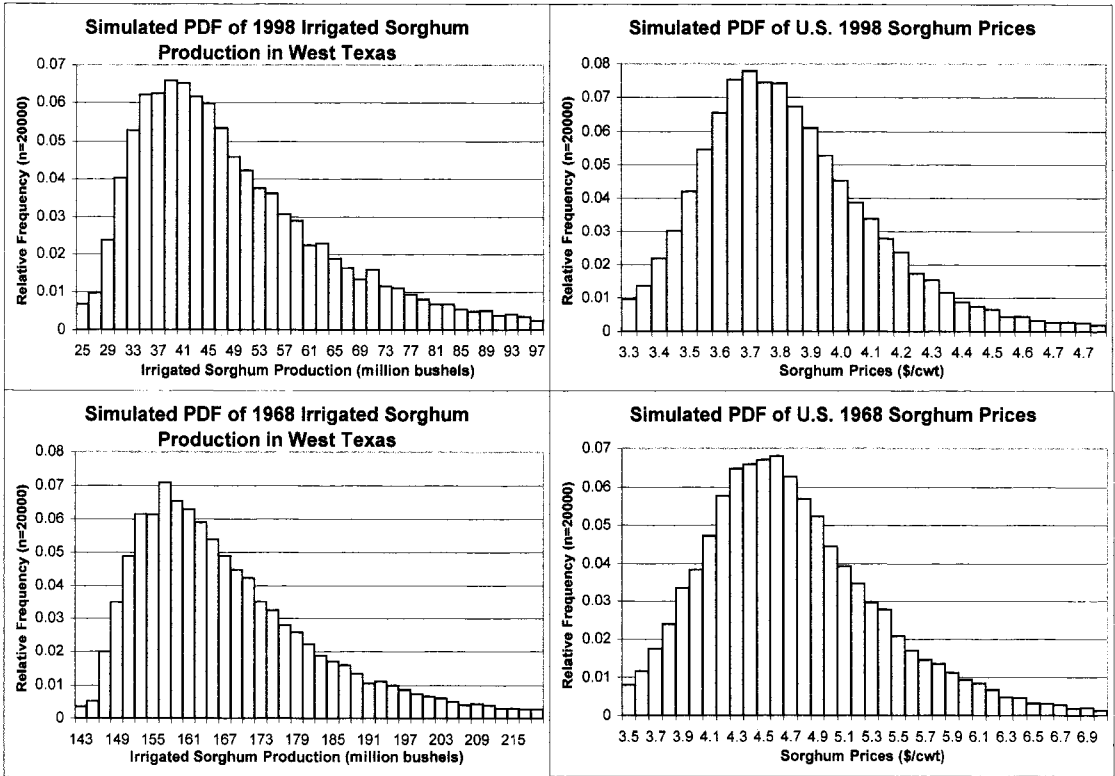
Figure 2.

(7) $GR_s = CTPR_sCTP_s + COPR_sCOP_s$

        $+ SPR_sSP_s + WPR_sWP_s$

    $= CTPA_sCTY_sCTP_s$

        $+ COPA_sCOY_sCOP_s + SPA_sSY_sSP_s$

        $+ WPA_sWTY_sWTP_s$

where CTPR, COPR, SPR, WPR, CTP, COP, SP, WP, CTPA, COPA, SPA, WPA, CTY, COY, SY and WY denote 1998 cotton, corn, sorghum and wheat production, price, planted acres and yields (measured in relation to planted acres), respectively, and the subscript s denotes a realization or simulated value of each of these random variables. In other words, the estimated/simulated aggregate production distribution for any of the crops can be viewed as estimating/simulating the product of the planted acres distribution times the yield distribution for that crop. It follows that a percent change in the

planted acreage or in the yield distribution of a given crop would result in the same percent change in the aggregate production distribution for that crop.

