A Comparison of Nominal and Real Historical Risk Measures

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Abstract

Previous studies of historical risk have used either nominal or real data to calculate risk measures for agricultural prices and income. However, the effects of using nominal and real data have not been evaluated. This study utilizes theoretical variance approximation relationships to examine variances from detrended real and nominal time series. The relationships between variances are derived for quarterly U.S. farm milk prices for 1960-72, 1973-80, and 1981-90. Contrary to common intuitive arguments, results indicate that variances of real time series can be larger than variances of nominal series. While definitive conclusions are not possible, several reasons for using nominal data in risk analysis are given.

Key Words: detrending, indices, nominal data, risk measurement

Variances and covariances of historical data are utilized extensively in agricultural economics as objective risk measures to provide information about risk in historical production and market environments to farm managers (Carter and Dean; Mathia; Walker and Lin) and policy makers (Hazell; Miranda). These measures also are used in risk programming and other simulation models for farm management (Mapp and Helmers) and policy analysis (Miranda and Glauber). Considerable agricultural economics literature evaluates choices of estimators and detrending methods used to calculate historical risk measures for output, income, and prices (Adams et al.; Fackler and Young; Ford et al.; Kramer et al.; Swinton and King; Young, 1984). Previous studies of risk measurement for agricultural prices and income used nominal and real data (Young, 1980). Although Ford et al. recognized the *ad hoc* nature of the choice to use real or nominal data, this choice and possible consequences have not been evaluated.

The purpose of this paper is to compare nominal and real historical risk measures for quarterly U.S. milk prices received by farmers for the 1960-1990 period, using the variance of a detrended data series as a measure of risk. Assumptions concerning the use of real versus nominal data are first discussed, and a statistical model of the process used to calculate real data is considered. Two alternative indices for calculating real prices are used in the study. One is a general index of prices paid for production inputs, and the other is an index of milk production costs. These alternatives also allow a comparison of specific versus general price indices in risk measurement.

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Conceptual Background

The underlying logic of using real or nominal data to estimate risk measures is usually not explicitly stated. Presumably, the decision to use real data is influenced by homogeneity conditions of standard neoclassical theory. However, zero homogeneity of output supply and input demand functions does not hold under expected utility without limiting assumptions (Paris; Pope; Coyle). Another argument for using real data is that risk measures will reflect current price levels after past inflation has been removed. However, detrending procedures can also remove past inflation from nominal data. This view is related to the rationale of earlier intuitive justifications for using nominal data in the literature. If unexpected inflation increases risk (Brake; Levy and Sarnat; White and Musser), nominal data should be used in calculating risk measures because detrending will remove systematic or expected inflation along with other secular trends, leaving the random component of inflation to be incorporated in the risk measure along with real random components of the data. According to this view, detrended real data is less variable than detrended nominal data so variances of real data will not reflect the increased risk in inflationary periods due to unexpected inflation. However, this conclusion implicitly assumes that expected inflation is less than actual inflation. A final consideration is that real or nominal data should reflect risk that is appropriate to the problem under study. Many short-run problems are only concerned with risk in prices or gross revenues. Nominal data are generally appropriate for these problems. In longer-run problems, risk in profits is often the concern. Real or nominal data may be appropriate for these problems. However, consistency is an issue with use of nominal data for long-run problems because both output and input prices are subject to inflation adjustments.

The relationship between nominal and real prices and risk measures can be examined using statistical formulas for variances and covariances of functions of random variables. Real prices in time t (P_t^R) are the product of nominal prices in time t (P_t^N) and D_t, where D_t equals $100/I_t$ and I_t is the index used to deflate/inflate nominal prices. The subscript t runs from $t = 1, 2, ..., Z$, where Z represents the current or most recent period. If $t = 1$ is the index base period (i.e., $I_1 = 100$), D_t deflates nominal prices to the first period, and if $t = Z$ is the index base period (i.e., $I_Z = 100$), D_t inflates nominal prices to the current period Z. Prices may also be deflated to some other base period between 1 and Z. Analysis in this paper assumes that third and higher moments of P_t^N and D_t are zero. Then variance of real prices can be defined as the variance of the product of the random variables P^N and D (Bohrnstedt and Goldberger):

$$V(P^R) = [E(D)]^2 V(P^N) + [E(P^N)]^2 V(D) + 2E(D)E(P^N)COV(D,P^N) + V(D)V(P^N) + [COV(D,P^N)]^2 \quad (1)$$

where E, V, and COV are the expectation, variance, and covariance operators, respectively. Subtracting $V(P^N)$ from both sides yields

$$V(P^R) - V(P^N) = \{V(D) + [E(D)]^2 - 1\}V(P^N) + [E(P^N)]^2 V(D) + 2E(D)E(P^N)COV(D,P^N) + [COV(D,P^N)]^2 \quad (2)$$

The relationship between nominal and real variances can be explored by examining the right-hand side of Equation (2). The components $V(D)$, $V(P^N)$, $E(D)$, $E(P^N)$, $[E(D)]^2$, $[E(P^N)]^2$, and $[COV(D,P^N)]^2$ are strictly positive. If nominal prices are inflated to base period Z, then $E(I) < 100$, $E(D) > 1$, and $\{V(D) + [E(D)]^2 - 1\}$ is positive. If nominal prices are deflated to base period 1 then $E(I) > 100$, $E(D) < 1$, and $[V(D) + [E(D)]^2 - 1]$ may be negative. If nominal prices are inflated to the current period Z, $COV(D,P^N) > 0$ is a sufficient condition for the variance of real prices to exceed the variance of nominal prices. However, because the covariance between nominal prices and the index used to deflate/inflate will often be positive, the sign of $COV(D,P^N)$ likely will be negative. $COV(D,P^N) < 0$ is a necessary condition for the variance of nominal prices to exceed the variance of real prices when prices are inflated to the current period. General conditions concerning the variance relationship between nominal and real prices are complex when this covariance is negative. Earlier intuitive views reviewed above are too simplistic. In part, these views do not recognize that the index (I_t) is stochastic and that creating real data introduces this additional stochastic element.

