Seasonality of Class I Price Differential Estimates for the Southeastern United States

Carlos E. Testuri, Richard L. Kilmer, and Thomas Spreen

ABSTRACT

This study provides insight into the seasonality of Class I price differentials in the southeastern dairy industry. This is accomplished by analyzing monthly estimates of Class I price differentials obtained from the imputed price solution or dual solution of a generalized capacitated minimum cost network flow model of the dairy industry. A smooth seasonal pattern emerges through the monthly sequence with the lowest and highest estimated Class I price differentials occurring in April and September respectively. Miami and Jacksonville areas reach $ 5.40 and $ 4.36 per hundredweight in April and $ 6.79 and $ 5.53 per hundredweight in September.

Key Words: *dairy, Southeast, Class I differentials, network flow model, pricing, marketing*

Several factors, including seasonal production, seasonal consumption, and the geographic isolation of the Florida peninsula, affect the marketing of milk in the southeastern United States. Milk production peaks in the spring and early summer due to breeding patterns and weather conditions; moreover, consumer preference results in the lowest milk demand in the summer. These inversely seasonal factors drive deficit and surplus patterns in the region.

Because of milk's perishability, potential disease carrier characteristics, and past disorderly marketing conditions, the dairy industry is one of the most regulated agricultural industries by federal and state programs in the United States. The Federal Milk Marketing

Carlos E. Testuri is a former graduate student in the Dairy and Poultry Science Department at the University of Florida and Richard L. Kilmer and Thomas Spreen are professors in the Food and Resource Economics Department at the University of Florida. This paper is Florida Agricultural Experiment Station Journal Series No. R-07384

Order (FMMO) agreement is a complex set of rules regulating the price of milk between producers and processors. Particularly, it establishes a classified price support system for milk according to its use. The price for the milk used as fluid milk, the Class I price, is the highest milk price. This price is calculated as the sum of a monthly determined nationwide basic component, the Basic Formula Price (BFP), plus a stated time invariant Class I price differential that varies geographically. These differentials are intended to "pay" for the cost of transporting milk from surplus to deficit areas and establish a price incentive over milk destined for manufacturing.

When the FMMO regulation was implemented, isolated markets existed in areas that included a major city and milk seldom moved across these markets (Bailey). That situation changed, however, after technological advances in transportation and storability. Formerly separated markets started to interact and overlap. Consequently, price differentials started to

Table 1. Difference between Estimated April and September Class I Price Differentials in $/Cwt per Regions, 1997

Region	Name	April	Sep.	Sep.–April
AL15	Calhoun	3.50	4.40	0.90
AL69	Houston	4.14	5.29	1.15
AL73	Jefferson	3.68	4.57	0.89
AL89	Madison	3.41	4.25	0.84
AL97	Mobile	4.00	4.98	0.98
AL101	Montgomery	4.02	4.92	0.90
AL103	Morgan	3.40	4.30	0.90
FL11	Broward	5.32	6.72	1.40
FL17	Citrus	4.56	5.80	1.24
FL19	Clay	4.44	5.59	1.15
FL25	Dade	5.40	6.79	1.39
FL31	Duval	4.36	5.53	1.17
FL39	Gadsden	4.06	5.28	1.22
FL57	Hillsborough	4.82	6.06	1.24
FL95	Orange	4.79	6.02	1.23
FL103	Pinellas	4.81	6.05	1.24
FL105	Polk	4.87	6.11	1.24
FL127	Volusia	4.72	5.88	1.16
GA51	Chatham	4.03	5.07	1.04
GA57	Cherokee	3.45	4.54	1.09
GA59	Clarke	3.51	4.53	1.02
GA89	DeKalb	3.61	4.70	1.09
GA121	Fulton	3.58	4.67	1.09
GA157	Jackson	3.60	4.61	1.01
GA215	Muscogee	3.71	4.87	1.16
GA267	Tattnall	3.86	5.01	1.15
LA17	Caddo	3.21	3.78	0.57
LA33	East Baton Rouge	3.60	4.70	1.10
LA51	Jefferson	3.68	4.78	1.10
LA55	Lafayette	3.86	4.59	0.73
LA61	Lincoln	3.43	4.00	0.57
LA71	Orleans	3.67	4.77	1.10
LA73	Ouachita	3.56	4.13	0.57
LA97	St. Landry	3.74	4.49	0.75
LA105	Tangipahoa	3.42	4.52	1.10
LA117	Washington	3.43	4.53	1.10
MS7	Attala	3.65	4.29	0.64
MS63	Jefferson	3.65	4.48	0.83
MS71	Lafayette	3.63	4.17	0.54
MS73	Lamar	3.64	4.65	1.01
MS89	Madison	3.68	4.46	0.78
NC21	Buncombe	3.60	4.28	0.68
NC49	Craven	4.00	4.15	0.15
NC67	Forsyth	3.45	4.05	0.60
NC81	Guilford	3.49	3.97	0.48
NC107	Lenoir	3.88	4.10	0.22
NC119	Mecklenburg	3.46	4.17	0.71
NC183	Wake	3.59	3.93	0.34

Table 1. (Continued)

Region	Name	April	Sep.	Sep.–April
SC7	Anderson	3.67	4.38	0.71
SC19	Charleston	4.05	4.88	0.83
SC21	Cherokee	3.64	4.35	0.71
SC35	Dorchester	4.02	4.87	0.85
SC45	Greenville	3.80	4.51	0.71
SC83	Spartanburg	3.71	4.42	0.71
SC85	Sumter	3.71	4.54	0.83
TN37	Davidson	3.31	3.96	0.65
TN65	Hamilton	3.52	4.31	0.79
TN93	Knox	3.37	4.06	0.69
TN107	McMinn	3.56	4.28	0.72
TN149	Rutherford	3.34	4.03	0.69
TN157	Shelby	3.45	3.89	0.44
TN163	Sullivan	3.44	4.04	0.60
TN167	Tipton	3.43	3.90	0.47

play a major role in the determination of the final price, since the process of setting Class I price differentials evolved from isolated or local characteristics towards a more integrated one.

