FINANCING PRODUCTION WITH LIQUIDITY CONSTRAINTS: THE ROLE OF TRADE CREDIT IN AGRO-FOOD SUPPLY CHAINS

JEL classification: E32, L14, Q14.

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Abstract. *This paper focuses on the role of trade credit in agri-food supply chains, with particular reference to a context of financial turmoil and credit rationing. Trade credit enhances the resilience of firms to liquidity shocks and creates systemic risk. These features of trade credit are investigated with the aim of pinning down their effects on the financing of working capital investments of liquidity-constrained firms. To this end, we put forward a simple model of trade credit connections in supply chains and use the model to measure the degree of exposure of these investment decisions to unexpected liquidity constraints arising from liquidity risk and systemic risk. We do so by characterising the impact of an exogenous liquidity shock on the investment and output of firms in agri-food supply chains in terms of threshold values of such a shock.*

Keywords: *trade credit, credit rationing, systemic risk, agro-food supply chain.*

1. Introduction

Financing the production of agricultural firms is an issue that constantly attracts the attention of both researchers and policy makers, especially so during periods when the institutional framework of the relations between banks and firms is undergoing substantial changes (e.g. Basle Agreements II and III). The issue becomes particularly relevant in periods of economic downturn associated with a financial crisis (like the current one), when the traditional concerns for the difficulties faced by agricultural firms in accessing bank credit are reinforced by the emergence of liquidity shortages and the detrimental effects that such constraints can have on the investment decisions and, consequently, on the production and earnings of agricultural firms.[1]

An analysis of the coverage of the financial needs arising from investment in working capital requires a careful evaluation of the role played by the sources of funding, such as self-financing and trade credit, that do not come from banks or other financial intermediaries. Regardless of the actual severity of liquidity constraints due to the rationing of bank credit, such sources of funds are relevant in as much as they are intertwined with the contractual terms (timing of payments, discounts, pricing, etc.) of the commercial links that firms have with their suppliers and

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[1] See, inter alia, Romano (2010).

buyers. Moreover, trade credit is particularly relevant for liquidity- constrained firms in that the default of a buyer does not imply the end of the commercial relation between the supplier and the defaulting client. It is well established by empirical evidence, and explained by economic theory, that it is more convenient for a supplier to concede a deferral of payment to a defaulting client, rather than to push for the liquidation of its assets, i.e. its bankruptcy. Thus, the use of trade credit as a source of funding improves the resilience of a liquidity-constrained firm to unexpected financial shortages, i.e. it helps an illiquid firm to stay solvent. In other words, trade credit contracts embed an insurance coverage against liquidity risk. The cost of this insurance is incorporated in the pricing policies set by suppliers (who often use trade credit terms to discriminate among their clients) and, as a consequence, affect the allocation of earnings among the firms that belong to a supply chain. On the other hand, the fact that -- in a trade credit contract -- a supplier shares part of the liquidity risk run by its client, implies that a liquidity shock suffered by a defaulting firm is transmitted to its supplier and from the latter to its own suppliers and so forth, generating a systemic liquidity risk affecting most of the firms in a supply chain. In brief, the use of trade credit in supply chains provides funding and the sharing of liquidity risk but, at the same time, it creates the grounds for financial contagion.

In general, it is well established that liquidity constraints limit the investment decisions of firms and, consequently, their output. In the agri-food industry, given the relative rigidity of agricultural production in the short run, liquidity shocks are bound to affect not only the levels of investment and output, but also the profitability of firms (or the implicit wages paid in family firms).

The relevance of these issues is underlined by recent attempts to introduce laws, in Italy[2] as well as in other European countries, that regulate the terms of transactions in agricultural and agri-food markets, imposing legal terms for the time elapsing between the delivery and the payments of supplies.

In a similar spirit, some ‘High Level Groups’, created by the European Commission, have expressed recommendations to Member States concerning competition in the agri-food sector and for specific sub-sectors (e.g. milk and wine); these include the compulsory use of written contracts between farmers and processors and the regulation of some terms of these contracts, such as the timing and modes of payments.[3]

This paper focuses on the role of trade credit in agri-food supply chains, with the aim of pinning down the effects of the two features of trade credit discussed above (enhancement of the resilience of firms to liquidity shocks and creation of systemic risk) on the financing of working capital investments of liquidity-constrained firms. To this end, we put forward a simple model of trade credit connections in supply chains and use the model to measure the degree of exposure of such investment decisions to unexpected liquidity constraints arising from liquidity risk and systemic risk. We do so by characterising the impact of an exogenous liquidity shock on the investment and output of firms in agri-food supply chains in terms of threshold values of such a shock.

[2] The new regulations recently introduced in Italy set a maximum deferral of payments of 30 days for perishable agri-food products, and of 60 days for other agri-food products, with penalties for those who do not respect these terms. See. art. 62 of the DL 24 january 2012, n. 1 - Disposizioni urgenti per la concorrenza, lo sviluppo delle infrastrutture e la competitività e successive modifiche.

[3] See the report of the Group of ‘High Level Experts’ on Milk (15 June 2010) and the subsequent Regulations that implement their recommendations (also known as ‘The Milk Package’).

2. Supply chains and trade credit in economic theory

The widespread use of trade credit, despite its high cost, has attracted the attention of economic theorists who, over the last fifteen years, have provided convincing and exhaustive answers to such a phenomenon.