Several scenarios involving shifts in the probability distributions of planted acres of cotton, corn, sorghum, and wheat in West Texas can be analyzed using the previous result. The most recent data (TASS) indicates that, of the four million acres of irrigated crops planted in West Texas in 1998, 1.82 million (44.6 percent) were cotton, 976 thousand (23.9 percent) where corn, 840 thousand (20.6 percent) were wheat, and 443 thousand (10.9 percent) were sorghum. These crop acreage figures are unbiased estimators for the means and for the mean percentage shares of the 1998 acreage distributions. Therefore, the 1998 gross revenue distribution under a change in the mean percentage acreage-share can be estimated/ simulated as:
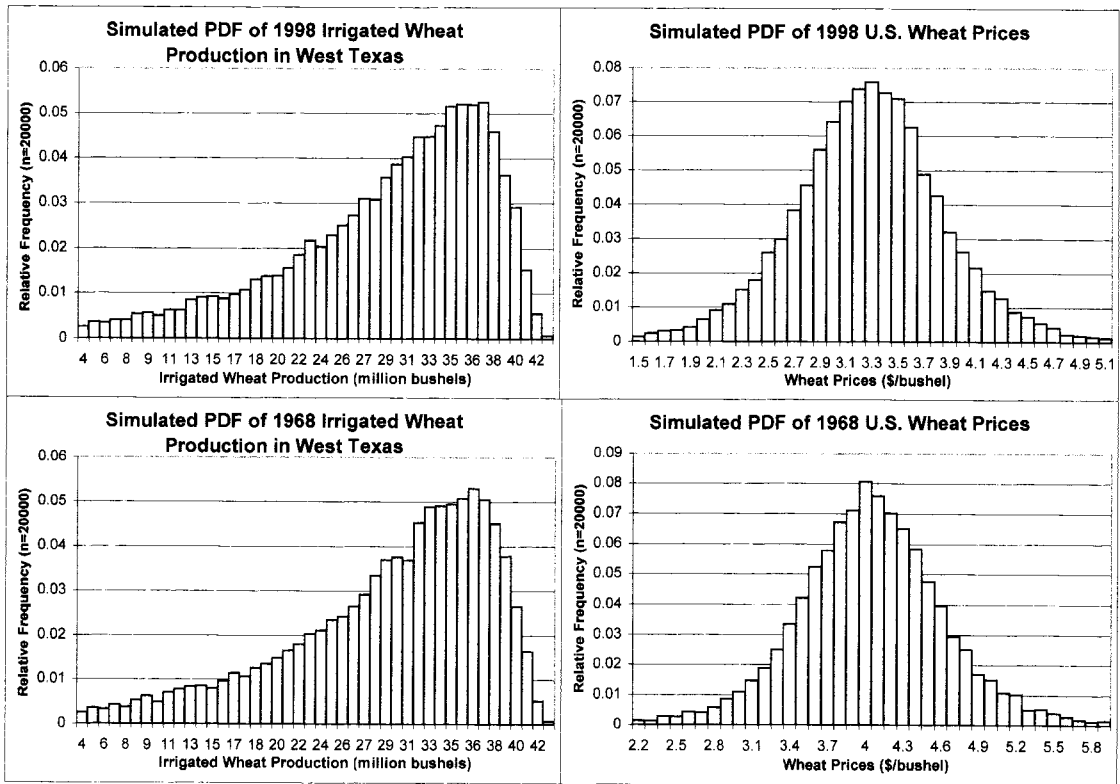
Figure 3.

$$(8) \quad GR_s = (MPSCA/44.6) \times (CTPA_sCTY_sCTP_s)$$
$$+ (MPSCOA/23.9)$$
$$\times (COPA_sCOY_sCOP_s)$$
$$+ (MPSSA/10.9) \times (SPA_sSY_sSP_s)$$
$$+ (MPSWA/20.6)$$
$$\times (WPA_sWTY_sWTP_s)$$
$$= (MPSCA/44.6) \times (CTPR_sCTP_s)$$
$$+ (MPSCOA/23.9) \times (COPR_sCOP_s)$$
$$+ (MPSSA/10.9) \times (SPR_sSP_s)$$
$$+ (MPSWA/20.6) \times (WPR_2WTP_s)$$

where MPSCA, MPSCOA, MPSSA and MPSWA are the assumed (new) mean percentage shares of cotton, corn, sorghum and wheat acreage, respectively. If the mean percentage share of cotton acreage increases to 50 percent at the expense of a reduction in the sorghum share to 5.5 percent, for example, equation (8) shifts the entire cotton acreage

and thus the cotton production distribution by $50/44.6 = 1.12$, and the sorghum production distribution by $5.5/10.9 = 0.50$.

As discussed earlier, gross revenue distributions are suitable for evaluating the risk and return implications of West Texas irrigated crop production and price trends/shifts on the overall economic activity in the region, since they are closely related to agricultural production expenditures. The financial wellbeing of the farm sector, however, is more closely linked to net revenue distributions. The 1998 net revenue distributions under alternative mean percentage shares of cotton, corn, sorghum and wheat acreage can be approximated from the previously discussed gross revenue distributions by assuming a fixed average cost of production for each of the crops in the analysis.