Following the standard detrending assumptions, this paper assumes that P_t^N and D_t are random variables with deterministic and random components. In the case of risk measurement, the primary concern is with the variance of random components. Random components can be isolated by detrending the data series of interest. Using standard detrending specifications, P_t^N and D_t are defined as follows:

$$P_t^N = a_0 + a_1 T + e_t \tag{3}$$

$$D_t = b_0 + b_1 T + w_t, \tag{4}$$

where a_0, a_1, b_0, and b_1 are parameters, T is a time trend, and e_t and w_t are iid error terms with $E(e_t) = E(w_t) = 0$ for all t. Using the definition of real prices, $P_t^R = P_t^N D_t$ and assuming the relationships in Equations (3) and (4), real prices are defined as:

$$\begin{aligned} P_t^R &= (a_0 + a_1 T + e_t)(b_0 + b_1 T + w_t). \\ &= a_0 b_0 + (a_0 b_1 + b_0 a_1)T + a_1 b_1 T^2 + P^N_t w_t + D_t e_t - w_t e_t \\ &= a_0 b_0 + (a_0 b_1 + b_0 a_1)T + a_1 b_1 T^2 + u_t \\ &= c_0 + c_1 T + c_2 T^2 + u_t . \end{aligned} \tag{5}$$

Thus, real prices are approximated in this study as the sum of a constant term, a linear trend, a quadratic trend, and an error term. This approximation is consistent with the standard assumption of linear trends in nominal data. The risk measure of interest is the variance of the random component of real prices in Equation (5), $V(u_t)$. This random component equals $P_t^N w_t + D_t e_t - w_t e_t$. Omitting the t subscripts, the risk measure for approximate real prices is $V(P^N w + De - we)$. This variance can be decomposed using statistical formulas for the sum and product of random variables (Bohrnstedt and Goldberger; Mood et al.). Assuming third and higher order terms are zero and $E(e) = E(w) = 0$ from above:

$$\begin{aligned} V(P^N w + De - we) &= V(P^N w) + V(De) \\ &+ V(we) + 2COV(P^N w, De) \\ &- 2COV(P^N w, we) - 2COV(De, we) \end{aligned} \tag{6}$$

As shown in Appendix A, Equation (6) can be decomposed into moments of variables in Equations (3) and (4):

$$\begin{aligned} V(P^N w + De - we) &= [E(P^N)]^2 V(w) \\ &+ a_1^2 V(T) V(w) + [E(D)]^2 V(e) \\ &+ b_1^2 V(T) V(e) + V(w) V(e) \\ &+ [COV(w,e)]^2 + 2COV(w,e)[E(D)E(P^N) \\ &+ a_1 b_1 V(T)]. \end{aligned} \tag{7}$$

Subtracting the variance of detrended nominal prices, V(e), from both sides yields

$$\begin{aligned} V(P^N w + De - we) - V(e) &= [E(P^N)]^2 V(w) \\ &+ a_1^2 V(T) V(w) + b_1^2 V(T) V(e) \\ &+ V(w) V(e) + [COV(w,e)]^2 + \{[E(D)]^2 \\ &- 1\} V(e) + 2COV(w,e)[E(D)E(P^N) \\ &+ a_1 b_1 V(T)]. \end{aligned} \tag{8}$$

Similar to the analysis of Equation (2), the relationship between the variance of approximated real prices $(V(P^N w + De - we))$ and the variance of detrended nominal prices $(V(e))$ can be explored by examining the right-hand side of Equation (8). The first five terms in Equation (8) are positive because $[E(P^N)]^2$, $V(w)$, $V(e)$, a_1^2, b_1^2, $V(T)$, and $[COV(w,e)]^2$ are strictly positive. The components $E(D)$ and $E(P^N)$ are also positive. If nominal prices are inflated to base period Z then $\{[E(D)]^2 - 1\}$ is positive and the sixth term in Equation (8) is positive. If nominal prices are deflated to base period 1 then $\{[E(D)]^2 - 1\}$ is negative. The choice of base period therefore affects the relationship between nominal and real risk measures. In addition, risk measures for detrended real prices calculated using the same index but different base periods will differ in magnitude because the dependent variable is rescaled by a constant as shown in Appendix B. The base period should be chosen so that magnitudes of variances reflect the time period of interest for risk measurement.

If nominal prices are inflated to the current period Z, the variance relationship for detrended nominal and real prices will depend on the signs and magnitudes of $COV(w,e)$ and $[E(D)E(P^N) + a_I b_I V(T)]$ from the last term in Equation (8). $COV(w,e) > 0$ and $a_I b_I > 0$ are sufficient conditions for real variances to exceed nominal variances. However, $a_I b_I$ will generally be negative because trends in prices and trends in indices used to deflate/inflate prices will generally be positively correlated so that the coefficient a_I is positive and the coefficient b_I is negative (because the dependent variable D_t equals $100/I_t$). Alternatively, if $COV(w,e) < 0$, then $[E(D)E(P^N) + a_I b_I V(T)] > 0$ is a necessary condition for nominal variances to exceed real variances. If $COV(w,e) > 0$, then $[E(D)E(P^N) + a_I b_I V(T)] < 0$ is a necessary condition for nominal variances to exceed real variances. In this latter case, $a_I b_I < 0$ is necessary because $E(D)$, $E(P^N)$, and $V(T)$ are strictly positive. Although general conditions concerning the relationship between nominal and real variances are complex, Equation (8) can be used to calculate approximate real variances that can be compared with actual detrended real variances and detrended nominal variances. Signs and magnitudes of $COV(w,e)$ and $[E(D)E(P^N) + a_I b_I V(T)]$ from Equation (8) also may provide insight into differences between nominal and real risk measures.

Nominal and Real Price Series

The nominal price series used in this analysis is the quarterly U.S. price received by farmers for all milk, measured in dollars per hundredweight (USDA, NASS), for the thirty-one-year period from 1960 to 1990. These data are illustrated in figure 1. In a recent study, Ford et al. noted that three distinct patterns characterize nominal milk prices. From 1960 to 1972, prices had a moderate trend and definite seasonal variation. The 1960-72 period reflects low inflation and a consistent government price support policy. Prices from 1973 to 1980 were highly volatile and trended strongly upward. Prices increased approximately seven dollars over the 1973-80 period, reflecting high inflation rates for all prices. Prices generally trended downward from 1981 to 1990.

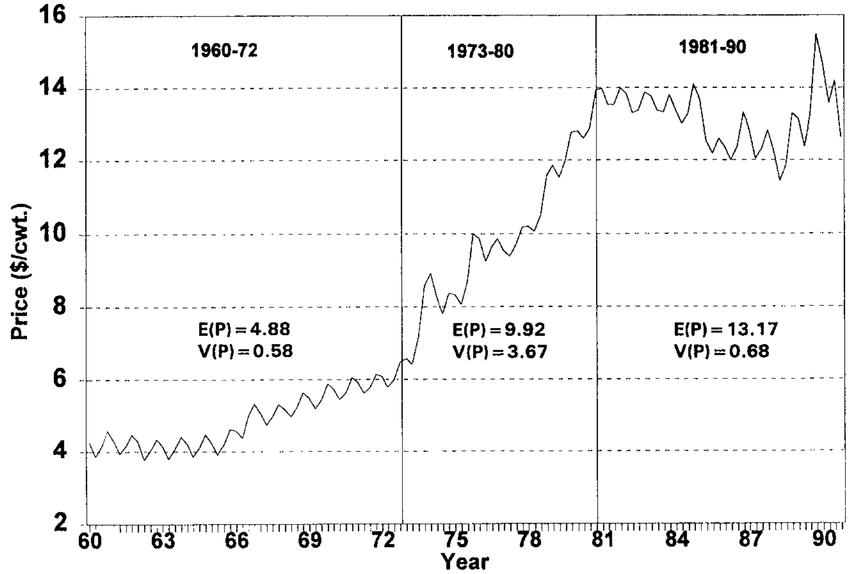
Real prices used in this study were calculated for 1960-90 using two production cost indices, both with 1990 as the base period. One series was calculated using the Index of Prices Paid by Producers for Production Items, Interest, Taxes, and Wage Rates (*IPP*) (USDA, NASS); this index is often used to calculate real prices for risk analysis. The *IPP* is an aggregate index of prices paid by both crop and livestock producers for farm inputs. A second series of prices was calculated using an index of U.S. milk production costs so that real prices would reflect constant purchasing power for farm inputs used only in milk production. This milk production cost index (*MPC*) was constructed using total cash costs of U.S. milk production (USDA, ERS) for 1972-90. These costs include variable cash costs of feed (concentrates, byproducts, hay, silage, and pasture/forage), other variable cash costs (milk hauling and marketing, veterinary, medical and breeding costs, livestock hauling, fuel and electricity, machinery and building repairs, hired labor, livestock supplies, rent/leasing costs, and fees), and fixed cash costs (general overhead, taxes and insurance, and interest). Indices of U.S. hay and corn prices (USDA, NASS) and an index of fuel/energy costs (USDA, NASS) were used to convert the hay, concentrate, and fuel/energy components of annual milk production costs to quarterly costs because these input prices were expected to vary from quarter to quarter. The indices of U.S. hay and corn prices were constructed using quarterly averages of monthly farm prices received during 1972-90.

