MEASURING PRE-COMMITED QUANTITIES THROUGH CONSUMER PRICE FORMATION

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Abstract:

We investigate how to theoretically and empirically measure pre-committed quantities through price formation utilizing translating in the consumer distance function. The translated consumer distance function is defined as a dual to the translated utility, indirect utility, and expenditure functions. Translating procedures also provide more general analytical means to incorporate pre-committed quantities (and other shift or demographic variables) into inverse demand systems. This approach yields a class of inverse demand functions that can nest most known functional forms. For example, the Inverse Generalized Almost Ideal Demand (IGAI) model can be formed by applying translating procedures to the Inverse Almost Ideal Demand model. An empirical example of the IGAI model with inferences on the translating parameters themselves is provided for illustrative purposes.

JEL Classification: C10, D11, D12

Key words: duality, distance function, price formation, food demand, translating, inverse demand system, inverse generalized almost ideal demand model

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Introduction

In this article we investigate the case in which consumers pre-commit to goods in a market characterized by fixed supply.[1] To do so we define a translated consumer distance function that is a natural dual to the translated utility, indirect utility, and expenditure functions and that can nest most known functional forms. The translated distance function is of interest for its role in the theory of dual functions and for its flexibility to incorporate "pre-committed quantities" or "necessary quantities" into complete inverse demand systems. This research is further motivated by a general interest in developing a better understanding of consumer price formation through inverse demand functions, which is particularly relevant in food and resource markets.

Pre-committed quantities incorporated into the direct utility, expenditure, and indirect utility functions have been previously investigated in the economic literature dating back to Samuelson (1947-1948), resulting in extensive applications on a wide range of topics. In this case, consumers pre-commit in markets with fixed prices. For example, the well known Stone-Geary utility function integrates pre-committed quantities into the Cobb-Douglas utility function yielding the linear expenditure system (LES). This specification generalizes the Cobb-Douglas to recognize pre-committed levels of consumption, such that any expenditure over the pre-committed expenditure is allocated according to Cobb-Douglas preferences.[2] Pollak and Wales (1978) provided dual relationships between the translated utility, indirect utility, and expenditure functions (but not the distance function) and pointed out that the LES also arises from the

[1] Pre-commitment is taken to be some irreversible act or choice by an economic agent, such as pre-commitment to quantities by a consumer. Pre-commitment arises in broad array of topics across the economic literature, including (but not limited to) subsistence consumption (Samuelson, 1947-1948; Stone, 1954) as well as in the context of demographic translating (Pollak and Wales, 1978) and food safety (Piggott and Marsh, 2004).

[2] See Kakwani (1977) or Deaton and Muellbauer (1980b) for background reading on the Stone-Geary utility function. A recent search on EconLit finds that the term linear expenditure system itself arises in 91 articles.

translated Cobb-Douglas expenditure function. The translated distance function, and its application to measuring pre-committed quantities, represents novel contributions to the economic literature.[3]

Consumer distance functions that yield inverse demand systems are relevant when attempting to better understand price formation at the market level. They have been used to derive price and scale flexibilities that are informative economic measures of price formation, as well as exact welfare measures in quantity space (Palmquist 1988, Kim 1997, Holt and Bishop 2002). Demand system modeling that specifies prices as a function of quantities is a growing literature in food, agricultural, environmental, and natural resource economics, wherein perishability and biological production lags are often inherent characteristics. For example, price formation has been previously studied for meat demand (Eales and Unnevehr 1994; Holt and Goodwin 1997; Holt 2002) and fish (Barten and Bettendorf 1989; Holt and Bishop 2002; Kristofersson and Rickertsen 2004, 2007). These studies provide significant contributions to the economic literature, but they do not theoretically investigate nor empirically test for pre-committed quantities through price formation. Translated consumer distance functions yield generalized inverse demand systems that naturally include pre-committed quantities and offer opportunities to empirically test for their statistical significance using straightforward inference methods. Pollak and Wales (1980) demonstrated that translated utility, indirect utility, and expenditure functions are important functional alternatives yielding demand systems that include pre-committed quantities and that are useful for empirical applications. This suggests that

[3] A suggestion to the authors has been to address the concept of pre-commitment in prices. While intriguing, our interest is in extending translating (and pre-committed quantities) to inverse demand relationships. The micro-foundations of pre-commitment in quantities are well established by Samuelson (1947-1948), Stone (1954), and Geary (1950-1951). It is outside the scope of the current study to conceptualize and investigate pre-commitment in prices.

generalizing inverse demand models by translating is also a plausible alternative that is important when investigating empirical questions of price formation and pre-commitment.

Several studies have generalized specific functional forms of demand systems (i.e., quantities as a function of prices and expenditure) using translating procedures. Pollak and Wales (1980) developed the generalized translog (GTL) model by introducing pre-committed quantities to the basic translog model of Christensen, Jorgenson, and Lau (1975). Bollino (1987) introduced the generalized almost ideal (GAI) model by incorporating pre-committed quantities into the almost ideal demand system (AI) of Deaton and Meullbauer (1980a, b). Bollino and Violi (1990) generalized the almost ideal and translog (GAITL) model by including pre-committed quantities into the almost ideal translog model of Lewbel (1989). Following this theme, and motivated by our interest in empirical applications−but in the context of inverse demand models−we introduce the inverse generalized almost ideal demand (IGAI) system by applying translating procedures to the inverse almost ideal (IAI) model of Eales and Unnever (1994).

