The Impact of Inventory on Tuna Price: An Application of Scaling in the Rotterdam Inverse Demand System

Fu-Sung Chiang, Jong-Ying Lee, and Mark G. Brown

Abstract
This study adopted the scaling approach to examine the impacts of inventories on tuna auction prices in Japan using the Rotterdam inverse demand system. The inclusion of two inventory variables in the model only increases the number of parameters by two. Results indicate that frozen tuna is more likely to be close substitute, fresh and frozen tuna of the same species are more likely to be substitutes, and inventory had significant impacts on auction prices.

Key Words: scaling, Rotterdam inverse demand system, tuna, inventory.

Japan is the world's leading producer and consumer of sashimi grade tuna. Sashimi grade tuna is produced from three types of tuna: yellowfin tuna (Thunnus albacares), bigeye tuna (Thunnus obesus), and bluefin tuna (Thunnus thynnus) (Williams, 1966). Most fresh and frozen tuna fisheries are used for high-value-added sashimi tuna consumption in Japan. Fresh tuna is more expensive and ordinarily available only in fine restaurants. Because of the scarcity of fresh bluefin tuna in Japan, substituting among different tuna species and between fresh and frozen tuna is common (Yamamoto, 1994; Owen and Troedson, 1994; Bose and McIlroy, 1996). For example, fresh bluefin tuna, with high quality and controlled quantity of production, is the most expensive type of tuna. In contrast, frozen tuna is cheaper and available in supermarkets nationwide for the purpose of household consumption.

The landing and prices of frozen sashimi grade tuna at Japan's markets have been steady and firm in the past several years. However, consumers' preferences for sashimi products seem to have changed in 1997 and 1998, apparently as a result of the Asian financial crisis. During this period Japanese consumers shifted their demands toward cheaper products, such as frozen yellowfin and bigeye that are red meat species, instead of expensive fresh/chilled sashimi grade tuna. Because of the economic recovery in 1999, all sashimi grade tuna products are expected to be more in demand in Japan.

In Japan, tuna are harvested by ice-chilled or deep-frozen longline fishing boats, and most tuna are landed at 42 major fishing ports and sold immediately at these wholesale auction markets. From January 1984 through September 1996, yellowfin tuna, bigeye tuna, and bluefin tuna accounted for 18.8 percent, 21.7 percent, and 6.0 percent, respectively.
51.8 percent, and 31.4 percent of the revenue from all tuna sold at those local wholesale fish markets, respectively. During this period the average landing was about 10,686 tons per month. Prices at wholesale markets as producing localities are determined by the interaction between broker demand and landings. In this study prices were calculated as the monthly mean auction auction value divided by the monthly total auction quantity sold in the markets. Price differences among the three types of tuna are due to quality, quantity, and consumer preference.

Over the 1984-91 period, the average monthly frozen tuna inventory was 30,780 tons, about 288 percent of the average quantity sold at wholesale fish markets at producing localities, or about three months' supply. Note that the wholesale markets are not able to hold fresh tuna as inventory.

The Rotterdam inverse demand system (RIDS) has been used to study the formation of fish prices at Belgian fishing ports (Barten and Betten) and the price formation of citrus fruits in the United States (Brown et al.). However, the RIDS applied by Barten and Betten assumed that prices are functions of quantities supplied and did not consider the possibility of other price-influencing variables such as inventory levels. The current study extends Barten and Betten's analysis and uses the scaling approach to incorporate inventory levels in the RIDS.

The Rotterdam Inverse Demand System

In a study of price formation of fish, Barten and Betten developed a RIDS using the direct utility function and World-Hotelling identity. The RIDS used in Barten and Betten's study can be written as

\[ w_d d \ln \tau = \beta D + \sum \beta_i d \ln x_i. \]

where subscript \( t \) represents time; \( \pi \) is the normalized price \( p_i = p_i/m \) of good \( i \); with \( p_i \) and \( m \) being the price and total expenditure, respectively; \( x_i \) is the quantity of good \( i \); \( w_d = x_i \pi_c \) is the budget share of \( x_i \); \( \delta \ln \pi = \log(p_i/m) ; d \ln x_i = \log(x_i/x_{i-1}) ; \delta = \omega \lambda x_i^\alpha \), with \( \lambda \) being the compressed-quantity elasticity; \( \delta = \omega \lambda \), with \( \lambda \) being the scale elasticity; and \( D = \xi_i d \ln x_i \) is the Division volume index. For simplicity, subscript \( t \) will be deleted in the following discussion. The above inverse-demand system satisfies the following demand restrictions

\[ \sum \delta = -1 \quad \text{and} \quad \sum \delta_i = 0 \quad (\text{adding-up}) \]

\[ \delta_i = 0 \quad (\text{homogeneity}) \]

\[ h_i = h_i \quad (\text{Arnotell's symmetry}) \]