Specifically, the total cost of production subtracted from each of the 20,000 simulated gross revenue realizations is obtained by multiplying the 1998 average (per unit) costs of
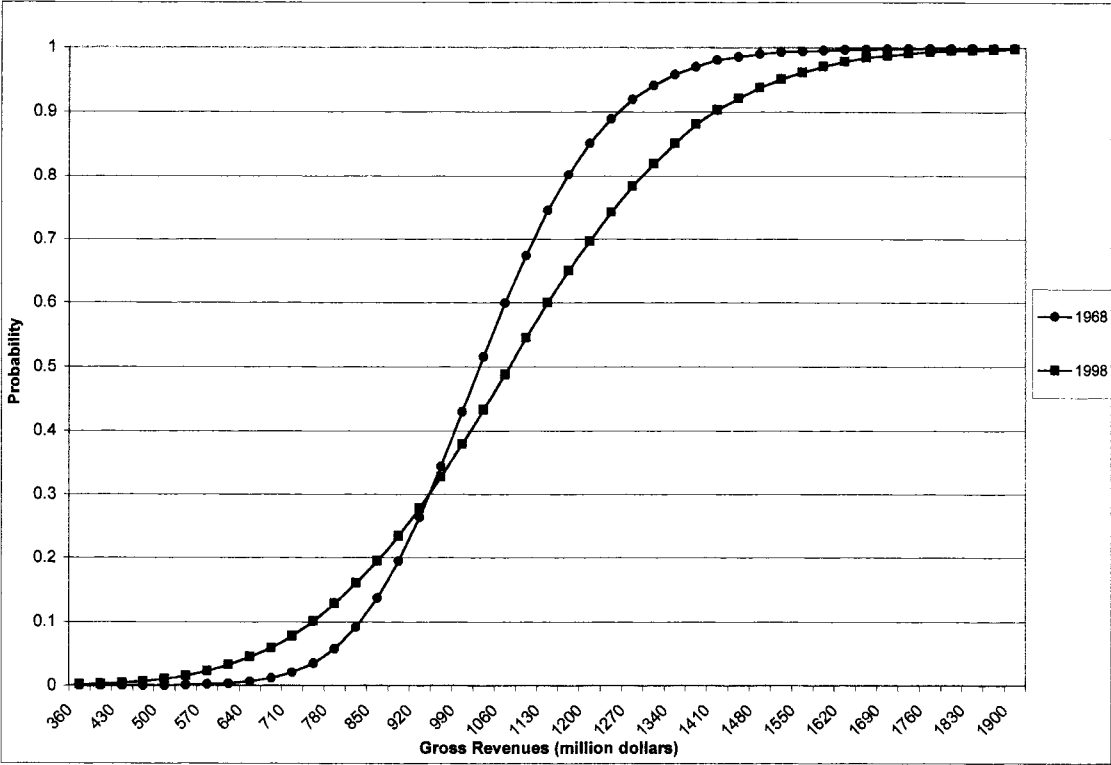
Figure 4. Simulated 1968 and 1998 Cumulative Gross Revenue Distributions

Table 3. Mean, Standard Deviation, Skewness and Kurtosis Coefficients of the Estimated/ Simulated 1998 Probability Distributions of Gross and Net Revenues from Irrigated West Texas Crops Under Different Acreage Shift Scenarios, and Their Implied Probabilities of Gross Revenues Below $800 and $1000 Million/Year and Net Revenues Below $0 and $30 Million/Year, Respectively.

Acreage Shifts	Gross Revenue Distributions					
	Mean	S.D.	Skew.	Kurt.	P<800	P<1000
1968	1028	170	0.36	0.74	0.075	0.455
1998 Baseline	1072	260	0.16	0.20	0.146	0.394
Cotton to Sorghum (10%)[1]	1050	212	0.17	0.22	0.117	0.412
Sorghum to Cotton (10%)[1]	1095	319	0.15	0.18	0.177	0.388
All Corn and Sorghum to Cotton[2]	1027	450	0.14	0.18	0.307	0.484

Acreage Shifts	Net Revenue Distributions					
	Mean	S.D.	Skew.	Kurt.	P<0	P<30
1998 Baseline	53.9	87.7	0.35	1.00	0.261	0.405
Cotton to Sorghum (10%)[1]	35.7	82.5	0.33	1.01	0.335	0.490
Sorghum to Cotton (10%)[1]	72.0	95.2	0.36	1.20	0.208	0.327
All Corn and Sorghum to Cotton[2]	55.0	111.5	0.45	1.90	0.293	0.436