Most of the southeastern United States and particularly Florida are characterized by a marked seasonal deficit of fluid milk and some geographic isolation. Florida production does not satisfy consumption during the summer and fall seasons (Kilmer, DeLorenzo, Rahmani). During the deficit phase imports from nearby markets are used to augment local production. In some years imports must be obtained from distant markets when nearby markets face the same stage. Also, the Southeastern market has high Class I utilization, ranging from 70 and 90 percent, compared with the FMMO market average of between 35 and 55 percent (USDA, 1998). Consequently, for this region, higher-than-average differential values are expected during the year and even higher during the deficit phase of the year.

In 1999, a set of amendments was introduced to the FMMO agreement (USDA, 1999) in order to accomplish the Federal Agriculture Improvement and Reform Act of 1996 (USDA, 1996). At the same time a new formulation of the Class I price differentials was proposed to eliminate market inefficiencies

that were generated by discrete adjustments and changes in the market since its previous establishment in 1986. The newly proposed price differential surface is based upon an economic model of the dairy industry (Pratt, *et al.* 1997).

Several studies have addressed the spatial organization of the dairy industry. Snodgrass and French did an early study (1958). Riley and Blakley analyzed the impact on the fluid milk industry of optional pricing schemes and structural changes on producer prices and revenues and on consumer prices and expenditures by using a general spatial equilibrium model of the fluid milk industry (Riley and Blakley).

In the late 1970s, Babb et al. and Novakovic et al. designed and developed a dynamic model to simulate the dairy industry under cost-minimization criteria (Babb, *et al.*; Novakovic, *et al.*). The model was specified as a sequence of transshipment problems minimizing the cost of significant market activities, where producers and consumers are modeled with short-run supply-and-demand functions.

A detailed analysis of the spatial organization of the U.S. northeast dairy industry was done by Pratt *et al.* in 1986. A transshipment model of the dairy industry was used to analyze the least-cost spatial organization of past and forecasted activity of the sector. Different scenarios were used to evaluate potential production facility locations and their overall cost impact.

In 1989 Schiek and Babb used a network model that minimizes the cost of interregional movements to analyze the impact of reverse osmosis on Florida markets.

In 1997 a highly detailed spatial model of the U.S. dairy industry was developed (Pratt, *et al.*) as a direct descendant of previous works (Novakovic, *et al.*; Pratt, *et al.* 1986). Formulated as a single time period transshipment model, it models production, processing, and consumption sectors in a processing and transporting cost-minimization problem. The model is highly partitioned in spatial and structural aggregation, which allows high precision response, however, with considerable effort in parameter specification. The resulting dual solution or shadow prices were used to estimate Class I price differentials under model assumptions. From the analysis of two model solutions (May and October 1995), it was concluded that the model estimates indicate several discrepancies with actual differentials (Pratt, *et al.*).

The main objective of this study is to obtain insight into the seasonality of the price components of milk and milk products related to transportation costs in the southeastern dairy industry. This is accomplished by estimating monthly Class I price differentials from the solution of a dairy market model that establishes the monthly spatial transportation price component variation of products among production, processing, and consumption sectors. Specifically, the differentials are estimated from the dual solution of a minimum cost capacitated-generalized network flow model. The model is an extension of the work done by Pratt *et al.* (1997). It gives a dynamic application to a static model, incorporates processing capacities, is run monthly for 1997, and highly disaggregates the southeastern dairy industry in the US.

Model Framework

Since the model is specified as a network flow problem formulation and related properties, it is described by using a general framework first and its details are introduced later.

Define a general network consisting of a set of nodes N and a set of arcs A, where a directed arc is an ordered pair (i, j) of distinct nodes. Associated to each node i there is a quantity b_i that represents the amount that enters or leaves the network from the environment. If b_i is positive, the node i is called a *source* and b_i is the amount supplied; if b_i is negative, the node i is called a *sink* and $|b_i|$ is the amount demanded; otherwise, b_i is zero and the node i is denominated as a *transient node*.

The minimum cost network flow problem may then be formulated algebraically as

(1)   minimize $\sum_{(i,j) \in A} c_{ij} x_{ij}$

(2)   subject to $\sum_{\{j \mid (i,j) \in A\}} x_{ij} - \sum_{\{j \mid (j,i) \in A\}} x_{ji} = b_i,$

$$\forall i \in N,$$

(3) $x_{ij} \geq 0, \qquad \forall(i, j) \in A,$

where the decision variables x_{ij} and the parameters c_{ij} denote level of flow and per-unit cost through arc (i, j) respectively. Expression (1) establishes the problem as the minimization of a linear cost function. The set of expressions (2) imposes the law of flow conservation at each node, i, by establishing the flow relationship between outgoing and incoming nodes of node i. Finally, the set of expressions (3) establishes flows nonnegativity; no backward flows are allowed. Since all expressions are linear in terms of the decision variables, the formulation belongs to the class of linear programming problems (LP).

Duality

Duality is a property of algebraic structures which states that in a given system two concepts are interchangeable, asserting that results applicable in one formulation, called the *primal*, also holds in its associated other problem, the dual.

Associated with each linear programming problem, the primal, there exists another problem called the *dual*. Following a technique similar to the Lagrange multiplier method, the dual problem is specified by associating a price variable with each constraint in the primal. A solution that allows the constraints to not affect the optimum is then obtained for all prices. Also, this price or dual solution can be obtained by solving a new linear programming problem, the dual of the original problem.