The existing data show that trade credit constitutes a relevant share of the balance sheet of companies in developed countries and an even larger share in developing countries. Rajan and Zingales (1995) show that trade credit, as a percentage of total credit granted to companies, amounts to 11,5% in Germany, to 17% in France, 15% in the United States, 13,3% in Canada, 13,7% in Great Britain, 14,7% in Italy, and to 15,4% in Japan. These authors also argue that the trade credit granted by companies, as a percentage of their total assets, goes from 13% in Canada to 29% in France and Italy. Cuñat (2007) shows that in Great Britain, trade credit amounts to 17% of total assets, 43% of debts and 52% of short term debts of companies, while in the US these percentages are 18%, 34% and 58%, respectively. This author sustains that these percentages grow during periods when buyers suffer temporary liquidity shortages, and that such an increase in trade credit occurs through defaults on existing debts, where suppliers allow lenders to postpone payments. Cannari *et al*. (2004) study trade credit terms in Italy on the basis of two surveys carried out by the Bank of Italy. The authors show that, on average, 80-90 per cent of sales of the Italian companies surveyed are paid on a deferred payment basis of 90 days and that the cost of such trade credit is normally very high and well above market interest rates.

This evidence made economists wonder why such a large share of companies' credit is not provided by banks and financial intermediaries, which are specialised in credit services. The question becomes even more puzzling because of the cost of trade credit, which is much higher than the cost of bank credit. Ng *et al.* (1999) find that, in the United States, the most common type of trade credit contract is the "2-10 net 30": the buyer obtains a 2% discount if he pays within 10 days, otherwise he can pay within 30 days with no discount. In such a case the buyer gets a loan for twenty days at 2% interest rate, which corresponds to 44% on a yearly base. The second most common contract in the US, according to these authors, is the "8-30 net 50", analogous to the previous one, but corresponding with an annual interest rate of 358%. Considering that trade credit is so expensive, why do companies with no liquidity shortage resort to trade credit? Why do firms with binding liquidity constraints grant trade credit to their clients? Economic theory has responded to these questions with arguments that can be classified in four categories:

1. Information advantage (monitoring costs): sellers are better informed than banks about their own clients; the receipt of trade credit is a signal of creditworthiness for banks. [Biais and Gollier (1997)];
2. Liquidation value: the collateral assets have a larger liquidation value for the suppliers than for the banks;
3. Moral hazard: i) delayed payments eliminate the risk that suppliers sell goods of a quality inferior to that contracted with the buyer; ii) Diversion theory: trade credit is in kind, suppliers lend goods while banks lend money. Trade credit makes it more difficult for the managers of firms to divert resources from purposes which are consistent with the interests of their creditors [(Burkart and Ellingsen (2004)];
4. Coverage of liquidity risk: suppliers, in the face of default of a client, are better off allowing postponement of payments rather than resorting to suing the debtor and possibly contribut-

ing to his bankruptcy. The trade credit contract embeds an insurance against liquidity shocks, and this can also explain its costs (Cuñat, 2007).

These types of rationale for the use of trade credit are clearly traceable in the functioning of specific supply chains in the agri-food industry. Suppliers of machinery, of intermediate goods and of business advice are usually better informed than banks about the liquidation value of the assets of their clients.[4]

When supply chains are controlled by large retail chains and private labels, supplies of intermediate goods and the corresponding trade credit come from downstream, where the processing industry and large retailers are particularly interested in guaranteeing rigorous qualitative standards for agricultural products. Moreover, the relevance of trade credit in the agri-food industry is increased by the fact that large retailers, given their negotiating power with respect to their suppliers of agricultural products, can impose deferral of payments that dramatically affect the liquidity needs and the profit margins of agricultural firms.

Many authors have worked on the above issues in the last twenty years. Here only the contributions which are most important and most relevant to our aims are cited, with special focus on the papers that have investigated the relationship between the use of trade credit and the rationing of bank credit.

Petersen and Rajan (1997) analyse the relevance of trade credit for companies of different size and age, using data from Compustat and from the Survey of Small Business Finances of the Federal Reserve. The authors find that: i) the amount of trade credit granted by companies is directly correlated with the size and with the age of companies; large and mature companies are often net suppliers of trade credit; ii) in general, companies prefer to resort to bank credit, when available; iii) companies endowed with liquid reserves and with long term relations with banks use less trade credit than companies with opposite features. Gustafson (2004) studies the role of trade credit in the agri-food industry of the US and obtains results which are similar to the results presented by Petersen and Rajan (1997).

Burkart and Ellingsen (2004) present a model of trade credit relations with asymmetric information and argue that trade credit reduces the scope for moral hazard on the part of debtors. For this reason, according to the authors, the weaker the legal protection of creditors, the larger the use of trade credit. They also argue that trade credit and bank credit are complementary for firms subject to liquidity constraints, while they are substitutes for firms with sufficient 'debt capacity', i.e. firms that have access to external sources of funding. Nilsen (2002) presents data in favour of the thesis that firms that face credit rationing use trade credit as a substitute for other sources of funding, and this occurs more markedly during periods of restrictive monetary policies. For Biais and Gollier (1997), bank credit and trade credit are complementary because the granting of trade credit by the suppliers of a firm reveals favourable information about the firm, a positive signal for other potential lenders.

While, on one hand, trade credit does provide the above-mentioned benefits -- including the attenuation of liquidity constraints and the coverage of liquidity risk -- on the other hand, trade credit relations create the grounds for the emergence of a systemic risk: the risk that through its default, the illiquidity or the insolvency of a firm can be transmitted to its suppliers, and by

[4] Some recent contributions that focus on the issues related to access to bank credit on the part of agricultural firms, point to an improvement in the capacity of the banking system to evaluate the creditworthiness of such firms. See, inter alia, Adinolfi F., Capitanio F., Sgroi F., (2012) and Adinolfi F., Capitanio F. (2009).

