A Uniform Substitute Demand Model with Varying Coefficients

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ABSTRACT

This study extends Barten's synthetic demand modeling approach to increase the flexibility of the uniform substitute specification of the Rotterdam demand system. Marginal propensities to consume (MPC) vary with budget shares and Slutsky coefficients are defined in terms of varying MPCs. An application of the model to orange-juice products shows that the pattern of income and price elasticities over time is much different than when MPCs are restricted to be constant.

Key Words: *Demand, varying coefficient, Rotterdam model, orange juice.*

Various flexible forms have been proposed for specifying demand systems, including the Almost Ideal Demand System or AIDS (Deaton and Muellbauer 1980a, 1980b) and the Rotterdam model (Theil 1965; Barten 1966). A demand system, referred to as a *synthetic model,* that combines the features of the AIDS and Rotterdam model has also been proposed by Barten (1993). In this paper we consider extending the synthetic modeling approach to increase the flexibility of a particular version of the Rotterdam model, the uniform substitute specification (Theil 1980). A key parameter in our analysis is the marginal propensity to consume (MPC) out of total expenditure. In the Rotterdam, the MPC for a good is treated as a constant, while in the AIDS the implied MPC is equal to a constant plus the budget share for the good in question. In Barten's synthetic model and the model we develop in this

paper, a good's MPC is a linear combination of MPCs for the Rotterdam and AIDS. This specification allows the MPCs to vary over the sample with budget shares. Our model differs from Barten's synthetic model in specification of Slutsky coefficients (price effects). In our model, Slutsky coefficients are defined in terms of the varying MPCs. In Barten's synthetic model, Slutsky coefficients are constant with respect to MPCs, although variable with respect to budget shares.

Allowing model parameters to vary can be important for estimating sensitivity of demand to income and prices. For example, in non-varying parameter specifications of the Rotterdam model, income and compensated price elasticities for a good are the good's constant MPC and Slutsky coefficients divided by the good's budget share, respectively. Thus, if a good is experiencing a declining sales trend, its income and compensated price elasticities are increasing in absolute value, to the extent its budget share is decreasing. Such a result may inaccurately depict the changing demand situation, as there is no reason why income and price elasticities can not be decreasing in absolute value with decreasing budget shares.

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Varying parameter specifications allows estimation of such possible demand responses.

The price coefficient structure for uniform substitutes is similar to that for the Rotterdam model under preference independence. In both cases, Slutsky coefficients are specified parsimoniously in terms of MPCs. This specification greatly reduces the parameter space, a potentially useful result for estimating large demand systems that include some products that are close substitutes (e.g., brands). Our proposed specification of varying Slutsky coefficients increases this model's flexibility.

We apply our specification of the uniform substitute model to a group of close substitutes—different types of orange juice products. In the rest of this paper we formally develop the model and discuss our application. The model is based on the traditional consumer allocation problem and we begin with a review of this problem.

Models

A basic problem confronting consumers is how to allocate their income over available goods. A solution to this problem is offered by consumer behavior theory which assumes a consumer chooses from the set of affordable bundles of goods that one which yields the greatest utility. Formally, this problem can be written as maximization of $u = u(q)$ subject to $p'q = x$, where u is utility; $p' = (p_1, \ldots, p_n)$ and $q' = (q_1, \ldots, q_n)$ are price and quantity vectors with p_i and q_i being the price and quantity of good i, respectively; and x is total expenditures or income. The first-order conditions for this problem are $\partial u/\partial q = \lambda p$ and $p'q = x$, where λ is the Lagrange multiplier which is equal to $\partial u/\partial x$. The solution to the first-order conditions is the set of demand equations $q = q(p, x)$, and the Lagrange multiplier equation $\lambda = \lambda(p, x)$. The Rotterdam demand model is an approximation of this set of demand equations and the demand model developed in this paper is a variant of this approximation. Analyses by Barnett, Byron and Mountain show the Rotterdam approximation is comparable to other popular flexible functional forms such as the AIDS.

