Economics of Trade-off Between Urea Nitrogen and Poultry Litter for Rice Production

Ramu Govindasamy, Mark J. Cochran,
David M. Miller and Richard J. Norman*

Abstract

This paper identifies optimal combinations of nitrogen in the form of urea, fresh litter and composted litter for rice production. Traditional cost minimization techniques using data from experimental results conducted at three sites in Arkansas during 1991 have been employed. Comparisons between different scenarios indicate that the trade-off between the use of poultry litter and urea nitrogen depends on such factors as soil fertility, the yield response to litter application and the relative prices of nitrogen and litter. The use of litter is more economical at high target yields than at low target yields.

Key Words: poultry litter, nitrogen, rice, programming

Introduction

The growth of the poultry industry in Arkansas has exploded in the past decade with an aim to meet the growing demand for poultry meat and egg products. In 1992, approximately 25 million chickens, 25 million turkeys and 1.02 billion broilers were produced in the state. As a result, approximately 1.05 million tons of poultry litter are produced per year.[2] Most of this poultry litter is used as fertilizer on nearby pasture lands consisting of bermudagrass and tall fescue (Buchberger; Buchberger et al.). The fertilizer contributions of poultry litter also greatly enhance the profitability of the beef cattle industry in the area. High concentrations of poultry litter may result in litter applications to the pasture that exceed nutrient requirements. This leads to the potential contamination of groundwater as well as surface water in the nearby areas (Steele et al.; and Madison and Brunett). One proposed solution to this problem is the export of litter to areas which concentrate on row crop production (Winrock International).[3] The litter has been found to restore productivity of rice lands due to precision grading (Govindasamy et al.). It is thought that precision grading can reduce productivity by introducing infertile sub-soil material into the root zone. For the export of the litter to be successful, there is a need to assess the economic value of poultry litter and its optimal input usage for crop production.

Although litter is a valuable agronomic resource for crop production, its optimal use depends on the crop response to litter application, the use of other inputs such as nitrogen and phosphorous, the nutrient content of the litter, the prices of the output and litter, and the fertility of the land. Malone estimated the total annual United States broiler nutrient production at 227,368, 249,050, and 157,048 metric tons of N, P_2O_5, and K_2O, respectively. However, he indicates that there are substantial

---

*R. Govindasamy is a research associate in Agricultural Economics, M.J. Cochran is a professor of Agricultural Economics, D. M. Miller is an associate professor of Agronomy. All are at the University of Arkansas at Fayetteville. R. J. Norman is at the Rice Research and Extension Center, Stuttgart, Arkansas.

J Agr. and Applied Econ. 26 (2), December, 1994: 552-564

variations in both litter production rates and composition values. Poultry litter in Arkansas has been measured with the composition displayed in Table 1 (Danforth et al.). There is growing interest in the feasibility of using poultry litter and poultry litter compost in the production of row crops such as rice, cotton, and soybeans. Rainey et al. and Danforth et al. have examined the derived demand for litter as a soil amendment in rice and cotton. The results indicate that, in general, rice has a higher value of marginal product than cotton for litter application. One component of successful litter application in rice production will be the determination of optimal rates in conjunction with conventional nitrogen applications.

This paper examines the optimal trade-off between urea nitrogen and poultry litter application for rice production using a traditional cost minimization technique with the data from experimental results conducted at three sites in Arkansas during 1991. The analysis examines the optimal use of urea nitrogen, poultry litter and poultry litter compost, given that litter and compost are imperfect substitutes for urea and perfect substitutes for one another. Section 1 describes the methodology used to determine the optimal trade-off between nitrogen and poultry litter. Section 2 describes the experimental design and the data, while section 3 discusses the results and section 4 provides the conclusions of the study.

Methodology

Assume the three factors of production are nitrogen (x_1), poultry litter (x_2) and poultry litter compost (x_3) with input prices at w_1, w_2 and w_3, respectively. Let y represent the target level of rice output. Yield response data can be used to fit a curvilinear relationship between rice production and the use of nitrogen, litter and composted litter. The quadratic rice production function can be represented as

$$f(x_1, x_2, x_3) = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_1^2 + a_5 x_2^2 + a_6 x_3^2 + a_7 x_1 x_2 + a_8 x_1 x_3 + a_9 x_2 x_3 \tag{1}$$

The quadratic production function was chosen primarily for two reasons. First, it is characterized by the diminishing marginal productivity and second, marginal rate of technical substitution (*MRTS*) can be derived directly from marginal products of inputs. The least cost combination of inputs (Varian) that produces a given level of output can be solved for as

$$\min_{x_1, x_2, x_3} \quad c = w_1 x_1 + w_2 x_2 + w_3 x_3 \tag{2}$$

subject to the output constraint

$$f(x_1, x_2, x_3) \geq y\,. \tag{3}$$

That is,

$$\min_{x_1, x_2, x_3} \quad c = w_1 x_1 + w_2 x_2 + w_3 x_3 + \lambda[y - f(x_1, x_2, x_3)]$$

where λ is the Lagrangian multiplier. The cost function can be represented as

$$c(w_1, w_2, w_3, y) \tag{4}$$

which measures the minimal cost of producing y units of output, given the factor prices at (w_1, w_2, w_3). Suppose that we want to plot all of the combinations of inputs, i.e., combinations of x_1, x_2 and x_3, that have some given level of cost, c. This can be written as

$$w_1 x_1 + w_2 x_2 + w_3 x_3 = c \tag{5}$$

Rearranging (5) yields

$$x_1 = (c/w_1) - (w_2/w_1)\, x_2 - (w_3/w_1)\, x_3 \tag{6}$$

The combinations of x_1 and x_2, for a given level of x_3 can be plotted as a straight line with an intercept of $(c/w_1) - (w_3/w_1)\, x_3$ and a slope of (w_2/w_1). Similarly, the combinations of x_1 and x_3, for a given level of x_2 can be plotted as a straight line with an intercept of $(c/w_1) - (w_2/w_1)\, x_2$ and a slope of (w_3/w_1). The first-order conditions for cost minimization are:

