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Assessing Spatial Break-even Variability in Fields with Two or More Management Zones

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ABSTRACT

Farmers are interested in knowing whether applying inputs at variable rates across a field is economically viable. The answer depends on the crop, the input, their prices, the cost of variable rate technology (VRT) versus uniform rate technology (URT), and the spatial and yield response variability within each field. Methods were investigated for determining the range of spatial variability over which the return to VRT covers its additional cost compared with URT in fields with multiple management zones. Models developed in this article, or variants thereof, could be used to help farmers make the VRT adoption decision.

Key Words: *management zones, nitrogen, precision farming, site-specific management, spatial break-even variability proportions, spatial variability, variable rate technology, yield response variability.*

Agricultural fields consist of numerous areas that differ from one another with respect to the factors that condition crop growth (Carr *et al.*; Hannah, Harlan, and Lewis; Hibbard *et al.*; Malzer *et al.*; Sawyer; Spratt and McIver). Precision farming uses a set of technologies to gather information about the heterogeneous makeup of a farm field and uses that information to make management decisions that address site-specific crop needs within the field (Swinton and Lowenber-DeBoer). Its component technologies enable farmers to understand the changing plant-growth environment across a field, estimate input requirements for relatively homogeneous smaller-than-field-size units, and apply inputs on a site-specific basis. Two important benefits of precision farming are claimed to be increased profits to farmers and reduced environmental harm resulting from more precise placement of inputs (Kitchen et al.; Koo and Williams; Sawyer; Watkins, Lu, and Huang). The key, however, to the acceptance of site-specific farming is the profitability of using these technologies (Daberkow; Reetz and Fixen; Sawyer).

Lowenberg-DeBoer and Swinton reviewed 17 precision farming studies conducted before 1998 and found inconclusive evidence about the profitability of site-specific management in field crops. Of the studies reviewed, 12 used empirical yields and five used simulated yields to determine profitability. At least nine additional studies have been conducted since Low-

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enberg-DeBoer and Swinton's review, one of which used empirical yields (Lowenberg-DeBoer and Aghib), while eight used simulated or hypothetical yields (Babcock and Pautsch; Bongiovanni and Lowenberg-DeBoer, 1998; Bullock et al.; English, Roberts, and Mahajanashetti; Lowenberg-DeBoer; Roberts, English, and Mahajanashetti; Thrikawala et al.; Watkins, Lu, and Huang). With these additional studies the profitability of site-specific input management is still inconclusive. The disparity in results stems from differences in assumptions about costs, yield response, and the value of the crop (Lowenberg-DeBoer and Swinton).

Another reason for finding different profitability results across fields is differences in spatial variability, where *spatial variability* is defined as the distribution across a field of management zones with different crop yield responses to an input (Roberts, English, and Mahajanashetti). Within-field variability in soil physical and chemical characteristics is a necessary condition for the economic viability of using variable rate technology (VRT) (English, Roberts, and Mahajanashetti; Forcella; Hayes, Overton, and Price; Roberts, English, and Mahajanashetti; Snyder). Relationships among crop yields, the level of input applied, and soil characteristics determine spatial variability within a field. These relationships also determine yield response variability, where *yield response variability* is defined as the differences in magnitudes of yield response among management zones (English, Roberts, and Mahajanashetti; Forcella; Roberts, English, and Mahajanashetti). Spatial and yield response variability, along with the crop price, the input price, and the additional cost of using VRT versus uniform rate technology (URT) factor into the economic decision to adopt VRT.

Roberts, English, and Mahajanashetti developed a theoretical model for evaluating the economic viability of VRT for fields with two management zones. Frequently, however, a decision-maker is faced with more than two management zones within a given field. The research presented in this article extends their model to multiple management zones. The objective of this research was to investigate methods for determining the range of spatial variability over which the return to VRT covers its additional cost compared with URT in fields with two or more management zones. The methods are presented in theoretical form and evaluated with sensitivity analyses using hypothetical examples.

Theoretical Model

Assume farmers are profit maximizers who can classify their fields into m management zones and have knowledge of the management-zone-specific yield response functions for a given crop and input. Suppose further that yield responses can be represented by concave functions (diminishing marginal physical product) and that fields can include any of these m management zones in any proportions. Assume the cost of obtaining knowledge about the management zones and their yield response functions is a sunk cost with regard to the decision of whether to use VRT instead of URT. Let the response functions be represented by equations (1).

$$(1) \qquad Y_i = Y_i(X_i) \qquad i = 1, 2, \ldots, m$$

where Y_i is crop yield per acre for the i^{th} management zone and X_i is the amount of input applied per acre to the i^{th} management zone.

A farmer using VRT on a particular field determines the optimal application rates for the m management zones by equating the marginal physical products of the respective response functions with the input-to-crop price ratio. Optimal return above input cost per acre for the field under VRT (R^*_{VRT}) is then calculated from the following profit function (Nicholson),

$$(2) \qquad R^*_{VRT} = \sum_{i=1}^{m} \lambda_i [P_Y Y_i(X_i^*) - P_X X_i^*]$$

$$= \sum_{i=1}^{m} \lambda_i (\pi_i^*)$$

$$= R^*_{VRT}(\lambda_1, \lambda_2, \ldots, \lambda_{m-1}, P_Y, P_X)$$

where P_Y is the crop price; P_X is the input price; X_i^* is the optimal input application rate for the i^{th} management zone; π_i^* is optimal return above input cost for the i^{th} management zone; and λ_i is the proportion of the field in the i^{th} management zone such that $\Sigma_{i=1}^{m} \lambda_i = 1$. Thus, R^*_{VRT} is the weighted average over λ_i of the optimal returns above input costs per acre obtained from each management zone. The proportion of the field in management zone m (λ_m) is not included as an argument in the R^*_{VRT} function because $\lambda_m = 1 - \Sigma_{i=1}^{m-1} \lambda_i$.