Because these U.S. milk production cost data were available only for the 1972-90 period, the constructed *MPC* index was regressed on the *IPP* over this period in order to obtain estimated index values for 1960-71. Results for the *MPC* regression with standard errors in parentheses are:

$$MPC\ index = \underset{(5.79)}{14.50} + \underset{(0.033)}{0.854}(IPP\ \text{index});\quad R^2 = 0.90 \qquad (9)$$

The regression has a good fit -- the R^2 and t-ratios are high. *MPC* real prices for 1960-71 were therefore estimated with Equation (9). Despite the

Figure 1. Quarterly U.S. Milk Prices Received by Farmers, 1960-90.



statistical properties of Equation (9), it is not possible to fully evaluate the estimation procedure. The *IPP* and *MPC* indices are illustrated in figure 2. The average value of *MPC* real prices, E(*MPC*), is greater than E(*IPP*) for all three periods. Differences are less than one point in 1981-90, but are approximately seven points in 1973-80 -- the differences are greater for 1973-78 than for 1979-80. The positive estimated intercept and the estimated coefficient for the *IPP* index being less than one in Equation (9) reflect the convergence of the two indices over time. Projecting this convergence backwards results in E(*MPC*) being about ten points higher than E(*IPP*) during 1960-72. Lack of data for this period prevents evaluation of the method used to estimate the *MPC* index. The decrease in the *MPC* index from the estimated 1971 value to the actual 1972 value suggests that the index estimates for 1960-71 may be biased upward. Thus, subsequent analyses of *MPC* real prices must be interpreted with caution, particularly for the 1960-72 period.

The *MPC* and *IPP* price indices in figure 2 follow patterns similar to those observed for nominal prices. Both *IPP* and *MPC* indices increase slightly from 1960 to 1972. These minor increases are consistent with low levels of overall input price inflation that occurred during 1960-72. The *MPC* index is approximately 50% larger than the *IPP* index for 1960-72. As discussed above, this difference may reflect biased estimates. The *MPC* index also generally exceeds the *IPP* index for the 1973-80 period, reflecting higher dairy input prices relative to general crop/livestock inputs. Large increases in both *IPP* and *MPC* indices reflect high inflation rates during 1973-80. Although both indices generally increase from 1981 to 1990, the increases are small relative to those observed during 1973-80. Real prices calculated using the *IPP* and *MPC* indices are illustrated in figure 3. *IPP* real prices exceed *MPC* real prices during 1960-72, a reflection of the difference in index magnitude noted above. *IPP* and *MPC* real prices converge in 1978, with both real price series trending downward from this year through 1990.

Measuring Historical Risk

The risk measures used in this analysis are variances of estimated residuals from detrended data. The previous discussion of figures 1, 2, and 3 suggests that seasonality and trends are the deterministic components of the price series and the indices. The distinct sub-period patterns for the prices and indices also suggest that piecewise linear regression methods (Johnston) are appropriate for detrending the data. Risk measures calculated from the detrended data are specific to each sub-period, *i.e.*, estimated residuals for each sub-period are used

Figure 2. IPP and MPC Indices, 1960-90 (1990 Base)

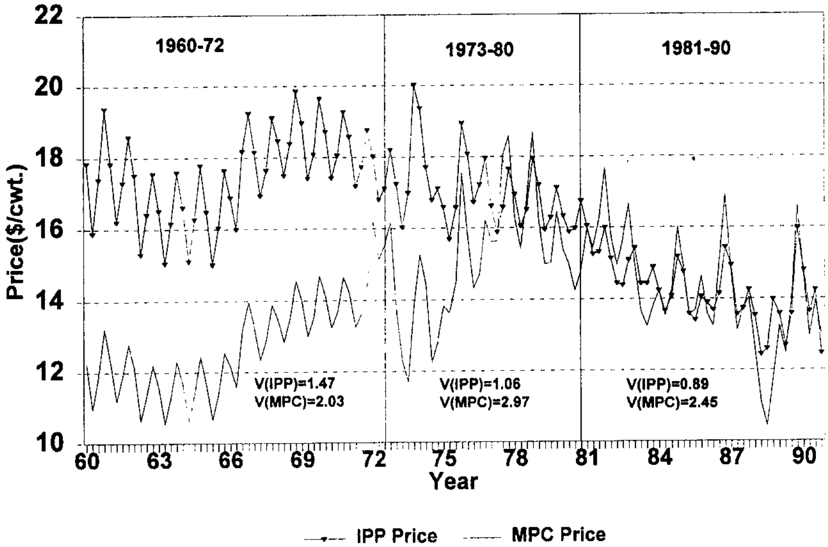
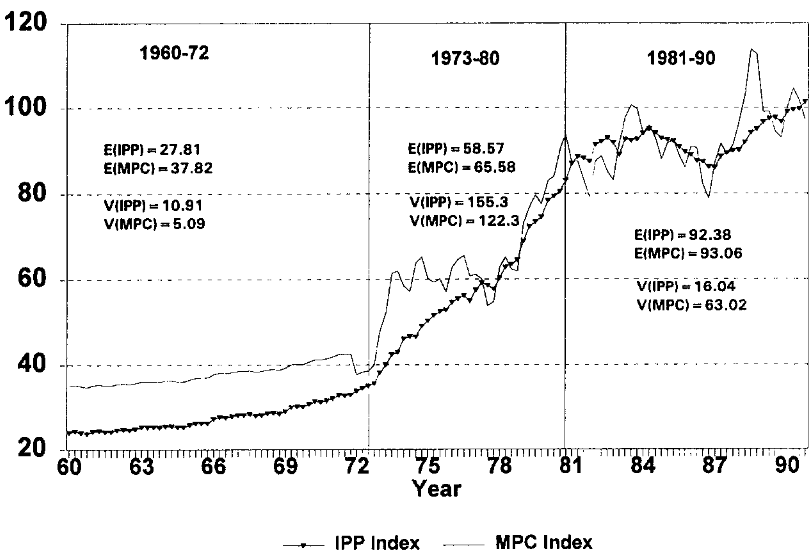


Figure 3. IPP and MPC Real Prices, 1960-90



in calculating sub-period variances. Although only detrended nominal and real prices are required for risk measurement, the *IPP* and *MPC* indices were also detrended to provide information about the relationship between nominal and real price variances. Both indices were transformed to calculate approximate real variances in Equations (1) and (7). The transformation created new dependent variables, D^{IPP} and D^{MPC}, where $D^{IPP} = 100/IPP$ and $D^{MPC} = 100/MPC$.