The purpose of this article is three-fold. First, we provide selected dual relationships for translation to the consumer distance function. [4] In this manner, we can specify translating into inverse demand functions in a more theoretically general manner to facilitate the study of price formation and translating. This allows flexibility between pre-commitment in quantities and market structure. Methodologies to measure marginal effects and flexibilities are also derived. Second, we provide illustrative examples of the translated consumer distance function with two different functional forms (the Cobb-Douglas and almost ideal functional forms) and a framework to accommodate other functional forms. In the latter, we extend the work of Eales

[4] Note that we do not intend to provide a complete taxonomy of dual relationships for the translated distance function, but rather provide those relationships that facilitate sufficient specification and derivations to complete the examples and empirical applications in the paper.

and Unnever (1994) on the inverse almost ideal demand system to define an inverse generalized almost ideal demand system that includes pre-committed quantities as elements of the parameter space. Third, we provide empirical applications demonstrating how to apply the IGAI to a complete translated inverse demand system and how to augment the translation parameter to include other shift variables. Also, we compare alternative forms of the IAI model. Our empirical application focuses on estimating retail price formation of U.S. food demand (food-at-home (FAH), food-away-from-home (FAFH), and alcoholic beverages (AB)) and relevant hypothesis tests. Retail price formation for food-at-home, food-away-from-home, and alcoholic beverages and the impacts of pre-committed quantities have not been addressed in previous empirical studies. Finally, concluding comments are provided.

Translating in Dual Functions

The direct utility maximization problem is

(1) $$\max_{x} \left\{ U(x) \ \text{st} \ p'x = M \right\}.$$

In (1), U is the utility function with classical properties, $x = (x_1, \ldots, x_n)' \geq 0$ is a $(n \times 1)$ nonnegative vector of goods, $p = (p_1, \ldots, p_n)' > 0$ is a $(n \times 1)$ vector of given prices, and M is total fixed expenditure. From the Hotelling-Wold Identity the uncompensated inverse demand system can be expressed as $\frac{p_\ell}{M} = \frac{\partial U}{\partial x_\ell} \Big/ \sum_{j=1}^{n} x_j \frac{\partial U}{\partial x_j}$.

Translating of x for some $(n \times 1)$ constant pre-committed consumption vector $c = (c_1, \ldots, c_n)' \in R^n$ is defined as the linear mapping $x_* = x - c$. The translated utility function is specified as

(2) $U^{*} = U(x - c)\,.(x)$

The transformed primal problem can be expressed as

(3) $\max_{x^*}\left\{U^{*}(x)\ \text{st}\ p'x^{*} = M^{*},\ x^{*} > [0]\right\}$

where $M^{*} = M - p'c$ is supernumerary expenditure. Because c is pre-committed and p fixed then $p'c$ can be interpreted as pre-committed expenditure. Samuelson (1947-1948) interprets $p'c$ as the minimum expenditure to which the consumer commits herself to attain a minimum subsistence level. [5]

It is well known that the transformed dual indirect utility function is $V = V(p, M^{*})$, which is a function of prices and supernumerary expenditure, and the transformed dual expenditure function is $E = p'c + E^{*}(p, u)$, which decomposes total expenditure into an additive relationship of pre-committed and supernumerary expenditure functions (e.g., Pollak and Wales 1978). Moreover, and for example, Shephard's Lemma applied to the transformed expenditure function yields total demand $x = c + x^{*}(p, u)$ that is interpreted as the sum of pre-committed quantities and compensated supernumerary demand. [6] Finally, the translated utility, indirect utility, and expenditures functions nest original specifications and become equivalent only if the translating vector c=0.

The Distance Function

The standard consumer distance function can be defined by

(4) $D(x, u) = \sup_{d}\left\{d > 0 \mid (x/d) \in S(u),\ \forall u \in R_{+}^{1}\right\}.$

[5] Since the linear mapping is a diffeomorphism the standard economic properties still hold for x*.
[6] The translated expenditure function provided the basis for studies by Bollino (1987), Bollino and Violi (1990), Piggott (2003), Piggott and Marsh (2004), and Tonsor and Marsh (2007).

In (4), u is a (1×1) scalar level of utility, $x = (x_1,\ldots,x_n)'$ is a $(n \times 1)$ vector of predetermined goods and $S(u)$ is the set of all vectors of goods $x \in R^n$ that can produce the utility level $u \in R^1_{++}$. The underlying behavioral assumption is that the distance function represents a rescaling of all goods consistent with a target utility level u. Intuitively, d is the maximum value by which one could divide x and still produce u. The value d places x / d on the boundary of $S(u)$ and on a ray through x.

Compensated inverse demand equations may be obtained by applying Gorman's Lemma

(5) $$\frac{\partial D(x, u)}{\partial x} = \tilde{p}(x, u),$$

where $M = \sum_{i=1}^{n} p_i x_i$ and $\tilde{p} = (\tilde{p}_1,\ldots,\tilde{p}_n)$ is a $(n \times 1)$ vector of expenditure normalized prices or $\tilde{p}_i = p_i / M$. If x is a bundle for which $U(x) = u$ then $D(x, u) = 1$, and the share form of the expression in (5) is given by $\frac{\partial \ln D(x, u)}{\partial \ln x} = w(x, u)$. The Hessian (or Antonellei) matrix is given by the second order derivatives of the distance function

(6) $$H = \begin{bmatrix} \frac{\partial^2 D(x, u)}{\partial x \partial x'} & \frac{\partial^2 D(x, u)}{\partial x \partial u} \\ \frac{\partial^2 D(x, u)}{\partial u \partial x'} & \frac{\partial^2 D(x, u)}{\partial u \partial u} \end{bmatrix}.$$

The properties of a distance function are that it is homogenous of degree one, nondecreasing, and concave in quantities x, as well as nonincreasing and quasi-concave in utility u (Shephard 1970; Cornes 1992). Because the distance function is homogenous of degree one in quantities, it follows that the compensated inverse demand function is homogenous of degree zero in quantities. Uncompensated inverse demand functions can be obtained applying the dual identity $\tilde{p}(x) = \tilde{p}(x, U(x))$.