Numerous decision rules could be assumed for URT application of the input, two of which are explored as examples below. The first rule assumes farmers base URT decisions on the profit-maximizing input level obtained from a weighted average yield response function, with the proportions of the field in each management zone serving as weights. The second rule assumes farmers determine the uniform rate for the entire field as the profit-maximizing level of input obtained from one management zone (eg., the highest or medium response management zone). These two examples are presented to demonstrate that the return to variable rate technology (RVRT) is a nonlinear or linear function of λ_i depending on the decision rule used for URT rather than depending on the shape of the response functions assumed in equations (1).

Weighted Average Response Function

Determining the optimal uniform rate based on the weighted average response function is analogous to some methods used to develop fertilizer recommendations. For example, receiving a recommendation from a soil test laboratory based on a soil sample that mixes soil cores drawn at random across a field (VanEck and Collier) is similar to weighting the recommendations for the management zones by the proportions of the field in each management zone. In addition, soil-test laboratories and the Extension Service often base their fertilizer recommendations on yield goals developed by farmers (Savoy and Joines). These yield goals can be formed in a variety of ways (O'Neal et al.). If the farmer forms the field yield goal by implicitly averaging yield goals across management zones, the field yield goal and the fertilizer recommendation would be approximately weighted by the proportions of the field in each management zone.

Assume the farmer determines the optimal uniform application rate based on the field average response function expressed as

$$(3) \qquad Y_u(X_u) = \sum_{i=1}^{m} \lambda_i Y_i(X_u)$$

where $Y_u(X_u)$ is the weighted average crop yield response function and X_u is the uniform input application rate. The optimal return above input cost per acre for URT (R^*_{URT}) is calculated from the following profit function:

$$(4) \qquad R^*_{URT} = P_Y \sum_{i=1}^{m} \lambda_i Y_i(X^*_u) - P_X X^*_u$$
$$= R^*_{URT}(\lambda_1, \lambda_2, \ldots, \lambda_{m-1}, P_Y, P_X)$$

where X^*_u is the optimal uniform application rate obtained by equating the marginal physical product of the average yield response function in equation (3) with the input-to-crop price ratio. Again λ_m is excluded as an argument because $\Sigma_{i=1}^{m} \lambda_i$ equals 1.

The difference between R^*_{VRT} and R^*_{URT}, which is the optimal return to VRT (RVRT*), can be specified as a profit function:

$$(5) \qquad RVRT^* = R^*_{VRT} - R^*_{URT}$$
$$= RVRT^*(\lambda_1, \lambda_2, \ldots, \lambda_{m-1}, P_Y, P_X)$$

where all variables are defined earlier.

Equation (5) is concave in λ_i. Its concavity can easily be understood by considering fields with only two management zones—Management Zones 1 and 2. For fields that are uniformly Management Zone 1 ($\lambda_1 = 1$ and $\lambda_2 = 0$), RVRT* = 0 because the weighted average response function and the response function for Management Zone 1 are the same. Fields with a positive λ_2 ($\lambda_1 < 1$) have both management zones and farmers can consider using VRT. Since optimization of input use with VRT is more suited to the site-specific yield response functions than to the average response function, RVRT* now becomes positive and continues to increase to a maximum as λ_2 increases (λ_1 decreases) over some range.

Eventually, RVRT* begins to decline until it reaches zero for fields with only Management Zone 2 ($\lambda_1 = 0$ and $\lambda_2 = 1$). At this point, the average response function and the response function for Management Zone 2 are the same. The above discussion can be generalized to m management zones for all concave functional forms, including the linear-plus-plateau function, which is not strictly concave.

Spatial Break-even Variability Proportions (SBVPs) (English, Roberts, and Mahajanashetti; Mahajanashetti; Roberts, English, and Mahajanashetti) for two particular management zones, say Management Zones m-1 and m, are defined as the lower and upper limits of λ_{m-1} and λ_m for given levels of $\lambda_1, \lambda_2, \ldots, \lambda_{m-2}$, P_Y, P_X, and V such that RVRT* = V, where V is the additional cost of using VRT compared to URT. The SBVPs for λ_m vary inversely with the SBVPs for λ_{m-1} because $\Sigma_{i=1}^{m} \lambda_i$ equals 1. Mathematically, equation (5) can be modified as follows and used to locate the SBVPs for λ_{m-1} and λ_m.

$$(6)\quad \begin{aligned} &RVRT^* \\ &= RVRT^*(\lambda_{m-1} | \bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_{m-2}, \bar{P}_Y, \bar{P}_X) \\ &= \bar{V} \end{aligned}$$

where $\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_{m-2}, \bar{P}_Y, \bar{P}_X$, and $\bar{V}$ are given levels of the respective variables and $\lambda_m = 1 - \lambda_{m-1} - \Sigma_{i=1}^{m-2}$.

