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Hedging with Futures and Options under a Truncated Cash Price Distribution

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ABSTRACT

Many agricultural producers face cash price distributions that are effectively truncated at a lower limit through participation in farm programs designed to support farm prices and incomes. For example, the 1996 Federal Agricultural Improvement Act (FAIR) makes many producers eligible to obtain marketing loans which truncate their cash price realization at the loan rate, while allowing market prices to freely equilibrate supply and demand. This paper studies the effects of truncated cash price distributions on the optimal use of futures and options. The results show that truncation in the cash price distribution facing an individual producer provides incentives to trade options as well as futures. We derive optimal futures and options trading rules under a range of different truncation scenarios. Empirical results highlight the impacts of basis risk and yield risk on the optimal futures and options portfolio.

Key Words: *farm programs, futures, hedging, options, truncation.*

The Agricultural Adjustment Act of 1933 was the first in a series of government programs designed to support and stabilize farm prices and incomes. While program details have changed considerably over the years, the basic policy measures have generally included nonrecourse loans; some form of production control, usually paying farmers to "set aside" some proportion of their acreage; and government acquisition and removal of commodity surplus (Cochrane).

In more recent years, the programs have included both a deficiency payment and nonrecourse loan component. The deficiency payment schemes specified a "target price" and made payments to farmers equal to any difference between the target price and the actual market price of the commodity. The maximum deficiency payment was restricted to be the difference between the target price and the "loan rate" established for nonrecourse loans. However, farmers could also receive loans from the government for the commodity at a value set by the loan rate. Then if the commodity price fell below the loan rate the farmer could turn the commodity over to the government and keep the loan money. If the commodity price moved above the loan rate, the farmer could sell the commodity on the market, pay off the loan, and keep the remaining proceeds from the sale. The combined result was that farmers who participated in the program faced a price distribution that was effectively truncated at the target price. Historically this target price, or truncation point, was set somewhere near the long-run average price level for the commodity.

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The Federal Agricultural Improvement and Reform Act (FAIR) altered the deficiency payment and nonrecourse loan provisions that had previously existed. The 1996 Act provides farmers with nonrecourse marketing loans and a loan deficiency payment option for eligible crops. Farmers are given two choices. First, they can obtain a nonrecourse marketing loan at the loan rate. Farmers have the option of repaying the loan within nine months or forfeiting the commodity. The loan repayment rate is the lesser of the loan rate plus interest or a loan repayment rate (Posted County Price) established for the commodity. The loan repayment rates are established at each county Farm Service Agency office based on current prices and locations. The second choice allows farmers to forgo the marketing loan and obtain a deficiency payment equal to the difference between the loan rate and the loan repayment rate. The net result of the program under the FAIR act is that farmers who participate in the program face a cash price for the commodity that is effectively truncated near the loan rate.

A primary difference between the farm bill under the FAIR Act and the previous farm bills is the level at which commodity prices are truncated. Under the previous Act, the commodity price was effectively truncated at the target price level while the FAIR Act truncates the commodity price near the marketing loan rate, which has been set below historical levels of the target price. This means the level of price support and protection under the FAIR Act is lower than in most previous farm bills. To compensate for the lower level of price support, the Act provides for a series of fixed, but declining, payments over time.

Although the marketing loan rate truncation point is currently, and has historically been, set well below the old target price levels, both current and historical prices suggest truncating commodity prices at the loan rate still provides important price protection to farmers. For example, the loan rate in 1998 is set at $1.89 per bushel for corn while the average price received by farmers in August of 1998 was $1.89 per bushel. Historically from 1960 to 1996, the *average* corn price paid to farmers during the year was actually below the loan rate seven times (19%) and less than 10 percent above the loan rate 16 times (43%) (USDA). Clearly there have been many instances in which cash prices have fallen down to the loan rate level in the past. The evidence suggests the current farm bill under the FAIR Act still plays an important role in putting a floor under prices for eligible farmers.