Our principal interest is to extend duality theory to incorporate translation in the distance function. Letting $U^* = U(x - c)$, we define a translated distance function through the dual(x) relationship

$$D(x, u; c) = \arg_d \{ U(x^*/d)\ u \}, \tag{7}$$

where $x^* \in R^n_+$ and $c \in R^n$. The translated distance function in (7) is a natural generalization of the standard distance function defined in (4), where $D(x, u; c) = D(x, u)$ only if c=0. [7]

A modified Gorman's Lemma can be derived applying the Envelope Theorem and a dual identity that defines the distance function through the normalized expenditure function

$$D(x, u; c) = \min_{\hat p} \{ \hat p' x^* \text{ such that } E^*(\hat p, u) = 1 \}$$

(8a) $$\frac{\partial D(x, u; c)}{\partial x^*} = \hat p(x, u; c),$$

where $\hat p = (\hat p_1, \ldots, \hat p_n)$ is a n × 1 vector of prices normalized by supernumerary expenditure, or $\hat p_i = p_i / M^*$. [8] The compensated $\hat p_i(x, u; c)$'s are functions of the supernumerary quantities x^* and the utility level u. The consumer's marginal willingness to pay for x^* is represented by the uncompensated $\hat p_i(x, U^*(x); c)$'s which are formed on the level of pre-committed quantities.

The second order derivatives of the translated distance function yield

(8b) $$H = \begin{bmatrix} \dfrac{\partial^2 D(x, u; c)}{\partial x^* \partial u} & \dfrac{\partial^2 D(x, u; c)}{\partial x^* \partial x^{*\prime}} \\ \dfrac{\partial^2 D(x, u; c)}{\partial u \partial u} & \dfrac{\partial^2 D(x, u; c)}{\partial u \partial x^{*\prime}} \end{bmatrix}.$$

[7] Luenberger (1992) introduced the benefit function and Chambers, Chung and Färe (1996) demonstrated that the benefit function is equivalent to a directional distance function. As pointed out by Luenberger (1992) the consumer distance function, and hence the translated form of it, and the benefit function are distinctly different specifications.

[8] Note that the expenditure value normalizing prices is the supernumerary expenditure M^*, which leads to a modified Gorman's Lemma.

where H^* is concave in supernumerary portion of consumption x^* and quasi-concave in u. H^* is equivalent to H in (6) when the pre-committed quantities are all equal to zero. Hence, while the mathematical properties for H^* are consistent with H, the relevant economic properties deserve further discussion. For instance, it is straightforward to demonstrate that symmetry conditions hold. However, the second order partial derivatives of $D(x, u; c)$ with respect to x does not necessarily yield to a negative semi-definite matrix.[9]

Translated share equations also can be derived. If x^* is a bundle of goods chosen such that $U^*(x) = u$ then $D(x, u; c) = 1$, and the compensated, supernumerary share expression can be derived as $\frac{\partial \ln D(x,u;c)}{\partial \ln x_i^*} = p_i(x,u;c)\,x_i^* = w_i^*(x, u; c)$.[10] The uncompensated, supernumerary share expression is $w_i^* = p_i(x_i - c_i)/M^*$ that can be rewritten as $w_i^* = \frac{p_i x_i}{M}\left(\frac{M}{M^*}\right)\left(\frac{x_i^*}{x_i}\right) = w_i\left(\frac{M\,(x_i - c_i)}{M^*\,x_i}\right)$.

Following Christensen et al (1975), the logarithmic form of the Hotelling-Wold Identity yields $\frac{p_\ell x_\ell}{M} = \frac{\partial \ln U / \partial \ln x_\ell}{\sum_{j=1}^{n} \partial \ln U / \partial \ln x_j}$. Hence, the general share expression for a translated inverse demand function can be represented by

$$w_i = \frac{w_i^*\left(\frac{M^*}{M}\right)\left(\frac{x_i}{x_i - c_i}\right)}{\sum_{j=1}^{n} w_j^*\left(\frac{M^*}{M}\right)\left(\frac{x_j}{x_j - c_j}\right)} = \frac{w_i^*\left(\frac{x_i}{x_i - c_i}\right)}{\sum_{j=1}^{n} w_j^*\left(\frac{x_j}{x_j - c_j}\right)} \tag{9}$$

Translating the distance function introduces a new class of functions completing the quadrality of dual functions that also includes the translated utility, indirect utility, and expenditure functions. Moreover, the translated distance functions provides the analytical

[9] Pollak and Wales (1980) point out that for the translated demand system the Slutsky symmetry conditions are satisfied, but that the substitution matrix need not be negative semi-definite except if c=0.

[10] It is termed the supernumerary share expression because it is a function of the supernumerary quantity.

framework with which to specify translated inverse demand systems in (8a) that nest their original counterparts defined in (5). For example, and as illustrated below, two generalized inverse demand systems arise by defining the $w_{i^*}(x; c)$ as the Cobb-Douglas and Almost Ideal Demand functional forms. Other logical candidates for the $w_{i^*}(x; c)$ are the translog, the normalized quadratic, and variations of them.[11] As discussed in more detail ahead, translating also introduces the flexibility to augment each c_i as linear or nonlinear function of a vector s of pre-committed, demographic, conditioning, or other shift variables that arise in empirical applications (i.e., $c_i = \varsigma_i(s)$). Hence, while the mapping on x by $x_*(s) = x - \varsigma(s)$ is linear, the pre-committed and supernumerary quantities may have a linear or nonlinear relationship with s.