Rotterdam Model

Following Theil (1975, 1976, 1980), the Rotterdam model can be written as

$$(1) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_i \pi_{ij} d(\log p_i)$$

$$i = 1, \ldots, n,$$

where $w_i = p_i q_i / x$ is the budget share for good i; $\theta_i = p_i(\partial q_i/\partial x)$ is the MPC for good i; $d(\log Q) = \Sigma_i\ w_i d(\log q_i)$ is the Divisia volume index[1]; and $\pi_{ij} = (p_i p_j / x)s_{ij}$ is the Slutsky coefficient, with $s_{ij} = (\partial q_i/\partial p_j + q_j \partial q_i/\partial x)$ being the i, jth element of the substitution matrix S. The Rotterdam model is obtained from the total differential of the first-order conditions (see, e.g, Theil 1975).

The Slutsky coefficient can be written as (e.g., Theil 1980)

$$(2) \quad \pi_{ij} = \phi(\theta_{ij} - \theta_i \theta_j),$$

where $\theta_{ij} = ((p_i p_j \lambda)/(x\phi))u^{ij}$, with u^{ij} being the (i, j)th element of the inverse of the Hessian matrix, $[u^{ij}] = [\partial^2 u/\partial q_i \partial q_j]^{-1}$. The parameter ϕ is referred to as the income flexibility or the reciprocal of the elasticity of the marginal utility of income with respect to income; ϕ is negative based on the assumption that U is negative definite for utility maximization. The term $\phi\theta_{ij}$ captures the specific substitution effect while the term $-\phi\theta_i\theta_j$ captures the general substitution effect (Theil 1975).

The general restrictions on demand (1) are (e.g., Theil 1975, 1976, 1980)

$$(3a) \quad \text{adding up:} \quad \sum_i \theta_i = 1; \quad \sum_i \pi_{ij} = 0;$$

$$(3b) \quad \text{homogeneity:} \quad \sum_j \pi_{ij} = 0;$$

$$(3c) \quad \text{symmetry:} \quad \pi_{ij} = \pi_{ji}.$$

From (1) and (3), the restrictions on Slut-

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[1] The Divisia volume index is a close approximation of $d(\log x) - \Sigma\ w_i d(\log p_i)$, the income term found in deriving the Rotterdam model, as shown by Theil (1971). The term $d(\log Q)$ is used instead of $d(\log x) - \Sigma\ w_i d(\log p_i)$ to ensure adding-up.

sky coefficient specification (2) are $\Sigma_i \; \theta_{ij} = \theta_j$; $\Sigma_j \; \theta_{ij} = \theta_i$; $\Sigma_j \; \Sigma_i \; \theta_{ij} = 1$; and $\theta_{ij} = \theta_{ji}$.

Uniform Substitute Model

Theil (1980) proposed the uniform substitute model to analyze the demands of closely related products such as a group of brands (for an application, see Brown). Beginning with assumptions on how the marginal utilities of closely related products behave, Theil shows the Slutsky coefficients for these products take a form that is essentially the same as that in the preference independent model. Below, we review the assumptions underlying the uniform substitute model and their parameterization implications.

Consider the Rotterdam model specific substitution term θ_{ij} specified in equation (2). This term equals the factor of proportionality, $\lambda/(x\phi)$, times $p_i p_j u^{ij}$. Given u^{ij} is the (i, j)th element of the inverse of the Hessian matrix, the term $p_i p_j u^{ij}$ is the (i, j)th element of the matrix $[\partial^2 u/\partial(p_i q_i)\partial(p_j q_j)]^{-1}$. That is, the inverse of $p_i p_j u^{ij}$ is $\partial^2 u/\partial(p_i q_i)\partial(p_j q_j)$, which indicates how the marginal utility of a dollar spent on good i changes in response to another dollar spent on good j.

Let G denote a group of goods—different types of orange juice products in this study. If the goods in this group were identical, we would expect the above marginal utility changes for these goods to be the same, say k_0. That is, we would have $\partial^2 u/\partial(p_i q_i)\partial(p_j q_j) = k_0$, for i, j $\in$ G. Instead of being exactly identical goods, assume the goods are nearly identical with respect to most attributes but unique with respect to some. The nearly identical nature of goods i and j is assumed to result in generic type changes in the marginal utilities, as indicated by k_0, while the unique nature of the goods are assumed to result in product specific changes in the marginal utilities. These two concepts can be expressed by $\partial^2 u/\partial(p_i q_i) \cdot \partial(p_j q_j) = k_0 + \Delta_{ij} k_i$, where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if i = j, otherwise $\Delta_{ij} = 0$), and both k_0 and k_i are negative. This specification of changes in marginal utilities underlies the uniform substitute model.