$$\partial c/\partial x_1 = w_1 - \lambda a_1 - 2\lambda a_4 x_1 - \lambda a_7 x_2 - \lambda a_8 x_3 = 0 \tag{7}$$

$$\partial c/\partial x_2 = w_2 - \lambda a_2 - 2\lambda a_5 x_2 - \lambda a_7 x_1 - \lambda a_9 x_3 = 0 \tag{8}$$

$$\partial c/\partial x_3 = w_3 - \lambda a_3 - 2\lambda a_6 x_3 - \lambda a_8 x_1 - \lambda a_9 x_2 = 0 \tag{9}$$

Table 1. Litter Composition (percent)[1]

	1991			1992		
	N	P	K	N	P	K
Fresh Litter	4.68	2.61	3.64	4.28	1 81	2.73
Litter Compost	3.33	2.90	4.05	3 38	2 59	3.87

[1] On average, 5-6 flocks were raised between cleaning out the litter.

$$\partial c/\partial\lambda = [y - f(x_1, x_2, x_3)] \quad = 0, \tag{10}$$

$$x_1, x_2, x_3 \geq 0. \tag{11}$$

The four equations (7), (8), (9) and (10) can be used to solve for the unknowns as (Chiang)

$$x_i^* = f_i(w_1, w_2, w_3, a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, y) \quad i=1,2,3 \tag{12}$$

The solutions x_i^* provide the cost minimizing input levels, given the level of output.

Now consider the MRTS between the two inputs, nitrogen and poultry litter. The slope of the isoquant between nitrogen and poultry litter is called MRTS. From equation (1), the marginal product of nitrogen can be derived as

$$MP_{x1} = a_1 + 2\,a_4\,x_1 + a_7\,x_2 + a_8\,x_3 \tag{13}$$

The marginal product for fresh litter can be derived as

$$MP_{x2} = a_2 + 2\,a_5\,x_2 + a_7\,x_1 + a_9\,x_3 \tag{14}$$

The marginal product for composted litter can be derived as

$$MP_{x3} = a_3 + 2\,a_6\,x_3 + a_8\,x_1 + a_9\,x_2 \tag{15}$$

As can be seen from equations (13), (14), and (15) the marginal product of one input is a function of not only its own input level but also the level of other inputs. The MRTS between nitrogen and fresh litter can be derived as

$$MRTS_{x1x2} = MP_{x1}/MP_{x2} \tag{16}$$

Similarly, the MRTS between nitrogen and composted litter can be derived as

$$MRTS_{x1x3} = MP_{x1}/MP_{x3} \tag{17}$$

The Data

The data were collected from experiments conducted on rice during 1991 at three locations in Arkansas; Rice Research Experimental Station (RREC) at Stuttgart and Jackson county on graded soil and Lawrence county on ungraded soil. The soils had been precision graded for better water management and tillage operations. At the RREC rice variety Allen was planted on 23 May on a Crowley silt loam that had been put to grade in 1989 and harvest was on 18 September. At the Jackson County site, rice variety Tebonnet was planted on 29 May on a Foley silt loam soil that has been put to grade in 1989. At the Lawrence county site, rice variety Tebonnet was planted on 19 May on an ungraded Foley-Calhoun complex. The Jackson County site was harvested on 7 October while the Lawrence County site was harvested on 27 September. The experiments were conducted with different rates of nitrogen, phosphorus, fresh litter and composted litter to measure the yield response of rice. The experiments were designed in such a way that either fresh litter or composted litter is applied for each treatment. These experiments were conducted to reflect the actual litter application conditions of the farmers.

Results

The results are discussed in three subsections: 1) estimation of the production function, 2) marginal rate of technical substitution between the nitrogen and fresh litter inputs as well as between nitrogen and composted litter inputs, and 3) the non-linear programming model in terms of the base scenario and the price sensitivity scenario.

Estimation of the Production Function

A quadratic production function was estimated using the experimental data on rice production as

$$\begin{array}{llll} y = & 4353.765 + 0.952141\,FL - 0.000072\,FL^2 & & \\ & (13.94) \qquad (11.10) \qquad (-7.08) & & \\ & + 0.833094\,CL - 0.000058\,CL^2 & & \\ & (7.59) \qquad (-4.53) & & \\ & + 32.71769\,N - 0.0846\,N^2 - 0.00342\,N{\cdot}FL & & \\ & (7.98) \qquad (-2.65) \qquad (-7.43) & & \\ & - 0.00329\,N{\cdot}\mathrm{CL} & & \\ & (-5.15) & & \\ & - 769.182\,L2 - 2944.120\,L3 & & \\ & (-12.78) \qquad (-4.04) & & (18) \end{array}$$