In addition to the government program, farmers have access to a variety of alternative risk-management instruments such as futures contracts, forward contracts, options on futures, and minimum price contracts. A large literature is devoted to managing price risk with these alternative pricing instruments; however, in most cases the government program is not included in the farmer's portfolio of risk-management instruments. In this paper we explore the farmer's optimal use of futures and options contracts when the farmer is eligible to participate in a government program that truncates the cash price distribution at the loan rate. The optimal hedging strategies shown in the paper suggest a previously unrecognized motive for using options to manage price risk.

Previous Studies on Hedging

The implications of using futures and forward contracts to manage risk have been explored for a variety of different cases. Danthine, as well as Feder, Just, and Schmitz show that with futures but no basis risk the optimal output level of a firm is not affected by price risk. In addition, if the futures price is unbiased the optimal hedging level is the full hedge while a biased futures price will result in a partly speculative position. Numerous other studies have extended these results including Antonovitz and Nelson who allow for basis risk, Rolfo who allows for production uncertainty, Chavas and Pope who account for both production uncertainty and hedging costs, and Myers and Hanson who consider the problem in a dynamic setting.

More recently, the role of options in managing risks has received increasing attention. It is now well known that when output price is the only source of risk, cash and futures

prices are linearly related at maturity, and futures and options prices are perceived as unbiased, then an expected utility maximizing producer has no incentive to trade options (Lapan, Moschini, and Hanson). Futures enter the optimal portfolio because futures prices are perfectly correlated with the diversifiable component of cash price risk. However, the return on options is truncated and so provides a less complete hedge than futures. Thus, if there is no expected profit from trading options then there is no incentive to trade them. This result creates somewhat of a paradox because farmers seem to actually use options at least as much as they use futures (Sakong, Hayes, and Hallam).

Extensions of the Lapan, Moschini, and Hanson (LMH) analysis have shown that options do play a role in a producer's optimal portfolio under more general conditions. In their original article LMH showed that positive expected returns from holding futures and/or options generate an incentive to trade options, assuming the underlying cash price distribution is symmetric. For example, suppose futures prices are expected to rise so there is a positive expected return from buying futures. Then the producer might buy futures in order to increase expected profits (rather than sell them to reduce risk), and then buy unbiased put options to manage the additional risk. This creates an incentive to include options as well as futures in the optimal portfolio. More recently, Vercammon has demonstrated that a similar result holds under skewed cash price distributions.

Yield risk also creates an incentive to trade options, even when futures and option prices are perceived as unbiased (Sakong, Hayes, and Hallam). The reason is that, under yield risk, a producer who has fully hedged *expected* output by selling futures remains exposed to the risk that lower than expected yields can still reduce revenue. This residual revenue risk can then be partially hedged by trading options. Finally, Moschini and Lapan have shown that even if there is no yield risk, and futures and options prices are unbiased, there may be an incentive to trade options if some input allocation decisions are made after the output price has been realized.

Throughout the large body of literature on risk management with commodity contracts, little attention has been given to how the use of futures and options contracts might change if the cash price distribution is truncated by participation in government programs. In this paper we explore impacts of government program truncation of the cash price on a farmer's hedging decision. The results suggest that cash price truncation from a marketing loan/deficiency payment program provides a previously unrecognized incentive to hedge with commodity options. In particular, we show that if the cash price facing a producer is truncated by being eligible for a marketing loan or deficiency payment, then there is incentive to trade options even when the futures and options prices are unbiased. The reason is that truncation of the producer's cash price distribution means that diversifiable risk is no longer perfectly correlated with the futures price. In this case, options can improve hedging performance by providing a more complete hedge than futures alone.