At the network problem formulation, define a dual variable p_i associated with each constraint. The final values of these prices provide an optimal valuation of the resources in the primal constraints. By applying dual construction rules to the network problem the network dual problem is

maximize $\sum_{i \in N} b_i p_i$

subject to $p_i - p_j \leq c_{ij}, \qquad \forall(i, j) \in A.$

The dual problem may be visualized as a competitive assignment of prices to each node in such a way that the revenue associated to the resources is maximized. The fundamental dual theorem of linear programming states that its optimal value of the objective function of the dual problem is equal to the optimal value of the objective function in the primal (Intrilligator, p. 82).

Furthermore, the aggregated problem can be interpreted as a market equilibrium problem, where the nodes are associated with the markets and the dual variables with the equilibrium price at each node. The system of primal equations, dual constraints, and its associated complementary slackness conditions without minimization or maximization operators defines the market equilibrium problem (Takayama and Judge).

Multiple Optimal Solutions

It is well known that the network flow problem is a member of a class of linear programming problems known as *transportation problems*. When a transportation problem is balanced, that is the sum of the quantity supplied (available) is equal to the quantity demanded (required), then the solution to the primal problem is degenerate and the corresponding dual has multiple optimal solutions. The presence of multiple optima in the dual problem presents a problem regarding economic interpretation of the dual variables (the shadow prices of the supply-and-demand constraints.)

For the dual network formulation, given a constant k, if $(p_1, \ldots, p_n)$ is a solution then $(p_1 k, \ldots, p_n k)$ is also a solution. This result can be verified by first substituting the translated solution for the original solution into the dual problem:

maximize $\sum_{i \in N} b_i(p_i + k)$

subject to $(p_i + k) - (p_j + k) \leq c_{ij},$

$$\forall(i, j) \in A,$$

By the law of flow conservation it is known that

$$\sum_{i \in N} b_i = 0.$$

Hence, the original dual problem can be written as

maximize $\quad \sum_{i \in N} b_i p_i + k \sum_{i \in N} b_i = \sum_{i \in N} b_i p_i$

subject to $\quad p_i - p_j \leq c_{ij}, \quad \forall (i, j) \in A.$

This property allows one to modify the dual solution by a constant without affecting optimality. This implies that adding a constant to the imputed price solution does not affect dual feasibility or optimality. Consequently, there is an infinite number of imputed price solutions for this problem; these solution components are related to each other by a constant across solutions. This states the relative nature of the imputed price solution formulation in a network problem. On the whole, the imputed price solution is distinguished by the relative dispersion of its components rather than by its absolute values.

Sensitivity of the Imputed Price Solution

Associated with each constraint in the primal formulation is a dual variable called its *imputed price*. This imputed price provides a measure of the change in the optimal value of the objective function associated with a unitary change in its corresponding constraint resource. It provides a sensitivity measure of cost with respect to a change in the right-hand-side value of a constraint. If a unit change is applied to any constraint resource, an equivalent and opposite change has to be correlated to one or more other constraints in order to satisfy the law of flow conservation. In the network formulation problem, select two nodes s and d that increase its supply b_s and demand b_d by ϵ units respectively, so they change to $b_s + \epsilon$ and $b_d - \epsilon$. Then, the total change in cost can be analyzed by applying the change to the dual problem, whose objective function changes from

$$\sum_{i \in N} b_i p_i$$

to

$$\sum_{i \in N, i \neq s, i \neq d} b_i p_i + (b_s + \epsilon) p_s + (b_d - \epsilon) p_d,$$

which can be rewritten as

$$\sum_{i \in N, i \neq s, i \neq d} b_i p_i + b_s p_s + \epsilon p_s + b_d p_d - \epsilon p_d.$$

hence

$$\sum_{i \in N} b_i p_i + \epsilon (p_s - p_d).$$

Therefore, the resultant cost change is equal to the unit change times the difference between the imputed prices. It is then concluded that the marginal cost of shipping an additional unit between any two regions is equal to the difference of their respective imputed prices.

Model Formulation: A Special Case of the Generalized Network Flow Problem

The dairy market includes several non-homogeneous products that are constituted from the same components. Raw milk flows from the production to the processing sector and milk products flow from the processing into the consumption sector. As a result, product transformations take place at the processing sector. These transformations are formulated in the model as three unit conversions; one from raw milk into its components, another from components into interplant products, and the third from components into final products.

These unit conversions categorized the final model as a generalized network flow problem, where the law of flow conservation is questioned. However, the unit conversions are stated in a way that the material balance or law of flow conservation is still valid for milk components. Therefore, the actual model formulation can be reduced to one with milk components alone. Consequently, it can be deduced that all the network-flow-problem properties previously stated are valid for this generalized network-flow problem.

The Empirical Model

The framework structure of the model is the transshipment problem formulation. Economic activity at production (farms), processing (plants), and consumption (markets) dairy sectors is modeled in a transportation-cost minimization problem by using a linear programming formulation. Milk and milk-products are modeled as flowing through the network of production, processing, and consumption activities in terms of its milk components. Regions, which were designated with representative locations, are used to delineate the geographic distribution of activities aggregated into state and county levels. Actual economic activities were specifically assigned to each modeled region.

Milk products which reach the consumption sector are classified in five categories: fluid milk products (FM); soft products (ST); cheese products (CH); butter products (BT); and condensed, evaporated, and dry products (PD). Cream (CR), skim milk (SK), and non-fat dry milk (NDM) model the exchange of dairy interplant products in the processing sector. Fat and solids-not-fat (SNF) were established as the components for raw-milk and milk products, and they are used to establish material balance on the conversion of raw-milk into different milk products.

The model primal solution represents the efficient allocation of milk and milk products among the activities in terms of transportation efficiency. Its associated dual solution or imputed price solution depicts the transportation composition price valuation to each activity for a given milk product and region.

Algebraic Formulation

Entities—*e.g.* regions, final products, etc.—included in the model are defined as sets. An upper-case character and its lower case are used to identify a set and its index respectively. Also, an index prime notation is used for subsequent appearances of the set in a given expression.