Note that the adding-up condition \( \sum h_i = \Sigma \)

\[ \omega \Delta = -1 \quad (\text{based on the reference quantity vector or the reference quantity vector has a scale factor } \kappa = 1 \quad (\text{Anderson}). \]

For scaling, let \( x^* = \phi x, \delta^* = \rho x, m^* = m \); and \( \phi = \phi(z) \) (Barten, 1964; Brown et al.). The impact of non-quantity, net-income variables, \( z \), is introduced through parameters \( \phi \). In this study, variables \( z \) are defined as the beginning inventory of various types of tuna. It is assumed that the existence of tuna inventories may affect the auction price. For example, high inventory levels may have an impact on minimum auction prices and buyers' willingness to offer high prices, i.e., \( \phi > 1 \), or in other words, \( \rho < 1 \). The existence of inventories may also affect buyers' perceptions of quantities available for sale, i.e., it amplifies buyers' perceptions of total physical quantities available for sale, or \( x^* > x \), thus affecting the auction prices of tuna.

The general form of the inverse demand equations (1) for scaling is

\[ w_d d \ln \pi^* = \beta_d d \ln \pi^* + \sum h_i d \ln x_i. \]

Given the scaling definitions of \( x^* \) and \( \delta^* \), let \( d \ln x^* = d \ln x + d \ln \phi, d \ln \pi^* = d \ln \pi, -d \ln \rho, \text{ and } d \ln x^* = \xi_i d \ln x_i \) (note that \( \xi_i = \phi^2 d^2 \phi/dz^2 \)). Also, we can write \( d \ln x_i = (d \ln x_i + d \ln \phi) \) in \( x_i \) and \( \xi_i \) in \( x_i \).

Therefore, (2) can be written as
(3) \[ \frac{\partial \ell}{\partial \theta} = \frac{\partial \ell}{\partial \theta} \mid \tau \]
\[ + h_{0}(\delta \ln \tau + \sum \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]
\[ + \sum h_{0}(\alpha \ln \tau + \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]

where \( \eta = \frac{\partial \ell}{\partial \theta} \mid \theta \). Equation (3) shows that variable \( \tau \) has three impacts on the normalized price \( \hat{\tau} \): a direct impact, \( \frac{\partial \ell}{\partial \hat{\tau}} \theta \), an indirect impact through the scale factor \( h_{0} \), and an indirect impact through the Autonelli coefficients \( h_{0} \). The total impact of inventory \( \tau \) on the normalized price \( \hat{\tau} \) equals \( \frac{\partial \ell}{\partial \hat{\tau}} \theta + h_{0} \theta + k_{0} \frac{\partial \ell}{\partial \tau} \mid \tau \). Note that when all \( \eta_{i} \)s are zero, (3) collapses to (1).

In order to examine the impacts of financial crisis on tuna prices during 1997 and 1998, a dummy variable, \( fc \), was added to (3)

(4) \[ \frac{\partial \ell}{\partial \theta} = \frac{\partial \ell}{\partial \theta} \mid \tau \]
\[ + h_{0}(\delta \ln \tau + \sum \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]
\[ + \sum h_{0}(\alpha \ln \tau + \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]

where \( fc = 1 \) for 1997 and 1998 and \( fc = 0 \) otherwise.