Flexibilities

Marginal effects and price flexibilities can also be derived in the case of a nonzero translation vector c. The uncompensated price flexibilities $f_{i\ell} = \frac{\partial \ln p_i(x;c)}{\partial \ln x_\ell}$ are defined by

(10a) $$\frac{\partial \ln p_i}{\partial \ln x_\ell} = -\delta_{i\ell} + \frac{\partial w_i}{\partial x_\ell}\frac{x_\ell}{w_i}.$$

where

(10b) $$\frac{\partial w_i}{\partial x_\ell} = \{A_{i\ell} - B_{i\ell}\} / C^2$$

and

(11) $$-\delta\ c\ w_* \quad \sum \quad x_j x_i\ \partial w_{i^*} \quad n\, i\ell\ i\ i$$
$$= \quad + \quad A_{i\ell} w_* \quad (x - c)^2\ x_i - c_i\ \partial x_\ell \quad j=1\ x_j - c_j\ j\ j \quad i\ i$$

11 Piggott (2003) provides a discussion of generalized demand systems.

$$B_{i\ell} = \left(\frac{x_i}{w_i^*}\right)^2 \left(-c_\ell \frac{w_\ell^*}{x_\ell} + \sum_{j=1}^{n} x_j \frac{\partial w_j^*}{\partial x_\ell}\right)$$

$$C = \sum_{j=1}^{n} \frac{(x_i - c_i)(x_\ell - c_\ell)\,(x_j - c_j)}{x_j^*}\,\frac{\partial x_\ell}{w_\ell^*}$$

The compensated flexibilities $f_{i\ell h} = \frac{\partial \ln p_i(x, u; c)}{\partial \ln x_\ell}$ can be recovered using the expression $f_{i\ell h} = f_{i\ell} - f_i w_j$. Scale flexibilities $f_i = \frac{\partial \ln p_i(\lambda x; c)}{\partial \ln \lambda}$ can be derived by

(12) $$\frac{\partial \ln p_i(\lambda x; c)}{\partial \ln \lambda} = \sum_{j=1}^{n} f_{ij}.$$ [12]

The price flexibility expressions for the translated inverse demand system are considerably more complicated than those of the IAI model. If the $c_i = 0\ \forall i$, or pre-committed consumption is zero for each good, then (10-11) yields the standard flexibility expression.

A Simple Example: The Cobb-Douglas Functional Form

A simple example demonstrating the dual relationships is the Cobb-Douglas functional form. Consider the utility function $U(x) = \left(x_1^{\alpha_1} x_2^{\alpha_2}\right)$ with two goods where $\alpha_1 + \alpha_2 = 1$. The translated Cobb-Douglas utility function can be defined as $U^*(x) = (x_1 - c_1)^{\alpha_1}(x_2 - c_2)^{\alpha_2}$. Following standard relationships the following dual functions can be derived: a) the indirect utility function $V(p, M^*) = \left(\alpha_1 \frac{M^*}{p_1}\right)^{\alpha_1}\left(\alpha_2 \frac{M^*}{p_2}\right)^{\alpha_2}$ and b) the expenditure function $E(p, u) = p'c + u\left(\left(\frac{p_1}{\alpha_1}\right)^{\alpha_1}\left(\frac{p_2}{\alpha_2}\right)^{\alpha_2}\right)$.

[12] From Anderson (1980), the flexibilities must satisfy the aggregate demand restrictions $\sum_{j=1}^{n} f_{ij} = f_i$ (homogeneity), $\sum_{i=1}^{n} w_i f_{ij} = -w_j$ (Cournot), and $\sum_{i=1}^{n} w_i f_i = -1$.

From (7) the translated distance function is $D(x, u; c) = \left(\frac{(x_1 - c_1)^{\alpha_1}(x_2 - c_2)^{\alpha_2}}{u}\right)$, which also can be derived from other dual relationships. Further, and considering good 1 for convenience, applying Roy's Identity to the translated distance function yields the uncompensated demand function $x_1^m = c_1 + \alpha_1 \frac{M^*}{p_1}$ which is composed of the pre-committed quantity c_1 and the supernumerary component of demand $\alpha_1 \frac{M^*}{p_1}$ and the compensated demand function $x_1^h = c_1 + u\left(\frac{p_1\alpha_2}{p_2\alpha_1}\right)^{\alpha_1 - 1}$ (from Shephard's Lemma). The uncompensated inverse demand function $p_1^m = \frac{\alpha_1 M^*}{(x_1 - c_1)}$ (by the Hotelling-Wald Identity) and compensated inverse demand function $\tilde{p}_1^h = \frac{\alpha_1}{u}\left(\frac{x_1 - c_1}{x_2 - c_2}\right)^{\alpha_1 - 1}$ (from Gorman's Lemma) all include pre-committed components that nest the original Cobb-Douglas functions and are equivalent only if c=0.[13] From (9) the uncompensated inverse share equation is $w_i^m = \left(\frac{\alpha_i x_i}{x_i - c_i}\right) \Big/ \sum_{j=1}^{n}\left(\frac{\alpha_j x_j}{x_j - c_j}\right)$ which is not the same as the uncompensated share equation $w_1^m = \frac{p_1 c_1}{M} + \alpha_1 \frac{M^*}{M}$.[14]

The Almost Ideal Functional Form

The almost ideal functional form is pervasive in the consumer demand literature. Choosing to generalize the IAI model by translating allows one to compare and contrast results (theoretical and empirical) to past research on price formation. Moreover, it provides an interesting

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[13] Note that with the Cobb-Douglas specification, it is straight forward to derive the inverse uncompensated demand function directly from the uncompensated demand function. However, as shown ahead with the almost ideal functional form, solving for the inversed demand function directly from the demand function is not always possible further motivating the usefulness of duality relationships.

[14] This uncompensated inverse share expression for the LES is identical equation (6) in Moschini and Vissa (1992).

comparison to the GAI model and applications of it. For example, while standard theory would suggest that pre-committed quantities specified in a demand system or inverse demand system are the same, this remains an open empirical question to be examined. Next we review the inverse (IAI) demand system and then specify a generalized version of the IGAI demand system.