Under the assumption that Group G is block independent of other goods, the specific substitution terms for the uniform substitute model can be written as

$$(4) \quad \theta_{ij} = (1/(1 - k\theta_G))\theta_i(\Delta_{ij} - k\theta_j), \quad i, j \in G,$$

where k is a positive parameter reflecting the commonality of the uniform substitutes in affecting utility; and θ_G is the MPC for Group G (Theil 1980; Brown).

Substituting (4) into (2), the Slutsky coefficients for uniform substitutes can be written as

$$(5) \quad \pi_{ij} = \phi_1\theta_i(\Delta_{ij} - \phi_2\theta_j), \quad i, j \in G,$$

where $\phi_1 = \phi/(1 - k\theta_G)$ and $\phi_2 = k + \phi/\phi_1$.

Conditional Rotterdam Model

Rotterdam model (1) is an unconditional specification showing the allocation of total consumer expenditure (x) across goods. Conditional Rotterdam models can also be specified showing how total expenditures on goods in a group are allocated across the goods in that group (e.g, Theil 1976). Below, we develop a conditional demand system for the goods in Group G (orange juice products) (for an application, see Pana-Cryan and Seale).

First, we obtain an expression for aggregate demand for Group G by summing (1) over the goods in G, i.e.,

$$(6) \quad d(\log Q_G) = \theta_G d(\log Q) + \sum_j \pi_{Gj} d(\log p_j),$$

where $d(\log Q_G) = \Sigma_{i \in G} \; w_i d(\log q_i)$; $\theta_G = \Sigma_{i \in G} \; \theta_i$; and $\pi_{Gj} = \Sigma_{i \in G} \; \pi_{ij}$.

Rearranging (6), we find $d(\log Q) = [d(\log Q_G) - \Sigma_j \; \pi_{Gj} d(\log p_j)]/\theta_G$; and substituting this result into (1) we find

$$(7) \quad w_i d(\log q_i) = \theta_i^* d(\log Q_G) + \sum_j \pi_{ij}^* d(\log p_j),$$

where $\theta_i^* = \theta_i/\theta_G$; and $\pi_{ij}^* = \pi_{ij} - \theta_i^*\pi_{Gj}$.

At this point, the j subscript in (7) runs across all goods (j = 1, . . . , n). However, under appropriate conditions, result (7) becomes a conditional demand system for group G (i.e.,

for i, j ∈ G). For block independence, the assumption underlying our uniform substitute specification, the Hessian matrix is group or block independent and $(\partial^2 u/\partial q_i \partial q_j) = 0$ for i and j belonging to different groups. Thus, for i and j in different groups, the inverse element $u^{ij} = 0$ and hence $\theta_{ij} = 0$. This result means that $\pi_{i,j} = -\phi\theta_i\theta_j$, i ∈ G, j ∉ G. As a result $\pi_{ij}^* = 0$, i ∈ G, j ∉ G (i.e., $\pi_{ij}^* = -\phi\theta_i\theta_j - (\theta_i/\theta_G)[-\phi \sum_{i \in G} \theta_i\theta_j] = -\phi\theta_i\theta_j + (\theta_i/\theta_G)\phi\theta_G\theta_j = 0$.) Hence, under block independence, equation (7) is the conditional demand for a good in Group G.

For uniform substitute specification (5), the conditional Slutsky coefficient in equation (7) is

$$(8) \quad \pi_{ij}^* = \phi_i\theta_i(\Delta_{ij} - \phi_2\theta_j)$$
$$- \theta_i^* \sum_{i \in G} \phi_i\theta_i(\Delta_{ij} - \phi_2\theta_j)$$

where $\phi^* = (\phi\theta_G)/(1 - k\,\theta_G)$. The parameter ϕ^* is negative given ϕ is negative and $0 < \theta_G < 1$.