We begin the analysis by assuming no yield risk and no basis risk because this is the simplest possible case and allows the main point of the paper to be explained in a very intuitive way. Next we continue to assume no yield risk but do allow basis risk. Here we study a special type of cash price truncation mechanism which allows closed form, analytical solutions for futures and options hedging under basis risk but no yield risk. Finally, we allow for basis risk, yield risk, and a more general form of cash price truncation, in order to examine the effects of truncation in a more general setting. Unfortunately, closed-form solutions cannot be obtained for this general setting and so we use a numerical model to investigate optimal futures and options portfolios for this case. Throughout the analysis we assume unbiased futures and options prices because this seems like a reasonable approximation for most cases of interest. Furthermore, the unbiasedness assumption allows us to study the effects of truncation in the absence of additional

profit-seeking incentives for trading futures and options.

Hedging With No Yield or Basis Risk

The effect of truncation on futures and options trading is easiest to see in a simple model with no yield or basis risk. We begin with the basic LMH model which defines a producer's end-of-period profit as:

$$(1) \quad \tilde{\pi} = \tilde{b}y - c(y) + (f - \tilde{p})x + (r - \tilde{v})z$$

where $\tilde{b}$ is the random output price; y is the nonstochastic output level; $c(y)$ is a strictly convex cost function; f is the initial futures price; $\tilde{p}$ is the random ending futures price; x is futures quantity sold (purchased if negative); r is the initial put option premium; $\tilde{v}$ is the random ending put option value; and z is the quantity of put options sold (purchased if negative). The random put option value is defined by:

$$(2a) \quad \tilde{v} = 0 \qquad \text{if } p \geq k$$

$$(2b) \quad \tilde{v} = k - \tilde{p} \quad \text{if } p < k$$

where k is the strike price on the option. The producer chooses y, x, and z to maximize the expected utility of end-of-period profit.

Assuming no basis risk then end-of-period cash and futures prices are always equal, $\tilde{b} = \tilde{p}$. Then as long as there is no yield risk, and futures and options prices are perceived as being unbiased, we get the standard Lapan, Moschini, and Hanson result that the optimal hedging strategy is to set $x = y$ and $z = 0$, which allows no role for options in the optimal portfolio.

Now suppose that the producer's cash price is truncated, say through participation in a marketing loan program which ensures the producer's effective cash price is at least equal to some support level s. If the cash price falls below s then the producer receives a payment equal to the difference between the support price and the cash price, thus effectively truncating the producer's effective price at s. End-of-period profit for the producer can now be expressed:

$$(3) \quad \tilde{\pi} = (\tilde{b} + \tilde{d})y - c(y) + (f - \tilde{p})x + (r - \tilde{v})z$$

where $\tilde{d}$ is a deficiency payment defined by:

$$(4a) \quad \tilde{d} = 0 \qquad \text{if } b \geq s$$

$$(4b) \quad \tilde{d} = s - \tilde{b} \quad \text{if } b < s.$$

We now examine the effects of this truncation on hedging decisions. For simplicity, we study optimal hedging decisions conditional on a given level of output. However, output could be included as a decision variable without altering any of the following results, so this assumption is for expositional purposes only and does not affect the results of the analysis. First-order conditions for choosing x and z to maximize the expected utility of profit are:[1]

$$(5a) \quad E[u'(\tilde{\pi})(f - \tilde{p})] = 0, \quad \text{and}$$

$$(5b) \quad E[u'(\tilde{\pi})(r - \tilde{v})] = 0$$

where $u(\cdot)$ is a strictly concave von Neumann-Morgenstern utility function. If futures and option prices are perceived as unbiased then $E(f - \tilde{p}) = E(r - \tilde{v}) = 0$, and any pair of values for x and z which make $\tilde{\pi}$ independent of $\tilde{p}$ and $\tilde{v}$ will satisfy (5a) and (5b), and therefore be optimal.