where y is the yield of rice in pounds per acre, FL is the fresh litter applied in pounds per acre, CL is the composted litter applied in pounds per acre, N is the nitrogen applied in pounds per acre, $L1$ is the location dummy variable for Lawrence county ungraded soil, $L2$ is the location dummy variable for RREC graded soil, and $L3$ is the location dummy variable for Jackson county graded soil. The number of observations used in the analysis is 255. The estimated regression model has an R^2 of 0.67 (the adjusted R^2 was 0.66), which implies that 67 percent of the variation in rice yield was explained by equation (18) (Johnston). A number of other specifications of the model such as linear dependent variables, interaction between nitrogen and phosphorous, slope adjusters on each of the locations and quadratic specification of slope adjusters were also examined. These alternative specifications not only resulted in non-significant variables but also the R^2 was lower than the current specification of the model. It is unusual for phosphorus to be applied to rice, and our intention is to identify a least cost combination for conditions common in Arkansas, where soils tend to be high in phosphorus. The Rice Production Handbook (United States Department of Agriculture) further indicates that rice seldom responds to fertilizer phosphorus. Although we estimated equations that both included and excluded phosphorus as an explicit variable, the equation presented here excludes the phosphorus variable. The equation presented here we believe conforms to the soil conditions found in most Arkansas rice-producing areas where soils tend to have enough phosphorus to make additional fertilizer application unnecessary. The F-statistic for the entire model was 78.25 which is significant at the 1 percent level. The figures in parenthesis are the t-values. The parameter estimates associated with the location dummy variables reflect differences in soil fertility, management, weather, etc., between the indicated sites and the ungraded test at Lawrence. The negative term associated with the interaction of litter and nitrogen can be attributed to the fact that nitrogen is one of the major components of the litter and therefore litter and nitrogen act as substitutes. The yield maximizing levels of the inputs, fresh litter and composted litter can be derived from equation (18). Since there are interactions between the urea nitrogen and the litter, yield-maximizing levels of each input are dependent upon the level of other inputs applied. Consider the first derivative of equation (18) with respect to inputs fresh litter and composted litter.

$$\partial y/\partial \mathrm{FL} = FL_{max} = 0.952141 - 0.000144\,FL - 0.00342\,N \quad (19)$$

$$\partial y/\partial \mathrm{CL} = CL_{max} = 0.833094 - 0.00012\,CL - 0.00329\,N \quad (20)$$

By equating equations (19) and (20) to zero, and evaluating them at the mean nitrogen application of 100 pounds per acre, the yield maximizing level of fresh litter and composted litter can derived as 4358 pounds and 4201 pounds per acre.

Marginal Rate of Technical Substitution and the Isoquants

Consider the marginal rate of technical substitution (*MRTS*) between nitrogen and poultry litter. Tables 2, 3, and 4 provide the *MRTS* between nitrogen and fresh litter and between nitrogen and composted litter for three locations in Arkansas. In general, the *MRTS* between nitrogen and fresh litter was lower than the *MRTS* between nitrogen and composted litter. This indicates that rice yields are more responsive to fresh litter than composted litter. Table 2 depicts all combinations of nitrogen and fresh litter and nitrogen

Table 2. Marginal Rate of Technical Substitution (lbs/acre) Between Nitrogen and Poultry Litter, RREC: Graded soil

Nitrogen	Yield of Rice in pounds per acre							
	5000	5000	6000	6000	6500	6500	7000	7000
	Poultry Litter							
	Fresh	Compost	Fresh	Compost	Fresh	Compost	Fresh	Compost
0	38.07	43.47	45.74	53.56	63.11	107.10	IF	IF[a]
5	37.26	42.64	44.12	51.61	57.89	84.97	IF	IF
10	36.49	41.84	42.64	49.86	53.89	73.83	IF	IF
15	35.74	41.08	41.28	48.27	50.66	66.70	IF	IF
20	35.02	40.34	40.01	46.81	47.96	61.56	IF	IF
25	34.32	39.62	38.83	45.46	45.63	57.59	IF	IF
30	33.64	38.93	37.72	44.20	43.60	54.36	IF	IF
35	32.98	38.25	36.67	43.02	41.78	51.65	116.89	IF
40	32.34	37.60	35.67	41.91	40.14	49.31	76.53	IF
45	31.71	36.95	34.72	40.86	38.63	47.24	62.78	IF
50	31.09	36.33	33.80	39.85	37.24	45.40	54.98	IF
55	30.65	35.91	32.92	38.88	35.93	43.71	49.65	IF
60	IF	IF	32.07	37.96	34.71	42.17	45.63	IF
65	IF	IF	31.24	37.06	33.55	40.73	42.39	IF
70	IF	IF	30.44	36.19	32.44	39.38	39.67	187.89
75	IF	IF	29.65	35.35	31.37	38.11	37.30	77.61
80	IF	IF	28.88	34.53	30.34	36.89	35.19	61.07
85	IF	IF	28.13	33.72	29.35	35.73	33.26	52.62
90	IF	IF	27.39	32.93	28.37	34.60	31.48	46.99
95	IF	IF	26.65	32.15	27.42	33.51	29.79	42.71
100	IF	IF	25.92	31.38	26.48	32.45	28.18	39.18
105	IF	IF	IF	IF	25.56	31.40	26.62	36.11
110	IF	IF	IF	IF	24.63	30.37	25.10	33.32
115	IF	IF	IF	IF	23.72	29.34	23.58	30.69
120	IF	IF	IF	IF	22.80	28.33	22.06	28.13
125	IF	IF	IF	IF	21.88	27.31	20.52	25.55
130	IF	IF	IF	IF	20.94	26.28	18.94	22.89
135	IF	IF	IF	IF	20.00	25.25	17.30	20.04
140	IF	IF	IF	IF	19.04	24.20	15.58	16.87
145	IF	IF	IF	IF	IF	IF	13.73	13.18
150	IF	IF	IF	IF	IF	IF	11.73	8.63
155	IF	IF	IF	IF	IF	IF	9.51	2.46
160	IF	IF	IF	IF	IF	IF	7.00	IF
165	IF	IF	IF	IF	IF	IF	4.07	IF

[a] IF represents the points where MRTS is not available due to the specification of the production function and the form of estimated equations (i.e., response data is interpreted that these rice yields are unattainable on these soils). In other words, it is not possible to solve for the level of litter use for a given level of nitrogen use, given the target rice yield and estimated production from the experimental data.