The Inverse Almost Ideal Demand System

Following Eales and Unnevehr (1994) the logarithmic distance function may be specified as:

(13) $$\ln D(x, u) = (1 - u) \ln a(x) + u \ln b(x)$$

The IAI expenditure system is obtained by substituting equations (14) and (15) below into (13) above:

(14) $$\ln a(x) = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln x_i \ln x_j$$

and

(15) $$\ln b(x) = \beta_0 \prod_{i=1}^{n} x_i^{-\beta_i} + \ln a(x) .$$

Applying Gorman's Lemma and substituting in the direct utility function $U(x) = -\ln a(x) / \{\ln b(x) - \ln a(x)\}$, which is obtained by inverting the distance function at $D(x, u) = 1$, the share form of the inverse demand function can be derived as

(16) $$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln x_j + \beta_i \ln Q$$

where

(17) $$\ln Q = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln x_i \ln x_j .$$

In (16) and (17), w_i = expenditure share of meat type i ($w_i = \frac{p_i x_i}{M}$) and $\gamma_{ij} = \frac{1}{2}\left(\tilde{\gamma}_{ij} + \tilde{\gamma}_{ji}\right)$.

Necessary demand conditions that lead to parameter restrictions of the distance function specification are as follows:

(18a) $\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \beta_i = 0$ adding up

(18b) $\sum_{i=1}^{n} \gamma_{ij} = 0$ homogeneity

(18c) $\gamma_{ij} = \gamma_{ji}$ symmetry.

Price and scale flexibilities provided in Eales and Unnevehr are defined by

(19a) $f_{i\ell} = \frac{\partial \ln p_i(\tilde{x})}{\partial \ln x_\ell} = \frac{1}{w_i}\left[\gamma_{i\ell} + \beta_i\left(\alpha_\ell + \sum_{j=1}^{n} \gamma_{j\ell} \ln(\tilde{x}_j)\right)\right] - \delta_{i\ell}$

and

(19b) $f_i = \frac{\partial \ln p_i(\lambda \tilde{x})}{\partial \ln \lambda} = -1 + \beta_i / w_i$,

where the last equality simplifies due to imposition of general demand restrictions with reference vector $\tilde{x}$.

The Inverse Generalized Almost Ideal Demand System

Using the translation identity $x_* = x - c$ and equations (7) and (13), we specify a generalized logarithmic distance function as[15]

(20) $\ln D(x, u; c) = (1 - u)\ln a(x_*) + u \ln b(x_*)$

The inverse generalized almost ideal (IGAI) expenditure system is defined by substituting

[15] Note that the translated distance function can also be derived from the translated direct utility function $U(x) = -\ln a(x_*) / \{\ln b(x_*) - \ln a(x_*)\}$ applying the dual relationship in (7) and imposing general demand restrictions.

(21) $$\ln a(x^*) = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln\left(x_j - c_j\right) + .5\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln\left(x_i - c_i\right) \ln\left(x_j - c_j\right)$$

and

(22) $$\ln b(x^*) = \beta_0 \prod_{i=1}^{n} \left(x_i - c_i\right)^{-\beta_i} + \ln a(x^*) ,$$

into equation (20). The supernumerary share expression of the inverse demand functions is then

(23a) $$w_i^* = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln\left(x_j - c_j\right) + \beta_i \ln Q^* ,$$

where $w_i^* = \dfrac{p_i x_i^*}{M}$ and

(23b) $$\ln Q = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln\left(x_j - c_j\right) + .5\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln\left(x_i - c_i\right) \ln\left(x_j - c_j\right) .$$

From (9) the inverse share equation can be expressed as

(24) $$w_i = \frac{\dfrac{x_i^*}{(x_i - c_i)}\left(\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln\left(x_j - c_j\right) + \beta_i \ln Q^*\right)}{\sum_{j=1}^{n} \dfrac{x_j^*}{(x_j - c_j)}\left(\alpha_j + \sum_{k=1}^{n} \gamma_{jk} \ln\left(x_k - c_k\right) + \beta_j \ln Q^*\right)}$$

where $w_i = \dfrac{p_i x_i}{M}$. For a n good system, including the translating constants c_i creates an additional n parameters to estimate with each translating constant appearing in each expenditure equation. The parameter restrictions from homogeneity, symmetry, and adding up conditions are consistent to those of the IAI model in (18a)-(18c).[16]

[16] The role of translating procedures in dual functions is not limited to incorporating pre-committed quantities. In addition, translating parameters can be augmented to be functions of demand shift variables to account for other factors impacting demand aside from prices and income. This includes the universe of non-price and non-income variables thought to impact demand, including seasonal dummy variables, time trends, advertising expenditures, food safety information, conditioned variables, and lagged quantities to capture potential habit effects to name a few candidates. Introducing non-price and non-income variables into demand functions in this manner avoids potential pitfalls of other commonly used approaches (such as augmenting intercept terms of demand or share equations), which yield economic measures that are not necessarily invariant to units of measurement (Alston, Chalfant, and

Applying (24) the uncompensated price flexibilities $f_{i\ell} = \frac{\partial \ln p_i(x;c)}{\partial \ln x_\ell}$ are defined in (10)-(11) with

$$(25)\quad \frac{\partial w_i^*}{\partial x_\ell} = \left(\frac{1}{x_\ell - c_\ell}\right)\left[\gamma_{i\ell} + \beta_i\left(\alpha_\ell + \sum_{j=1}^{n} \gamma_{j\ell} \ln\left(x_j^*\right)\right)\right]$$

For the case that the $c_i = 0$, $\forall i = 1,...,n$, the IGAI model in (24) becomes identical to the IAI model in (16). Moreover, the price and scale flexibilities collapse to those for the IAI model in 19(a) and 19(b).

Empirical Application

Following Eales and Unnevehr (1994), Holt and Goodwin (1997) and Holt (2002), we apply the IAI and IGAI model to quarterly U.S. meat consumption data as an empirical application.[17] Moreover, this allows us to compare outcomes from the IGAI model to results from previous studies. As an illustration of linear translating, the pre-committed parameters, $c_i = \tilde{\varsigma}_i(s)$'s, are modified to depend linearly upon seasonal variables shift variables. Using notation from equation (26), the $s_m = qd_m$ (m=1, 2, and 3) are seasonal quarterly dummies with the parameters to be estimate being the c_{i0}'s and the φ_{im}'s. This IGAI model with linear translating involving seasonal dummy variables will be denoted as IGAI $\varsigma(s)$ in the discussion ahead.