Hence, for uniform substitutes, conditional demand equation (7) can be written as

$$(9) \quad w_i d(\log q_i)$$
$$= \theta_i^* d(\log Q_G)$$
$$+ \sum_{j \in G} \phi^*\theta_i^*(\Delta_{ij} - \theta_j^*)d(\log p_j),$$
$$= \theta_i^* d(\log Q_G)$$
$$+ \phi^*\theta_i^*\left(d(\log p_i^*) - \sum_{j \in G} \theta_j^* d(\log p_j)\right).$$

The term $\sum_{i \in G} \theta_j^* d(\log p_j)$ is known as the *Frisch price index* for Group G (e.g., Theil 1980).

By dividing (9) by $w_G = \sum_{i \in G} w_i$, we obtain an alternative conditional demand specification,

$$(10) \quad w_i^* d(\log q_i)$$
$$= \theta_i^* d(\log Q_G^*)$$
$$+ \phi^{**}\theta_i^*\left(d(\log p_i^*) - \sum_{j \in G} \theta_j^* d(\log p_j)\right),$$

where $w_i^* = w_i/w_G$; $d(\log Q_G^*) = \sum_{i \in G} w_i^* d(\log$

$q_i)$, a conditional Divisia volume index; and $\phi^{**} = \phi^*/w_G$.

Synthetic Model

Consider using the AIDS to model how total expenditure on goods in Group G is allocated to the goods in the group. For this allocation problem, the AIDS can be written as

$$(11) \quad w_i^* = \alpha_i + \sum_j \gamma_{ij}\log(p_j) + \beta_i\log(x^*/P),$$
$$i, j \in G,$$

where $x^* = \sum_{j \in G} p_j q_j$; P is a price index, and α_i, γ_{ij}, and β_i are coefficients.

For AIDS model (11), the MPC for good i is $p_i(\partial q_i/\partial x^*) = w_i^* + \beta_i$. This result can be verified by noting that by definition $p_i q_i = x^* w_i^*$; hence, $p_i(\partial q_i/\partial x^*) = w_i^* + x^*(\partial w_i^*/\partial x^*) = (w_i^* + \beta_i)$. We can see then that the MPC for the AIDS varies with w_i^*, while the MPC for the Rotterdam model is constant. Linearly combining the MPCs for the AIDS and Rotterdam models, we obtain

$$(12) \quad p_i(\partial q_i/\partial x^*) = b_i + \rho w_i^*,$$

where $b_i = \rho\beta_i + (1 - \rho)\theta_i^*$, and ρ is between zero and one. Result (12) is the MPC for Barten's (1993) synthetic model.

Consider substituting the right hand side of equation (12) for θ_i^* in equation (10). This substitution would yield a model similar to the CBS model (Barten 1993), except for specification of Slutsky coefficients. In the CBS model, Slutsky coefficients are specified as constants as in equation (1), while the proposed substitution here would make the Slutsky coefficients functions of budget shares through the MPCs. Note that the budget shares in the specifications of MPCs and Slutsky coefficients are endogenous parts of the model. To account for this endogeneity, we replace w_i^* in (12) with its lag value. Our resulting model can then be written as

Table 1. Sample Means

Variable	OJ[a] Category			
	FCOJ[b]	NFC[c]	RECON[d]	Other
Weekly Per Capita Gallons	0.020	0.013	0.024	0.001
	(0.005)[e]	(0.004)	(0.002)	(0.000)
Price: $/Gallon	3.030	4.951	3.560	5.071
	(0.322)	(0.343)	(0.361)	(0.246)
Budget Share	0.282	0.290	0.407	0.021
	(0.074)	(0.081)	(0.015)	(0.004)

[a] OJ = orange juice.
[b] FCOJ = frozen concentrated orange juice.
[c] NFC = not-from-concentrate orange juice.
[d] RECON = reconstituted orange juice.
[e] Standard deviations in parentheses.