Models for the nominal price series (P^N) and each transformed index $(D^{IPP}$ and $D^{MPC})$ were specified as follows:

60-72 sub-period: $$Y_{1t}^{k} = a_{11} + b_{11}T + b_{12}s2 + b_{13}s3 + b_{14}s4 + e_{1t}^{k}; \quad t \leq t_1 \tag{10}$$

73-80 sub-period: $$Y_{2t}^{k} = a_{21} + b_{21}T + b_{22}s2 + b_{23}s3 + b_{24}s4 + e_{2t}^{k}; \quad t_1 < t \leq t_2 \tag{11}$$

81-90 sub-period: $Y_{3t}^k = a_{3t} + b_{3t}T + b_{32}s2 + b_{33}s3$

$$+ b_{34}s4 + e_{3t}^k; \quad t_2 < t, \tag{12}$$

where k indicates the model (P^N, D^{IPP}, and D^{MPC}) and Y_{jt}^k is the price or transformed index in time t and sub-period j ($j = 1, 2, 3$). The parameters t_1 and t_2 are used as endpoints for the 60-72 and 73-80 sub-periods, respectively, where $t_1 = 52$ for the fourth quarter of 1972 and $t_2 = 84$ for the fourth quarter of 1980. T is a time trend starting at times $t = 1$, $(t_1+1) = 53$, and $(t_2+1) = 85$ and ending at times $t_1 = 52$, $t_2 = 84$, and $t = 124$ for sub-periods $j = 1, 2$, and 3, respectively. The variables $s2$, $s3$, and $s4$ are dummy variables that equal one for the second, third, and fourth quarters, respectively, and zero otherwise, and e_{jt}^k is an error term specific to each sub-period j model. If standard statistical assumptions are met, e_{jt}^k will have a zero mean and constant variance for each sub-period j. Real price models (P^R_{IPP}, and P^R_{MPC}) were also specified as in Equations (10), (11), and (12), with a squared trend term (T^2) also included; these models are consistent with the theoretical formulation in Equation (5). The following restrictions are required to force Equations (10), (11), and (12) to join at points $t1$ and $t2$:

$$a_{11} + b_{11}t1 + b_{12}s2 + b_{13}s3 + b_{14}s4 =$$

$$a_{21} + b_{21}t1 + b_{22}s2 + b_{23}s3 + b_{24}s4; \tag{13}$$

$$a_{21} + b_{21}t2 + b_{22}s2 + b_{23}s3 + b_{24}s4 =$$

$$a_{31} + b_{31}t2 + b_{32}s2 + b_{33}s3 + b_{34}s4. \tag{14}$$

For real price models, additional terms $b_{15}(t1)^2$ and $b_{25}(t1)^2$ are included on the right and left sides of restriction (13), respectively, and $b_{25}(t2)^2$ and $b_{35}(t2)^2$ are included on the right and left sides of restriction (14), respectively.

The detrending models described by Equations (10)-(14) are quite naive so specification errors may exist. With time series data as in this study, model specification error may be manifested in autocorrelation so that coefficient estimates are inefficient and standard errors are biased. While most previous studies have used ordinary least squares (OLS) methods for detrending, the resulting variance estimates are biased when the data are autocorrelated (Judge et al.). Fackler and Young suggest using feasible generalized least squares (FGLS) methods to detrend data in these cases. Ford et al. found that FGLS methods performed better than OLS methods in calculating risk measures for milk prices. Thus, the analysis tests and corrects for autocorrelation rather than using the traditional approach of ordinary least squares to detrend historical data. Parameters for each of the five detrending models (P^N, P^R_{IPP}, P^R_{MPC}, D^{IPP}, and D^{MPC}) were estimated with OLS, imposing the restrictions in (13) and (14). Residuals were tested for autocorrelation using the Durbin-Watson statistic. Because the null hypothesis of no autocorrelation was rejected for all five models, the models were re-estimated with the restrictions to correct for autocorrelation using the Cochrane-Orcutt iterative least squares (ILS) procedure (Judge et al.).

Detrending Model Results

ILS regression estimates for detrended prices are reported in table 1. The detrending models for nominal and real prices fit well, with R^2 values for the nominal price, *IPP* real price, and *MPC* real price models equal to 0.88, 0.92, and 0.84, respectively. Trend and seasonal effects are significant components of nominal prices; the trend coefficient and all but one of the seasonal coefficients for the detrended nominal prices are statistically significant at the 0.05 level of probability. All seasonal effects are statistically significant in both *IPP* and *MPC* real price models. However, trend effects are not as significant in the real price models as in the nominal price models. In the *IPP* model, only T^2 is significant for 1981-90. In the *MPC* model, T^2 is significant for 1960-72 and both T and T^2 are significant for 1981-90. These results are consistent with limited trends in the real data as shown in figure 3. In addition, the quadratic trend may have introduced some multicollinearity into the estimation. However, this specification was retained for consistency with the theoretical specification in Equation (5).

The strong trend in nominal prices as compared with real prices suggests that expected inflation may be a substantial component of this trend. Results for the transformed index models (D^{IPP} and D^{MPC}) in table 2 support this view. Trend

Table 1. Regression Estimates for Nominal and Real Prices (1990 Base)[a,b]