Solving equation (6) for λ_{m-1} provides the SBVPs for λ_{m-1} and λ_m that bound the range over which RVRT* ≥ $\bar{V}$. However, for certain $\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_{m-2}$, $\bar{P}_Y$ and $\bar{P}_X$, RVRT* may be less than $\bar{V}$ for all possible levels of λ_{m-1}, implying that SBVPs do not exist and that economic losses from using VRT would occur at all levels of λ_{m-1}. In some cases, RVRT* may be greater than $\bar{V}$ for all possible levels of λ_{m-1}, implying that SBVPs do not exist and that economic gains would occur from using VRT regardless of the level of λ_{m-1}. Finally, in the remaining cases, only an upper or a lower SBVP exists, but not both. If RVRT* > $\bar{V}$ for $\lambda_{m-1} = 0$, and RVRT* = $\bar{V}$ for $0 < \lambda_{m-1} \leq (1 - \Sigma_{i=1}^{m-2} \bar{\lambda}_i)$, only an upper SBVP exists. In this case, the maximum this upper SBVP can be is $1 - \Sigma_{i=1}^{m-2} \bar{\lambda}_i$ when $\lambda_m = 0$. However, if RVRT* > $\bar{V}$ for $\lambda_{m-1} = 1 - \Sigma_{i=1}^{m-2} \bar{\lambda}_i$ and RVRT* = $\bar{V}$ for $0 \leq \lambda_{m-1} < (1 - \Sigma_{i=1}^{m-2} \bar{\lambda}_i)$, only a lower SBVP exists. In this case, the minimum this SBVP can be is 0 when $\lambda_m = 1 - \Sigma_{i=1}^{m-2} \bar{\lambda}_i$.

As a more specific example using a concave functional form, assume three management zones and express equations (1) as quadratic yield response functions. Given these assumptions, the functional forms of equations (2), (4), (5), and (6) can be determined and the SBVPs can be identified. Let the respective management-zone proportions be λ_1, λ_2, and λ_3, and let equations (1) be represented by equations (7), (8), and (9).

$$(7)\quad Y_1 = a_1 + b_1X_1 + c_1X_1^2$$

$$(8)\quad Y_2 = a_2 + b_2X_2 + c_2X_2^2$$

$$(9)\quad Y_3 = a_3 + b_3X_3 + c_3X_3^2$$

where Y_i and X_i are as defined in equations (1) for Management Zones 1, 2, and 3.

For VRT, set the first derivative of each function equal to P_X/P_Y and solve for X_1^*, X_2^*, and X_3^*. Substitute these optimal input rates into equation (2) to get equation (10), which is the profit function for VRT.

$$(10)\quad \begin{aligned} R_{vrt}^* = &\left(1 - \sum_{i=1}^{2} \lambda_i\right)\left\{\left[P_Y\left(a_3 - \frac{(P_X/P_Y)^2 - b_3^2}{4c_3}\right)\right] - P_X\left(\frac{(P_X/P_Y) - b_3}{2c_3}\right)\right\} \\ &+ \sum_{i=1}^{2} \lambda_i\left\{\left[P_Y\left(a_i - \frac{(P_X/P_Y)^2 - b_i^2}{4c_i}\right)\right] - P_X\left(\frac{(P_X/P_Y) - b_i}{2c_i}\right)\right\} \end{aligned}$$

For URT, substitute equations (7), (8), and (9) into equation (3) and set $X_1 = X_2 = X_3 = X_u$. Set the first derivative of the resulting field average yield response function equal to P_X/P_Y and solve for X_u^*. Substitute this optimal uniform input application rate into equation (4) to get equation (11), which is the profit function for URT.

Table 1. Maximum Return to Variable Rate Technology and Spatial Break-even Variability Proportions for Hypothetical Corn Fields with Three Management Zones with the Proportion of the Field in the High-Yield Response Management Zone Held Constant, Weighted Average Response Function

Percentage of Field in High-Response Management Zone ($\bar{\lambda}_1$)	RVRT*[a] Maximizing Percentage of Field in Low-Response Management Zone (λ_3)	Maximum RVRT*[a] for Percentage of Field in Low-Response Management Zone (λ_3)	SBVPs[a] for Percentage of Field in Low-Response Management Zone ($\hat{\lambda}_3$)		SBVPs[a] for Percentage of Field in Medium-Response Management Zone (λ_2)	
			Lower	Upper	Lower	Upper
%		$/ac		%		
0	58	1.95	[b]	[b]	[b]	[b]
20	79	5.22	22	80[c]	0[c]	58
40	60	7.03	8	60[c]	0[c]	52
60	40	6.38	7	40[c]	0[c]	33
80	20	3.89	12	20[c]	0[c]	8

[a] RVRT* is the return-to-variable-rate technology defined in equation (12) and the SBVPs are spatial break-even variability proportions found by solving equation (13).

[b] Because the maximum RVRT* attainable by varying λ_3 is less than the additional custom charge for VRT of $3.00/ac, break-even values for λ_3 and λ_2 do not exist.

[c] This number is the maximum or minimum for λ_3 or λ_2, respectively. Upper or lower SBVPs do not exist because RVRT* is greater than the additional custom charge of $3.00/ac when λ_3 or λ_2 are at their constrained maximum or minimum.