(13) $\quad w_{i,t}^* d(\log q_{i,t})$

$$= (b_i + \rho w_{i,t-1}^*) d(\log Q_{G,t}^*)$$

$$+ \phi^{**}(b_i + \rho w_{i,t-1}^*)$$

$$\times \left[d(\log p_{i,t}) - \sum_{j \in G} (b_j + \rho w_{j,t-1}^*) d(\log p_{j,t}) \right],$$

where subscript t has been added to the variables to denote time. Equation (13) is our proposed model—it extends the synthetic and CBS models by allowing the Slutsky coefficients to vary consistently with the MPCs. The coefficients in equation (13) to be estimated are b_i, ρ, and ϕ^{**}, which we will refer to as the MPC constant for good i, the MPC slope and the (conditional uniform substitute) income flexibility, respectively.

Given that the adding-up condition requires the conditional MPCs sum to one, coefficients b_i and ρ obey

(14a) $\quad \sum_{i \in G} (b_i + \rho w_{i,t-1}^*) = 1 \quad$ or

(14b) $\quad \sum_{i \in G} b_i = 1 - \rho.$

Based on the underlying assumptions of the uniform substitute model (Theil, 1980), all MPCs must be positive, and, given the adding-up property, each MPCs is required to be in the zero-one interval. Given ϕ^{**} is negative,

these MPC restrictions guarantee negativity of the Slutsky matrix. These results indicate that the additional flexibility of model (13) has come at a cost: namely, some estimates of b_i and ρ may result in MPCs outside the zero-one interval, depending on the size of the budget shares.

Application

We take model (13) as our maintained hypothesis and apply it to ACNielsen grocery-store scanner sales data on four orange juice (OJ) categories—frozen concentrated orange juice (FCOJ), not-from concentrate orange juice (NFC), reconstituted orange juice (RECON), and other orange juice including canned and aseptic products (OTHER). The ACNielsen data are from retail chains doing $2 million or greater annual business; these chains represent roughly 80% of total U.S. OJ sales. The data are weekly running from the week ending January 9, 1988 through the week ending November 28, 1998 (569 weekly observations). The raw data were comprised of gallon and dollar sales. In our application, quantity demanded was measured by per-capita gallon sales which was obtained by dividing gallon sales by the U.S. population; prices were obtained by dividing dollar sales by gallon sales. Sample mean-per-capita gallon sales, prices and budget shares are shown in Table 1. The infinitely small changes in quantities and prices in model (13) were measured

Table 2. Full Information Maximum Likelihood Estimates of Equation (13)

	OJ[a] Category				MPC[e]	Income
	FCOJ[b]	NFC[c]	RECON[d]	Other	Slope	Flexibility
MPC Constant	−0.131	0.140	−0.105	−0.007	1.103	−1.251
	(0.031)[f]	(0.025)	(0.039)	(0.002)	(0.093)	(0.029)
Trend	−0.023	0.025	−2E-04	−0.001		
	(5E-4)	(8E-4)	(7E-4)	(6E-5)		
R-Square	0.868	0.743	0.821	0.380		

[a] OJ = orange juice.
[b] FCOJ = frozen concentrated orange juice.
[c] NFC = not-from-concentrate orange juice.
[d] RECON = reconstituted orange juice.
[e] MPC = marginal propensity to consume.
[f] Asymptotic standard errors in parentheses.

by discrete differences (Theil 1975, 1976). To account for seasonality (Duffy; Brown and Lee), the quantity and price logarithms were 52nd differenced (for the 52 weeks in a year)— $d(\log q_{i,t}) = \log q_{i,t} - \log q_{i,t-52}$ and $d(\log p_{i,t}) = \log p_{i,t} - \log p_{i,t-52}$; and average budget share values over the 52-week period underlying the differencing were used in constructing the model variables—$w_{i,t}^*$ was replaced by $(w_{i,t}^* + w_{i,t-52}^*)/2$.

In addition to prices and income, generic and brand advertising levels may impact OJ demand but were not included in the model due to lack of data. Based on work by Brown and Lee, generic advertising was expected to have little impact on conditional demand by product form (generic advertising is aimed at expanding the OJ category and not changing the demand for specific OJ product forms). On the other hand, trends in consumption related to preferences for convenience were expected to impact OJ demand by product form (NFC and RECON are convenient ready-to-serve, chilled juice products, as opposed to FCOJ which must be reconstituted by the consumer). To capture possible trend effects, OJ category demands were viewed as being dependent on time, which required adding an intercept (C_i) to model (13), given the variables of the Rotterdam model are in differences. The adding-up condition requires that these coefficients sum to zero (i.e, $\Sigma_{i \in G} C_i = 0$).