Table 3. Marginal Rate of Technical Substitution (lbs/acre) Between Nitrogen and Poultry Litter, JACKSON: Graded soil

Yield of Rice in pounds per acre

Nitro-gen	3500	3500	4000	4000	4500	4400	5000	4900[b]
				Poultry Litter				
	Fresh	Compost	Fresh	Compost	Fresh	Compost	Fresh	Compost
0	41.83	48.10	48.38	57.75	90.95	228.45	IF	IF[a]
5	40.67	46.84	46.38	55.12	74.07	106.49	IF	IF
10	39.58	45.66	44.60	52.84	64.80	84.10	IF	IF
15	38.55	44.55	42.98	50.83	58.62	72.95	IF	IF
20	37.58	43.50	41.51	49.04	54.07	65.85	IF	IF
25	36.65	42.51	40.15	47.41	50.48	60.73	IF	IF
30	35.76	41.56	38.89	45.91	47.54	56.78	IF	IF
35	34.91	40.65	37.71	44.53	45.04	53.57	IF	IF
40	34.09	39.78	36.59	43.25	42.87	50.87	IF	IF
45	33.30	38.94	35.54	42.04	40.95	48.53	IF	IF
50	32.53	38.13	34.53	40.90	39.21	46.47	IF	IF
55	31.79	37.35	33.57	39.82	37.62	44.63	IF	IF
60	31.06	36.58	32.64	38.78	36.16	42.94	93.33	IF
65	30.35	35.84	31.75	37.79	34.79	41.39	64.58	IF
70	29.66	35.11	30.88	36.84	33.50	39.95	53.47	IF
75	28.97	34.40	30.04	35.91	32.27	38.59	46.81	IF
80	28.30	33.70	29.21	35.01	31.10	37.30	42.08	IF
85	IF	IF	28.40	34.13	29.97	36.07	38.37	155.60
90	IF	IF	27.61	33.28	28.88	34.88	35.27	67.63
95	IF	IF	26.82	32.43	27.81	33.74	32.56	53.15
100	IF	IF	26.05	31.60	26.77	32.62	30.10	45.38
105	IF	IF	25.28	30.78	25.73	31.53	27.80	39.92
110	IF	IF	24.51	29.97	24.71	30.45	25.60	35.53
115	IF	IF	IF	IF	23.69	29.38	23.44	31.65
120	IF	IF	IF	IF	22.68	28.32	21.26	27.98
125	IF	IF	IF	IF	21.65	27.26	19.02	24.28
'30	IF	IF	IF	IF	20.61	26.19	16.66	20.31
35	IF	IF	IF	IF	19.56	25.11	14.10	15.70
40	IF	IF	IF	IF	18.48	22.89	11.24	9.82
45	IF	IF	IF	IF	17.37	IF	7.90	1.00
50	IF	IF	IF	IF	16.22	IF	3.79	IF
55	IF	IF	IF	IF	15.03	IF	IF	IF
60	IF	IF	IF	IF	13.78	IF	IF	IF
65	IF	IF	IF	IF	IF	IF	IF	IF

IF represents the points where MRTS is not available due to the specification of the production function and the form of estimated equations (i.e., response data is interpreted that these rice yields are unattainable on these soils). In other words, it is not possible to solve for the level of litter use for a given level of nitrogen use, given the target rice yield and estimated production from the experimental data.

There is a difference in maximum attainable yield levels between fresh litter and composted litter due to the nature of the experimental data and the specification of the model. (e.g., 5000 pounds of rice yield is not attainable with any amount of nitrogen and composted litter at Jackson.)

Table 4. Marginal Rate of Technical Substitution (lbs/acre) Between Nitrogen and Poultry Litter, LAWRENCE: Ungraded soil

	Yield of Rice in pounds per acre							
	6000	6000	7000	7000	7500	7300	8000	7800[b]
Nitrogen	Poultry Litter							
	Fresh	Compost	Fresh	Compost	Fresh	Compost	Fresh	Compost
0	38.99	44.56	49.66	59.98	152.29	109.88	IF	IF[a]
5	38.10	43.64	47.46	56.93	91.27	86.07	IF	IF
10	37.25	42.76	45.51	54.35	73.68	74.44	IF	IF
15	36.44	41.92	43.77	52.10	64.22	67.10	IF	IF
20	35.66	41.11	42.20	50.12	57.97	61.85	IF	IF
25	34.91	40.33	40.75	48.34	53.39	57 80	IF	IF
30	34.18	39.58	39.42	46.72	49.79	54.53	IF	IF
35	33.47	38.85	38.17	45.24	46.84	51.78	IF	IF
40	32.79	38.14	37.01	43.87	44.33	49.42	IF	IF
45	32.12	37.45	35.90	42.59	42.15	47.34	IF	IF
50	31.47	36.78	34.85	41.38	40.22	45.47	IF	IF
55	30.83	36.13	33.85	40.24	38.48	43.78	IF	IF
60	IF	IF	32.89	39.16	36.88	42.23	IF	IF
65	IF	IF	31.97	38.12	35.40	40.78	IF	IF
70	IF	IF	31.07	37.12	34.02	39.43	158.88	IF
75	IF	IF	30.20	36.16	32.71	38.14	67.82	83.09
80	IF	IF	29.35	35 23	31.47	36.92	52.75	62.96
85	IF	IF	28.52	34.32	30.27	35.75	44.90	53.61
90	IF	IF	27.70	33.43	29.12	34 62	39.57	47.59
95	IF	IF	26.90	32.56	28.00	33.53	35.46	43.10
100	IF	IF	26.10	31.70	26.90	32.46	32.01	39.45
105	IF	IF	25.31	30.86	25.82	31.41	28.94	36.29
110	IF	IF	24.53	30.02	24.75	30.37	26.08	33.43
115	IF	IF	23.75	29.18	23.68	29.35	23.30	30.74
120	IF	IF	IF	IF	22.62	28.33	20.50	28.12
125	IF	IF	IF	IF	21.54	27.30	17.56	25.49
130	IF	IF	IF	IF	20.46	26.28	14.34	22.77
135	IF	IF	IF	IF	19 35	25.24	10.64	19.84
140	IF	IF	IF	IF	17.04	24.18	6.07	16.58
145	IF	IF	IF	IF	15.83	IF	IF	12.75
150	IF	IF	IF	IF	14.56	IF	IF	7.95
155	IF	IF	IF	IF	13.22	IF	IF	1.32
160	IF	IF	IF	IF	11.79	IF	IF	IF
165	IF	IF	IF	IF	10.27	IF	IF	IF