(11)
$$R^*_{URT} = P_Y\left[\left\{a_3 + \sum_{i=1}^{2}[(a_i - a_3)\lambda_i]\right\} - \left((P_X/P_Y)^2 - \left\{b_3 + \sum_{i=1}^{2}[(b_i - b_3)\lambda_i]\right\}^2\right) \div \left(4\left\{c_3 + \sum_{i=1}^{2}[(c_i - c_3)\lambda_i]\right\}\right)\right] - P_X\left[\left((P_X/P_Y) - \left\{b_3 + \sum_{i=1}^{2}[(b_i - b_3)\lambda_i]\right\}\right) \div \left(2\left\{c_3 + \sum_{i=1}^{2}[(c_i - c_3)\lambda_i]\right\}\right)\right]$$

The optimal return to variable rate technology is given by

(12)
$$RVRT^* = R^*_{VRT} - R^*_{URT}$$

Setting λ_1, P_Y, and P_X equal to $\bar{\lambda}_1$, $\bar{P}_Y$, and $\bar{P}_X$, setting equation (12) equal to $\bar{V}$, and consolidating terms gives the following quadratic function in λ_2, which can be solved using the quadratic formula for the lower and upper SBVPs for λ_2 if they exist:

(13)
$$\begin{aligned} 0 = \{&-4\bar{V}[c_3 + (c_1 - c_3)\lambda_1] \\ &+ 4\pi^*_3[c_3 + (c_1 - c_3)\bar{\lambda}_1] \\ &- 4\pi^*_3\bar{\lambda}_1[c_3 + (c_1 - c_3)\bar{\lambda}_1] \\ &+ 4\pi^*_1\bar{\lambda}_1[c_3 + (c_1 - c_3)\bar{\lambda}_1] \\ &+ 2\bar{P}_X(\bar{P}_X/\bar{P}_Y) - 2\bar{P}_X[b_3 + (b_1 - b_3)\bar{\lambda}_1] \\ &- (\bar{P}^2_X/\bar{P}_Y) + \bar{P}_Y[b^2_3 + 2b_3(b_1 - b_3)\bar{\lambda}_1] \\ &+ (b_1 - b_3)^2\bar{\lambda}_1]\} \\ &+ \{-4\bar{V}[(c_2 - c_3)\bar{\lambda}_1] \\ &\quad + 4\pi^*_2[c_3 + (c_1 - c_3)\bar{\lambda}_1] \\ &\quad + 4\pi^*_3(c_2 - c_3) \\ &\quad - 4\pi^*_3[c_3 + (c_1 - c_3)\bar{\lambda}_1] \\ &\quad - 4\pi^*_3(c_2 - c_3)\bar{\lambda}_1 + 4\pi^*_1(c_2 - c_3)\bar{\lambda}_1 \\ &\quad - 2\bar{P}_X(b_2 - b_3) \\ &\quad + 2\bar{P}_Y[b_3(b_2 - b_3) \\ &\qquad + (b_2 - b_3)(b_1 - b_3)\bar{\lambda}_1]\}\lambda_2 \\ &+ [4\pi^*_2(c_2 - c_3) - 4\pi^*_3(c_2 - c_3) \\ &\quad + \bar{P}_Y(b_2 - b_3)^2]\lambda^2_2 \end{aligned}$$

The SBVPs for λ_3 are found from the restriction $\lambda_3 = 1 - \bar{\lambda}_1 - \lambda_2$. Equation (13) demonstrates that equation (12) is concave (quadratic in this case) in λ_2. This concavity results from assuming the farmer uses the weighted-average yield response function to choose the uniform rate. More specifically it results because the average response function approaches the response function for Management Zone i as λ_i approaches 1 and diverges from that response function as λ_i approaches 0.

The RVRT* maximizing λ_2 is found by setting the partial derivative of equation (12) with respect to λ_2 equal to zero and solving for λ_2 (given $\bar{\lambda}_1$). The resulting λ_2 is substituted into equation (12) to find the maximum RVRT*. If this maximum RVRT* is less than $\bar{V}$, SBVPs for λ_2 and λ_3 do not exist and a farmer would have no economic incentive to use VRT on the field in question, given $\bar{\lambda}_1$.

Response Function for One Management Zone

Using the response function for one management zone to determine the uniform input application rate is a less appealing criterion than the aforementioned criterion, but anecdotal evidence suggests that some farmers make uniform-rate decisions based on this criterion. For example, some farmers who use URT for fertilizer application may fertilize the entire field based on the yield goal for the "best land." Obviously, a farmer would use this method only if a considerable proportion of the field were in the targeted management zone. Nevertheless, for illustrative purposes, the example presented below explores the entire range of possible proportions of the field in the targeted management zone.

Assume the farmer determines the optimal uniform input application rate based on the response function for a single management zone, say Management Zone m. The uniform application rate is now determined as X_m^* using $Y_u(X_u) = Y_m(X_m)$ instead of equation (3). Substitute X_m^* for X_u^* in equation (4) to get the new profit function for URT. Subtract the new R_{URT}^* from equation (2) to get the new RVRT* function in equation (14).

$$(14)\quad RVRT^* = \sum_{i=1}^{m} \lambda_i[\pi_i^*(X_i^*) - \pi_i(X_m^*)]$$

where $\pi_i^*(X_i^*)$ is optimal return above input cost per acre for Management Zone i and $\pi_i(X_m^*)$ is return above input cost per acre for Management Zone i when X_m^* is applied to it. The expression in brackets is zero for i = m because applying the input to Management Zone m at its optimal rate gives the same return above input cost under VRT and URT.