		Nominal Price	IPP Real Price	MPC Real Price
60-72	Constant	4.0830**	17.4786**	12.6166**
		(0.5379)	(0.6126)	(0.8524)
	Trend	0.0374**	-0.0032	-0.0723
		(0.0199)	(0.0555)	(0.0806)
	Trend^2		0.0003	0.0026**
			(0.0010)	(0.0015)
	Quarter 2	-0.3735**	-1.3325**	-1.0103**
		(0.0922)	(0.1371)	(0.1919)
	Quarter 3	-0.1471	-0.3709**	-0.3800*
		(0.1202)	(0.1687)	(0.2444)
	Quarter 4	0.2060**	0.9986**	0.5223**
		(0.0951)	(0.1411)	(0.1977)
73-80	Constant	4.2411**	17.7289**	12.2131**
		(0.6537)	(0.8353)	(1.184)
	Trend	0.0787**	0.0303	0.0302
		(0.0119)	(0.0398)	(0.0579)
	Trend^2		-0.0005	0.0002
			(.0005)	(.0007)
	Quarter 2	-0.4409**	-1.0423**	-0.7406**
		(0.1162)	(0.1742)	(0.2429)
	Quarter 3	-0.3026**	-0.6278**	-0.5630**
		(0.1523)	(0.2158)	(0.3119)
	Quarter 4	0.2709**	0.7149**	0.8392**
		(0.1227)	(0.1839)	(0.2577)
81-90	Constant	4.5351**	17.7305**	12.2920**
		(0.6447)	(0.7889)	(1.116)
	Trend	0.0858**	-0.0087	0.0680**
		(0.0075)	(0.0197)	(0.0274)
	Trend^2		-0.0002*	-0.0005**
			(0.0002)	(0.0002)
	Quarter 2	-0.7111**	-0.8394**	-1.0643**
		(0.1044)	(0.1553)	(0.2162)
	Quarter 3	-0.4029**	-0.4883**	-0.4020*
		(0.1367)	(0.1897)	(0.2737)
	Quarter 4	0.3474**	0.3709**	0.9229**
		(0.1090)	(0.1615)	(0.2262)
Durbin-Watson		1.989	1.999	1.998
R^2		0.876	0.920	0.838

[a] ** and * denote significance at the 0.05 and 0.10 levels of probability, respectively.
[b] Standard errors are in parentheses.

estimates for D^{IPP} and D^{MPC} are statistically significant at the 0.05 level of probability for 1960-72, 1973-80, and 1981-90. Although figure 2 shows positive trends in the *IPP* and *MPC* production cost indices, trend estimates for the transformed index models are negative because the dependent variables are inverse transformations of the original indices. Seasonal effects for the indices are generally insignificant. R^2 values for the D^{IPP} and D^{MPC} models are 0.96 and 0.93, respectively.[1]

Variance Results

Undetrended Price Variances

Variances of undetrended nominal and real prices are presented in table 3. Undetrended nominal price variances for 1960-72 and 1981-90 are close in magnitude (0.58 and 0.68, respectively). However, the variance for 1973-80 (3.67) is approximately six times larger than variances for the

Table 2. Regression Estimates for Transformed *IPP* and *MPC* Indices (1990 Base)[a,b]

		Transformed *IPP* Index	Transformed *MPC* Index
60-72	Constant	4.2104**	2.8738**
		(0.0783)	(0.0735)
	Trend	-0.0238**	-0.0081**
		(0.0027)	(0.0025)
	Quarter 2	-0.0013	-0.0159
		(0.0162)	(0.0173)
	Quarter 3	0.0186	-0.0137
		(0.0205)	(0.0221)
	Quarter 4	0.0412**	-0.0116
		(0.0168)	(0.0179)
73-80	Constant	4.2820**	2.8473**
		(0.1001)	(0.0988)
	Trend	-0.0357**	-0.0183**
		(0.0017)	(0.0016)
	Quarter 2	-0.0255	-0.0112
		(0.0204)	(0.0217)
	Quarter 3	-0.0096	-0.0295
		(0.0259)	(0.0279)
	Quarter 4	0.0219	0.0260
		(0.0216)	(0.0230)
81-90	Constant	4.2085**	2.7649**
		(0.0983)	(0.0967)
	Trend	-0.0302**	-0.0163**
		(0.0011)	(0.0010)
	Quarter 2	0.0117	-0.0133
		(0.0184)	(0.0196)
	Quarter 3	0.0257	0.0261
		(0.0233)	(0.0251)
	Quarter 4	0.0229	0.0548**
		(0.0192)	(0.0205)
Durbin-Watson		1.999	1.999
R^2		0.963	0.930

[a] ** and * denote significance at the 0.05 and 0.10 levels of probability, respectively.
[b] Standard errors are in parentheses.

other periods. The magnitude of the 1973-80 variance reflects large deviations of observed prices at the beginning and the end of this period from the average price in this period. The large deviations used in the variance calculation result from the strong upward trend in undetrended nominal prices (figure 1). Variation associated with trends in data would not usually be considered risk, thus supporting the need for detrending data to evaluate risk faced by producers. The undetrended *MPC* real variances were higher for all periods than the undetrended *IPP* real variances. The undetrended *MPC* real price variance was highest during 1973-80, and the undetrended *IPP* real price variance was highest during 1960-72. Use of estimated *MPC* index values limits comparison of *MPC* real prices during 1960-72 with other prices. The large variance observed for *IPP* real prices during the relatively stable 1960-72 period also suggests a need for detrending data in order to obtain meaningful risk estimates.

Comparisons of Undetrended Nominal and Real Price Variances

Variances of undetrended real prices are larger than variances of undetrended nominal prices for 1960-72 and for 1981-90, while the variance of undetrended nominal prices is larger than the variance of undetrended real prices for 1973-80. Again, some caution is warranted when interpreting the 1960-72 *MPC* real price variances. Approximate variances in table 3 show the same pattern with respect to undetrended nominal variances, although the 1973-80 approximation for *IPP* prices is substantially larger than the actual

Table 3. Nominal and Real Milk Price and Transformed Price Indices Variances

Model/Price	1960-1972	1973-1980	1981-1990
Undetrended Price Series			
Variance:			
Nominal Prices	0.5797	3.6732	0.6830
IPP Real Prices (1990 Base)	1.4665	1.0619	0.8889
MPC Real Prices (1990 Base)	2.0321	2.9697	2.4057
Approximate Variance:			
IPP Real Prices (1990 Base)	1.8918	3.1193	0.9036
MPC Real Prices (1990 Base)	2.1462	3.4627	2.4849
Detrended Price Series			
Variance:			
Nominal Prices	0.0080	0.1688	0.3859
IPP Real Prices (1990 Base)	0.2174	0.2598	0.3482
MPC Real Prices (1990 Base)	0.1342	1.0501	0.7636
Approximate Variance:			
IPP Real Prices (1990 Base)	0.2209	0.4189	1.1709
MPC Real Prices (1990 Base)	0.1290	1.2151	0.8129
Transformed Price Index[a]			
Undetrended Variance			
D^{IPP}	0.1652	0.1423	0.0021
D^{MPC}	0.0238	0.0558	0.0080
Detrended Variance			
D^{IPP}	0.0043	0.0027	0.0071
D^{MPC}	0.0025	0.0111	0.0041

[a] Reciprocal of price index.

variance for this period. Insight into these results can be obtained by examining covariance components of the variance relationship in Equation (2). These covariances are listed in table 4. The pattern observed for undetrended nominal and real variances illustrates the importance of the covariance between nominal prices and the transformed indices $(COV(D,P^N))$. The covariance between nominal prices and each transformed index is negative during 1973-80, a necessary condition for the variance of undetrended nominal prices to exceed the variance of undetrended real prices. Absolute values of these covariances are also large relative to those for 1960-72 and 1981-90.