Notes: [1] Ten percent of the total 1998 acreage is shifted from one crop to the other, assuming one-to-one acre shifts.
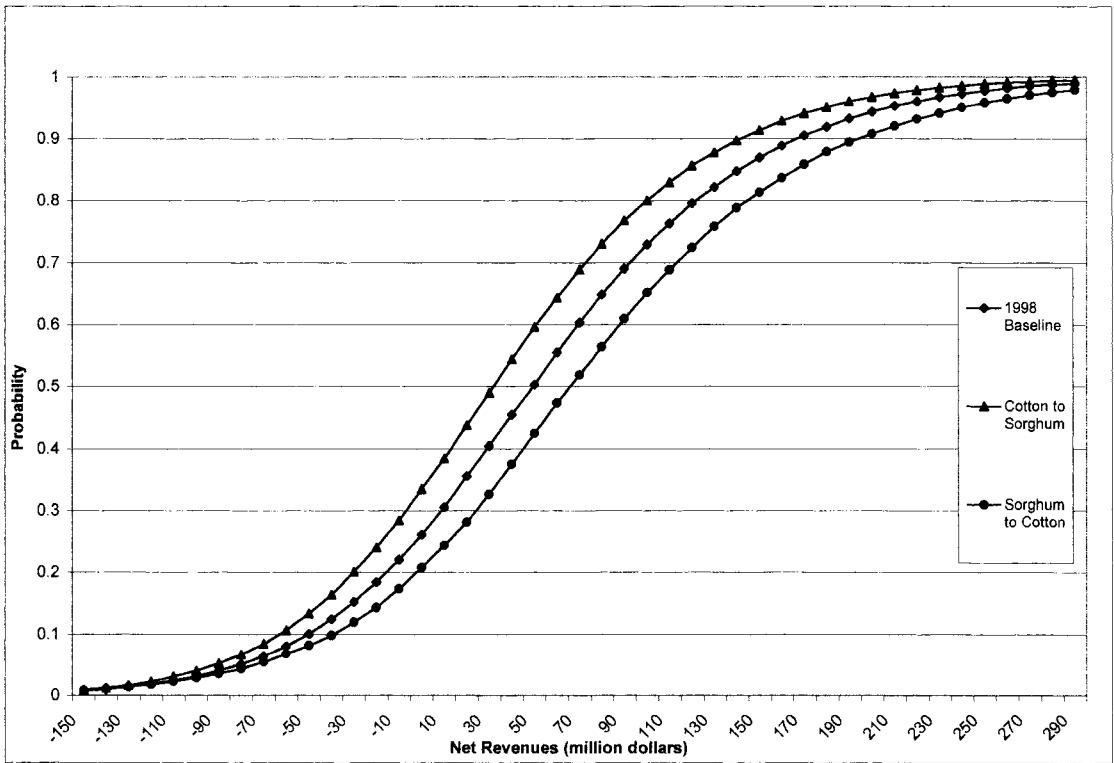[2] Assuming one-to-one acre shifts from corn and sorghum to cotton.

Figure 5. Simulated Cumulative Net Revenue Distributions under the 1998 Baseline and Alternative Acreage Shift Scenarios

production reported in the Texas A&M crop budgets for irrigated cotton, corn, sorghum, and wheat in the South Plains times the corresponding 20,000 × 4 simulated crop production values. The average costs used to simulate the net revenue distributions ($0.579/lb. for cotton, $2.145/bu. for corn, $2.354/bu. for sorghum and 3.163/bu. for wheat) were calculated without considering the cost of the land.

The current (baseline) 1998 net revenue distribution has a mean of $53.9 million and a standard deviation of $87.7 million. It is slightly right-skewed and moderately kurtotic. It implies a 0.261 probability of negative net revenues and a 0.405 probability of net revenues below $30 million, which might be problematic for agricultural producers in the region (Table 3 and Figure 5).

Cotton and sorghum have similar irrigation water requirements and acreage shifts from one crop to the other are relatively easy. Thus,

the following evaluation will focus on this more feasible acreage shift scenarios. The current 1998 net revenue distribution stochastically dominates the distribution resulting from the hypothetical alternative of shifting 10 percent of the total 1998 acreage from cotton to sorghum (Figure 5). The alternative exhibits much lower average net revenues of 35.7 million/year with an only marginally reduced standard deviation. As a result, the probabilities of net revenues below $0 and $30 million/year swell to 0.335 and 0.490, respectively.