the latter, to other firms, across the network of contacts linked by trade credit. Kiyotaki and Moore (1997) analyse this risk of contagion in trade credit chains. One of the results obtained by these authors, which points to the negative externalities of trade credit contagion, is particularly important for the present paper. The authors argue that the common habit of granting postponement of payments to the defaulting buyer, as an alternative to claiming the liquidation of his assets through bankruptcy procedures, exacerbates the systemic effects of a liquidity shock because the liquidation of a defaulting debtor would inject liquidity into the supply chain while the postponements of payments does not. Thus while, on one hand, the deferral of payments is optimal for both the defaulting buyer and the supplier,[5] on the other, such behaviour amplifies the transmission of a liquidity shortage along the chains of trade credit. Kiyotaki and Moore also comment on the role played by firms with no liquidity constraints: the so called 'deep pockets'. These firms absorb the liquidity shortage which is transferred upstream in the chains of trade credit -- as a consequence of an initial shock -- and, in so doing, inject liquidity into the system. Boissay and Gropp (2007) empirically test the implications of the model by Kiyotaki and Moore using a large data-set of defaults of French firms.[6] Their results show that: i) there is a high probability that firms that face a liquidity shock do not honour their debts; ii) this probability of default is larger for unexpected shocks and for small firms subject to liquidity constraints; iii) on average, such firms transfer to their suppliers one quarter of the amount of the liquidity shock; iv) the chain of defaults, conveyed by the chain of trade credit, stops when it reaches a 'deep pocket' firm.[7]

In the next section we put forward a model of trade credit chains inspired to the work of Kiyotaki and Moore. The difference between their paper and the present one lies in the fact that Kiyotaki and Moore analyse the features of the optimal trade credit contract, while we forego such theoretical issues (that we take for granted) and aim to evaluate the role of trade credit in improving the resilience of firms to liquidity shocks and in exposing firms to systemic risk.

3. A model of trade credit chains

Unexpected liquidity shortages can have binding effects on the funding of the working capital of a firm and, consequently, on its production levels. Such effects are transmitted from firms facing a liquidity crisis to their suppliers to an extent that depends on the size of their trade credit obligations. In what follows, we analyse those sorts of effects of liquidity shortages which are caused by adverse events that generate unanticipated costs. The model described below lends itself, with simple adaptations, to characterise also the effects of liquidity shocks generated by other causes, such as a credit crunch, fluctuations of the product price and/or of the exchange rate (for exporting firms), etc.

The model is composed by two group of agents: banks and firms. Firms can be buyers, suppliers, or both buyers and suppliers of a homogeneous good y.[8] Moreover, we have two kinds of firms: ordinary firms which are subject to liquidity constraints due to credit rationing, and

[5] This feature of trade credit contracts has been proved by several authors. See, inter alia, Cuñat (2007) e Wilner (2000).

[6] The authors use more than 1.800.000 observations of default cases, data collected by the Chambers of Commerce of France.

[7] These results are in accordance with the model of trade credit contract by Cuñat (2007), as underlined by the authors.

[8] For our aims, the distinction between intermediate and final goods, between agricultural firms or other parts of the supply chain, is irrelevant.

'deep pocket' firms which have no liquidity constraints, firms that have access to external and/or internal funding.[9]

Banks behave competitively and are willing to lend and borrow at an interest rate equal to r. Banks face problems of asymmetric information and, as a consequence, they ration the amount of credit granted to the firms which are not 'deep pocket'. Apart from this point, we assume that the firms in the model are identical to one another.

For the sake of tractability, and without altering the nature of the results we obtain, we resort to the following simplifying hypotheses:

1. time is divided in periods; the length of a period corresponds to a production cycle. At the beginning of each period, a firm inherits the stock of good produced in the previous period. This stock is sold at the beginning of the period and paid by buyers only at the end of the period. In other words, each supplying firm grants trade credit to its clients for the entire amount of their purchases and for the duration of a production cycle;
2. on the basis of the theoretical and empirical results cited above, we assume that a supplier never asks for the liquidation of a defaulting buyer and always accepts to defer the payment to the next period;
3. given that the focus of the model is on the funding of production, we characterise the production function as a function of the investment in working capital only, foregoing the stock of fixed capital, that we assume to be adequate to the production level in the steady state equilibrium described below.[10] Moreover, we assume that the production function exhibits constant returns to scale and that the combination of the variable factors of production, namely labour and inventories, is fixed.

3.1. The steady state

Let N be a set composed by n firms. Let us assume that such firms form a supply chain, where each firm has just one supplier and one buyer,[11] and that they are linked to one another by trade credit obligations. Let us define the steady state of such a supply chain as the state in which firms operate at full capacity and produce the optimal amount of output y^* We assume that the price of output is fixed and we take it as numeraire, setting it equal to unity. As mentioned above, we assume a technology of production such that: i) labour and inventories are combined in a fixed proportion equal to $k{:}I = kl$, where I is the stock of good (inventories) that is used in the production process, and l is the amount of labour; ii) the production function, expressed in terms of value of output, is linear in the amount of the current investment in working capital: $y = \alpha\ (I + wl) = I\alpha\ (1{+}kw)$, where w is the (real) wage and $\alpha > 1$ is a scalar.

At the beginning of each period, each firm does the following: i) sells to its client the entire output produced in the previous period; ii) buys from its suppliers the inventories of good I^* to be used in the production of the current period, where I^* is such that $I^*\ \alpha\ (1{+}kw) = y^*$; iii) obtains an amount of bank credit equal to B, paying an interest rate equal to r; iv) purchases the amount of labour that is necessary to produce the desired output. The transactions sub (i) and

[9] The opportunity cost of such liquid reserves, that can be lent to banks, is r, the interest rate paid by banks.

[10] This assumption is compatible with technologies which have sufficient flexibility in the utilisation of fixed capital to allow maintenance of the optimal combination of fixed and working capital for all output levels.

[11] This assumption is more realistic than it might appear at first glance. As Boissay puts it, "Trade credit is in general not well diversified at the firm level, as firms' customers tend to belong to a specific sector. It is indeed not rare for a company to have one large trade credit vis-a-vis one main client on its books, which may represent the entire profit of the year." [Boissay (2006), page 5].