The relationships between nominal prices and the transformed indices are more easily interpreted using the covariances between nominal prices and the untransformed indices $(COV(I,P^N))$ in the second column of table 4. For example, the large negative covariance between nominal prices and each transformed index $(COV(D,P^N)$ in the first column of table 4) during 1973-80 suggests that nominal milk prices and both general production costs and dairy production costs were increasing rapidly in a closely related manner. Covariances between nominal prices and the untransformed *IPP* and *MPC* indices for 1973-80 are 22.3716 and 17.2957, respectively. These large, positive

Table 4. Selected Components of Variance Approximation and Price-Index Correlations

	$[COV(D,P^N)]$	$[COV(I,P^N)]$
Undetrended Series:		
IPP Real Prices (1990 Base)		
1960-72	-0.2788	2.2571
1973-80	-0.6651	22.3716
1981-90	-0.0095	0.8362
MPC Real Prices (1990 Base)		
1960-72	-0.0976	1.4188
1973-80	-0.3657	17.2957
1981-90	0.0100	-0.9049
	$[COV(w,e)]$	$E(P^N)E(D) + a_1b_1V(T)$
Detrended Series:		
IPP Real Prices (1990 Base)		
1960-72	0.0003	17.5641
1973-80	-0.0116	17.4355
1981-90	-0.0205	13.9227
MPC Real Prices (1990 Base)		
1960-72	0.0005	12.8705
1973-80	-0.0099	15.3692
1981-90	-0.0131	14.0528

covariances indicate that nominal prices and production costs moved together closely during 1973-80. Covariances between nominal milk prices and the untransformed *IPP* and *MPC* indices for 1981-90 are 0.8362 and -0.9049, respectively. These small covariances indicate that prices and production costs were unrelated during 1981-90.

Comparisons of Undetrended Real Price Variances

Magnitudes of undetrended real price variances also differ substantially depending upon which index was used to calculate real prices. Variances of undetrended *MPC* real prices are consistently larger than variances of undetrended *IPP* real prices. Equation (1) indicates that the source of this pattern is complex. In a comparison of Equation (1) for *IPP* and *MPC* real prices, $V(P^N)$ and $E(P^N)$ remain unchanged because the same P^N is used to calculate both real series. The first term, $[E(D)]^2V(P^N)$, is larger for *IPP* than for *MPC* for all periods because $E(D^{IPP}) > E(D^{MPC})$, which results from $E(I^{IPP}) < E(I^{MPC})$ in figure 2. The second and fourth terms, $[E(P^N)]^2V(D)$ and $V(D)V(P^N)$, respectively, are larger for *IPP* than for *MPC* for 1960-72 and 1973-80 because undetrended $V(D^{IPP})$ is larger than undetrended $V(D^{MPC})$ for these periods. The fifth term, $[COV(D,P^N)]^2$, is also larger for *IPP* than for *MPC* for 1960-72 and 1973-80 because $COV(D^{IPP},P^N)$ is larger in absolute value than $COV(D^{MPC},P^N)$ in these periods. In the third term $(2E(D)E(P^N)COV(D,P^N))$, $E(D^{IPP}) > E(D^{MPC})$ for all periods and $COV(D^{IPP},P^N)$ is more negative than $COV(D^{MPC},P^N)$ for all periods so the third term will be smaller for *IPP* real prices than for *MPC* real prices for all periods. Except for the third term, all terms in Equation (1) are larger for *IPP* real prices than for *MPC* real prices during 1960-72 and 1973-80. However, variances of undetrended *MPC* real prices are larger than variances of undetrended *IPP* real prices for all periods. The magnitude of the third term, and in particular, the magnitude of $COV(D,P^N)$, determines the relationship between undetrended *IPP* and *MPC* real price variances. Thus, the amount of variability in the price index used to form a real price series and its relationship with the nominal price variability jointly influence the magnitudes of the variance estimates for the real price series.

Detrended Price Variances

Variances of detrended nominal and real prices are presented in table 3. Variances of nominal and *IPP* real prices increase in each sequential time period. Variances of nominal prices increase from 0.008 in 1960-72 to 0.3859 in 1981-90; variances of *IPP* real prices increased from 0.2174 to 0.3482 in the same period. Both absolute and percentage increases in nominal price variances are higher than increases in *IPP* real price variances. *MPC* real price variances increase from 0.1342 in 1960-72 to 0.7636 in 1981-90. While the absolute increase in *MPC* real price variances is larger than the absolute increase for nominal prices, the corresponding percentage increase is much smaller. In addition, the largest *MPC* real price variance occurs during 1973-80. All variances are consistent with an industry perception that risk in milk prices increased dramatically from 1960-72 to 1981-90 (Ford et al.). However, the greater percentage increase for nominal price variances and the decrease in variances between the second and third periods for *MPC* real prices suggest that detrended nominal prices are most consistent with this perception.

Comparisons of Detrended Nominal and Real Price Variances

Variances in table 3 for both detrended real price series are larger than variances of detrended nominal prices for 1960-72 and for 1973-80. This result for variances of detrended prices is opposite the pattern for undetrended prices in 1973-80. Earlier observations of weak trends in both *IPP* and *MPC* real prices and a strong trend in nominal prices in figures 1 and 3, and in the regression analysis are consistent with the outcome for this period. Contrary to the assumption implicit in Brake, Levy and Sarnat, and in White and Musser, expected inflation reflected in the nominal price trend may exceed actual inflation. In 1973-80, detrending removes more variation from nominal prices than from real prices so the variance of detrended nominal prices is smaller than the variance of detrended real prices. Approximate variances calculated using Equation (8) provide further evidence on this relationship between nominal and real prices. The variance relationship for detrended prices depends on covariances between errors from the nominal price and transformed index models $(COV(w,e))$ and on $[E(D)E(P^N) + a_l b_l V(T)]$ in Equation (8). The covariances between nominal errors and transformed *IPP* and *MPC* errors in table 4 are positive for 1960-72. Similarly, the term $[E(D)E(P^N) + a_l b_l V(T)]$ is positive for 1960-72. These two conditions are sufficient for real variances to exceed nominal variances. Although signs of these covariances are negative and the signs of $[E(D)E(P^N) + a_l b_l V(T)]$ are positive for 1973-80, real variances exceed nominal variances because magnitudes of $2COV(w,e)[E(D)E(P^N) + a_l b_l V(T)]$ are outweighed by the remaining terms in Equation (8). This result also holds for detrended *MPC* real prices for 1981-90. However, the variance of detrended nominal prices is larger than the variance of detrended *IPP* real prices for 1981-90. This variance relationship appears to result from the smaller (larger absolutely) magnitude of the covariance between nominal errors and transformed *IPP* index errors in comparison with magnitudes of other covariances for 1981-90.