For given crop and input prices, the expressions in brackets are constants for each management zone; therefore, RVRT* is linear in λ_i. When all management-zone proportions except two are fixed, only one SBVP can exist for λ_i. If the expression in brackets is greater (less) than V regardless of the level of λ_i (for $i \neq m$), no SBVP exists for λ_i and VRT is more (less) profitable than URT. Also, because the expression in brackets is zero for Management Zone m, the larger (smaller) λ_m the smaller (larger) RVRT* and an SBVP will exist for λ_m only if the expression in brackets is greater than V for Management Zone $i \neq m$. The SBVPs for any pair of λ_is can be found by setting equation (14) equal to $\bar{V}$, holding prices and all other λ_is constant, and solving for λ_i. Finally, as a more specific example for concave functions, the parameters of the quadratic yield response functions in equations (7) through (9) can be substituted into equation (14) as in the previous case and solved for the SBVP if it exists.

Equation (14) is linear in λ_i because the uniform rate is constant and independent of λ_i. Even if the uniform rate were chosen as a constant, R, determined by family tradition, for example, equation (14) would still be linear in λ_i after substituting R for X_m^*.

Illustrative Example

To illustrate the concepts presented above, assume hypothetical fields suited to corn production can be classified into three management zones and that the following quadratic functions represent corn yield response to fertilizer nitrogen for the management zones:

(15) $Y_1 = 120 + 1.11N_1 - 0.0023N_1^2$

(16) $Y_2 = 100 + 1.05N_2 - 0.0026N_2^2$

(17) $Y_3 = 75 + 0.5N_3 - 0.0014N_3^2$

where Y_1, Y_2, and Y_3 are corn yields (bu/ac) and N_1, N_2, and N_3 are nitrogen application rates (lb/ac) for high-, medium-, and low-response management zones, respectively.

Equations (15)–(17) are plausible corn yield response functions chosen for illustrative purposes. They were not estimated from site-specific field data, but were assumed for ease of exposition and because similar ones have been used historically to represent corn yield response to nitrogen (eg., Arce-Diaz et al.; Agrawal and Heady; Mjelde et al.; Vanotti and Bundy; Schlegel and Havlin). Their use facilitates exposition of the aforementioned concepts because they are continuous and exhibit diminishing marginal physical productivity throughout, and because a mathematical solution to equation (6) exists as expressed in equation (13). The latter cannot be said when equations (1) are expressed in semi-log form (also concave), for example. Even when they are expressed as quadratic-plus-plateau or Mitscherlich-Baule functions (Bullock and Bullock; Cerrato and Blackmer; Frank, Beattie, and Embleton; Llewelyn and Featherstone; Stecker et al.), which were shown for those cases to more accurately represent corn yield response, mathematical solutions would be difficult. Also, if quadratic response functions overstate nitrogen use at the economic optima (Cerrato and Blackmer; Llewelyn and Featherstone) for all management zones, the effects on RVRT* may be mitigated somewhat. Consequently, the less complicated quadratic functional form was used in this article. Even when mathematical solutions do not exist for other functional forms, the concepts presented above still hold and iterative procedures can be used to find approximate solutions for the SBVPs by adjusting λ_{m-1} (λ_2 for this specific example) until the left-hand side of equation (6) equals $\bar{V}$.

After defining λ_1, λ_2, and λ_3 as the proportions of the field in high-, medium-, and low-yield response management zones, spatial break-even analyses were conducted. The average Tennessee corn price received by farmers ($\bar{P}_Y$ = \$2.79/bu) and the average nitrogen price ($\bar{P}_N$ = \$0.26/lb) over the 1993–1997 period (Tennessee Department of Agriculture) were used in the analysis.

The additional custom charge for variable rate nitrogen application compared to uniform rate application was assumed to be $\bar{V}$ = \$3.00/ac. This additional charge was close to the mean of \$3.08/ac (range \$1.50 to \$5.50/ac) obtained from personal telephone interviews with firms providing precision farming services to Tennessee farmers in 1999 (Roberts, English, and Sleigh). Responding firms indicated that the additional charge would include the difference in application costs for VRT versus URT and a charge to create a nitrogen application map based on soil survey maps in conjunction with the consultant's knowledge about corn response on various soils, a visit to the field to observe conditions, and an interview with the farmer about historical yields.

Sensitivity analyses examined the effects on the SBVPs of 10-percent increases and decreases in $\bar{P}_Y$, $\bar{P}_N$, and the linear (b_3) and squared (c_3) terms of equation (9) as found in equation (17) (low-response management zone). Sensitivity of the SBVPs to changes in $\bar{V}$ was examined by decreasing $\bar{V}$ by \$1.50/ac and increasing $\bar{V}$ by \$2.50/ac, which is the range in cost differences found by Roberts, English, and Sleigh. These analyses were conducted for the weighted-average-response-function case and for the case where the uniform rate is determined as the optimal rate for the high-response management zone.

Weighted Average Response Function

The maximum RVRT* for example fields with no land in the high-response management zone ($\bar{\lambda}_1$ = 0 percent) was \$1.95/ac (Table 1). This maximum RVRT* occurred in fields with 58 percent of their area in the low-response management zone and 42 percent in the medium-response management zone. Thus, a farmer with a field containing only low- and medium-response management zones would not be able to cover the additional custom

Table 2. Maximum Return to Variable Rate Technology and Spatial Break-even Variability Proportions for Hypothetical Corn Fields with Three Management Zones with the Proportion of the Field in the Low-Yield Response Management Zone Held Constant, Weighted Average Response Function

Percentage of Field in Low-Response Management Zone ($\bar{\lambda}_3$)	RVRT*[a] Maximizing Percentage of Field in Medium-Response Management Zone (λ_2)	Maximum RVRT*[a] for Percentage of Field in Medium-Response Management Zone (λ_2)	SBVPs[a] for Percentage of Field in Medium-Response Management Zone (λ_2)		SBVPs[a] for Percentage of Field in High-Response Management Zone (λ_1)	
			Lower	Upper	Lower	Upper
%		$/ac	%			
0	48	2.33	[b]	[b]	[b]	[b]
20	21	4.38	0[c]	58	22	80[c]
40	0	6.37	0[c]	50	10	60[c]
60	0	7.03	0[c]	33	7	40[c]
80	0	5.22	0[c]	12	8	20[c]

[a] RVRT* is the return-to-variable-rate technology defined in equation (12) and the SBVPs are spatial break-even variability proportions found by solving equation (13).