(ii) are cleared at the end of each period, when each firm receives, from its clients, sale revenues equal to $y^* (1 + \delta)$, and uses such cash flows to pay $B (1 + r)$ to the bank and $I^* (1 + \delta)$ to its supplier, where $\delta > r$ is the interest rate paid on trade credit. The profits π^*, that remunerate the proprietors of the firm, are equal to $y^* (1 + \delta) - B (1 + r) - I^* (1 + \delta)$, and we assume that, in the steady state, they are entirely distributed to shareholders. The flow-of-funds equation of a firm in N, during the steady state, is then equal to:

$$y^* (1 + \delta) = B (1 + r) + I^* (1 + \delta) + \pi^*$$

3.2. The effects of a shock on a single firm

In this section we proceed to characterise the effects of a shock on a firm $\alpha \in N$. To this end, we consider a succession of production cycles, i.e. a succession of time periods $t = 1, 2, 3, \ldots$, and perturbation of the steady state of the supply chain by an exogenous shock -- such as a livestock epidemic, adverse weather, the breakdown of a plant, etc. -- that affects a firm a in N, inflicting on it a loss of σ. In the first period, when the shock occurs, the loss is absorbed (partially or totally) by the current profits, π^*. Let us assume that the loss is sufficiently large to induce the default of the firm: $\sigma > \pi^*$. In such a case, the revenues coming from the goods sold at the beginning of the period are insufficient to cover the operating expenses:

$$(1) \qquad y^* (1 + \delta) < B (1 + r) + I^* (1 + \delta) - \sigma,$$

and the firm does not honour its trade debt with its supplier b for an amount equal to $\lambda = \delta - \pi^*$, i.e. equal to the liquidity shortage suffered by a. Given the time structure of the model, the effects of such a liquidity shortage start occurring in the second period, when firms have to pay back the deferred trade debt. As assumed above, the supplying firm b does not claim the liquidation of the assets of its debtor and accepts a deferral of the payment to the next period. Thus, in the second period, the firm must pay $\lambda (1 + \delta)$ for the debt backorder and may not have sufficient liquidity to fund the notional level of production y^*. The impact of a binding liquidity constraint on the investment in working capital, hence of the production level, depends on the degree of flexibility that a firm has in choosing the amount of labour to purchase in each period.

3.3. Non flexible labour contracts

If the amount of labour bought by a firm -- at the beginning of a period -- is fixed by previously signed contracts, a reduction in the investment in working capital, caused by a liquidity deficit, affects solely the amount of inventories purchased for the current production cycle. In this case it is relatively simple to characterise the impact of a shock on the investment in inventories and, consequently, on production levels. At the beginning of period 2 -- i.e. the period subsequent to the default of firm a -- firm b is willing to grant trade credit to its client a only insofar as a can fully pay back its debt. The capability of a to honour its debt with b depends on the relative magnitude of the exogenous shock and of the profits of the firm. If $\lambda (1 + \delta) < \pi^*$, then the surplus yielded by the production inherited from the previous period enables a to purchase the notional amount of inventories I^* and to achieve the notional level of production y^*. If, vice versa, $\lambda (1 + \delta) > \pi^*$, i.e. if:

$$\sigma > \pi^* \ \ 1 + \ \ 1/ \ 1 + \delta$$

then the revenues accruing in the second period are not sufficient to absorb the liquidity shortage and the firm faces an upper bound in the current investment in working capital. The supplier b, being aware of the actual conditions of its client a, is willing to grant trade credit up to an amount which is smaller than or equal to the spending power of a, which is equal to $y^* (1 + \delta) - B (1 + r) - \lambda (1 + \delta)$, i.e. the revenues from the beginning-of-the-period sales minus the outstanding debt towards the bank and the supplier. Thus, in the case at hand, the investment in inventories in period 2, I_2 is strictly smaller that the notional level I^* and is equal to:

$$I_2 (1 + \delta) = y^* (1 + \delta) - B (1 + r) - \lambda (1 + \delta)$$

hence:

$$\text{(2)} \qquad I_2 = y^* B ((1 + r) / (1 + \delta)) - \lambda.$$

The end-of-the-period output, obtained with this stock of inventories, is equal to:

$$\text{(3)} \qquad y_2 = I_2 \alpha (1 + kw)$$

which is strictly smaller than y^*. The reduction in investment, with respect to the steady state level, is equal to:

$$I^* - I_2 = \lambda - ((\pi^*) / (1 + \delta));$$

and, since we assumed constant returns to scale, the decrease in production is proportional to the decrease in investment:

$$\text{(4)} \qquad y^* - y_2 = (\lambda - \left(\frac{\pi^*}{1 + \delta}\right)) \alpha (1 + kw).$$

Finally, the amount of labour that remains idle in firm a is equal to:

$$l^* - l_2 = (1/k) \Delta_2 I = (1/k) (\lambda - \left(\frac{\pi^*}{1 + \delta}\right)).$$

The decrease in production in the second period induces a proportional reduction in revenues of the third period, $y_2 (1 + \delta)$ that can be rewritten as:

$$\text{(5)} \qquad y_2 (1 + \delta) = [y^* - \left(\frac{\pi^*}{1 + \delta}\right)) \alpha (1 + kw)] (1 + \delta).$$

If:

$$y_2 (1 + \delta) < B (1 + r) + I^* (1 + \delta)$$

then the investment in inventories in the third period is bound by the liquidity shortage. Recalling that $y^* (1/(\alpha(1 + kw))) = I^*$, let us rewrite this inequality as:

$$[y^* - (y - \left(\frac{\pi^*}{1+\delta}\right)) \alpha (1 + kw)] (1 + \delta) < B (1 + r) + y^* \left(\frac{1}{\alpha (1 + kw)}\right) (1 + \delta)$$

and:

$$(y - \left(\frac{\pi^*}{1+\delta}\right)) \alpha (1 + kw) > y^* (1 - \left(\frac{1}{\alpha (1 + kw)}\right) - B \left(\frac{1+r}{1+\delta}\right).$$