Comparisons of Detrended Real Price Variances

A final comparison evaluates variances of detrended *IPP* real prices and detrended *MPC* real prices. As with undetrended real prices, variances differ substantially depending upon which index was used to calculate real prices. The *MPC* variance is larger than the *IPP* variance in 1973-80 and 1981-90, and smaller than the *IPP* variance in 1960-72. The latter result reflects the smaller variance of the estimated *MPC* index relative to the *IPP* index in this period. Except for 1981-90, magnitudes of approximate variances also follow this pattern. Although the approximation in Equation (7) is not helpful in understanding the 1981-90 period, variances for the first two time periods can be evaluated. The variances of detrended D^{IPP} and D^{MPC} (from table 3) and covariances of D^{IPP} and D^{MPC} with P^N $(COV(w,e)$ from table 4) explain the patterns of variances in 1960-72 and 1973-80. $V(w)$ is in the first, second, third, and fourth terms of Equation (7). Estimates of $V(w)$ are higher for D^{IPP} in the first period (0.0043 vs. 0.0025) and for D^{MPC} in the second period (0.0111 vs. 0.0027), which is consistent with the patterns in the variances of detrended real prices. However, the regression coefficients for the trend terms in D_t are also in the second and fourth terms of Equation (7). The absolute values of these coefficients from table 2 are greater for D^{IPP} than D^{MPC} in 1960-72 and 1973-80 (0.0238 vs. 0.0081 and 0.0357 vs. 0.0183, respectively), which removes part of the differential influence of *V(w)* in 1973-80. *COV(w,e)* is in the final two terms of Equation (7). Although these covariances are higher for *MPC* real prices in both periods (table 4), the multiplicative factor on these estimates has the opposite pattern, being greater for *IPP* real prices. However, the product of these two terms is larger for *MPC* real prices than for *IPP* real prices in both periods--0.0053 and 0.0064 in 1960-72 and -0.2023 and -0.1522 in 1973-80 for *IPP* and *MPC* real prices, respectively. Therefore the differences in *V(w)* account for the different patterns in this case. In 1960-72, the detrended variance for D^{IPP} is higher than the detrended variance for D^{MPC}, while the opposite pattern holds in 1973-80. Similarly, the detrended variance for *IPP* real prices is higher than the detrended variance for *MPC* real prices in 1960-72 and lower in 1973-80.

Conclusions

This paper examined the issue of using variances of real versus nominal data for measuring historical risk. Approximation formulas of theoretical variances and covariances were used to demonstrate that the variance of the index used to create real data and its covariance with nominal prices can cause the variance of real data to be either larger or smaller than the variance of nominal data. These relationships were derived for both original and detrended real price series. The results are inconsistent with the basic presumption of some risk literature that converting nominal data to real data reduces the variance and therefore the measure of risk.

The theoretical relationships discussed above were illustrated with variances estimated for U.S. quarterly milk prices for three different periods during 1960-90. Two real price series were calculated with the Index of Prices Paid by Farmers for all Production Items and an index of milk production costs constructed from historical costs of production. The data were detrended using feasible generalized least squares estimates of a piecewise linear regression to accommodate three time periods. Variances of the detrended price data were higher for real prices than for nominal prices in two of the three periods. Estimates of the theoretical variance approximations were helpful in explaining differences in patterns of variances for the nominal and real price series. Because quarterly U.S. milk prices were used in this research, the results may not hold for annual prices or prices of other commodities. Results may also fail to apply to net returns and gross margins, which include production risk. More research on risk measures of other time series would therefore be helpful.

Use of the specific milk cost of production index rather than the general index gave mixed results. The variance of real prices is influenced by the variance of the index reciprocal and its covariance with nominal prices. No general pattern emerged in the variances of detrended real data for the different indices. Furthermore, data were unavailable to calculate milk production costs for a portion of the time period. Estimates from a regression model were used for most of the 1960-72 period, which may have contributed to the mixed model results. At least two reasons for using a general index are apparent. First, data may be unavailable to estimate specific indices, as occurred in this research. Second, estimation of specific indices may be beyond the scope of research with several risky price series.

More generally, this paper identifies issues concerning the use of real or nominal data in measuring historical risk. Many risk analyses have often used real historical data to estimate price or income variance. Real data have been used either (1) because neoclassical theory assumes that prices are homogenous of degree zero or (2) to reflect current risk measures. Other authors cited in this paper suggest that the homogeneity condition does not generally hold under expected utility theory. In addition, current price levels may also be reflected in detrended nominal data. Results from this paper illustrate that using real data can have unpredictable effects on risk estimates because an additional stochastic variable (the index used to create real data) is introduced. Risk estimates from nominal data also were more consistent with industry perceptions reported in previous research. While beyond the scope of this study, analysis of the effects of nominal versus real data in risk programming or simulation models would be helpful to provide further evidence on this issue.

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Appendix A. Approximated Variances of Real Prices

The following formulas from Bohrnstedt and Goldberger and Mood et al. were used to calculate variances of real prices in Equations (1) and (7):

(A.1) $V(x \pm y) = V(x) + V(y) \pm 2COV(x,y)$
(A.2) $V(xy) = [E(x)]^2V(y) + [E(y)]^2V(x) + 2E(x)E(y)COV(x,y) + V(x)V(y) + [COV(x,y)]^2$.
(A.3) $COV(xy,uv) = E(x)E(u)COV(y,v) + E(x)E(v)COV(y,u) + E(y)E(u)COV(x,v) + E(y)E(v)COV(x,u) + COV(x,u)COV(y,v) + COV(x,v)COV(y,u)$

These formulas assume that third and higher-order moments of the random variables x, y, u, and v are equal to zero. Using (A.2), Equation (1) is derived as follows:

$$V(P^R) = V(DP^N) = [E(D)]^2V(P^N) + [E(P^N)]^2V(D) + 2E(D)E(P^N)COV(D,P^N) + V(D)V(P^N) + [COV(D,P^N)]^2.$$

To derive Equation (7), (A.1) is used to specify: $V(P^Nw + De - we) = V(P^Nw) + V(De) + V(we) + 2COV(P^Nw,De) - 2COV(P^Nw,we) - 2COV(De,we)$. For expository purposes, rewrite this equation as $V(P^Nw + De - we) = A + B + C + 2D - 2E - 2F$. Using (A.2) and $E(w) = 0$, $A = [E(P^N)]^2V(w) + [E(w)]^2V(P^N) + 2E(P^N)E(w)COV(P^N,w) + V(P^N)V(w) + [COV(P^N,w)]^2 = [E(P^N)]^2V(w) + V(P^N)V(w) +$

$[COV(P^N,w)]^2$. Under the maintained hypothesis of this study in Equation (3), $P_t^N = a_0 + a_1T + e_t$. Using this definition of P^N (omitting t subscripts) and definitions of COV and V, $A = [E(P^N)]^2V(w) + V(w)[E(a_0 + a_1T + e - a_0 - a_1E(T))^2] + [E(a_0 + a_1T + e)w]^2$. Simplifying, $A = [E(P^N)]^2V(w) + V(w)[E(a_1(T-E(T)) + e)^2] + [E(a_0w + a_1Tw + ew)]^2$. Expanding the second term $V(w)[E(a_1(T-E(T)) + e)^2]$ yields $V(w)[E(a_1^2(T-E(T))^2 + e^2 + 2a_1(T-E(T))e)]$. After taking expectations and using the standard Gauss-Markov assumption that $COV(T,e) = COV(T,w) = 0$, the second term equals $V(w)[a_1^2V(T) + V(e)]$. Again expanding and using the Gauss-Markov assumption to simplify the third term yields $[COV(e,w)]^2$. Finally, $A = [E(P^N)]^2V(w) + a_1^2V(w)V(T) + V(w)V(e) + [COV(w,e)]^2$. Using similar procedures as for A, it can be shown that $B = [E(D)]^2V(e) + b_1^2V(e)V(T) + V(w)V(e) + [COV(w,e)]^2$.