[a] IF represents the points where MRTS is not available due to the specification of the production function and the form of estimated equations (i.e , response data is interpreted that these rice yields are unattainable on these soils). In other words, it is not possible to solve for the level of litter use for a given level of nitrogen use, given the target rice yield and estimated production from the experimental data.

[b] There is a difference in maximum attainable yield levels between fresh litter and composted litter due to the nature of the experimental data and the specification of the model. (e.g., 8000 pounds of rice yield is not attainable with any amount of nitrogen and composted litter at Lawrence.)

and composted litter that can produce 5000, 6000, 6500, and 7000 pounds of rice per acre. The table indicates that as more nitrogen is applied, the litter replaced by one more pound of nitrogen decreases. Due to the nature of the quadratic production function and the experimental results at different locations, some yield levels are un-attainable with specific combinations of nitrogen and litter. As the target rice yield increases, the amount of fresh or composted litter replaced by a pound of nitrogen also increases. If we compare Tables 2, 3, and 4 across locations, the MRTS between nitrogen and litter varies depending on the target rice yields. Consider the MRTS for a target yield of 5000 pounds of rice per acre in Table 2. When the nitrogen application is 0 pound per acre, each pound of nitrogen can be replaced by 38 pounds of fresh litter to achieve a target yield of 5000 pounds. When the nitrogen application increases to 55 pounds per acre, each pound of nitrogen can be replaced by about 31 pounds of fresh litter. That is, as the level of nitrogen application increases, the number of pounds of fresh litter replacing each pound of nitrogen decreases. Consider a target yield of 6500 pounds of rice per acre in Table 2. When the nitrogen application level is 0, each pound of nitrogen can be replaced by approximately 63 pounds of fresh litter to attain a target yield of 6500 pounds of rice per acre. The significant yield differences across locations also causes differences in MRTS across locations.

Non-linear Programming Model

A non-linear programming model (Brooke et al.) was constructed using GAMS to minimize the cost of the inputs subject to the quadratic production function.[4] The non-negativity constraints on the use of inputs were also introduced. Two scenarios, base scenario and price sensitivity scenario, were analyzed to capture the effects of changes in the input prices.

Base Scenario

The base scenario has results from three locations where the experiments were conducted; RREC graded soil, Jackson graded soil, and Lawrence ungraded soil. The cost minimizing inputs at each of the locations vary depending on the intrinsic nature of the soils such as the soil fertility level, and yield response of rice to litter. The base scenario assumes a cost of $0.21 per pound of nitrogen as urea, $40 per ton of fresh litter and $140 per ton of composted litter. The target yields ranged from 3500 pounds per acre to 8000 pounds per acre, depending on the location. Tables 5 and 6 present the results on optimal combinations of inputs to achieve cost minimization for a series of output levels.

First, consider the RREC graded soil with base prices. As can be seen from the tables, for a target yield of 6000 pounds of rice per acre, it is uneconomical to use fresh litter or composted litter as a soil amendment. The minimized input cost up to the target yield of 6000 pounds is the same in Table 5 and Table 6 because fresh litter as well as composted litter are uneconomical to use for rice production. For a target yield of 7000 pounds of rice, the cost minimizing inputs are 153 pounds of nitrogen per acre in addition to 1225 pounds of fresh litter or 155 pounds of nitrogen in addition to 1892 pounds of composted litter. The minimized cost for the target rice yield of 7000 pounds with the nitrogen and fresh litter combination is $56.6 whereas with the nitrogen and composted litter combination it is $164.9. This is due to the fact that the composted litter is almost four times more expensive than the fresh litter with approximately the same yield response.

The use of fresh litter or composted litter depends on the relative size of the input price ratio compared to that of the MRTS between the two inputs. The input price ratio (IPR) for the base prices represented as IPR1 between the nitrogen and fresh litter can be derived as

$$\begin{aligned} IPR1_{N,FL} &= (\text{price/lb of nitrogen})/(\text{price/lb of fresh litter}) \\ &= 0.21/.02 = 10.5 \qquad (21) \end{aligned}$$

Likewise, the IPR1 between nitrogen and composted litter can be derived as

$$\begin{aligned} IPR1_{N,CL} &= (\text{price/lb of nitrogen})/(\text{price/lb of comp. litter}) \\ &= 0.21/.07 = 3.00 \qquad (22) \end{aligned}$$

From the traditional theory of cost minimization, the cost minimizing input levels are given by the tangency between the isocost line and the isoquant of the two inputs. That is, the point where the slope of the isocost line equals the slope of the isoquant (which is *MRTS*). This can be represented as

$$IPR_{N,FL} = MRTS_{N,FL} \qquad (23)$$

Given that the $IPR1_{N,FL}$ is 10.5, as shown in Table 2, with a target yield of up to 6500 pounds per acre,

Table 5 Cost Minimizing Input Combinations of Nitrogen and Fresh Litter (pounds per acre).