Consider a candidate solution of the form $x = z = y$. Substituting this candidate solution back into the profit function (3), letting $\tilde{b} = \tilde{p}$ (no basis risk), and choosing a strike price of $k = s$, then end-of-period profit reduces to:[2]

[1] Second-order conditions for a maximum are satisfied by the concavity of the utility function.

[2] This implicitly assumes that there is a continuum of option strike prices to choose from when, in reality, there are a discrete number of strike prices available and the producer can only choose the strike price that is closest to s. However, if we allow the strike price k to differ from s then there is no analytical solution to the problem because the producer's portfolio risk can never be completely eliminated through hedging. Since the aim in this section is to derive a simple, analytical, closed-form solution, we continue to assume that the producer can always find an option with a strike price that equals the support level s.

(6) $\pi = fy - c(y) + rz$

which is completely deterministic. Because all profit risk has been eliminated then profit must be independent of $\tilde{p}$ and $\tilde{v}$, which proves that the candidate solution is optimal. Thus we have the following result.

Result 1. If there is no yield or basis risk, and futures and options prices are perceived as unbiased, then truncation of the producer's cash price distribution at a support level s will cause the producer to fully hedge on the futures market, $x = y$, and sell put options with strike price s to cover his/her entire output level, $z = y$.[3]

The intuition for this result is straightforward. Truncation of the producer's cash price distribution at s provides a free put option to the producer with strike price s. Given that the producer owns this free put option, then the futures market alone is no longer sufficient to fully hedge his/her price risk. A fully hedged futures position would leave the producer exposed to residual price risk whenever the realized price is below s (whenever the implicit option provided by the truncation is valuable). Selling put options equal to y allows the producer to eliminate this residual price risk while still appropriating the expected value of the implicit put option provided by truncation. Alternatively, the producer could just sell y call options with strike price s, which would fully hedge all of the truncated price risk and again allow appropriation of the expected value of the implicit put option provided by truncation. This is essentially a strategy of "locking in" the support price level which has been advocated by some extension agents and market commentators. In contrast to the LMH result that futures provide a complete hedge and there is no role for options in the producer's portfolio, truncation of the cash price distribution results in call options providing a complete hedge with no role for futures in the portfolio.

[3] The combined futures and options positions are equivalent to selling y call options at strike price s (Jarrow and Rudd).

Introducing Basis Risk

Suppose we continue to assume no yield risk but allow for basis risk. Following LMH and others we assume a linear basis relationship at maturity:

(7) $\tilde{b} = \alpha + \beta\tilde{p} + \tilde{\theta}$

where $\tilde{p}$ and the shock $\tilde{\theta}$ are independently distributed and $E(\tilde{\theta}) = 0$. Furthermore, we assume a special kind of truncation where the producer receives a payment if his/her "localized" futures price,

(8) $\tilde{l} = \alpha + \beta\tilde{p}$

falls below a specified support level, s. Notice that this "localized" futures price is obtained by taking the random futures price $\tilde{p}$ and adjusting it by the expected basis. Truncation of the producer's cash price distribution is achieved by paying the producer the difference between the support price and the localized futures price, $s - l$, whenever the localized futures price is below the support level. Thus, end-of-period profit continues to be defined by (3) but deficiency payments to the producer are now defined by,

(9a) $\tilde{d} = 0 \quad \text{if } l \geq s$

(9b) $\tilde{d} = s - \tilde{l} \quad \text{if } l < s.$

These payments truncate the producer's cash price distribution but do it in a very special way. This specification was chosen so that a simple closed-form solution could be obtained under basis risk. More general kinds of truncation schemes will be examined under basis risk in the next section using a numerical solution approach.

Consider a candidate solution to the producer's hedging problem of the form $x = z = \beta y$. Substituting the candidate solution back into the profit function (3), and using the basis relationship (7), gives:

(10) $\tilde{\pi} = (\alpha + \beta f + \tilde{\theta})y - c(y) + r\beta y + (\tilde{d} - \beta\tilde{\nu})y.$

Suppose the producer chooses a strike price $k = (s - \alpha)/\beta$. Then using the definition of $\tilde{l}$ in (8) the option value $\tilde{\nu}$ can be expressed:

(11a) $\tilde{\nu} = 0$ if $l \geq s$

(11b) $\tilde{\nu} = (s - \tilde{l})/\beta$ if $l < s$.