Demand specification (13) is conditional on expenditure or income allocated to the four OJ categories. Income allocated to the OJ group is measured by the conditional Divisia volume index term which was treated as independent of the error term added to each OJ category demand equation for estimation, based on the theory of rational random behavior (Theil 1980; Brown, Behr and Lee). As the data add up by construction—the left-hand-side variables in model (13) sum over i to the conditional Divisia volume index—the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten 1969). The parameters of the excluded equation can be obtained from the adding-up conditions or by re-estimating the model omitting a different equation. The equation error terms were assumed to be contemporaneously correlated and the full-information maximum-likelihood procedure (TSP) was used to estimate the system of equations.

The estimates of model (13) are shown in Table 2. All coefficient estimates are significant to the extent that they are twice or greater than their asymptotic standard error estimates, except for the trend coefficient estimate for RECON. The income flexibility is negative and consistent with theory. The trend coefficient estimates indicate the demands for FCOJ and OTHER OJ are declining over time while the demand for NFC is increasing. The MPC slope estimate is about equal to unity and the MPC constant estimates for FCOJ, RECON and OTHER are negative while that for NFC

Table 3. Conditional Uncompensated Elasticity Estimates at Sample Means

OJ[a] Category	Income	Price FCOJ[b]	NFC[c]	RECON[d]	Other
FCOJ	0.639	−0.835	0.182	0.015	−1E-04
	(0.025)[e]	(0.028)	(0.012)	(0.009)	(8E-4)
NFC	1.581	−0.089	−1.528	0.037	−3E-4
	(0.043)	(0.016)	(0.032)	(0.023)	(0.002)
RECON	0.845	−0.048	0.240	−1.037	−2E-04
	(0.023)	(0.008)	(0.016)	(0.024)	(0.001)
Other	0.792	−0.045	0.225	0.018	−0.990
	(0.047)	(0.008)	(0.021)	(0.011)	(0.058)

[a] OJ = orange juice.
[b] FCOJ = frozen concentrated orange juice.
[c] NFC = not-from-concentrate orange juice.
[d] RECON = reconstituted orange juice.
[e] Asymptotic standard errors in parentheses.

is positive. Application of the MPC constant and slope estimates to the range of sample budget share values showed that the MPC for each type of OJ was in the zero-one interval over the sample, satisfying theory.

Conditional income (e_i) and uncompensated price elasticity estimates (e_{ij}), calculated at sample mean budget share values, are shown in Table 3. The formulas for these elasticities are

$$(15a) \quad e_i = (b_i + \rho w^*_{i,t-1})/w^*_{i,t} \quad \text{and}$$

$$(15b) \quad e_{ij} = \phi^{**}(b_i + \rho w^*_{i,t-1})$$
$$\times (\Delta_{ij} - (b_j + \rho w^*_{j,t-1}))/w^*_{i,t} - w^*_{j,t}e_i.$$

NFC has the highest income elasticity at 1.6, followed by RECON and OTHER at .8 and FCOJ at .6 respectively. NFC also has the largest (in absolute value) own-price elasticity at −1.5, followed by RECON and OTHER at about −1 and FCOJ at −.8. Most of the cross-price elasticity estimates are positive, reflecting substitution; some of the cross-price elasticities are near zero.

The negative uncompensated cross-price elasticities are a result of the negative income terms ($-w^*_j e_i$) of these elasticities. Removing these income terms, the compensated or real-income-held-constant cross-price elasticities ($e^*_{ij} = e_{ij} + w^*_j e_i$) at the sample means are all positive and significant (Table 4). The Divisia volume index is a measure of real income,

and, hence, all goods are substitutes with d(log Q*) = 0.