	RREC GRADED SOIL				JACKSON GRADED SOIL				LAWRENCE UNGRADED SOIL			
TARGET RICE YIELD(lbs/ac)	5000	6000	7000	8000	4000	4500	5000	5500	6500	7000	8000	9000
Price of Nitrogen at $0.21 per pound and Fresh Litter $40 per ton												
Nitrogen	50.7	101.1	152.9	IF[a]	111.1	164.0	141.3	IF	83.7	115.2	135.2	IF
Fresh Litter	0.0	0.0	1224.5	IF	0.0	0.0	2037.4	IF	0.0	0.0	2460.5	IF
Minimized cost $/acre	10.7	21.2	56.6		23.3	34.4	70.4		17.6	24.2	77.6	IF
Price of Nitrogen at $0.2625 per pound and Fresh Litter $30 per ton												
Nitrogen	50.7	101.1	134.6	IF	111.1	144.4	128.6	IF	83.7	115.2	125.4	IF
Fresh Litter	0.0	0.0	1484.0	IF	0.0	297.9	2217.6	IF	0.0	0.0	2599.5	IF
Minimized cost $/acre	13.3	26.5	57.6		29.2	42.4	67.0		22.0	30.2	71.9	IF
Price of Nitrogen at $0.1575 per pound and Fresh Litter $50 per ton												
Nitrogen	50.7	101.1	161.3	IF	111.1	164.0	147.1	IF	83.7	115.2	139.7	IF
Fresh Litter	0.0	0.0	1153.2	IF	0.0	0.0	1988.0	IF	0.0	0.0	2422.4	IF
Minimized cost $/acre	8.0	15.9	54.2		17.5	25.8	72.9		13.2	18.1	82.6	IF

[a] IF represents the points where MRTS is not available due to the specification of the production function and the form of estimated equations (i.e., response data are interpreted that these rice yields are unattainable on these soils). In other words, it is not possible to solve for the level of litter use for a given level of nitrogen use, given the target rice yield and estimated production from the experimental data.

$MRTS_{N,FL}$ always stays above 10.5. As a result, nitrogen is more cost effective than the fresh and composted litter to attain the specified yield. That is, the use of fresh litter is economical only when the $MRTS_{N,FL}$ falls below 10.5 at the specified N application rate. From Table 2, in the case of a target yield of 7000 pounds of rice per acre, the *MRTS* falls below 10.5 when the nitrogen applied is between 150 and 155 pounds per acre. This implies that the cost minimizing N application rate lies between 150 and 155 pounds for a target yield of 7000 pounds as shown in Table 5. In the case of composted litter, given that the price ratio is 3, the MRTS falls below 3 when the nitrogen applied lies between 150 and 155 as shown in Table 2. This in turn implies that the cost minimizing N application rate also lies between 150 pounds and 155 pounds per acre as shown in Table 6 for a target yield of 7000 pounds per acre. It is not feasible to attain a yield of above 8000 pounds per acre at the RREC graded soil due to the specification of the production function, the estimated regression equation and the given inputs.

Second, consider the results on Jackson graded soil. This is the least productive soil among all the three experimental sites. The maximum attainable yield at this site is 5000 pounds of rice per acre with fresh litter and 4900 pounds of rice per acre with composted litter. There is a difference in maximum attainable yield levels between fresh litter and composted litter due to the inherent nature of the soil. Neither the fresh litter nor the composted litter enter as a cost minimizing input up to a target yield of 4500 pounds per acre. To attain a yield of 5000 pounds per acre, the cost minimizing input levels are 141 pounds of nitrogen per acre and 2037 pounds of fresh litter per acre. As can be seen from Table 3, for a target yield of 5000 pounds, the $\text{MRTS}_{N,FL}$ falls below 10.5 when the nitrogen application lies between 140 and 145. In the case of composted litter,

Table 6. Cost Minimizing Input Combinations of Nitrogen and Compost Litter (pounds per acre)

		RREC GRADED SOIL			JACKSON GRADED SOIL				LAWRENCE UNGRADED SOIL			
TARGET RICE YIELD(lbs/ac)	5000	6000	7000	8000	4000	4500	4900	5500	6000	7000	8000	9000
Price of Nitrogen at $0.21 per pound and Compost Litter $140 per ton												
Nitrogen	50.7	101.1	154.7	IF[a]	111.1	164.0	144.2	IF	59.5	115.2	IF	IF
Compost Litter	0.0	0.0	1892.0	IF	0.0	0.0	2463.6	IF	0.0	0.0	IF	IF
Minimized cost $/acre	10.7	21.2	164.9		23.3	34.4	202.7		12.5	24.2	IF	IF
Price of Nitrogen at $0.2625 per pound and Compost Litter $105 per ton												
Nitrogen	50.7	101.1	153.2	IF	111.1	164.0	143.1	IF	59.5	115.2	IF	IF
Compost Litter	0.0	0.0	1898.0	IF	0.0	0.0	2467.8	IF	0.0	0.0	IF	IF
Minimized cost $/acre	13.3	26.5	139.9		29.2	43.1	167.1		15.6	30.2	IF	IF
Price of Nitrogen at $0.1575 per pound and Compost Litter $175 per ton												
Nitrogen	50.7	101.1	155.5	IF	111.1	164.0	144.7	IF	59.5	115.2	IF	IF
Compost Litter	0.0	0.0	1890.1	IF	0.0	0.0	2462.3	IF	0.0	0.0	IF	IF
Minimized cost $/acre	8.0	15.9	189.9		17.5	25.8	238.2		9.4	18.1	IF	IF

[a] IF represents the points where MRTS is not available due to the specification of the production function and the form of estimate equations (i.e., response data are interpreted that these rice yields are unattainable on these soils). In other words, it is not possib to solve for the level of litter use for a given level of nitrogen use, given the target rice yield and estimated production from the experimental data

the cost minimizing input levels for a target yield of 4900 pounds of rice are 144.2 pounds of nitrogen and 2463.6 pounds of composted litter.