Comparing (11) with (9) we see that $d = \beta\nu$ at every realization and so the last term in (10) drops out to leave:

(12) $\tilde{\pi} = (\alpha + \beta f + \tilde{\theta})y - c(y) + r\beta y.$

The only random component left in (12) is the unhedgable basis risk $\tilde{\theta}$ which is independent of $\tilde{p}$ (and, hence, independent of $\tilde{\nu}$). If $\tilde{\pi}$ is independent of $\tilde{p}$ and $\tilde{\nu}$ the first-order conditions (5) are immediately satisfied under unbiased futures and options, and the candidate solution is optimal. Thus we have the following result.

Result 2. If there is no yield risk, cash and futures prices at maturity are linearly related, futures and options are perceived as being unbiased, and the truncation scheme is based on supporting the "localized" futures price above a minimum support level, then the optimal hedge requires selling an equal amount of futures and put options, $x = z = \beta y$, with the option strike price chosen to be $k = (s - \alpha)/\beta$.[4]

With basis risk, the truncation scheme continues to provide a free put option to producers. Because of basis risk, however, a payment based on the "localized" futures price does not put a firm floor under the cash price received by producers. Nevertheless, this free option remains valuable because it does reduce the producer's downside price exposure substantially by limiting risk to bad draws on $\tilde{\theta}$ rather than bad draws on $\tilde{b}$. The producer can appropriate this option value, and at the same time eliminate all but the unhedgable basis risk represented by $\tilde{\theta}$, by simultaneously selling futures and put options. Equivalently, the producer can just sell call options to replicate the portfolio return from the futures and put options.

Yield Risk and a More General Truncation Scheme

In the previous section we assumed no yield risk and the localized futures price (futures price minus expected basis) was used to truncate cash prices. The advantage of this specification is that it provides a one-to-one correspondence between movements in the localized futures price and movements in the futures price itself. Thus, by selling an appropriate amount of put options with the correct strike price, any fluctuations in producer payments are completely counteracted by offsetting fluctuations in put option profits. This leads to a closed-form solution even in the presence of basis risk.

In this section we investigate a more general form of truncation scheme based on the producer's actual cash price realization. This form of truncation is more consistent with the way actual marketing loan programs, target price-deficiency payment schemes, and minimum price contracts work to support the effective price received by producers. We begin by continuing to assume no yield risk in order to isolate the effects of the more general truncation scheme. Then we allow both yield risk and the more generalized truncation scheme to investigate how yield risk influences the results. All of the results of this section are based on numerical solutions because no closed form, analytical results are available for the more general framework studied here. The model is parameterized using simple, convenient assumptions because the aim is to illustrate some possible effects of cash price truncation on hedging decisions under yield and basis risk, rather than make particular hedging recommendations to decision makers. We also undertake sensitivity analysis to show how

[4] The combined futures and options positions are equivalent to selling βy call options at strike price $k = (s - \alpha)/\beta$ (Jarrow and Rudd).

changes to the basis will influence the hedging decision.