**Data and Results**

The above three model specifications—(1), (3), and (4)—were applied to the Japanese monthly wholesale data on bluefin tuna, bigeye tuna, and yellowfin tuna. Note that (1) is nested in (3) and (3) is nested in (4); hence, (1) and (3) can be used to examine the importance of inventory variables and (3) and (4) for the impacts of financial crisis on tuna prices. The data cover from January 1964 through September 1999. Six types of tuna were considered: fresh and frozen yellowfin tuna, fresh and frozen bluefin tuna, and fresh and frozen bigeye tuna. The data were collected from various monthly issues of Annual Statistics of Fishery Products Marketing (Ministry of Agriculture, Forestry and Fisheries, Japan, 1984–1999), which are Japanese official fishery publications. The data were for monthly Japanese catches from 42 fishing areas in Japan and include the amounts of tuna sold at the respective wholesale fish markets in tons, the average monthly prices in yen per kilogram, and the amounts of tuna in inventory in tons. The inventory variables include frozen yellowfin and bigeye tuna.

Bluefin tuna is protected by the Convention on International Trade of Endangered Species of Wild Fauna and Flora. There are quotas for each fishing country and each bluefin tuna caught has to come with a certification of product of origin issued by the fishing country’s authority. Therefore, the landings of frozen bluefin tuna are limited and unstable (Table 1). As a result of the unstable and limited supply of bluefin tuna and its perceived quality and taste, bluefin tuna commands the highest price among the three tuna species. Also, due to the limited and unstable supply, bluefin tuna inventories are very small and not reported.

The price differences among the six types of tuna are due to differences in quality, supply, and consumer preferences. Based on Table 1, the average monthly prices of fresh tuna are higher than the prices of frozen tuna. In addition, the average monthly prices of bluefin tuna are higher than the prices of bigeye and yellowfin tuna. The average monthly prices of fresh and frozen bluefin tuna are about three and four times the prices of fresh and frozen bigeye and yellowfin tuna, respectively. Note that higher price levels also have higher standard deviations.

For the six types of tuna mentioned above, the following difference forms of (1), (3), and (4) were estimated:

(1') \[ \frac{\partial \ell}{\partial \theta} = \frac{\partial \ell}{\partial \theta} \mid \tau \]
\[ - h_{0}(\delta \ln \tau + \sum \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]
\[ + \sum h_{0}(\alpha \ln \tau + \eta_{i} \frac{\partial \ell}{\partial \tau} \mid \tau) \]

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Table 1. Sample Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Error</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipment Sold (metric tons)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fresh Yellowfin</td>
<td>1.199</td>
<td>854</td>
<td>172</td>
<td>6,178</td>
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<tr>
<td>Frozen Yellowfin</td>
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<td>513</td>
<td>5,295</td>
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<td>311</td>
<td>184</td>
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<td>522</td>
<td>1</td>
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<tr>
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<td>1.020</td>
<td>614</td>
<td>50</td>
<td>3,296</td>
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<tr>
<td>Average Ancient Price (pounds)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Fresh Yellowfin</td>
<td>807</td>
<td>193</td>
<td>390</td>
<td>1,337</td>
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<tr>
<td>Frozen Yellowfin</td>
<td>382</td>
<td>119</td>
<td>313</td>
<td>1,094</td>
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<tr>
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<td>1,477</td>
<td>497</td>
<td>479</td>
<td>2,777</td>
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<tr>
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<td>175</td>
<td>606</td>
<td>1,544</td>
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<tr>
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<td>2,160</td>
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<td>10,328</td>
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<tr>
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<td>3,411</td>
<td>1,518</td>
<td>641</td>
<td>6,871</td>
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<td>Inventory (metric tons)</td>
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<tr>
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<tr>
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<td>0.078</td>
<td>0.000</td>
<td>0.411</td>
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<td>0.263</td>
<td>0.118</td>
<td>0.017</td>
<td>0.663</td>
</tr>
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</table>

$w_i^2 \Delta \ln \tau_i = \alpha_i \tau_i + w_i^2 \eta_i \ln \tau_i + \sum \beta_i \Delta \ln \tau_i + \sum \gamma_i \ln \tau_i$ $+$ $w_i^2 \eta_i \Delta \ln \tau_i + \tau_i.$