Estimates showing how the MPCs and income and own-price elasticities vary over the sample are presented in Table 5. Mean (lagged) budget shares for three 52-week periods—the first period at the beginning of the sample, the second in the middle and the third at the end of the sample—were calculated and used in estimating the MPCs and elasticities. The budget shares for FCOJ and OTHER categories decrease over these select sample periods, while the budget share for NFC increases and that for RECON is relatively flat. With each MPC specified as a positive linear function of the lagged budget share, the estimated MPCs follow the budget share patterns. The income elasticity estimates for FCOJ, NFC and OTHER OJ decline over the sample by 52 percent, 23 percent and 21 percent, respectively, while that for RECON is relatively flat. Similarly, in absolute value, the own-price elasticity estimates for FCOJ, NFC and OTHER decline by 49 percent, 29 percent and 20 percent, while that for RECON is flat. Estimates of model (13) under the assumption of constant MPCs (restriction $\rho = 0$ imposed) yielded results that greatly differ from those reported in Tables 2 through 5; notably, the income and own-price elasticity estimates for FCOJ and OTHER OJ increase over time in contrast to the estimates for the varying coefficient specification.

Table 4. Conditional Compensated Price Elasticity Estimates at Sample Means

OJ[a] Category	FCOJ[b]	NFC[c]	RECON[d]	Other
FCOJ	−0.656	0.368	0.274	0.013
	(0.023)[e]	(0.013)	(0.014)	(9E-4)
NFC	0.355	−1.067	0.679	0.033
	(0.013)	(0.026)	(0.021)	(0.002)
RECON	0.190	0.486	−0.694	0.018
	(0.010)	(0.015)	(0.018)	(0.001)
Other	0.178	0.456	0.340	−0.974
	(0.012)	(0.031)	(0.021)	(0.057)

[a] OJ = orange juice.
[b] FCOJ = frozen concentrated orange juice.
[c] NFC = not-from-concentrate orange juice.
[d] RECON = reconstituted orange juice.
[e] Asymptotic standard errors in parentheses.

Changes in consumer demand responses over time may have various marketing implications. For example, if FCOJ and OTHER OJ have become more price inelastic as our study suggests, price promotions may be less effective in stimulating demands for these categories. Price promotions for NFC may also have become less effective, although NFC demand still appears to be price elastic. In our study, the estimated price elasticity trends and resulting marketing implications for FCOJ and OTHER are just the opposite that would have occurred if the MPCs were specified as constants as in the usual Rotterdam model.

Conclusions

This paper extends Barten's (1993) synthetic modeling approach which combines features of the Rotterdam and AIDS. Our extension increases the flexibility of the uniform substitute model, as well as that of the similar preference independent model, by specifying the MPC for each good as a linear combination of Rot-

Table 5. Conditional Uncompensated Elasticity Estimates at Select Budget Shares

		FCOJ[a]	NFC[b]	RECON[c]	Other
Mean Budget Share[d]	Beginning[e]	0.384	0.173	0.416	0.026
	Middle[f]	0.280	0.305	0.392	0.022
	End[g]	0.177	0.388	0.419	0.015
MPC	Beginning[e]	0.293	0.331	0.354	0.022
	Middle[f]	0.178	0.476	0.327	0.018
	End[g]	0.065	0.568	0.357	0.010
Income Elasticity	Beginning[e]	0.763	1.900	0.852	0.851
	Middle[f]	0.636	1.558	0.835	0.808
	End[g]	0.366	1.460	0.853	0.675
Own-Price Elasticity	Beginning[e]	−0.968	−1.921	−1.042	−1.063
	Middle[f]	−0.833	−1.497	−1.030	−1.011
	End[g]	−0.492	−1.357	−1.043	−0.846

[a] FCOJ = frozen concentrated orange juice.
[b] NFC = not-from-concentrate orange juice.
[c] RECON = reconstituted orange juice.
[d] Lagged budget share.
[e] 52-week period from 1/14/89 through 1/6/90.
[f] 52-week period from 6/26/93 through 6/18/94.
[g] 52-week period from 12/6/97 through 11/28/98.

terdam and AIDS MPCs. This specification straightforwardly affects the model's income elasticities, and indirectly affects the price elasticities as they are defined as functions of the MPCs.

A study of OJ demand by product category illustrates the model. Results show that the decreasing sales trend of FCOJ may be accompanied by less sensitive income and price responses, just the opposite that would be predicted using a constant MPC specification.

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