Data

Piggott 2001). Augmenting translating constants to incorporate demographic variables in this fashion has been coined as demographic translation (Pollak and Wales 1981). A natural and simple choice is to employ linear translating where the ci's are specified to be a linear function of demand shift variables and parameters.

[17] Scanner data information suggests consumers pre-commit to purchasing meat products relatively infrequently (only once or twice a month) compared to most food products.

Meat data used in the analysis are quarterly observations over the period 1982(1)-2005(4), providing a total of 96 observations. The basic quantity data are per capita disappearance data from the United States Department of Agriculture (USDA), Economic Research Service (ERS) supply and utilization tables for beef, pork, and poultry (broiler, other-chicken, and turkey) published in the Red Meats Yearbook and Poultry Yearbook with data after 1990 taken from updated revisions of these publications made available online. The beef price is the average retail choice beef price, the pork price is average retail pork price, and the poultry price was calculated by summing quarterly expenditures on chicken, using the average retail price for whole fryers, and quarterly expenditures on turkey, using the average retail price of whole frozen birds, divided by the sum of quarterly per capita disappearance on chicken and turkey. All of the price variables are published in the same USDA, ERS sources with the original sources identified as the ERS (Animal Products branch) for the beef and pork prices (variable names BFVRCCUS and PKVRCCUS, respectively) and the Bureau of Labor Statistics, U.S. Department of Labor for the whole fryers (chicken) and whole frozen bird (turkey) prices. Table 1 provides descriptive statistics for model variables.

Empirical Issues

Several important issues regarding parameter restrictions and differences in methodology need to be discussed. First, the necessary demand conditions that lead to parameter restrictions in (20) remain unchanged for the IGAI relative to the IAI expenditure system. As in the GAI model there are no necessary economic restrictions to be imposed on the individual pre-committed quantities c_i's (see Piggott and Marsh 2004)

Meat is aggregated into three goods: beef, pork, and poultry (chicken and turkey). Models were estimated using iterated non-linear seemingly unrelated estimation techniques. The parameter α_0 is restricted zero, which has been standard practice for the IAI model (see Eales and Unnevehr 1994, Holt 2002) due to problems of convergence in estimation. Because of the singular nature of the share system one of the equations must be deleted (poultry) with the remaining equations being estimated (beef and pork). Theoretical restrictions such as homogeneity and symmetry were imposed as maintained hypotheses.

Results and Discussion

Parameter estimates and asymptotic standard errors are presented for the IAI, IGAI, and IGAI $_{\varsigma(s)}$ in Table 2. Results for all three alternative models are reported for comparisons and robustness checks. In all three models, most of the coefficients are individually statistically significantly different from zero at the 0.05 level. For the IAI and IGAI models all of coefficients are individually statistically significantly different from zero at the 0.05 level except for β_b and β_p. Comparison of the IAI to IGAI reveals that generalization significantly enhances the model fit with R^2 for beef increasing from 0.721 to 0.980 and R^2 for pork increasing from 0.404 to 0.964. The translating parameters are all highly individually statistically significant and positive in the IGAI model. Results of nested hypothesis tests shown in Table 3 demonstrate that the null hypothesis of the IAI model is rejected at the 0.01 level against the IGAI model. All reported joint hypothesis tests are based on asymptotic chi-square likelihood ratio statistics (Mittelhammer et al 2000), which are adjusted for small-sample size as suggested by Bewley (1986).

Comparison of the estimated parameters of IGAI and IGAI $\varsigma(s)$ reveals that six of the nine coefficients on the seasonal dummy variables are individually statistically significantly different from zero at the 0.05 level. Results of nested hypothesis tests shown in Table 3 also reveal that the null hypothesis of the IGAI model is rejected at the 0.01 level against the IGAI $\varsigma(s)$ model. Thus there is strong empirical evidence to support not only the existence of pre-committed quantities of beef, pork, and poultry but also that of seasonality. These seasonal differences were mostly found to be on the order of 1 pound but were as large as 2 pounds in a given quarter.

Uncompensated price and scale flexibilities for the IAI, IGAI, and IGAI $\varsigma(s)$ models are reported in Table 4. The own-flexibilities and scale flexibilities are negative as expected across all models. The majority of the cross-flexibilities are negative, indicating gross-substitutes, with exception for the cross-flexibilities for beef and poultry prices with pork quantities being positive indicating gross-complements. The scale flexibilities for beef and poultry are all less than 1 and for pork greater than 1. For the statistically preferred IGAI $\varsigma(s)$ model, the own-flexibilities for beef (-0.607) and poultry (-0.606) are inflexible whereas the own-flexibility for pork (-1.567) is flexible. It is noteworthy that own-flexibility for pork is not robust across model specifications with estimates of -0.607 (IAI model), -0.912 (IGAI model) and -1.567 (IGAI $\varsigma(s)$ model). The own-price flexibilities for beef and poultry are much more robust across model specifications. The scale flexibilities for IGAI $\varsigma(s)$ reveals that the marginal value of meats in consumption declines by 0.6% for beef, 2.1% for pork, and 0.3% for poultry.

The estimated price and scale flexibilities can be compared with previous results from Eales and Unnevehr (NL/IAIDS model, Table 3). Eales and Unnevehr own-price flexibilities for beef (-0.750) and poultry (-0.611) are comparable but their pork estimate (-0.785) is much more inflexible. Their cross-flexibilities were all negative, indicating gross-substitutes, compared with mix of positive and negative estimates from the IGAI ς (s) model. Finally, there are significant differences in the scale flexibilities between the two studies with the most notable being for pork and poultry.