[b] Because the maximum RVRT* attainable by varying λ_2 is less than the additional custom charge for VRT of $3.00/ac, break-even values for λ_2 and λ_1 do not exist.

[c] This number is the maximum or minimum for λ_2 or λ_1, respectively. Upper or lower SBVPs do not exist because RVRT* is greater than the additional custom charge of $3.00/ac when λ_2 or λ_1 are at their constrained maximum or minimum.

charge of $3.00/ac, implying that the adoption of VRT would lead to economic losses on that field. The maximum RVRT* ($2.33/ac) for example fields having only medium- and high-response management zones ($\bar{\lambda}_3$ = 0 percent) also was less than the additional custom charge (Table 2), suggesting that adoption of VRT would not be profitable. For fields with only low- and high-yield response management zones ($\bar{\lambda}_2$ = 0 percent), SBVPs were clearly identified at 15 and 90 percent of the field in the low-response management zone, with the maximum RVRT* ($7.07/ac) occurring at 56 percent in the low-response management zone (Table 3). Thus, for fields with only high- and low-response management zones, farmers would have an economic incentive to adopt VRT on those fields with between 15 and 90 percent of their area in the low-response management zone or between 85 and 10 percent in the high-response management zone.

When the percentage of a field in the high-response management zone ($\bar{\lambda}_1$) was specified at 20, 40, 60, or 80 percent, economically viable ranges of spatial variability in the low- and medium-response management zones were identified (Table 1). These ranges, however, had only lower SBVPs for the low-response management zone and upper SBVPs for the medium-response management zone. No upper or low SBVPs existed for these management zones because RVRT* was greater than $3.00/ac when λ_3 reached its maximum and λ_2 reached its minimum. A similar kind of result occurred when the percentage of a field in the low-response management zone ($\bar{\lambda}_3$) was set at 20, 40, 60, and 80 percent (Table 2).

When the share of an example field in the medium-response management zone was specified at 60 or 80 percent ($\bar{\lambda}_2$ = 60 or 80 percent), no economically viable mix of Management Zones 1 and 3 could be found (Table 3). However, given $\bar{\lambda}_2$ = 20 or 40 percent, VRT could be employed more profitably than URT on fields provided they had land in all three management zones. For example, for $\bar{\lambda}_2$ = 20 percent, fields with between 9 and 73 percent of their area in the low-response management

Table 3. Maximum Return to Variable Rate Technology and Spatial Break-even Variability Proportions for Hypothetical Corn Fields with Three Management Zones with the Proportion of the Field in the Medium-Yield Response Management Zone Held Constant, Weighted Average Response Function

Percentage of Field in Medium-Response Management Zone ($\bar{\lambda}_2$)	RVRT*[a] Maximizing Percentage of Field in Low-Response Management Zone (λ_3)	Maximum RVRT*[a] for Percentage of Field in Low-Response Management Zone (λ_3)	SBVPs[a] for Percentage of Field in Low-Response Management Zone (λ_3)		SBVPs[a] for Percentage of Field in High-Response Management Zone (λ_1)	
			Lower	Upper	Lower	Upper
%		$/ac		%		
0	56	7.07	15	90	10	85
20	43	5.68	9	73	7	71
40	31	4.28	7	53	7	53
60	18	2.89	[b]	[b]	[b]	[b]
80	5	1.50	[b]	[b]	[b]	[b]

[a] RVRT* is the return-to-variable-rate technology defined in equation (12) and the SBVPs are spatial break-even variability proportions found by solving equation (13).

[b] Because the maximum RVRT* attainable by varying λ_3 is less than the additional custom charge for VRT of $3.00/ac, break-even values for λ_3 and λ_1 do not exist.

zone (λ_3) and between 7 and 71 percent in the high-response management zone (λ_1) would be considered for VRT instead of URT.

Illustrative sensitivity-analysis results are presented in Table 4 for example fields with 20 percent of their area in the medium-response management zone ($\bar{\lambda}_2$). As the difference increases between the upper and lower SBVPs with changes in a parameter, a particular field would be more likely to have RVRT* $\geq \bar{V}$, increasing the economic incentive for the farmer to use VRT on that field. Ten-percent increases in prices result in only slightly wider ranges of spatial break-even variability, implying for this example that economic incentives to use VRT are relatively insensitive to price changes.

The model seems quite sensitive to changes in response function parameters. As the yield response functions for high- and low-response management zones become more similar in slope (b_3 or c_3 increases), spatial break-even variability decreases, decreasing the economic incentive to use VRT. Sensitivity to changes in these parameters suggests that accurate estimation of the management-zone yield response functions is critical to obtaining accurate estimates of RVRT* and the SBVPs.