The objective function to minimize is specified as

(1)　minimize

$$\sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \text{AsCost}_{ij} \times As_{ijp}$$

$$+ \sum_{j \in J} \sum_{p \in P} \sum_{j' \in J} \sum_{p' \in P} \sum_{n \in N} \text{IntCost}_{jj'n} \times Int_{jpj'p'n}$$

$$+ \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \text{DistCost}_{jkp} \times Dist_{jkp}$$

Subject to the following constraints:

(2)　$\displaystyle RMSup_i \geq \sum_{j \in J} \sum_{p \in P} As_{ijp}, \qquad \forall i \in I$

(3)　$\displaystyle \sum_{i \neq I} RMComp_{ic} \times As_{ijp} \geq Rec_{jpc},$

$$\forall j \in J, \qquad \forall p \in P, \qquad \forall c \in C$$

(4)　$\displaystyle Rec_{jpc} + \sum_{j' \in J} \sum_{p' \in P} \sum_{n \in N} Int_{j'p'jpn} \times \text{IPComp}_{nc}$

$$\geq \sum_{j' \in J} \sum_{p' \in P} \sum_{n \in N} Int_{jpj'p'n} \times \text{IPComp}_{nc}$$

$$+ Proc_{jpc},$$

$$\forall j \in J, \qquad \forall p \in P, \qquad \forall c \in C$$

(5)　$\displaystyle Proc_{jpc} \geq \sum_{k \in K} \text{DemComp}_{kpc} \times Dist_{jkp},$

$$\forall j \in J, \qquad \forall p \in P, \qquad \forall c \in C$$

(6)　$\displaystyle \sum_{j \in J} Dist_{jkp} \geq \text{DemQty}_{kp},$

$$\forall k \in K, \qquad \forall p \in P$$

(7)　$\displaystyle \sum_{j \in J} \sum_{p \in \{BT, PD\}} As_{ijp} \geq \alpha \times RMSup_i, \qquad \forall i \in I$

(8)　$\displaystyle \beta \times \left(\sum_{j' \in J} \sum_{p' \in P} \sum_{n \in N} Int_{jFMj'p'n} + \sum_{k \in K} Dist_{jkFM} \right)$

$$\geq \sum_{j' \in J} \sum_{p' \in P} \sum_{n \in N} Int_{jFMj'p'n}, \qquad \forall k \in K$$

(9a)　$\displaystyle \sum_{k \in K} Dist_{jkp} \leq \text{PlanCap}_{jp},$

$$\forall j \in J, \qquad \forall p \in P, \qquad p \neq FM,$$

$$p \neq PD,$$

(9b)　$\displaystyle \sum_{k \in K} Dist_{jkFM} + \sum_{j' \in J} \sum_{p' \in P} Int_{jFMj'p'CR}$

$$+ \sum_{j' \in J} \sum_{p' \in P} Int_{jPDj'p'CR} + \sum_{j' \in J} \sum_{p' \in P} Int_{jBTj'p'SK}$$

$$\leq \text{PlanCap}_{jFM}, \qquad \forall j \in J$$

(9c)　$\displaystyle \sum_{k \in K} Dist_{jkPD} + \sum_{j' \in J} \sum_{p' \in P} Int_{jPDj'p'NDM}$

$$\leq \text{PlanCap}_{jPD}, \qquad \forall j \in J$$

(10)　$As_{ijp}, Rec_{jpc}, Int_{jpj'p'n}, Proc_{jpc}, Dist_{jkp} \geq 0$

$$\forall i \in I, \quad \forall j, j' \in J, \quad \forall k \in K,$$

$$\forall p, p' \in P, \quad \forall c \in C, \quad \forall n \in N,$$

where, the entities are

- R, regions,
- I, production regions (aggregation of dairy farms); such that $I \subseteq R$,
- J, processing regions (aggregation of dairy plants); such that $J \subseteq R$,
- K, consumption regions (aggregation of population); such that $K \subseteq R$,
- $P = \{FM, ST, CH, BT, PD\}$, final products,
- $N = \{CR, SK, NDM\}$, interplant products, and
- $C = \{FAT, SNF\}$, milk components.

The parameters in the model are named using a regular font style as

- $AsCost_{ij}$, Assembly cost of raw milk from production region i to processing region j,
- $IntCost_{jj'n}$, Shipment cost of interplant product n from processing regions j to processing region j',
- $DistCost_{jkp}$, Distribution cost of final product p from processing region j to consumption region k,
- $RMSup_i$, Raw milk supply at production region i,
- $RMComp_{ic}$, Raw milk composition in component c at production region i,
- $IPComp_{nc}$, Interplant product n composition in component c,
- $DemComp_{kpc}$, Final product p composition in component c at consumption region k,
- $DemQty_{kp}$, Demand of final product p at consumption region k,
- α, Operating reserve factor at production regions for manufactured products,
- β, Ratio between interplant products and FM final products,
- $PlantCap_{jp}$, Processing capacity for final product p at region j.

Decision variables in the model are named using italic font style as

- As_{ijp}, Amount of raw milk shipped from pro-

duction region i to processing region j which will be used for final product p,
- Rec_{jpc}, Amount of component c received to be used for final product p at processing region j,
- $Int_{jpj'p'n}$, Amount of interplant product n shipped from the final product p at processing region j to final product p' at processing region j',
- $Proc_{jpc}$, Amount of component c used in the production of final product p at processing region j, and
- $Dist_{jkp}$, Amount of final product p shipped from processing region j to consumption region k.

Expression (1) states the objective of the problem—minimize the sum of all transportation cost. This is a linear formulation of the decision variables that represents transportation cost among regions on a per-unit basis. The objective is composed by the costs associated with each of the shipments among the production, processing, and consumption activities. Each of these terms is expanded for all possible combination of incoming regions, outgoing regions, products, and inter-plant products.