The estimated pre-committed quantities were highly significant and very robust across the IGAI and IGAI ς (s) models. Based on the IGAI model the pre-committed quantities are estimated to be 13.709 pounds of beef, 10.403 pounds of pork, and 11.357 pounds of poultry per quarter per person. When compared to the sample means (shown in Table 1) these estimates show that pre-committed quantities are a significant proportion of total consumption making up 78.8% for beef, 81.9% for pork, and 54.4% for poultry. The preferred IGAI ς (s) model yielded very similar estimates of pre-committed quantities (13.830 pounds of beef, 11.813 pounds of pork, and 12.617 pounds of poultry). Piggott and Marsh (2004), using the same quarterly data source but over a different period (from 1982 to 1999), estimated a GAI demand system (specifying quantities as a function of prices and expenditures) and reported pre-committed values of 15.170 pounds for beef, 7.294 pounds for pork, and 10.383 pounds for poultry. While the values from the IGAI and the GAI models are not identical, they were estimated over different time periods and are very close in magnitude. In all the inverse demand results provide strong statistical support for specification of the IGAI model in explaining price formation and offer further evidence for the existence of pre-committed quantities in U.S. meat demand.

Conclusion

This article investigates the case in which consumers pre-commit to goods in a market characterized by fixed supply. We demonstrate how to theoretically and empirically measure pre-committed quantities by incorporating translating in consumer distance functions. Translating the distance function completes the quadrality of dual functions that also includes the translated utility, indirect utility, and expenditure functions. The translated distance functions provides the analytical framework with which to specify translated inverse demand systems that nest most known functional forms.

Translating procedures are important when incorporating pre-committed quantities in the inverse demand system. Furthermore, translating parameters can be augmented to be functions of demand shift variables to account for other factors impacting demand other than prices and income (e.g., seasonality, advertising, health or food safety information) into distance functions to better understand price formation. Building upon the work of Deaton and Meullbauer on the almost ideal demand system, Eales and Unnevehr on the inverse almost ideal (IAI) demand system and of Pollak and Wales on translating dual functions, a new class of inverse demand systems is defined, including an inverse generalized almost ideal (IGAI) demand system. General results for marginal effects and price flexibilities are also derived. Further research can use the framework developed in this paper to examine alternative functional forms and for even more general inverse demand models.

For an empirical application the IAI and IGAI models are estimated using quarterly U.S. meat consumption data. The IAI model is rejected in favor of the generalized model supporting the idea of pre-committed quantities in beef, pork, and poultry. The goodness of fit statistics showed dramatic improvement for the IGAI over the IAI model; especially for pork. As an illustration of linear translating the pre-committed quantities are modified to depend linearly upon seasonal dummy variables. The IGAI model is rejected against the alternative model that includes linear translation IGAI $\varsigma(s)$ indicating the importance of seasonality. The own-flexibilities for beef (-0.607) and poultry (-0.606) are estimated to inflexible whereas the own-flexibility for pork (-1.567) is flexible. Most of the cross-flexibilities are negative, indicating the meats are gross-substitutes, with exceptions for the cross-flexibilities for beef and poultry prices with pork quantities being positive indicating gross-complements. In all the empirical results provide strong statistical support for specification of the IGAI model in explaining price formation, offer further evidence for the existence of pre-committed quantities in U.S. meat demand, and demonstrate the empirical applicability of generalized inverse demand systems from translated consumer distance functions.

Table 1: Summary Statistics of Annual Data, 1954-2007

Variables	Average	Std. Dev.	Minimum	Maximum
FAFH Expenditure ($/capita)	673.363	543.382	94.706	1,840.180
FAH Expenditure ($/capita)	871.985	519.071	281.831	1,932.630
Alcoholic Beverages ($/capita)	215.653	140.763	56.043	538.209
Total Expenditures ($/capita)	1,761.000	1,201.320	435.072	4,311.020
FAFH Price Index	92.557	60.472	21.900	206.659
FAH Price Index	94.238	56.995	29.500	201.245
Alcoholic Beverages Price Index	100.786	55.166	40.500	207.026
Share FAFH	0.336	0.074	0.217	0.428
Share FAH	0.537	0.066	0.447	0.654
Share Alcoholic Beverages	0.127	0.010	0.110	0.146

Sources

Food Expenditures are from USDA, Economic Research Service

http://www.ers.usda.gov/briefing/CPIFoodAndExpenditures/Data/table1.htm

Price Data are from US Bureau of Labor Statistics

http://data.bls.gov/PDQ/outside.jsp?survey=cu

Population Data are from US Census Bear

http://www.census.gov/popest/archives/1990s/popclockest.txt

http://www.census.gov/popest/states/NST-ann-est2007.html

Table 2. Estimated Coefficients for the Inverse Almost Ideal (IAI) and Inverse Generalized Almost Ideal (IGAI) Model