Since the right-hand-side of this inequality is equal to $y^* - I^* - B ((1 + r) / (1 + \delta))$ which, in turn, is equal to $\pi^* (1/(1 + \delta))$, we get:

$$(y - \left(\frac{\pi^*}{1+\delta}\right)) \alpha (1 + kw) > \left(\frac{\pi^*}{1+\delta}\right),$$

then:

$$\lambda\alpha (1 + kw) > \left(\frac{\pi^*}{1+\delta}\right) + \left(\frac{\pi^*}{1+\delta}\right) - \alpha (1 + kw),$$

and finally:

$$\lambda > \pi [\left(\frac{1}{1+\delta}\right) + \left(\frac{1}{(1 + \delta) \alpha (1 + kw)}\right)]. \tag{6}$$

If this equality does not hold, the revenues yielded by production in the previous period is sufficient to finance the purchase of the notional amount of inventories in the third period. In such a case the firm returns, in the third period, to the notional steady state production y^*. Conversely, if the above inequality is satisfied, i.e. if the exogenous shock is such that:

$$\sigma > \pi^* [1 + (1/(1 + \delta)) + (1/(1 + \delta) \alpha (1 + kw))],$$

then the investment is constrained by the liquidity deficit: $I_3 < I$^ {*}. In such a case we have:

$$I_3 (1 + \delta) = y_2 (1 + \delta) - B (1 + r)$$

$$= [y^* - (y - \pi^* \alpha (1 + kw)] (1 + \delta) - B (1 + r),$$

thus:

$$I_3 = y^* - (\lambda - \pi^*) \alpha (1 + kw) - B ((1 + r) / (1 + \delta))$$

and, finally, the decrease in investment and production in the third period are respectively equal to:

$$I^* - I_3 = I^* - y^* + (\lambda - \pi^*)\, \alpha\, (1 + kw) + B\, ((1 + r) / (1 + \delta))$$

$$= (\lambda - \pi^*)\, \alpha\, (1 + kw) - \pi$$

$$y^* - y_3 = \alpha\, (1 + kw)\, [I^* - I_3]$$

$$= \alpha\, (1 + kw)\, [(\lambda - \pi^*)\, \alpha\, (1 + kw) - \pi^*]. \tag{7}$$

Note that the investment and production suffer a further reduction in the third period, $I_3 < I_2$, if:

$$I_2 - I_3 = (\lambda - \pi^*)\, \alpha\, (1 + kw) - \lambda > 0$$

from which we get:

$$\lambda\, [\alpha\, (1 + kw) - 1] > \pi^*\, \alpha\, (1 + kw)$$

and finally:

$$\lambda > \pi^*\, (\alpha\, (1 + kw) / (\alpha\, (1 + kw) - 1)). \tag{8}$$

It can be checked by inspection[12] that this threshold value of the liquidity shortage, that we call *inter-temporal contagion threshold*, is larger than the one set by equation (4). This threshold shows that exogenous shocks such that:

$$\sigma > \pi^*\, [1 + (\alpha\, (1 + kw) / (\alpha\, (1 + kw) - 1))]$$

trigger a decelerator effect of the investment in inventories, caused by the liquidity shortage, that amplifies the reductions in production occurring in the periods subsequent to the repayment of the backorder debt.[13] In this scenario, and from the third period on, the investment of firm a returns progressively towards the steady state level, thanks to self-financing. The new liquid resources that became available in a period $t > 3$, are the ones generated by the production surplus produced in the previous period. As we assumed constant returns to scale, and a fixed proportion $\alpha > 1$ between the investment in working capital and the corresponding output, from period 3 onwards the resources available for self-financing are progressively larger and, period after period, they gradually fill the liquidity deficit and enable the return of the firm to the steady state optimal level of production.

[12] It is sufficient to check that $((\alpha\, (1 + kw)) / (\alpha\, (1 + kw) - 1)) - ((\alpha\, (kw + 1) + 1) / (\alpha\, (kw + 1)\, (1 + \delta))) > 0$.

[13] It is interesting to note that this threshold, keeping profits π^*constant, diminishes as the production surplus $\alpha\, (kw + 1)$ grows. For instance, for $\alpha\, (kw + 1) = 1{,}5$ we get $\lambda''' = 3\, \pi$, while for $\alpha\, (kw + 1) = 1{,}3$ we obtain $\lambda''' = 4{,}3\, \pi^*$ The rationale for this result lies in the fact that the larger the surplus accruing on the investment in inventories, the larger the opportunity cost -- in terms of missed production and revenues -- caused by the liquidity constraint to the investment in the second period.

In synthesis, we have three contagion thresholds of the exogenous shock that correspond to three different scenarios, in terms of the effects of the shock on the investment and output levels:

$$\sigma' = \pi^* [1 + (1/(1 + \delta))]$$
$$\sigma'' = \pi^* [1 + (1/(1 + \delta)) + (1/(1 + \delta) \alpha (1 + kw))]$$
$$\sigma''' = \pi^* [1 + (\alpha (1 + kw) / (\alpha (1 + kw) - 1))]$$

where:

1. For shocks $\sigma \geq \sigma'$, the earnings of the second period are not sufficient to absorb the liquidity deficit and investment and production in the second period are constrained: $I_2 < I^*$ and $y_2 < y^*$.
2. For shocks $\sigma \geq \sigma''$ investment and production are constrained also in the third period: $I_3 < I^*$ and $y_3 < y^*$.
3. Finally, for shocks $\sigma \geq \sigma'''$ the liquidity constraint on investment is sufficiently large to generate a decelerator effect: the peak in the decrease of activity occurs in the third period after the shock: $I_3 < I_2$ and $y_3 < y_2$.

These thresholds lends themselves to be used as measures of the exposure to liquidity risk of a firm that faces a liquidity constraint and resorts to trade credit.