For fields with $\bar{\lambda}_2$ = 20 percent, a decrease in the cost difference between VRT and URT ($\bar{V}$) widens the range of spatial break-even variability and an increase in $\bar{V}$ narrows it (Table 4). At the lower end of the range in $\bar{V}$ ($1.50/ac) found by Roberts, English, and Sleigh, a field would need to have between 0 and 79.8 percent of its area in the low-response management zone (λ_1) for VRT to at least break even with URT. The range of SBVPs for λ_1 narrows to between 35 and 51.6 percent at the upper end of the range in $\bar{V}$ ($5.50/ac).

Fertilize for the Highest Response Management Zone

Table 5 presents the SBVPs for URT farmers who are assumed to fertilize the entire field at the optimal nitrogen rate for the high-response management zone. Farmers with fields having high percentages of their areas in low- and medium-response management zones have economic incentive to use VRT. In general, VRT has its greatest economic advantage over URT in fields with smaller proportions of land in the high-response management zone because more can be gained from adopting VRT.

Table 4. Impacts of Changes in Nitrogen and Corn Prices, Response Function Parameters, and the Additional Cost of VRT on Spatial Break-even Variability Proportions for Hypothetical Corn Fields with Three Management Zones with 20 Percent of the Field in the Medium-Response Management Zone, Weighted Average Response Function

	SBVPs[a] for Percentage of Field in Low-Response Management Zone (λ_3)		SBVPs[a] for Percentage of Field in High-Response Management Zone (λ_1)	
Parameters that Change	Lower	Upper	Lower	Upper
Prices of nitrogen and corn		%		
Mean prices	9.1	73.2	6.8	70.9
Increase P_N by 10%	8.8	74.0	6.0	71.2
Decrease P_N by 10%	9.3	72.4	7.6	70.7
Increase P_Y by 10%	7.4	73.9	6.1	72.6
Decrease P_Y by 10%	11.9	72.4	7.6	68.1
Low response function				
Original parameter values	9.1	73.2	6.8	70.9
Decrease b_3 by 10%	5.4	80.0[b]	0.0[b]	74.6
Increase b_3 by 10%	20.0	56.7	23.3	60.0
Decrease c_3 by 10%	5.5	79.2	0.8	74.5
Increase c_3 by 10%	20.7	57.5	22.3	59.3
Additional cost of VRT				
Decrease $\bar{V}$ by \$1.50	0.0	79.8	0.2	80.0
Original $\bar{V}$ = \$3.00/ac	9.1	73.2	6.8	70.9
Increase $\bar{V}$ by \$2.50	35.0	51.6	28.4	45.0

[a] SBVPs are spatial break-even variability proportions found by solving equation (13).
[b] This number is the maximum or minimum for λ_3 or λ_1, respectively. Upper or lower SBVPs do not exist because RVRT* is greater than the additional custom charge ($\bar{V}$) when λ_3 or λ_1 are at their constrained maximum or minimum.

On these fields, URT greatly over fertilizes the low- and medium-response management zones, while VRT provides each management zone with its optimal level of nitrogen. Also, for a fixed proportion of a field in the high-response management zone, the larger the proportion of the field in the low-response management zone and the smaller the proportion in the medium-response management zone, the more profitable VRT is relative to URT. For example, for VRT to be profitable when 80 percent of the field is in the high-response management zone ($\bar{\lambda}_1$ = 80 percent), at least 8 percent of the field must be in the low-response management zone ($8 \leq \lambda_3 \leq 20$) and at most 12 percent can be in the medium-response management zone ($0 \leq \lambda_2 \leq 12$).

Table 6 shows sensitivity-analysis results for prices, low response function parameters, and changes in the additional cost of VRT versus URT. A 10-percent change in the nitrogen price (P_N) has imperceptible effects on the SBVPs and a 10-percent change in the corn price (P_Y) has only slightly larger impacts. The SBVPs also seem insensitive to changes in the low-response function parameters. Nevertheless, the SBVP for the high-response management zone (λ_1) increases slightly and the SBVP for low-response management zone (λ_3) decreases slightly when the low-response function parameters decrease by 10 percent. Thus, as the marginal physical product of the low-response function diverges from the marginal physical products of the other two response functions, more of the field can be in the high-response management zone for VRT to break even with URT. Alternatively, as the cost of VRT compared to URT ($\bar{V}$) changes over the range found by Roberts, English, and Sleigh, the minimum proportion of the field that must be in the low-response management zone (λ_3) increases from 0 to 15.7 percent,

Table 5. Spatial Break-even Variability Proportions with Farmers Fertilizing for the High-Response Management Zone for Hypothetical Corn Fields with Three Management Zones

Percentage of Field in High ($\bar{\lambda}_1$), Medium ($\bar{\lambda}_2$), and Low ($\bar{\lambda}_3$) Response Management Zones	Spatial Break-even Variability Proportions (SBVPs) for Low (λ_3) and Medium (λ_2) Response Management Zones	Spatial Break-even Variability Proportions (SBVPs) for Medium (λ_2) and High (λ_1) Response Management Zones
%	%	%
High Response ($\bar{\lambda}_1$)	Low Response (λ_3)	Medium Response (λ_2)
0	0[a]	100[b]
20	0[a]	80[b]
40	0[a]	60[b]
60	0[a]	40[b]
80	8	12
Medium Response ($\bar{\lambda}_2$)	Low Response (λ_3)	High Response (λ_1)
0	13	87
20	5	75
40	0[a]	60[b]
60	0[a]	40[b]
80	0[a]	20[b]
Low Response ($\bar{\lambda}_3$)	Medium Response (λ_2)	High Response (λ_1)
0	30	70
20	0[a]	80[b]
40	0[a]	60[b]
60	0[a]	40[b]
80	0[a]	20[b]

[a] An SBVPs does not exist because RVRT* is greater than the additional custom charge for VRT of \$3.00/ac when λ_3 or λ_2 are at their constrained minimum of zero.