	IAI Model			IGAI model		
	N-R matrix	D-R matrix	F-R matrix	N-R matrix	D-R matrix	F-R matrix
α_0	-277.506	412.968	342.243	1341.703	14442.860	7866.014
	(302.200)	(586.600)	(474.300)	(1560.100)	(15436.600)	(8039.200)
α_1	32.135	-25.092	-22.393	-57.624	-254.854	-172.833
	(25.425)	(26.400)	(22.848)	(44.754)	(179.900)	(118.800)
α_2	-20.514	30.502	29.417	40.757	174.376	121.529
	(14.807)	(30.364)	(28.701)	(32.241)	(126.100)	(82.755)
γ_{11}	-3.335	1.815	1.779	2.731*	4.719*	4.023*
	(1.835)	(1.156)	(1.056)	(0.970)	(1.579)	(1.388)
γ_{12}	2.167*	-2.059	-2.169	-1.925*	-3.241*	-2.839*
	(0.965)	(1.287)	(1.278)	(0.722)	(1.120)	(0.941)
γ_{22}	-1.345*	2.422	2.743	1.436*	2.306*	2.075*
	(0.583)	(1.522)	(1.618)	(0.541)	(0.851)	(0.687)
β_1	0.115*	0.061*	0.066*	0.043*	0.018*	0.022*
	(0.036)	(0.027)	(0.028)	(0.017)	(0.007)	(0.008)
β_2	-0.076*	-0.073*	-0.084*	-0.030*	-0.012*	-0.015*
	(0.031)	(0.033)	(0.036)	(0.011)	(0.004)	(0.005)
c_1	--	--	--	3.903*	3.818*	3.590*
	--	--	--	(0.088)	(0.248)	(0.339)
c_2	--	--	--	5.370*	5.010*	2.722
	--	--	--	(0.500)	(0.996)	(2.066)
c_3	--	--	--	1.200*	1.129*	1.052*
	--	--	--	(0.029)	(0.046)	(0.064)
ρ	--	0.966*	--	--	0.965*	--
	--	(0.011)	--	--	(0.010)	--
ρ_{11}	--	--	0.994*	--	--	1.134*
	--	--	(0.057)	--	--	(0.068)
ρ_{12}	--	--	0.850*	--	--	0.687*
	--	--	(0.105)	--	--	(0.127)
ρ_{21}	--	--	0.048	--	--	0.252*
	--	--	(0.075)	--	--	(0.088)
ρ_{22}	--	--	-0.067	--	--	-0.188*
	--	--	(0.080)	--	--	(0.093)
LL	397.371	503.218	504.895	404.715	501.255	504.447
R^2 FAFH	0.991	0.999	0.999	0.987	0.999	0.999
R^2_FAH	0.984	0.997	0.997	0.989	0.996	0.996
DW_FAFH	0.417	1.302	1.273	0.288	1.241	1.363
DW_FAH	0.377	1.403	1.329	0.483	1.407	1.371

Table 3: Hypothesis Tests of Alternative Models

	H_0: IAI Ha: IGAI	H_0: IGAI ς (s) Ha: IGAI
Statistic	415.592*	315.208*
df	3	9
$\chi_{0.01,df}$	11.35	21.67

Notes: $c_i = \varsigma_i(s)$ represents a function with that includes an intercept term and seasonal dummy variables using linear translation. df denotes degrees of freedom. Reported asymptotic chi-square test statistics are adjusted likelihood ratio tests calculated by adjusting the usual LR test statistic $LR=2*(LL_U-LL_R)$ according to following: $LR_s=[(M*T-k_u)/M*T]*LR$ as suggested by Bewley (1986) where LL_U and LL_R are the maximized likelihood value in the unrestricted and restricted models; M is the number of estimated equations; T is the sample size, k_u is the estimated number of parameters in the unrestricted model. A * denotes a significant test statistic at the 5% level.

Table 4. Estimated Coefficients for the Inverse Almost Ideal (IAI) and Inverse Generalized Almost Ideal (IGAI) Model

	IAI Model			IGAI model		
	N-R matrix	D-R matrix	F-R matrix	N-R matrix	D-R matrix	F-R matrix
f11	0.043	-0.390	-0.382	-0.247	-0.324	-0.325
f12	-0.452	-0.215	-0.219	-0.434	-0.431	-0.435
f13	-0.175	-0.142	-0.129	-0.123	-0.127	-0.107
f21	-0.445	-0.304	-0.326	-0.378	-0.335	-0.345
f22	-0.708	-0.841	-0.829	-0.650	-0.656	-0.662
f23	-0.041	-0.044	-0.055	-0.080	-0.082	-0.072
f31	-0.601	-0.372	-0.311	-0.405	-0.367	-0.323
f32	-0.196	-0.017	-0.032	-0.313	-0.308	-0.272
f33	-0.457	-0.473	-0.467	-0.338	-0.323	-0.412
f1	-0.584	-0.746	-0.731	-0.803	-0.881	-0.868
f2	-1.194	-1.188	-1.211	-1.107	-1.073	-1.080
f3	-1.253	-0.862	-0.810	-1.056	-0.998	-1.006

Note: f_{ij} represent the uncompensated price flexibilities of the ith good with respect to the jth quantity, and f_i is the scale flexibility of the ith good, where i,j = FAFH, FAH, AB. Estimates shown are sample means of flexibilities computed at every data point using predicted shares.

Table 3: Hypothesis Tests of Alternative Models

	H_0: IAI Ha: IGAI	H_0: IGAI ς (s) Ha: IGAI
Statistic	415.592*	315.208*
df	3	9
$\chi_{0.01,df}$	11.35	21.67

Notes: $c_i = \varsigma_i(s)$ represents a function with that includes an intercept term and seasonal dummy variables using linear translation. df denotes degrees of freedom. Reported asymptotic chi-square test statistics are adjusted likelihood ratio tests calculated by adjusting the usual LR test statistic LR=2*(LL_U-LL_R) according to following: LR_s= [(M*T- k_u)/M*T]*LR as suggested by Bewley (1986) where LL_U and LL_R are the maximized likelihood value in the unrestricted and restricted models; M is the number of estimated equations; T is the sample size, k_u is the estimated number of parameters in the unrestricted model. A * denotes a significant test statistic at the 5% level.

Table 4: Estimated Price and Scale Flexibilities

	IAI	IGAI	IGAI ς (s)
Price Flexibilities			
f_{bb}	-0.596	-0.650	-0.607
f_{bp}	-0.199	-0.129	0.136
f_{by}	-0.172	-0.141	-0.130
f_{pb}	-0.456	-0.453	-0.552
f_{pp}	-0.607	-0.912	-1.567
f_{py}	-0.110	0.002	-0.041
f_{yb}	-0.369	-0.224	-0.186
f_{yp}	-0.059	0.208	0.478
f_{yy}	-0.405	-0.642	-0.606
Scale Flexibilities			
f_b	-0.968	-0.919	-0.602
f_p	-1.173	-1.364	-2.161
f_y	-0.834	-0.658	-0.315

Notes: f_{ij} represent the uncompensated price flexibilities of demand for the i_{th} good with respect to the j_{th} price, and f_i are scale flexibilities expenditure for the i_{th} good, where i, j =b for beef, p for pork, and c for poultry. Estimates shown are calculated at the sample means..

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