3.4. Flexible labour contracts

Let us now assume that labour contracts are flexible, i.e. they get signed period by period. In this case the contraction of the investment in working capital $I + wl$, caused by an exogenous shock, involves both the purchase of inventories and the purchase of labour. From the assumption made above that production occurs with a fixed proportion between the inputs: $I = kl$, it follows that, for each euro spent, a portion equal to $(k/(k + w))$ cents is spent to buy inventories and a portion equal to $(w/(k + w))$ cents is used to buy labour.

Let us now consider again the firm a in N. As above, let us assume that a suffers unexpected costs equal to $\sigma > \pi^* [1 + (1/(1 + \delta))]$ in period 1, i.e. a shock larger than the threshold σ' defined above, and that its supplier *b* is willing to grant to *a* an amount of trade credit not larger than what *a* can actually pay back:

$$I_2 (1 + \delta) = y^* (1 + \delta) - B (1 + r) - \lambda (1 + \delta)$$

thus:

$$\hat{I}_2 = y^* - B ((1 + r) / (1 + \delta)) - \lambda, \qquad (9)$$

where $\hat{I}_2$ is the amount of inventories that a buys resorting to trade credit. The shock σ is large enough to constrain production in the second period below the steady state level y^* and, therefore, a purchases, in period 2, an amount of labour less than l^*. It follows that firm *a* can purchase inventories, paying them up-front, using the bank credit *B* which is not used to pay labour.[14] Then bank credit is used as follows:

[14] Given the simplifying assumptions made above, we have that, in the steady state, the bank credit B available to a firm is equal to the wages wl^* and inventories are bought using trade credit. This restriction can be easily removed, defining the steady state of the model for any initial liability structure of a solvent firm.

$$B = wl_2 - I_2 - \hat{I}_2$$

rewritten as:

$$(10) \qquad wl_2 = B - (I_2 - \hat{I}_2),$$

where $I_2 - \hat{I}_2$ is the amount of inventories paid up-front by firm *a*. Substituting (9) in (10), and recalling the technological restriction $kl_2 - I_2$, we get the values of the constrained investments in inventories and labour in the second period:

$$I_2 = (1/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda]$$

and:

$$I_2 = (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda]$$

The gap between the constrained investment in inventories and the notional level I^* is equal to:

$$I^* - I_2 = y^* (1/(\alpha(1 + kw))) - (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda]$$
$$= y^* [(1/(\alpha(1 + kw))) - (k/(w + k))] - (k/(w + k)) [B ((\delta - r) / (1 + \delta)) - \lambda]$$

The constrained investment I_2 yields an output equal to:

$$y_2 = \alpha (1 + kw) (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda]$$

and the gap with respect to the notional output y^* is equal to:

$$(11) \qquad y^* - y_2 = y^* - \alpha (1 + kw) (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda]$$

The reduction of the output produced in the second period diminishes the cash flows earned by the firm in the third period. As above, if such a decrease of earnings is larger than the steady state profits π^*, i.e. if $(y^* - y_2) (1 + \delta) > \pi^*$ the investment in inventories in the third period is bound by the insufficient liquidity $I_3 < I^*$. To characterise the threshold value of an exogenous shock large enough to generate such a scenario, rewrite $y^* - y_2 > (\pi^*/(1 + \delta))$ as:

$$y^* - \alpha (1 + kw) (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda] > (\pi^*/(1 + \delta))$$

then:

$$\lambda\alpha (1 + kw) (k/(w + k)) > (\pi^*/(1 + \delta)) - y^* [1 - \alpha (1 + kw) (k/(w + k))]$$
$$+ \alpha (1 + kw) (k/(w + k)) B ((\delta - r) / (1 + \delta))$$

and finally:

$$(12) \qquad \lambda > (\pi^*/(1 + \delta)) ((w + k) / (\alpha(1 + kw)k)) + y^* [1 - (w + k) / (k\alpha (1 + kw))]$$
$$+ B ((\delta - r) / (1 + \delta)),$$

that corresponds to the shock threshold:

$$\sigma'' = \pi^* + (\pi^*/(1 + \delta)) ((w + k) / (\alpha(1 + kw)k)) + y^* [1 - (w + k) / (k\alpha(1 + kw)))] + B((\delta - r) / (1 + \delta))$$

In other words, for shocks larger than σ'', the production in the third period is constrained by the lack of liquidity caused, in turn, by the sub-optimal investment realised in the second period. Knowing that the residual spending power is invested in inventories paid with trade credit: $y_2 (1 + \delta) - B (1 + r) = I_3 (1 + \delta)$, we have that:

$$I_3 = y_2 - B ((1 + r) / (1 + \delta)).$$

As in the second period, we have that $wl_3 = B - (I_3 - I_3)$ and since $kl_3 = I_3$ we obtain:

$$I_3 = (k/(w + k)) [y_2 + B ((1 + r) / (1 + \delta))].$$

and:

$$y_3 = \alpha(1 + kw) (k/(w + k)) [y_2 + B ((1 - r) / (1 + \delta))].$$

The production gap in the third period is equal to:

$$(13) \qquad y^* - y_3 = y^* - ((\alpha(1 + kw)k) / (w + k)) [y_2 + B ((1 - r) / (1 + \delta))].$$