The Numerical Model

The numerical model solved in this section is based on a producer profit equation of the form:

$$(13) \quad \tilde{\pi} = \tilde{b}\bar{y} + \tilde{d}\bar{y} - c(\bar{y}) + (f - \tilde{p})x + (r - \tilde{v})z$$

where $\tilde{d}$ is a deficiency payment defined by (4) and $\bar{y}$ is expected output. This specification for profit is different from that in previous sections in two important ways. First, output $\tilde{y}$ is now a random variable and cost is based on expected output $\bar{y}$. Second, truncation occurs by farmers receiving a payment equal to $(s - b)$ whenever the cash price $\tilde{b}$ falls below the support level s, but the payment is only made on the expected output level $\bar{y}$ not actual realized output $\tilde{y}$. This is consistent with the idea that truncation occurs through the producer participating in a marketing loan program, where the loan rate only applies to expected output, or by taking out minimum price contracts on the mean output level $\bar{y}$. This model is solved for the optimal futures and options portfolios using an expected utility maximizing objective and assuming a simple logarithmic utility function.[5]

To solve the model numerically, information is needed on the joint distribution of prices, basis, and yield. Here these distributions are estimated assuming a linear basis relationship and using a simple econometric model of corn futures prices, cash prices, and yields. The model specification is:

$$(14a) \quad p_t = f_{t-0.583} + \sigma_1 z_{1t}$$

$$(14b) \quad b_t = \alpha + \beta p_t + \sigma_2 z_{2t}$$

$$(14c) \quad y_t = \mu + \gamma y_{t-1} + \delta t + \rho(p_t - f_{t-0.583}) + \sigma_3 z_{3t}$$

where z_{1t}, z_{2t}, and z_{3t} are independent $N(0, 1)$ variables. Notice that σ_1 is the standard deviation of the error from predicting the harvest futures price based on the futures price at planting (seven months prior to harvest), and σ_2 is the standard deviation of the residual basis risk θ conditional on information available at planting.[6] Expected yields depend on last harvest's yield and a time trend, and yield prediction errors have a component that is correlated with price shocks, $(p_t - f_{t-0.583})$, and a component that is orthogonal to price shocks, $\tilde{z}_3$. The parameters ρ and σ_3 determine the degree of correlation between price and yield and the standard deviation of the orthogonal component of yield risk, respectively. The model is flexible enough to account for various levels of futures price risk, basis risk, yield risk, and price-yield correlation.

The model was estimated using cash price data from the Saginaw market in Michigan, futures price data from the Chicago Board of Trade, and NASS corn yield data for Gratiot County near Saginaw in Michigan. The results can therefore be interpreted in the context of a Gratiot County corn producer selling at Saginaw and hedging on the Chicago Board of Trade, although various sensitivity analyses are conducted as well. The futures price at planting was defined as the closing price for December corn futures on the first Wednesday in May, while the harvest price was defined as the average Wednesday closing price for December corn futures over a five-week period beginning the first Wednesday in October. Harvest cash prices in Saginaw are defined as the average Wednesday price over the same five-week harvest period beginning the first week of October. Individual farm yields may be more variable than the average county yields used here but reliable individual farm yield data were not available and county yield data are adequate for the present purpose. The model was estimated over the 23-year period from 1973 to 1995 and results are presented in Table 1. The model fits the data reasonably well and suggests a standard deviation of 55 cents for futures price prediction errors, a stan-

[5] Logarithmic utility features constant relative risk aversion of 1 and decreasing absolute risk aversion. The assumption has both theoretical and empirical support but is chosen here primarily for its simplicity.

[6] The future price at planting is observed seven months before harvest or, in notation in (14), 7/12 = 0.583 of a year before harvest.

Table 1. Price and Yield Distributions for a Representative Farmer in Saginaw, Michigan

Future price	$p_t = f_t + 0.55z_{1t}$	
Cash price	$b_t = -0.23 + 0.95p_t + 0.15z_{2t}$;	$R^2 = 0.93$
Yield	$y_t = 61.01 + 0.22y_{t-1} + 1.76t - 8.74(p_t - f_t) + 14.18z_{3t}$;	$R^2 = 0.62$
	(17.23) (0.092) (0.72) (5.96)	

Note: p_t is the future price at time t (December harvest); f_t is the futures price at planting for a contract that expires at t; b_t is the cash price at t; y_t is the farm yield at t; and z_{it} are standardized errors for equation i. Numbers in parentheses are standard errors. Because the cash price regression involves two cointegrated variables OLS standard errors are unreliable and have not been reported.

dard deviation of 15 cents for basis risk, and a price yield correlation of −.32.[7] These seem like reasonable values to use to parameterize the base model.