This cotton-to-sorghum alternative also reduces expected gross revenues by $22 million to $1050 million/year, decreases the standard deviation of the gross revenue distribution by $48 million to $212 million, and causes marginal increases in its right-skewness and kurtosis. The gross revenue distribution under this scenario can be considered less risky than the current 1998 distribution up to $925 million/year (Figure 6). Both distributions exhibit a
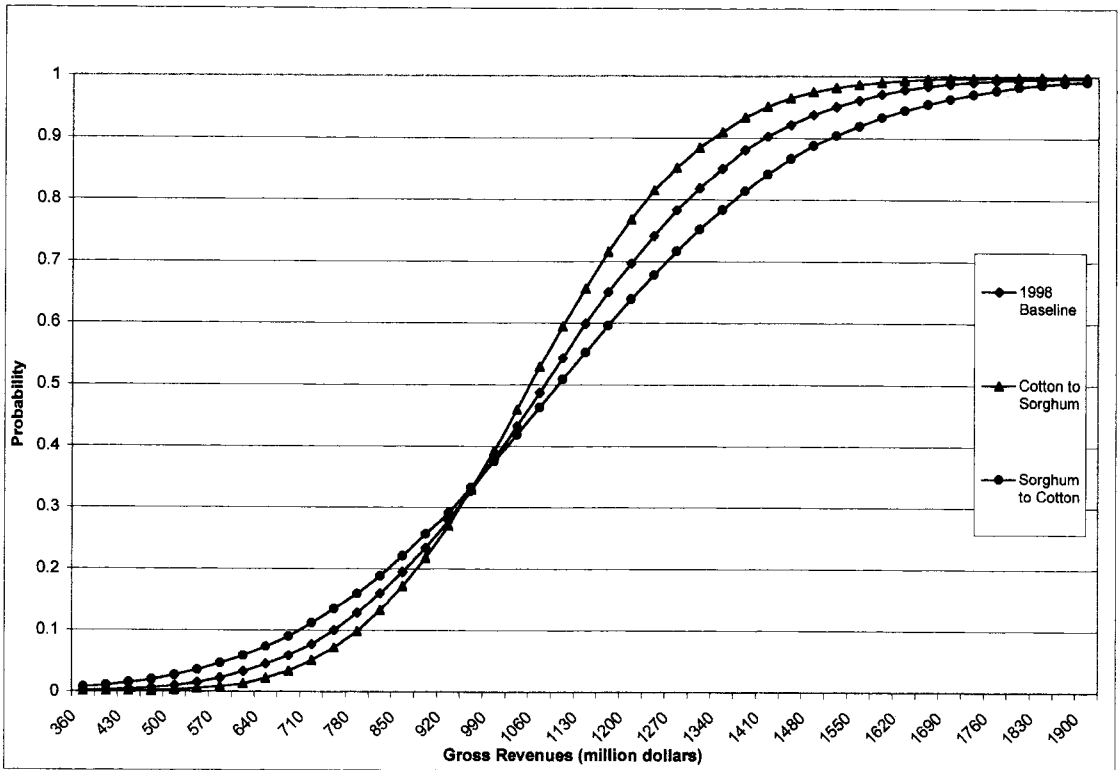
Figure 6. Simulated Cumulative Gross Revenue Distributions under the 1998 Baseline and Alternative Acreage Shift Scenarios

32-percent probability of gross revenue occurrences below that level.

The opposite scenario of a 10-percent acreage shift from sorghum to cotton substantially increases expected net revenues by 33.6 percent to $72 million/year, at the cost of a modestly higher standard deviation and kurtosis. This alternative distribution stochastically dominates the current 1998 distribution (Figure 5) with noticeably lower probabilities of net revenues below $0 and $30 million (Table 3). The former suggests that an alternative involving more cotton and less sorghum could be desirable from the net revenue, i.e. from the agricultural sector's welfare perspective.

Although this shifting of acreage from sorghum to cotton results in higher expected gross revenues ($1095 vs. $1072 million/year), it also induces a substantial increase in the standard deviation of the gross revenue distribution (from $260 to $319 million). The probability of annual gross revenues below

$800 million increases from 14.6 percent to 17.7. However, the probability of annual gross revenues below $1000 million remains the same (Figure 6 and Table 3).