To characterise the third contagion threshold of the exogenous shock, above which the production gap grows in the third period, i.e. $y_3 < y_2$, we start from the fact that $y_3 < y_2$ implies $I_3 < I_2$. For $I_3 = y^* + B ((1 - r) / (1 + \delta)) - (y^* - y_2)$ to be smaller than $I_2 = y^* - B ((1 - r) / (1 + \delta)) - \lambda$, it is necessary that $(y^* - y_2) - \lambda = 0$. Rewrite this inequality as:

$$y^* - \alpha(1 + kw) (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta)) - \lambda] - \lambda > 0$$

then:

$$y^* - \alpha(1 + kw) (k/(w + k)) [y^* + B ((\delta - r) / (1 + \delta))] + \lambda [\alpha(1 + kw) (k/(w + k)) - 1] > 0$$

$$y > ((y^* [\alpha(1 + kw) (k/(w + k)) - 1] + B ((\delta - r) / (1 + \delta)) \alpha (1 + kw) (k/(w + k))) / (\alpha(1 + kw) (k/(w + k)) - 1))$$

and finally:

$$y > y^* + B ((\delta - r) / (1 + \delta)) ((\alpha (1 + kw)k) / (\alpha (1 + kw) k - w - k))$$

from which we get the value of the third contagion threshold:

$$\sigma''' = \pi^* + y^* + B ((\delta - r) / (1 + \delta)) ((\alpha (1 + kw)k) / (\alpha (1 + kw) k - w - k))$$

Under the above assumptions, a shock larger than σ''', i.e. larger than the sum of steady state profits and output, causes the bankruptcy of the firm. Clearly such a shock is much larger than the shocks that can be absorbed thanks to the use of trade credit.[15]

The flexibility of the labour contracts, assumed in this case, prevents the occurrence of the decelerator effect shown in the previous section: it is impossible that a shock that makes the firm illiquid but not insolvent, i.e. shocks such that $\sigma < I^* (1 + \delta) + \pi^*$, can cause a further decrease of output in the third period.

As above, we have three threshold values of the exogenous shock that divide the set of possible shocks into three ranges to which correspond three levels of impact on investment and production:

$$\sigma' = \pi^* (1 + (1/(1 + \delta)))$$
$$\sigma'' > \pi^* + (\pi^*/(1 + \delta)) ((w + k) / (\alpha (1 + kw)/k)) + y^* [1 - ((w + k) / (k\alpha(1 + kw)))] + B ((\delta - r) / (1 + \delta))$$
$$\sigma''' > \pi^* + y^* B ((\delta - r) / (1 + \delta)) ((\alpha (1 + kw)/k) / (\alpha (1 + kw) k -w - k))$$

where:

1. For shocks $\sigma \geq \sigma'$, investment and production are constrained by the liquidity shortage: $I_2 < I^*$ and $y_2 < y^*$
2. For shocks $\sigma \geq \sigma''$, investment and production are constrained also in the third period: $I_3 < I^*$ and $y_3 < y^*$
3. For shocks $\sigma \geq \sigma'''$, the liquidity constraint on investment is large enough to generate the above described decelerator effect: $I_3 < I_2$ and $y_3 < y_2$

4. Contagion in a supply chain

We now proceed to analyse the dynamics of the financial contagion that travels along the network of vulnerable firms linked by trade credit exposure. Let us assume that i) firm *a* suffers an unexpected loss equal to $\sigma > \pi^*$ and, consequently, defaults on its supplier for an amount equal to its own liquidity shortage: $\lambda_a = \sigma - \pi^*$; and ii) firm *b* grants a deferral of such payments to the next period. The deferred earnings generate a liquidity shortage of firm *b* equal to $\lambda_b = \lambda_a - \pi^* = \sigma - 2\pi^*$. If firm *b* is a 'deep pocket', i.e. if it can acquire liquidity at an interest rate (or opportunity cost) equal to *r*, then *b* borrows from a bank and/or uses its own liquid reserves to cover the liquidity deficit. Thus, being a 'deep pocket', *b* injects liquidity into the supply chain and stops the contagion process. Conversely, if firm *b* faces a liquidity constraint, i.e. it is not a 'deep pocket', then also firm *b* is enforced to default on its own supplier, firm *c*, for an amount equal to λ_b. Then, if *c* is a 'deep pocket', the contagion process stops. If, vice versa, *c* is not a 'deep pocket', it transfers part of its own liquidity shortage on its suppliers *d*, defaulting on its trade credit for an amount equal to $\lambda_c = \lambda_b - \pi^* = \sigma - 3\pi^*$. And so forth: while the contagion process does not involve a 'deep pocket' firm, the propagation of the liquidity shock continues upstream in the supply chain. At each step of the process, the liquidity shortage is progressively absorbed by the retained profits of the firms involved in the contagion.

[15] It can be checked by inspection that all shocks such that π^* can be absorbed by the firm, thanks to trade credit, without causing the bankruptcy of the firm.

We assumed above that all firms in *N* are equal to one another and that they all earn, in the steady state, profits equal to π^*. Under these conditions, the number of firms involved in the contagion is equal to the smallest among the natural numbers which are larger than or equal to σ / π^*, as long as there is no 'deep pocket' among them. On the contrary, the contagion process stops when it reaches a 'deep pocket'. Let *D* be the succession of firms, along the supply chain, involved in the default contagion caused by a shock σ suffered by firm a, and let m be the number of firms in *D*. Let us index such firms with $j = 1, 2, 3, ..., m$, where $j - 1$ is the number of steps between firm a and the *j*-th firm along the contagion chain. Then, the *j*-th firm suffers the default of its client for an amount equal to $\sigma - (j - 1)\ \pi^*$, facing a consequent liquidity deficit equal to $\lambda_j = \sigma - j\pi^*$. The effects of such liquidity shortages for the investment and production of a single firm have been discussed in the above section. To complete the analysis, we now proceed to compute the overall effect of a shock on the activity levels of the firms affected by its propagation.