Optimal futures and options portfolios were computed for 1996 based on the estimated model and data available at planting time of that year. The planting period futures price was $3.41 and the expected yield was estimated at 130 bushels per acre. The joint distribution of futures price, cash price, and yield at harvest was simulated by making 25,000 draws from three independent N(0, 1) variables and using the parameter values estimated from the econometric model to simulate prices and yields. Having obtained an empirical distribution of futures prices at harvest it is easy to compute the distribution of harvest option prices. Simply take the maximum of $(k - p)$ or zero at every futures realization and this will give the corresponding option price realization, v. Averaging over realizations then gives the unbiased option premium, r, at planting. Profit and the utility of profit can be calculated under each draw, conditional on a given portfolio of futures and options. Profit was normalized by expressing it per acre and setting production costs to zero. Averaging utility over realizations then gives expected utility, again conditional on a given portfolio of futures and options. Optimizing over futures and options portfolios then returns the expected utility maximizing portfolio. We used the OPTMUM module of GAUSS to implement the optimization.

Results

We began by setting the expected basis to zero ($\alpha = 0$ and $\beta = 1$) and investigating the effects of various levels of basis risk and yield risk on optimal portfolio choice under truncation (Table 2). The top half of Table 2 shows the case of no yield risk while the bottom half shows yield risk with a price-yield correlation of −0.32. All other parameters are as in the base model. In each part of the table basis risk is varied between zero, its estimated value of $\sigma_2 = 0.15$, and an upper range value of $\sigma_2 = 0.3$. This is to illustrate the impact of increased basis risk.

With no yield or basis risk the numerical model faithfully reproduces Result 1 above under truncation—the optimal position is sell both futures and put options equal to 100 percent of output in order to become fully hedged and eliminate risk (see the first row of the table). However, as basis risk increases then the optimal futures and options positions begin to decline. Remember that in this table we have maintained the assumption that $\alpha = 0$ and $\beta = 1$ so all of this decline is due to the effect of basis risk, with the expected basis remaining at zero. Basis risk lowers optimal futures and options positions because with the general truncation scheme specified here options can be valuable when cash price truncation has not occurred and vice versa. The optimal response to this additional risk is to lower futures and options positions below 100 percent of output.

Turning to the bottom half of the table we see that yield risk also reduces the optimal fu-

[7] This correlation can be calculated as $\rho\sigma_1/(\rho^2\sigma_1^2 + \sigma_4^2)^{1/2}$. The model does assume normally distributed prices and yields which is questionable for corn price and yield data but adequate for the present purpose of illustrating the effects of truncation under alternative yield and basis risk configurations.

Table 2. Impacts of Basis and Yield Risk on Hedging Positions Under Truncation

Basis Risk	Optimal Futures Position	Optimal Put Option Position
No Yield Risk		
$\sigma_2 = 0.00$	1.00	1.00
$\sigma_2 = 0.15$	0.97	0.95
$\sigma_2 = 0.30$	0.90	0.83
With Yield Risk		
$\sigma_2 = 0.00$	0.67	0.84
$\sigma_2 = 0.15$	0.64	0.77
$\sigma_2 = 0.30$	0.56	0.66

Notes: The expected basis is kept at zero throughout (i.e. $\alpha = 0$ and $\beta = 1$) as basis risk (σ_2) is varied. The with yield risk case has a price-yield correlation of -0.32. Futures and options positions are reported as a proportion of expected output.