Raw milk production is modeled with expression (2). These constraints establish for each region that the total shipments of raw milk outgoing a region must not be greater that the total available at the region.

The reception of milk components at a processing region is designed with expression (3). For each processing region and final product destination, the set determines the amount of components contained in the raw milk, and it establishes that the amount coming into the processing region must not be greater than the total amount of each component received.

Interplant transfers among regions are modeled with the set of expressions (4). These constraints establish the flow balance at the processing regions. For each processing region and final product destination, the set states that the amount of each component, either from raw milk or inter-plant products, entering the region must not be less than the amount leaving the region.

The processing of final products at processing regions is modeled with the set of expressions (5). For each processing region and final product destination, the set determines the amount of the final product, and it establishes that the amount of each component leaving the processing region must not be less than the total amount in terms of final product outgoing the region.

The distribution of final products at consumption regions is modeled with the set of expressions (6). For each consumption region and final product destination, it states that the total amount of final product shipped into the region must not be less than the amount of the final product consumed.

An operational requirement is established at the assembly of raw milk with the set of expression (7). These constraints establish for each production region i that at least an $\alpha = 0.15$ proportion of the total shipments of raw milk leaving a region must be destined to either a BT or PD final product processing regions. This requirement reflects the actual processing of raw milk for manufacturing purposes even in fluid milk deficit regions.

A weight balance requirement is established at the processing regions with the set of expression (8). These constraints are necessary because not all milk components are used in the model. Therefore, they ensure that, on a weight basis, a processing region accomplishes reality. Particularly, the control is applied to FM processing regions were there is not a weight reduction due to processing. The constraints establish a requirement of no more than a proportion ($\beta = 0.1$) of interplant products to final products at FM processing regions. For instance, the weight of interplant products that leaves an FM processing regions must not exceed one-tenth of the fluid milk product leaving the region.

Processing capacity constraints are established by expressions (9a), (9b) and (9c). The requirement (9a) establishes capacity constraints for final products ST, CH and BT at the processing regions. The sets of expressions (9b) and (9c) establish capacity constraints for final products FM and PD, and they include the outgoing interplant products CR and SK

in the FM final product capacity and the interplant product NDM in the PD final product capacity account respectively.

Non-negativity constraints are established by the set of expressions (10). These constraints establish the direction of flow from production through processing to consumption region by imposing the non-negativity requirement, because negative values would mean backward or inverse shipments; these are not allowed in the formulation.

Specification of the Model's Parameters

Parameters in the model are classified as entities—data with static definitions—and relationships among entities—data with a dynamic nature (monthly basis). The model has a county- or parish-level aggregation for the southeastern states and a state-level aggregation for the remaining 40 conterminous states. The Southeast region consists of Alabama, Florida, Georgia, Louisiana, Mississippi, North Carolina, South Carolina, and Tennessee. The complete geographic delineation contains 723 regions: 40 states (all states except southeastern states), 680 southeastern counties, and three sites modeling the import sector.

Production, Processing and Consumption Entities Modeled at Regions

The production sector is modeled as the aggregation of raw milk production in each region. The processing sector is designed as the aggregation of the processing facilities for each milk product in each region. Consequently, in a given region there are as many processing sectors as milk products. Its processing capacity is assumed invariant during the year. Moreover, the average annual ratio between product throughput and capacity is approximately one half for each milk product (Testuri).

The consumption sector is modeled as the aggregation of milk product consumption in each region. Usually there is milk product consumption for each region, since it is estimated from the region's population. Because of the lack of definitive information about consumption, the estimated consumption of

each product class was accomplished by using information on production, population, consumption trends, net foreign trade, and net stock change. Levels in foreign trade and stock are modeled as consumption of a specific class of products without an explicit representative geographic region. Since the model's balance requirement between supply and demand is stated in terms of composition of raw milk supply and milk products, demand quantities—the most uncertain figures—were adjusted to satisfy the equalizing requirement of composition for each month.

Transportation Costs

Three main types of transportation cost were modeled among regions for the different classes of products. They are assembly cost of raw milk from production to processing regions, the shipment cost of interplant products among processing regions, and the distribution cost of final product from processing to consumption regions.

Transportation costs were assumed to be a linear function of product weight. Furthermore, unitary transportation costs (per unit of weight) between regions are a function of the distance between them and the transportation labor cost associated with the departure region. Unitary transportation costs involving distance (Pratt, et al. 1997) and a wage labor index were established for milk assembly, interplant shipment, and final product distribution. The labor index models the variation of labor costs among the regions.

Results

Twelve monthly instances of the model were evaluated for 1997. In the model context, the set of imputed prices defines the solution to the dual problem. The imputed price solution provides a measure of the change in the objective's optimal value associated with a unitary change of the constraint resources. Imputed prices can be established for each resource at each economic activity for each product class. The imputed prices obtained from the model do not include price compo-

nents associated with production or processing costs; moreover, they only represent prices associated with transportation costs.

Estimated Class I Price Differentials

The imputed prices associated to the fluid milk processing sector can be used to simulate the Class I price differentials (USDA 1989). From the dual properties of the network formulation dual properties, it is known that the imputed price solution levels are price offsets with respect to a given imputed price, which is set to zero. Usually this is the imputed price corresponding to the network formulation redundant constraint removed by the LP optimizer. Moreover, the differences among prices remain constant for different solutions.