4.1. Contagion with non flexible labour contracts

The above derived equation (3) shows the production gap of the *j*-th firm, in the period after the occurrence of a shock $\sigma > \sigma'$ as a function of the corresponding liquidity deficit:

$$y^* - y_2 = (\lambda_j - (\pi^*/(1 + \delta)))\ \alpha\ (1 + kw).$$

Assuming that there are no 'deep pocket' firms among those involved in the contagion, the total production gap in the whole supply chain in the second period is equal to:

$$\textstyle\sum_{y=1}^{m} (\sigma - j\pi^* - (\pi^*/(1 + \delta)))\ \alpha\ (1 + kw)$$

rewritten:

$$m\ (\sigma - (\pi^*/(1 + \delta)))\ \alpha\ (1 + kw) - \textstyle\sum_{y=1}^{m} j\pi^*\ \alpha\ (1 + kw).$$

Conversely, if the -th firm along the supply chain, for $h < m$, is a 'deep pocket', the total production gap is equal to:

$$\textstyle\sum_{y=1}^{h-1} (\sigma - j\pi^* - (\pi^*/(1 + \delta)))\ \alpha\ (1 + kw)$$

In period three, for shocks $\sigma > \sigma''$, the production gap of firm with respect to the steady state level of output is:

$$y^* - y_3 = \alpha\ (1 + kw)\ [(\lambda_j - \pi^*)\ \alpha\ (1 + kw) - \pi^*]$$

from which we get the total production gap for the whole supply chain:

$$\textstyle\sum_{y=1}^{m} \alpha\ (1 + kw)\ [(\sigma - (j + 1)\ \pi^*)\ \alpha\ (1 + kw) - \pi^*]$$

that, if there is in *D* a firm h that is 'deep pocket', becomes:

$$\textstyle\sum_{y=1}^{h-1} \alpha\ (1 + kw)\ [(\sigma - (j + 1)\ \pi^*)\ \alpha\ (1 + kw) - \pi^*].$$

4.2. Contagion with flexible labour contracts

In this case the production gap of the j-th firm in D is set by equation (11):

$$y^* - y_2 = y^* - \alpha\,(1 + kw)\,(k/(w + k))\,[y^* + B\,((\delta - r) / (1 + \delta) - \lambda]$$

then the total production gap for the whole supply chain is:

$$\textstyle\sum_{y=1}^{m}\ y^* - \alpha\,(1 + kw)\,(k/(w + k))\,[y^* + B\,((\delta - r) / (1 + \delta))] + \alpha\,(1 + kw)\,(k/(w + k))\,\lambda$$

that can be rewritten as:

$$m\,[y^* - \alpha\,(1 + kw)\,(k/(w + k))\,(y^* + B\,((\delta - r) / (1 + \delta)))] \\ +\textstyle\sum_{y=1}^{m}\,[\alpha\,(1 + kw)\,(k/(w + k))\,(\sigma - j\pi^*)].$$

If D in there is a firm h that is 'deep pocket', such a total gap is equal to:

$$\textstyle\sum_{y=1}^{h-1}\ y^* - \alpha\,(1 + kw)\,(k/(w + k))\,[y^* + B\,((\delta - r) / (1 + \delta))] + \alpha\,(1 + kw)\,(k/(w + k))\,\lambda\ .$$

In order to characterise the production gap in the third period, we start from equation (13):

$$y^* - y_3 = y^* - ((\alpha\,(1 + kw)k) / (w + k))\,[y_2 + B\,((\delta - r) / (1 + \delta)]$$

from which, substituting y_2 into it, we get

$$y^* - y_3 = y^* - ((\alpha\,(1 + kw)k) / (w + k))\,B\,((\delta - r) / (1 + \delta)) - ((\alpha\,(1 + kw)k) / (w + k))\,y_2$$
$$y^* - y_3 = y^* - ((\alpha\,(1 + kw)k) / (w + k))\,B\,((\delta - r) / (1 + \delta)) - [((\alpha\,(1 + kw)k) / (w + k))]^2 \\ [y^* + B\,((\delta - r) / (1 + \delta)) - y_j].$$

Finally, the overall gap for the supply chain is the sum of such gaps:

$$\textstyle\sum_{y=1}^{m}\ y^* - ((\alpha\,(1 + kw)k) / (w + k))\,B\,((\delta - r) / (1 + \delta)) - [((\alpha\,(1 + kw)k) / (w + k))]^2 \\ [y^* + B\,((\delta - r) / (1 + \delta)) - y_j]\}.$$

that can be expressed as:

$$m\ \ y^* - ((\alpha\,(1 + kw)k) / (w + k))\,B\,((\delta - r) / (1 + \delta)) - [((\alpha\,(1 + kw)k) / (w + k))]^2 \\ [y^* + B\,((\delta - r) / (1 + \delta))]\} + \textstyle\sum_{y=1}^{m}\,(\sigma - j\pi^*)\,[((\alpha\,(1 + kw)k) / (w + k))]^2.$$

5. Conclusions

The current economic and financial crisis, with its detrimental effects on the funding of productive activities, has once again brought into the foreground the problems related to the financing of agricultural production. This paper addresses one facet of this issue: the role of trade credit in determining liquidity provision and the liquidity and systemic risks of agri-food firms that operate in supply chains. To this end, we put forward a simple, benchmark model of trade credit chains. Being aware of the different specific features of different sub-sectors of the agri-food industry, we designed the model in general terms, in order to present an analytical tool that can be promptly adapted to the study of specific agri-food supply chains by imposing on it the corresponding restrictions (such as rigidities in the short-run production levels, different lengths of the production cycle, different degrees of access to bank credit, etc.). The main output of our analysis is the characterisation of three threshold values of an exogenous shock to which correspond three different contagion scenarios, namely three levels of impact on the investment in working capital, and hence on the activity levels, of the firms that belong to a supply chain. Such thresholds lend themselves to be used as measures of the resilience of supply chains to unexpected liquidity shocks. Thus, by calibrating the model presented above to the specific features of specific agri-food supply chains, the analyst can make an evaluation of the liquidity conditions of a supply chain that takes explicitly into account the liquidity and systemic risks to which that supply chain is exposed. This line of research, i.e. the application of the present framework to empirical data concerning agri-food supply chains, is on our agenda.

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