To account for seasonality, 12-month differences, as opposed to first differences, were taken in transforming the data as required by these three models. In the above three equations, $w_i^2 = \Sigma w_i^{2*} + \Sigma w_i^{2*}/2$ is the 12-month moving average of the share of good $i$ in total sales. $\Delta \ln \tau_i = \ln \tau_i - \ln \tau_{i-12}$ for $\tau_i$ being $\tau_{i}, \tau_{i},$ and $\tau_{i-12}$ for $\lambda_i,$ being $\lambda_i^*, \lambda_i^*,$ and $\lambda_i^*$ for $\ln \tau_i = \Sigma w_i^2 \ln \tau_i$. The $\beta_i^*, \gamma_i^*$ and $\eta_i$ are assumed to be constants. The disturbance terms, $\nu_i$, were assumed to be normally distributed with mean zero and the errors across equations are contemporaneously correlated. The sets of six equations for ($1^*$), ($3^*$) and ($4^*$) have been estimated jointly by an iterative seemingly unrelated regression procedure. As the data add up by construction, the error co-variance matrix is singular and the equation for frozen bluefin ton was excluded from the system for estimation. As shown by Barter, the estimates are invariant with respect to the equation excluded from the system. The likelihood ratio test (Chow) was used to test model ($1^*$) against unrestricted model ($3^*$) and model ($3^*$) against ($4^*$). The $\chi^2$ test statistic for ($1^*$) and ($3^*$) is $31.85$ (the tabulated value of $\chi^2 = 5.99$ at $a = 0.05$ level), an indication that the addition of inventory variables in the analysis improves the model's explanatory power. The $\chi^2$ test statistic for ($3^*$) and ($4^*$) is $12.65$. Note that $\Sigma, \eta_i^* = 1$; therefore, $\Sigma (\lambda_i^* + \eta_i^*) = 0$, or $\Sigma, \eta_i^* = 0$, or $\Sigma, \eta_i^* = 0$, or $\Sigma, \eta_i^* = 0$. The log likelihood function values for ($1^*$), ($3^*$), and ($4^*$) are $192.52, 195.45$, and $198.98$, respectively.
13.70 (the table value of \( \chi^2_{57} = 14.07 \) at \( \alpha = 0.05 \) level), an indication that financial crisis during 1997 and 1998 had significant impacts on tuna prices. Based on these test results, model (4') was used in the following discussion. Table 2 shows the estimates for \( a, b_0, h \), and \( h_0 \) together with their corresponding standard errors in parentheses for model (4').

As Table 2 shows, the financial crisis in 1997 and 1998 had a negative impact on frozen bigeye tuna price and a positive impact on fresh yellowfin tuna price while the financial crisis impact on other tuna prices was not statistically different from zero. The scale effects \( h_i \) are all negative and statistically different from zero at \( \alpha = 0.01 \) level. These negative scale effects are to be expected. With a fixed budget \( a \), a proportionate increase of all quantities causes a decrease in prices, p.s. hence a decrease in \( \pi = p/n \). The scale coefficient \( h_i \) can be divided by \( w_i \) and converted into scale elasticities. The estimatedapat sample means of \( w_i \) scale elasticities are presented in Table 3. Results show that the scale elasticities for the three types of frozen tuna are smaller than those for the three types of fresh tuna. The scale elasticities for frozen yellowfin and bigeye tuna are less than unity in absolute value, while the rest of the scale elasticities are greater than unity in absolute value. This result may reflect that fresh tuna is more perishable than frozen tuna; hence, fresh tuna prices are more responsive to scale changes than frozen tuna prices.

The own substitution effects \( h_{ij} \) are all negative and significantly different from zero except for the ones for frozen bigeye tuna and frozen yellowfin tuna. The estimated compensated own-quantity elasticities are derived by dividing \( h_j \) by the sample mean \( w_i \) and are presented in Table 3. Similar to the scale elasticity pattern, the own-quantity elasticities for fresh tuna prices are greater than unity in absolute value than those for frozen tuna prices.

The matrix \( H = [h_{ij}] \) reflects to a certain degree the interactions between the goods in their ability to satisfy \( w_i \). More of good \( i \) is generally sold at a lower price for \( i \). One may also say that a good is its own substitute. Extending the notion of substitution to all negative \( h_{ij} \), it is natural to consider a positive \( h_{ij} \) as an indication of complementarity between \( i \) and \( j \). Note that the adding-up condition \( \sum_i h_{ij} = 0 \) together with \( h_0 > 0 \) means that \( \sum_i h_{ij} > 0 \); therefore, for \( i \neq j \) complementarity may dominate in an inverse-demand system. The dominance does not come from the structure of preferences but from the condition \( \pi x = 1 \), which makes the \( h_{ij} \)s imperfect measures of the interaction of goods in their satisfaction of wants.