[b] An SBVP does not exist because RVRT* is greater than the additional custom charge for VRT of \$3.00/ac when λ_2 or λ_1 are at their maximum of $1 - \bar{\lambda}_i$.

while the maximum proportion allowed in the high-response management zone (λ_1) decreases from 80 to 64.3 percent.

Discussion

This hypothetical example emphasizes that obtaining information about a management zone's yield response potential is more important than obtaining information about its yield potential (maximum yield). This point can be generalize to all concave functional forms and is illustrated for the quadratic case by the absence of the intercept terms (a_1, a_2, and a_3) in equations (13) and (14). Even for linear-plus-plateau response functions, which do not exhibit diminishing marginal physical productivity (not strictly concave), RVRT* is determined by the yield responses for the management zones that are not at their respective yield plateaus when the uniform input rate is applied, rather than by the maximum yields themselves.

If a farmer can gain knowledge of the field-specific management zones for a particular crop and input and the parameters of the corresponding yield-response functions, the methods discussed above could be used in deciding whether to use VRT or URT on a field. Unfortunately, this knowledge is difficult to obtain with certainty, but farmers are currently using other precision farming technologies (eg., yield monitors, grid soil sampling, field mapping) that can be used to identify management zones and their yield-response potentials (English, Roberts, and Sleigh). Yield-monitor and grid-soil-sampling data can provide information about yield-response potential, especially when a historical database of those data is available. The uncertainty about yield-response potential can be further

Table 6. Impacts of Changes in Nitrogen and Corn Prices, Response Function Parameters, and the Additional Cost of VRT on Spatial Break-even Variability Proportions for Hypothetical Corn Fields with Three Management Zones with 20 Percent of the Field in the Medium-Response Management Zone, Farmers Fertilizing for the High-Response Management Zone

Parameters that Change	SBVPs[a] for Percentage of Field in Low-Response Management Zone (λ_3)	SBVPs[a] for Percentage of Field in High-Response Management Zone (λ_1)
	%	
Prices of corn and nitrogen		
Mean prices	4.5	75.5
Increase P_N by 10%	4.5	75.5
Decrease P_N by 10%	4.5	75.5
Increase P_Y by 10%	3.3	76.7
Decrease P_Y by 10%	5.9	74.1
Low response function		
Original parameter values	4.5	75.5
Decrease b_3 by 10%	2.9	77.1
Increase b_3 by 10%	8.7	71.3
Decrease c_3 by 10%	3.0	77.0
Increase c_3 by 10%	8.1	71.9
Additional cost of VRT		
Decrease $\bar{V}$ by \$1.50	0.0	80.0
Original $\bar{V}$ = \$3.00/ac	4.5	75.5
Increase $\bar{V}$ by \$2.50	15.7	64.3

[a] SBVPs are spatial break-even variability proportions.

reduced when data collected through precision technologies are combined with expert perceptions or knowledge, such as 1) the farmer's historical perceptions about yield response in different parts of the field and 2) recommendations from experts—such as soil-test laboratories, crop consultants, input suppliers, or extension personnel—who may implicitly or explicitly assume yield-response functions based on their knowledge when making recommendations about input application.

Researchers are exploring inexpensive methods for estimating management-zone-specific yield-response functions from yield monitor data (Bongiovanni and Lowenberg DeBoer, 2000). Other researchers are developing methods for estimating management-zone-specific meta-response models from crop-growth simulation models (Peeters and Booltink). As these estimation methods become more refined, the methods presented in this article will become increasingly important in the VRT-versus-URT decision.

Actual fields within a geographic area can contain a wide variety of soil types suited to producing several major crops. Over the years a limited number of field experiments have allowed estimation of a patchwork of yield-response functions for some geographic areas. The demand for VRT will probably increase in the future, requiring estimates of yield-response functions for a growing number of farmers. A concerted effort to estimate and document yield response for a variety of crops, soil series, and weather conditions would be beneficial to agribusiness firms who are interested in providing VRT services to farmers and to farmers who are contemplating adopting VRT. Estimation of meta-response functions for major crops and soil series within a particular geographic area could be used with the methods in this article until methods for estimating management-zone-specific response functions are improved and become less expensive for on-farm use. These meta-response functions could be made available to agribusiness firms and farmers in a user-friendly modeling framework that would al-low them to evaluate the VRT-versus-URT decision for a specific field.

Conclusions

Adoption of VRT depends to a large extent on the expected net economic benefits received by potential adopters. Fields generally exhibit yield variability; however, not all fields warrant VRT from an economic standpoint. Farmers are interested in knowing whether VRT is economically viable on their fields. The answer to this question varies from field to field depending on spatial variability as well as yield-response variability among management zones. The answer also varies with the crop, the input, prices, and the cost of using VRT relative to URT. In the end, no general formula exists for determining whether VRT or URT should be used on a particular field because each field presents a different case. What researchers can do, however, is provide agribusiness firms, extension personnel, and farmers with a consistent means for evaluating this decision based on the economic models presented above, or variants thereof. Required model inputs would include estimates of yield-response potentials and the proportions of the field in each management zone, expected crop and input prices, and the additional cost of VRT versus URT. Even with educated guesses for the model inputs, model outputs in the form of a field RVRT*s or SBVPs could be used as additional pieces of information in helping farmers make the VRT-versus-URT decision for a particular field.

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