Class I price differentials were established by the FMMO agreement to ensure the supply of milk from surplus areas into deficit areas. They include the transportation cost incurred in shipments and a premium over the price of milk with manufacturing destination. Therefore, the model's fluid milk imputed prices at the processing sector are equivalent to the Class I price differentials. In order to establish a comparison in absolute terms between Class I prices and imputed prices, imputed prices were shifted towards Class I prices (allowed by the multiple solution property). The imputed price solution was adjusted (translated by a constant, θ) in such a way that the weighted average of its adjusted values with respect to the fluid milk quantities processed was set equal to the 1997 Class I price differential weighted average, 2.59, at 3.5 percent butterfat and average Basic Formula Price (USDA, 1998). The constant used in the translation of the imputed price solution was obtained by solving for θ

$$2.59 = \frac{\sum_{j \in J} \sum_{k \in K} (FMProcImpPr_j + \theta) \times Dist_{jkFM}}{\sum_{j \in J} \sum_{k \in K} Dist_{jkFM}};$$

where $FMProcImpPr_j$ is the associated imputed price of node j specified in term of milk components.
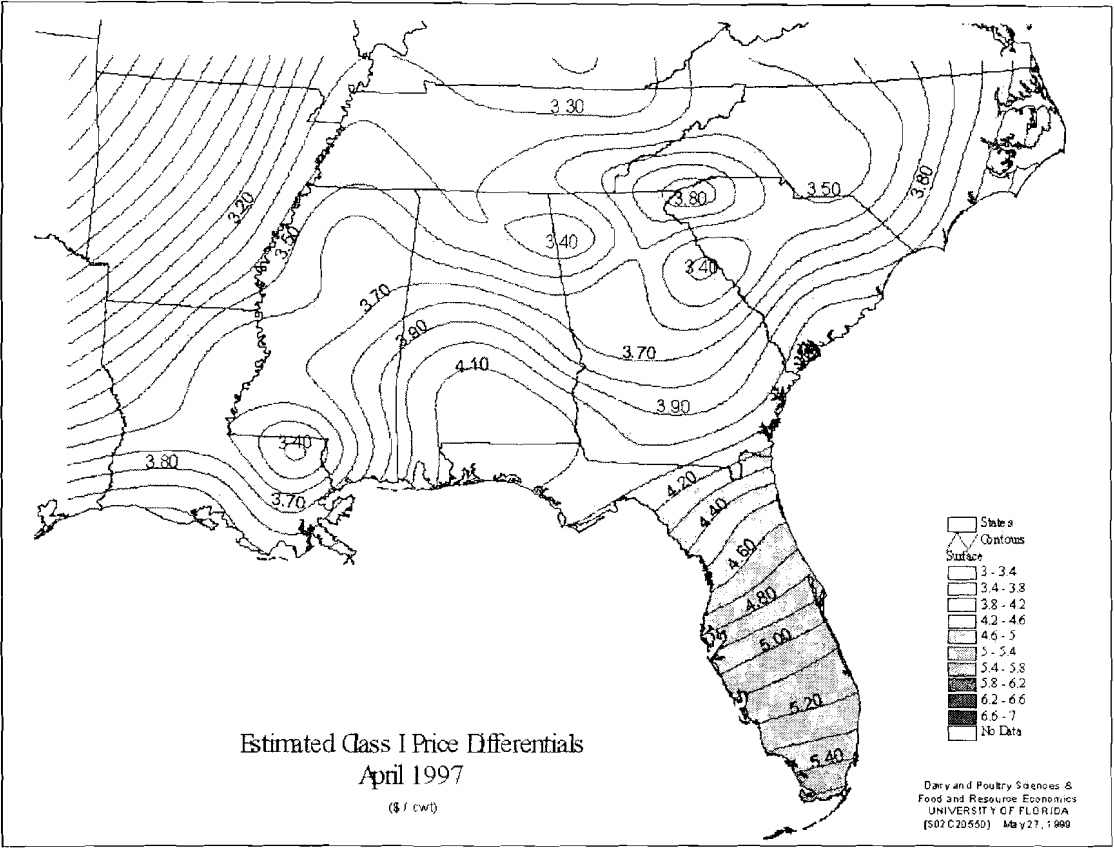
Figure 1. Model estimated Class I price differentials, April 1997

The Seasonality of Estimated Class I Price Differentials in the Southeast

In April and September the supply of raw milk in the Southeast reaches the highest and lowest rates respectively. Also, the lowest and highest estimated Class I price differentials are obtained for April and September solutions, respectively. Figures 1 and 2 show surface maps which depict the estimated Class I price differentials for April and September (Testuri). State level regions were not included in the surface maps since they represent extremely large aggregations and a low-density grid of input points to support surface interpolation and extrapolation.

The Class I price differential difference between September and April is shown in Figure 3. The figure depicts a difference map with a blue-red tone graduated scale ranging from −0.20 to 1.40 with a 0.20 $/cwt. (hundredweight = 100 lb.) interval and along with contours level features at a 0.10 $/cwt. interval.

The difference among the extreme value months presents an increasing pattern towards and along the Florida peninsula. For instance, it averages a $0.60 and $1.00 difference at the frontline and middle range Southeast region respectively and peaks with $1.40 difference at the Miami area. Also, it shows a smooth $1.10 peak at the New Orleans domain. The west Arkansas area and the negative difference display should not be considered since they are located at low-density point support areas for the surface generation.