Barten and Bettendorf worked with a transformation of the \( H \). Using the vectors \( h = [h_{ij}] \) and \( w = [w_i] \) they derived the counterpart of the Allais coefficients for the inverse-demand system. By selecting \( r \) and \( s \) as the standard pair of goods, the Allais coefficient for the inverse demand system can be defined as

\[
\begin{align*}
\phi_{ij} &= h_{ij}/w_i - h_{ij}/w_j - (h_{ij}/w_i - h_{ij}/w_j) + (h_{ij}/w_i - h_{ij}/w_j) \frac{w_j}{w_i} \\
\phi_{ij} &= \phi_{ij}/w_i \frac{w_j}{w_i}
\end{align*}
\]

In the definition of \( a = [a_{ij}] \), the subscripts \( r \) and \( s \) refer to some standard pair of goods \( r \) and \( s \). The above equation indicates that \( a_{ij} = 0 \). Thus \( a_{ij} > 0 \) indicates that \( i \) and \( j \) are more complementary than \( r \) and \( s \), while \( a_{ij} < 0 \) indicates that \( i \) and \( j \) are stronger substitutes than \( r \) and \( s \), and \( a_{ij} = 0 \) indicates that \( i \) and \( j \) have the same type interaction as \( r \) and \( s \). Based on the Allais coefficient the measure of the intensity of interaction can be defined as

\[
\phi_{ij} = a_{ij}/(a_{ij} + a_{ij})^{1/2}
\]

which for a negative definite matrix \( A = [a_{ij}] \), \( a_{ij} \) varies between \(-1\) (perfect substitution) and \(+1\) (perfect complementarity).

To apply this relation to the results in Table 2, one has to identify a standard pair of goods. Following Barten and Bettendorf, we have selected the interaction between fresh and frozen yellowfin tuna as the standard pair of goods for the simple reason that this makes all other Allais interactions negative. This expresses the intuitive idea that all the types of tuna considered here are substitutes in consumption. Sample means of \( w_i \) were used in the calculation of Allais interaction intensity. Results are pre-
### Table 2. Demand Parameter Estimates

<table>
<thead>
<tr>
<th>Financial Crisis Dummy</th>
<th>Scale Effect</th>
<th>Fresh Yellowfin</th>
<th>Frozen Yellowfin</th>
<th>Fresh Bigeye</th>
<th>Frozen Bigeye</th>
<th>Fresh Bluefin</th>
<th>Frozen Bluefin</th>
<th>Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresh Yellowfin</td>
<td>0.0102*</td>
<td>-0.0864*</td>
<td>-0.0189*</td>
<td>0.0064*</td>
<td>0.0001</td>
<td>0.0069*</td>
<td>0.0008</td>
<td>0.0039</td>
</tr>
<tr>
<td>Frozen Yellowfin</td>
<td>-0.0057</td>
<td>-0.0821*</td>
<td>-0.0147*</td>
<td>0.0058</td>
<td>0.0003</td>
<td>0.0037</td>
<td>0.0010</td>
<td>0.0024</td>
</tr>
<tr>
<td>Fresh Bigeye</td>
<td>-0.0043</td>
<td>-0.1589*</td>
<td>-0.0145*</td>
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<td>0.0023</td>
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<tr>
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<td>0.0017</td>
<td>-0.0009</td>
<td>0.0017</td>
<td>0.0027</td>
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<td>-0.0009</td>
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<td>-0.0224</td>
<td>0.0198</td>
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</tr>
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</table>

Numbers in parentheses are standard errors of parameter estimates.

* Statistically different from zero at α = 0.01 level.
sent in Table 3. Note that the diagonal entries are -1, which is consistent with the notion that a good is its own perfect substitute. Also, by construction, the Allais interaction intensity between fresh and frozen yellowfin tuna is zero. Of the 14 Allais intensity coefficients, only four are greater than 0.50 and absolute value.