Using (A.2) again, $C = [E(w)]^2V(e) + [E(e)]^2V(w) + 2E(w)E(e)COV(w,e) + V(w)V(e) + [COV(w,e)]^2$. Because $E(w) = E(e) = 0$, $C = V(w)V(e) + [COV(w,e)]^2$. From (A.3), $D = E(P^N)E(D)COV(w,e) + E(P^N)E(e)COV(w,D) + E(w)E(D)COV(P^N,e) + E(w)E(e)COV(P^N,D) + COV(P^N,D)COV(w,e) + COV(P^N,e)COV(w,D)$. Again using $E(w) = E(e) = 0$, $D = E(P^N)E(D)COV(w,e) + COV(P^N,D)COV(w,e) + COV(P^N,e)COV(w,D)$. Substituting for P^N and D (again, omitting t subscripts) using Equations (3) and (4) yields $D = E(P^N)E(D)COV(w,e) + COV(w,e)[E(a_0 + a_1T + e - a_0 - a_1E(T))(b_0 + b_1T + w - b_0 - b_1E(T))] + [E(a_0e + a_1Te + e^2)E(b_0w + b_1Tw + w^2)]$. Rearranging terms and using $E(w) = E(e) = 0$ gives $D = E(P^N)E(D)COV(w,e) + COV(w,e)[E(a_1(T-E(T)) + e)(b_1(T-E(T)) + w)] + [a_1E(Te) + E(e^2)][b_1E(Tw) + E(w^2)]$. Expanding and taking expectations yields $D = E(P^N)E(D)COV(w,e) + COV(w,e)[E(a_1b_1V(T) + a_1COV(T,w) + b_1COV(T,e) + COV(w,e)] + [a_1COV(T,e)b_1COV(T,w) + a_1COV(T,e)V(w) + b_1COV(T,w)V(e) + V(e)V(w)]$. Applying $COV(T,e) = COV(T,w) = 0$ gives $D = E(P^N)E(D)COV(w,e) + a_1b_1V(T)COV(w,e) + [COV(w,e)]^2 + V(e)V(w)$.

Using (A.3), $E = E(P^N)E(w)COV(w,e) + E(P^N)E(e)V(w) + [E(w)]^2COV(P^N,e) + E(w)E(e)COV(P^N,w) + COV(P^N,w)COV(w,e) + COV(P^N,e)V(w)$. Applying $E(w) = E(e) = 0$ gives $E = COV(P^N,w)COV(w,e) + COV(P^N,e)V(w)$. Using $COV(P^N,w) = COV(w,e)$ and $COV(P^N,e) = V(e)$ as previously calculated for terms A and D, respectively, the term E equals $[COV(w,e)]^2 + V(w)V(e)$. Using similar procedures, $F = [COV(w,e)]^2 + V(w)V(e)$.

Collecting terms, $V(P^R) = [E(P^N)]^2V(w) + a_1^2V(w)V(T) + V(w)V(e) + [COV(w,e)]^2 + [E(D)]^2V(e) + b_1^2V(e)V(T) + V(w)V(e) + [COV(w,e)]^2 + V(w)V(e) + [COV(w,e)]^2 + 2\{E(P^N)E(D)COV(w,e) + a_1b_1V(T)COV(w,e) + [COV(w,e)]^2 + V(w)V(e)\} - 2\{[COV(w,e)]^2 + V(w)V(e)\} - 2\{[COV(w,e)]^2 + V(w)V(e)\}$. Simplifying the expression gives $V(P^R) = [E(P^N)]^2V(w) + a_1^2V(w)V(T) + [E(D)]^2V(e) + b_1^2V(e)V(T) + 2[E(P^N)E(D)COV(w,e)] + 2[a_1b_1V(T)COV(w,e)] + V(w)V(e) + [COV(w,e)]^2$ as in Equation (7).

Appendix B. Variance Transformation With a New Index Base

Define the series I_t as an index with $t = 1, 2, ..., Z$, where $t = 1$ in 1960 and $t = 31$ in 1990. Define I_t^{90} as the index with 1990 as the base period (i.e., $I_{31}^{90} = 100$) and define I_t^{60} as the index with 1960 as the base period (i.e., $I_1^{60} = 100$). Then $I_t^{60} = 100(I_t^{90}/I_1^{90})$. Define D_t as $100/I_t$. Then $D_t^{60} = 100/I_t^{60}$ and $D_t^{90} = 100/I_t^{90}$. Define the real price as $P_t^R = D_tP_t^N$, where P_t^N is the nominal price. P_t^{R60} and P_t^{R90} are real prices in 1960 and 1990 dollars, respectively. Using these definitions the following holds:
$P_t^{R60} = P_t^ND_t^{60}$.

Using the definition of D_t^{60},

$P_t^{R60} = P_t^N(100/I_t^{60})$.

Substituting in $I_t^{60} = 100(I_t^{90}/I_1^{90})$ yields

$P_t^{R60} = P_t^N(100I_t^{90})/(100I_t^{90})$.

Rearranging the above equation gives

$P_t^{R60} = P_t^N(100/I_t^{90})(I_t^{90}/100)$.

Substituting for $100/I_t^{90}$ yields

$P_t^{R60} = P_t^N D_t^{90}(I_t^{90}/100)$.

Using the definition of real prices gives

$P_t^{R60} = P_t^{R90}(I_t^{90}/100)$.

The variance of a real price series in 1960 dollars can be calculated using the above definition:

$$V(P^{R60}) = V[P^{R90}(I_t^{90}/100)]$$

$$= [(I_t^{90}/100)]^2 V[P^{R90}].$$

Because I_t^{90} is a constant and $I_t^{90} < 100$ so $(I_t^{90}/100) < 1)$, the variance of a real price series in 1960 dollars is smaller than the variance of real prices in 1990 dollars. As $[(I_t^{90}/100)]^2$ is a constant, a constant relationship between the variances exists. The above time specific example can be generalized by shifting from base a to base b. Then $V(P^b) = [I_b^a/100]^2 V[P^a]$. If $a > b$, $I_b^a < 100$ and $V(P^b) < V(P^a)$. If $a < b$, $I_b^a > 100$ and $V(P^b) > V(P^a)$.

Endnotes

1. The effects of using estimated data for the *MPC* index on the regression results for the 1960-72 period in Table 1 and Table 2 are difficult to determine. Use of piecewise regressions limits the effects; however, the restrictions that require the regressions to meet at endpoints may cause intercepts and slope coefficients to be biased for the second and third periods. The magnitudes and signs of estimated coefficients are consistent with trends in actual observed data. Nevertheless, caution is warranted.