The correlation coefficient of the estimated values of Class I price differential at the milk processing regions between April and September is 0.95, indicating an almost uniform seasonal change for the Southeast region (Table 1).
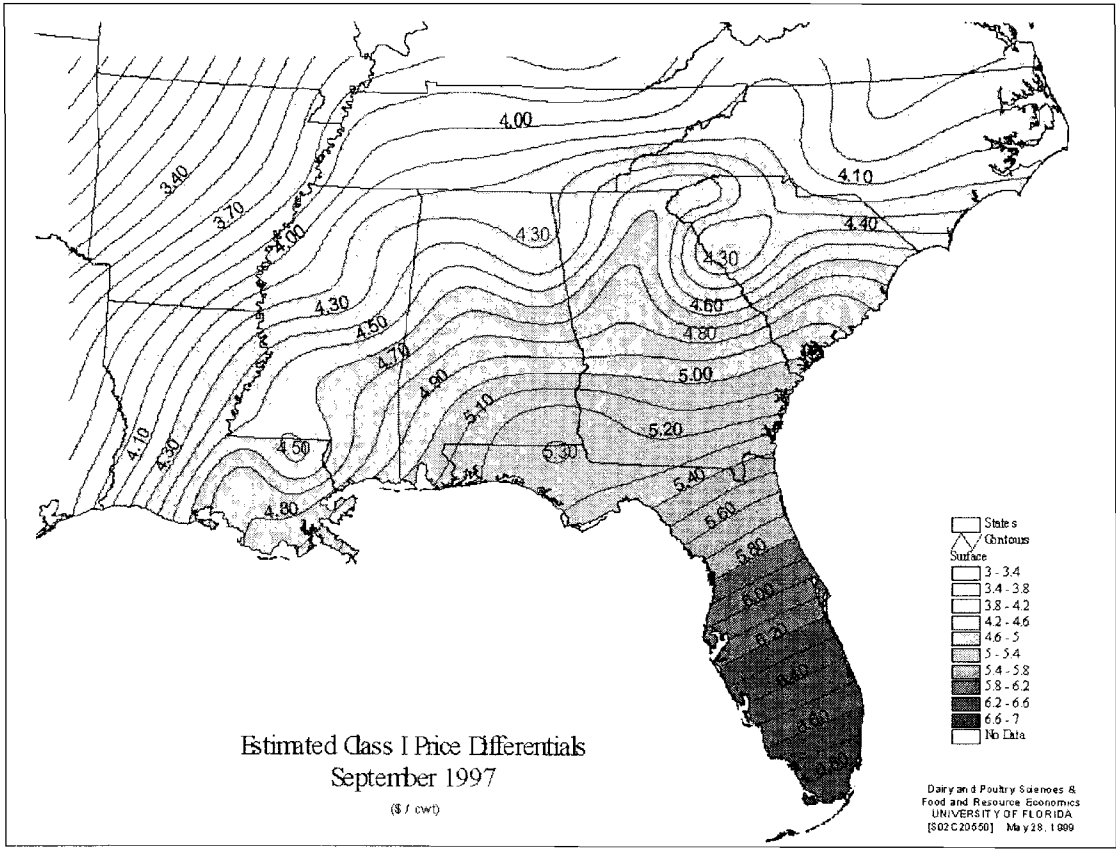
Figure 2. Model estimated Class I price differentials, September 1997

Monthly Variation of Selected Estimated Class I Price Differentials

Selected county regions with their main city were AL73-Jefferson/Birmingham, FL25-Dade/Miami, FL31-Duval/Jacksonville, FL105-Polk/Lakeland, GA121-Fulton/Atlanta, MS89-Madison/Jackson, LA71-Orleans/New Orleans, SC85-Sumter/Sumter, NC107-Lenoir/Kinston, and TN37-Davidson/Nashville. Monthly estimated Class I price differentials of selected regions were depicted in Figure 4.

Most regions show a smooth seasonal pattern with valley and peak at April and September respectively, on the whole reflecting the fluid milk surplus and the deficit months of the year. The exception is Kinston-NC that follows a near constant trend and other border locations of the Southeast region, including Nashville-TN, which depict a non-smooth sea-

sonal pattern. Moreover, it can be seen that the seasonal pattern increases its amplitude in Florida regions, indicating that the seasonal effect is more prominent there.

Florida's regions show the highest values during the year. Their values increase along the peninsula from Jacksonville through Lakeland to Miami. Birmingham-AL, Atlanta-GA, New Orleans-LA, Jackson-MS, and Sumter-SC display a package trend with very close values inside an amplitude range of 0.50 $/cwt. Finally, Nashville-TN shows the lowest values of the compared regions.

Summary and Conclusions

As a capacitated minimum cost network flow problem or transshipment problem, the model encompasses the minimum transportation cost of products among market activities that in-
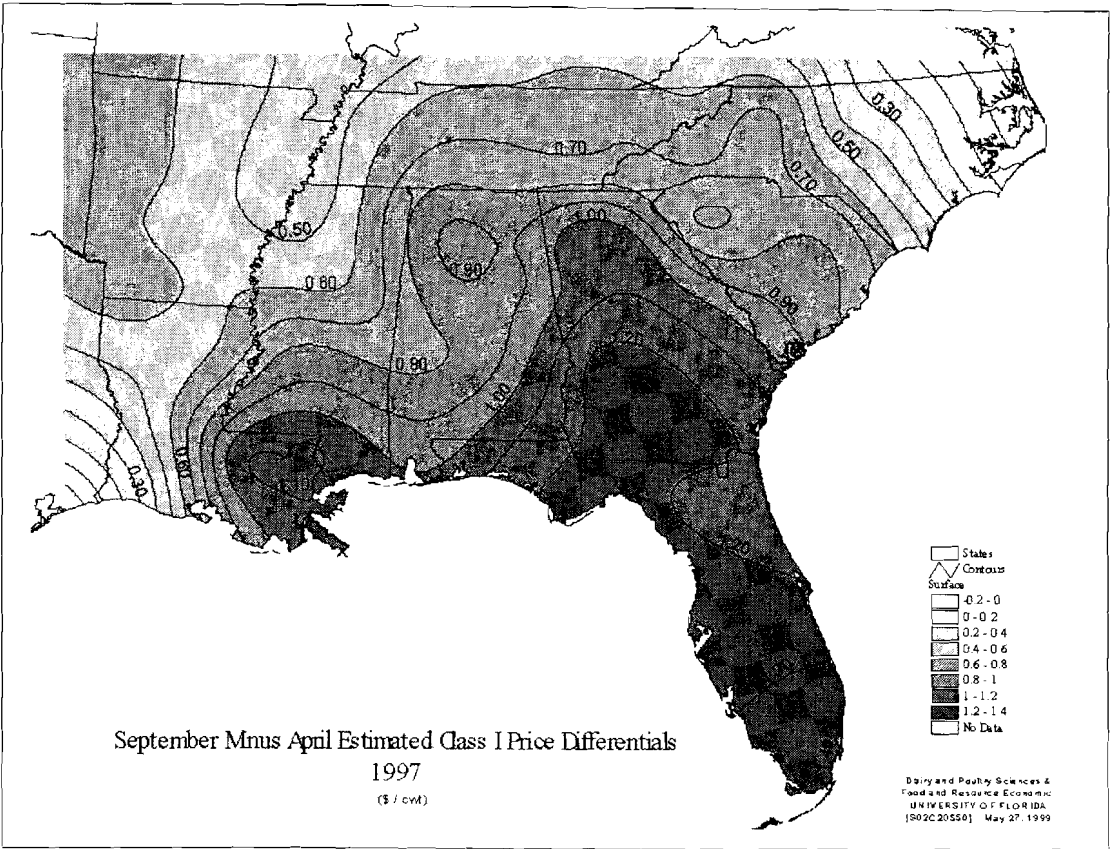
Figure 3. September minus April estimated Class I price differentials, 1997