Third, consider the results on Lawrence ungraded soil. The Lawrence ungraded soil has the highest yield potential among the three sites. The maximum attainable yield at this site is about 8000 pounds of rice per acre. It is uneconomical to use either fresh litter or composted litter to attain a yield of up to 7000 pounds per acre. The cost minimizing input level to produce 8000 pounds of rice per acre are 135.2 pounds of nitrogen and 2461 pounds of fresh litter. This is explained by the $MRTS_{N,FL}$ in Table 4 falling below 10.5 when the nitrogen application lies between 135 and 140 pounds for a target yield of 8000 pounds per acre.

It should be noted that with these prices and the conventional assumption (United States Department of Agriculture) that only one-half of the nitrogen in the litter is available in the first year, the available nitrogen in the litter costs about $1/lb so any substitution for urea is greatly influenced by the other characteristics of the litter. Since litter and compost produce relatively similar yield responses and litter is considerably cheaper than compost, litter tends to be more cost effective.

Price Sensitivity Scenario

Price sensitivity scenario comprises two components; 1) a low litter price-high nitrogen price scenario and 2) a high litter price-low nitrogen price scenario. The low litter price-high nitrogen price scenario was conducted with a simultaneous increase in nitrogen price by 25 percent and a decrease in litter price by 25 percent (both fresh litter as well as composted litter). The high litter price-low nitrogen price scenario was conducted with a simultaneous decrease in nitrogen price by 25 percent and an increase in litter price by 25 percent.

First, consider the low litter price-high nitrogen price scenario. The cost minimizing input combination depends on the new input price ratio. The new input price ratio $IPR2_{N,FL}$ between nitrogen and fresh litter can be derived as

$$IPR2_{N,FI} = (\text{price/lb of nitrogen})/(\text{price/lb of fresh litter}) = 0.2626/.015 = 17.5 \qquad (24)$$

$IPR2_{N,FI}$ is 67 percent higher than $IPR1_{N,FL}$ which implies that the cost minimizing input combination will shift towards use of more litter than nitrogen compared to the base scenario. From the traditional theory of cost minimization, as $IPR_{N,FL}$ goes up, the $MRTS_{N,FL}$ will also go up to meet the criteria given in equation (23). In the case of RREC graded soil, the cost minimizing nitrogen application levels stay the same as that of the base scenario for target yields of 5000 pounds and 6000 pounds. This is because the $MRTS_{N,FL}$ at the cost-minimizing nitrogen application of 101.1 pounds, 25.92, is above the price ratio of 17.5. The same applies for the target yields of 4000 pounds rice yield on the Jackson graded soil, and 6500 and 7000 pounds on the Lawrence ungraded soil.

Consider the case of a target yield of 7000 pounds of rice on the RREC graded soil. The cost-minimizing input levels are nitrogen at 135 pounds per acre and fresh litter at 1484 pounds per acre. Compared to the base scenario, the nitrogen use has declined by about 18 pounds and the fresh litter use has increased by about 260 pounds. Given that the new price ratio is 17.5 from equation (24), the $MRTS_{N,FL}$ shown in Table 2 for 7000 pounds of rice falls below 17.5 when the nitrogen application rate is 135 pounds per acre.

In the case of Jackson graded soil, for a target yield of 4500 pounds of rice, the optimal combination of inputs changed from just urea in the base scenario to urea and fresh litter with the low litter price-high nitrogen price scenario. The reason behind the use of fresh litter at 4500 pounds target yield is that the $MRTS_{N,FL}$ falls below 17.5 which is the new price ratio at 144 pounds of nitrogen. Also, in the case of Lawrence ungraded soil, the use of litter increases with a decrease in the use of nitrogen.

In the case of composted litter, the new price ratio with low litter price-high nitrogen price can be derived as

$$IPR2_{N,CI} = (\text{price/lb of nitrogen})/(\text{price/lb of composted litter}) = 0.2625/.0525 = 5.0 \qquad (25)$$

Now consider the optimal input combination for a target yield of 4900 pounds of rice yield at Jackson. The minimized input cost is \$167 with nitrogen application at 143 pounds and composted litter application at 2468 pounds per acre. Given that the price ratio is 5, Table 3 indicates that the optimal nitrogen application should lie between 140 and 145 pounds for a target yield of 4900 pounds of rice yield.

Second, consider the high litter price-low nitrogen price scenario. Once again, the price ratio will change and therefore the cost minimizing use of input combinations will also change. The new price ratio with high litter price can be calculated as follows.

$$IPR3_{N,FL} = (\text{price/lb of nitrogen})/(\text{price/lb of fresh litter}) = 0.1575/.025 = 6.3 \qquad (26)$$

The new price ratio is much lower compared to the base scenario. This implies that the optimal input combination will be determined at the high application of nitrogen, since the $MRTS_{N,FL}$ will decrease as the nitrogen application increases. Compared to the base scenario, there is no change in the optimal input combination at RREC graded soil for target yields of 5000 and 6000 pounds. This is due to the fact that the $MRTS_{N,FL}$ is much higher at the optimal input combination compared to the new price ratio. The optimal input combination will change only when the $MRTS_{N,FL}$ falls below the price ratio. The same applies to the Jackson graded soil with target yields of 4000 and 4500 pounds, and Lawrence ungraded soil with target yields of 6500 and 7000 pounds.

Consider the RREC graded soil with a target yield of 7000 pounds of rice. The optimal input combination dictates an increased use of nitrogen and a decreased use of fresh litter due to the decreased nitrogen price and increased fresh litter price. The optimal nitrogen application rate of 161.3 is determined by the $MRTS_{N,FL}$ at the target yield. Given that the $MRTS_{N,FL}$ at the target yield of 7000 pounds in the base scenario is about 10.5, the nitrogen application rate has to increase until the $MRTS_{N,FL}$ meets the new price ratio of 6.3. From Table 2, it can be seen that the $MRTS_{N,FL}$ falls below 6.3 when the nitrogen application rate lies between 160 and 165 pounds of nitrogen for a target yield of 7000 pounds of rice.