The next scenario evaluates the impact of a likely more reduced irrigation water availability in West Texas, which could make it impossible to grow irrigated corn. This scenario involves the shifting of all irrigated corn and sorghum acreage into irrigated cotton on a one-to-one acre basis. This does not substantially change expected net revenues in relation to the 1998 baseline, but increases the standard deviation, right-skewness, and kurtosis of the net revenue distribution to 111.5, 0.45, and 1.50, respectively. The somewhat higher probabilities of net revenues below $0 and $30 million/year only indicate a modest increase in the level of risk to be faced by agricultural producers as a whole. The net revenue distribution under this limited water alternative, however, is substantially inferior to the pre-

viously discussed sorghum-to-cotton scenario that maintains 23.9 percent of the irrigated acreage in corn production (Table 3).

The implications of this reduced water availability alternative for the gross revenue distribution, which is more relevant to evaluate the impact of irrigated crop production on the overall economic activity of West Texas, are of greater concern. Expected gross revenues fall by $45 million while their standard deviation nearly doubles to $450 million, resulting in much higher probabilities of gross revenues below $800 and $1000 million/year. This stresses the importance of water conservation and of a more rational, economically efficient use of water in West Texas.

Conclusions and Recommendations

The procedure used in this study provides an estimate of the joint probability distribution of U.S. cotton, corn, sorghum, and wheat price and West Texas irrigated cotton, corn, sorghum, and wheat production. In general, both the mean and the variance of the marginal distributions of crop production and prices are found to be shifting over time. All but the marginal distribution of cotton production exhibit positive first-order autocorrelation. Several marginal distributions are kurtotic and right-skewed. The marginal distribution of wheat production is highly kurtotic and left-skewed. All of the marginal price distributions are correlated with each other. The marginal probability distribution of sorghum production is also correlated with the cotton, corn, and wheat production distributions.

Using a procedure capable of modeling and simulating a multivariate probability distribution that presents such a complex combination of key statistical attributes enhanced the reliability of the subsequent risk analysis. In this and in many other potential applications using alternatives that ignore some of those critical distributional characteristics or that follow a piecemeal approach to their modeling and simulation would be inferior from the statistical standpoint and, therefore, increase the likelihood of incorrect conclusions.

Time trends were detected in the marginal

distributions of real crop prices, indicating decreasing means and standard deviations, except for the standard deviation of the wheat price distribution which has remained constant. Also, with the exception of wheat, the decrease in the standard deviations of the price distributions has been higher in relative terms than the reduction in their means. Real cotton, corn, and sorghum prices have come down during the last 90 years, but they have also become substantially less volatile. This conclusion, however, is likely affected by the farm policies of the 1980s and might not hold in the absence of those policies.

On average, irrigated cotton production in West Texas has increased by about 50 percent during the last 30 years, while the mean of the corn production distribution is now nearly four times higher than in 1968. Despite a fourfold increase in its mean, the standard deviation of the corn production distribution has remained stable, which implies a substantially lower relative variability. Average sorghum production has decreased almost four-fold since 1968, while its standard deviation has remained unchanged. In terms of average production and production variability, corn in 1968 was very similar to sorghum in 1998, while corn in 1998 resembles sorghum in 1968; i.e., the risk implications of the changes in the corn and sorghum production distributions through time could be offsetting each other. The wheat production distribution has remained stable.

Estimated/simulated net revenue distributions are used to evaluate risk and returns from the perspective of West Texas agricultural producers, while gross revenue distributions are used to assess the risk and return implications of irrigated crop production from the standpoint of the region's economic activity. On average, gross revenues are estimated to have increased by about 5 percent since 1968, while their variability, as measured by the standard deviation of the gross revenue distribution, has increased by more than 50 percent. The actual 1998 net revenue distribution is centered at $53.9 million/year with 26 percent of its mass below zero and 40 percent of its mass below $30 million.

An alternative to improve the net revenue

situation given the estimate of the joint 1998 crop price-production distribution obtained in this study is to shift 10 percent of the total irrigated acreage from sorghum to cotton. This increases expected net revenues to $72 million and reduces the probability of net losses to 21 percent. However, it also increases the standard deviation of the gross revenue distribution and the probability of low gross revenue realizations substantially. Higher gross revenue variability implies more risk from the perspective of the region's economic activity.

Finally, the model estimated in this study is used to assess the potential impact of a more reduced irrigation water availability on the West Texas agricultural and regional economy. It is concluded that net revenue risk could increase slightly, mainly due to a higher net revenue variability. Gross revenue risk, however, could grow substantially due to moderately lower expected gross revenues with a much higher degree of volatility.

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