clude production, processing, and consumption of milk and milk products. Market supply and demand at production and consumption sectors respectively were assumed fixed. Consequently, the market is assumed to be in equilibrium.

The problem was model using a network flow formulation. The United States was divided into 48 state regions, with the eight southeastern states further disaggregated into counties. Each region in the model is treated as a node in the network model. At each node an endowment is specified which represents either the milk available or the milk products required for consumption. Associated with each endowment is an imputed price that measures the change in the objective function or total transportation cost given a per-unit endowment change (duality property). These imputed prices, measuring the transportation cost

associated with each activity for each location, were used to calculate an estimate of the Class I price differentials for the corresponding fluid milk processing sector.

Monthly data of production, processing, and consumption of milk and milk products for each region was estimated for 1997. Non-homogeneous milk and milk products were related in an equilibrium market by balancing their milk components.

Model estimated Class I price differentials show an increasing concave pattern along the range from the southeast sector towards the Florida peninsula for all months of 1997. Also, the estimated differentials maintain an almost constant variation among months; the monthly surface maps show similar shape.

A smooth seasonal pattern is estimated across months with the lowest and highest estimated Class I price differentials for April and
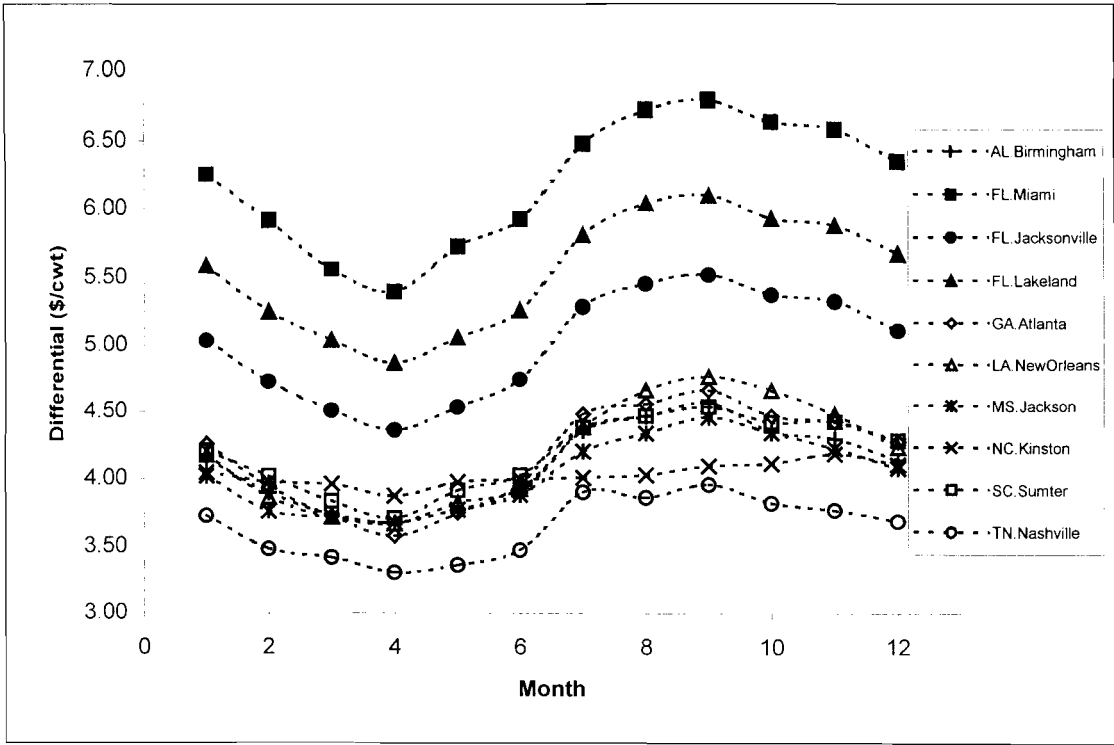
Figure 4.   Selected Southeast estimated Class I price differentials, 1997

September, respectively. For example, Florida's Miami and Jacksonville areas reach 5.40 and 4.36 $/cwt in April and 6.79 and 5.53 $/cwt in September, respectively. These extreme differences correspond to those months with the largest surplus and deficit of fluid milk. The estimated Class I price difference between September and April shows a concave increasing pattern through the Florida peninsula. It reaches values of 1.39 and 1.17 $/cwt in Miami and Jacksonville respectively. This implies that the seasonal impact grows from the Southeast and along the Florida peninsula.

The implication of this research is that the Class I price differentials should be changed from month to month instead of the same differential being use throughout the year as is the current practice. The seasonally adjusted price differentials would increase the price during the deficit months and decrease the milk price during the surplus months. This would increase the quantity produced during the deficit months and decrease the quantity produced during the surplus months.

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