Note that the base of comparison is the substitution relationship between fresh and frozen yellowfin tuna. As Table 3 shows, the highest Allais interaction intensity coefficient is the one between frozen bigeye tuna and frozen bluefin tuna (-0.98). The second highest Allais interaction intensity coefficient is that between fresh bigeye tuna and fresh bluefin tuna (-0.66), followed by the substitution relationships between frozen yellowfin tuna and frozen bluefin tuna (-0.56), and between fresh bluefin tuna and frozen bluefin tuna (-0.54). These findings—frozen tunas are more likely to be close substitutes and fresh and frozen tunas of the same species are more likely to be substitutes—seem to be quite reasonable.

The inventory effects are evaluated at sample means of \( w \), and presented in Table 4. The direct and indirect impacts of inventory are presented in the first three columns, and the total inventory impact and inventory elasticity estimates are presented in the last two columns. Results show that inventory had direct impacts on frozen yellowfin tuna and frozen bigeye tuna on normalized price, \( w_n \), are negative.

Equation (4.7) shows that a negative direct effect, \( \eta_n \), would reduce the quantity \((d \ln X + \sum \eta_i x_j \ln x_i)\), and, with the negative scale effect \( h_n \), the result is a positive indirect scale inventory effect. For the indirect substitution effect, a negative \( \eta_n \) is also equivalent to a decrease in \( d \ln x_i \); therefore, the impact of this indirect substitution effect depends on the sign of \( h_n \). When \( h_n \) is negative, the result is a positive effect; when \( h_n \) is positive, the indirect inventory effect would be negative.

Results presented in Table 4 show that the direct inventory effects for frozen yellowfin tuna and frozen bigeye tuna are negative. All
<table>
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<th>Inventory Effect and Inventory Elasticity</th>
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<tr>
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<td>Direct (1) (Δ%)/Scale (2) (Δ/Q)</td>
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<td>Bigeye Tuna Inventory</td>
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<td>Frozen Bluefin</td>
<td>0.0054*</td>
</tr>
</tbody>
</table>

Numbers in parentheses are standard errors. *Statistically different from zero at α = 0.01 level.

Indirect scale inventory effects are positive, indicating that inventory had positive scale impacts on prices. The own-substitution inventory effect for frozen yellowfin tuna is positive and not statistically different from zero for frozen bigeye tuna. The cross-substitution inventory effects are either negative or statistically not different from zero. As shown in Table 4, the indirect scale inventory effects dominated the total inventory effects in the six types of tunas studied.

The total own-inventory effects for frozen yellowfin and bigeye tunas are negative; in other words, when the inventories of these two types of tunas increase, their auction prices would decrease. Results show that for a 1-percent increase in the inventories of frozen yellowfin and bigeye tunas, the auction prices of these tunas would decrease by 0.12 percent and 0.07 percent, respectively. Results also show that when inventories of frozen yellowfin and frozen bigeye tunas increase, the auction price of other tunas would increase. The estimated inventory elasticities indicate that bigeye tuna inventory had larger impacts on auction prices than the inventory of yellowfin tuna. The total inventory elasticity estimates presented in Table 4 indicate that a 1-percent increase in frozen yellowfin tuna inventory would increase the prices of other tunas by less than 0.02 percent, while a 1-percent increase in frozen bigeye tuna inventory would increase the prices of other tunas by more than 0.04 percent.
Concluding Remarks

The addition of inventory variables to an in-verse demand system may result in a large in-crease in parameter space and make estimation difficult. The scaling approach adopted in this study does not add a large number of ad-di-itional parameters to be estimated. The inclu-sion of two inventory variables in the model only increases the number of parameters by two. In addition, the results found in the study seem reasonable. Although the results of this study are specific for the Japanese tuna mar-kets, the approach used in this study is easy to apply to other problems. One of these prob-lems could be whether generic advertising in-creases growers' retails.

Theoretically there should be other ways to incor-porate inventory variables in the inverse demand systems. A possible alternative ap-proach would be using a translation approach in the inverse demand system, a concept that inven-tories are necessary for orderly market-ing. Another approach is to assume use of the Tintner-Ichimura conditions (Tintner; Ichimu-ra). In the latter alternative approach, inven-tories would be considered as factors that in-fluence indirect utility. The empirical estima-tible models using these alternative ap-proaches need to be developed.

References

FAO, GlobalFish—Tuna Commodity Update, Food and Agriculture Organization of the Unit-ed Nations, 1997.