In the case of composted litter, the new price ratio with high litter price can be calculated as follows.

$$IPR3_{N,CL} = \text{(price/lb of nitrogen)/(price/lb of composted litter)} = 0.1575/.0875 = 1.8 \quad (27)$$

With the new prices, the solution indicates that the optimal level of nitrogen application is 144.7 pounds and that of composted litter is 2462 pounds per acre. Given that the price ratio is 1.8, Table 3 indicates that the optimal nitrogen application should lie between 140 and 145 pounds for a target yield of 4900 pounds of rice yield.

In general, compared to the base scenario, the results in the sensitivity scenario are robust. The changes in the input prices did not significantly affect the optimal input combinations. In fact, for both fresh and composted litter, there is no change in the optimal input combinations when nitrogen alone is applied.

Conclusions

This paper examines the optimal trade-off between use of urea-nitrogen and poultry litter for rice production using the traditional cost minimization technique based on data from experimental results conducted at three sites in Arkansas during 1991. The traditional cost minimization technique is coupled with non-linear programming to identify the optimal input combinations between nitrogen and fresh litter as well as between nitrogen and composted litter. The results of the base scenario indicate that it is uneconomical to use fresh or composted litter at low target yields of rice. Higher yields can be achieved by the application of nitrogen in addition to fresh litter. It is more economical to use the fresh litter than the composted litter, since composted litter costs four times that of fresh litter but yields are not significantly different from fresh litter applications.

Sensitivity analysis was also conducted to measure the robustness of the results. The low litter price-high nitrogen price scenario was conducted with a simultaneous increase in nitrogen price by 25 percent and a decrease in litter price by 25 percent. The high litter price-low nitrogen price scenario was conducted with a simultaneous decrease in nitrogen price by 25 percent and an increase in litter price by 25 percent. The results indicate that the optimal input combinations are robust to input price changes.

References

Brooke, A., D. Kendrick, and A. Meeraus. GAMS, A User's Guide. *The Scientific Press*: South San Francisco, 1988.

Buchberger, E. An Economic and Environmental Analysis of Land Application of Poultry Litter in Northwest Arkansas, *Unpublished thesis*, Department of Agricultural Economics and Rural Sociology, University of Arkansas, Fayetteville, 1991.

Buchberger, E., M. J. Cochran, and R. Govindasamy. Optimal Poultry Litter Management strategies for Better Environmental Quality. *Staff Paper* #SP0193, University of Arkansas, Department of Agricultural Economics and Rural Sociology, Fayetteville, 1993.

Chiang, A.C. *Fundamental Methods of Mathematical Economics*. McGraw-Hill Book Company: Auckland, 1984.

Danforth, D., M. J. Cochran, D. Miller, and S. McConnell. The Derived Demand for Poultry Litter and Poultry Litter Compost in Delta Cotton Production. *Staff Paper* #SP1393, University of Arkansas, Department of Agricultural Economics and Rural Sociology, Fayetteville, 1993.

Govindasamy, R., M. J. Cochran, and E. Buchberger. Efficiency Implications of Environmental Regulation on Poultry Litter Management. *Staff Paper* #SP0293, University of Arkansas, Department of Agricultural Economics and Rural Sociology, Fayetteville, 1993.

Johnston, J. *Econometric Methods*, Third Edition. McGraw-Hill Book Company: New York, 1984.

Madison, R.J. and J.O. Brunett. *Over View of the Occurrence of Nitrate in Groundwater in the United States*, U.S. Geological Survey - Water Supply Paper 2275, 1985, pp 93-106.

Malone, G.W. Nutrient Enrichment in Integrated Broiler Production Systems. *Poultry Science*, 71(1992): 1117-1122.

Rainey, A., M. J. Cochran, and D. Miller. Derived Demand for Poultry Litter as a Soil Amendment in Rice, *Arkansas Farm Research*, 41(1992): 10-11.

Steele, K.F., W. K. McCallister, and J.C. Adamski. *Nitrate and Bacterial Contamination of Limestone Aquifers in Poultry Cattle Production Areas of Northwestern Arkansas, U.S.A.*, 4th International Conference-Barcelona: Environmental Contamination, October, 1990.

United States Department of Agriculture and County Governments Cooperating. *Rice Production Handbook*, Cooperative Extension Service, University of Arkansas, MP 192.

Varian, R. H. *Microeconomic Analysis*. New York: W.W. Norton & Company, 1984.

Winrock International, SEEDS Planting Ideas for a Better Future, Fall 1993, Morrilton, Arkansas, 1993.

Endnotes

1. The authors would like to thank the anonymous referees for their helpful comments on an earlier draft. All remaining errors are, of course, our own.

2. The annual production of litter was estimated at one ton per 1,000 birds produced where litter is defined as the dry mixture of the base flooring materials such as wood shavings, broiler manure, spilled feed, feathers, moisture, soil and other ingredients.

3. Winrock International estimated that about 30,000 tons of litter were transported last year.

4. Rice uptake of P is normally about 30 lbs per year, very near to the amounts applied in the poultry litter at rates examined. The analysis focuses on the short run private costs and does not consider the long term external costs resulting from environmental loadings of P. However, Delta soils are borderline deficient in P so that poultry litter could be applied at these rates for decades before the P fixation capacity of these soils would be exceeded and significant environmental loadings would be observed. It should also be noted that in these studies, litter is incorporated into the top six inches of soil profile so the P fixation capacity is much greater than with surface applications.