Table 3. Impacts of the Size of Expected Basis on Hedging Decisions under Truncation

Basis Risk		Optimal Futures Position	Optimal Put Option Position
No Yield Risk			
$\sigma = 0$	$\beta = 1.00$	0.97	0.95
$\sigma = 0$	$\beta = 0.95$	0.92	0.89
$\sigma = 0$	$\beta = 0.90$	0.87	0.84
$\sigma = 0$	$\beta = 0.70$	0.66	0.63
$\sigma = -0.23$	$\beta = 1.00$	0.97	0.95
$\sigma = -0.23$	$\beta = 0.95$	0.92	0.89
$\sigma = -0.23$	$\beta = 0.90$	0.87	0.84
$\sigma = -0.23$	$\beta = 0.70$	0.66	0.63
With Yield Risk			
$\sigma = 0$	$\beta = 1.00$	0.64	0.76
$\sigma = 0$	$\beta = 0.95$	0.59	0.72
$\sigma = 0$	$\beta = 0.90$	0.56	0.69
$\sigma = 0$	$\beta = 0.70$	0.44	0.52
$\sigma = -0.23$	$\beta = 1.00$	0.64	0.76
$\sigma = -0.23$	$\beta = 0.95$	0.61	0.73
$\sigma = -0.23$	$\beta = 0.90$	0.57	0.68
$\sigma = -0.23$	$\beta = 0.70$	0.44	0.52

Notes: Basis risk is kept at $\sigma_2 = 0.15$ throughout as expected basis (α and β) are varied. The with yield risk case has a price-yield correlation of -0.32. Futures and options positions are reported as a proportion of expected output.

tures and options positions, even when there is no basis risk ($\sigma_2 = 0$). This is consistent with the Sakong, Hayes, and Hallam result that yield risk can impact optimal futures and options portfolios. Together, basis and yield risk can reduce optimal futures and options substantially below their theoretical 100 percent levels under truncation and no basis or yield risk. For example, with a price-yield correlation of -0.32, and a standard deviation of 30 cents for basis risk, the optimal futures position is reduced to 56 percent of expected output and the optimal options position to 66 percent of expected output.

Next we keep basis risk at its estimated value of $\sigma_2 = 0.15$ and investigate the effects of different levels of expected basis (α and β values) on the optimal futures and options positions under truncation (Table 3). Again, the top half of the table shows the case of no yield risk while the bottom half shows yield risk with a price-yield correlation of -0.32. All other parameters remain as in the base model. Results for two α values (zero and its estimated value of -0.23) and four β values (one, its estimated value of 0.95, 0.9, and 0.7) are shown to illustrate the effects of different assumptions about the expected basis.

As expected, the value of α has virtually no effect on the optimal portfolio irrespective of whether or not there is yield risk. On the other hand, smaller β values lead to smaller optimal futures and options positions—an effect that is consistent with Result 2 above in the case of no yield risk. The numerical results in Table 3 confirm that a similar result holds in the case of yield risk, in that even with a price-yield correlation of -0.32 reductions in the value of β continue to reduce optimal futures and options positions. Notice also that the size of the optimal futures and options positions fall dramatically in the presence of both yield risk and low β values. For example, with a price-yield correlation of -0.32 and $\beta = 0.7$ the optimal futures position falls to 44 percent of expected output while the optimal put option position falls to 52 percent. Combining the results from Tables 2 and 3 we have the following result.

Result 3. If futures and options are perceived as being unbiased, yield is stochastic, and the joint distribution of prices and yield is given by (14), then truncation of the producer's cash price distribution leads to optimal futures and options positions that decline as basis risk increases, as the expected basis parameter β declines, and as the negative correlation between price and yield increases. In general, optimal futures and options positions are no longer identical and so the optimal portfolio cannot be completely replicated by selling call options alone.

It should be emphasized that, unlike Results 1 and 2, Result 3 is developed from numerical results and so may be sensitive to the particular assumptions used to parameterize the model (e.g., logarithmic utility). However, the results clearly indicate that under truncation both futures and options play a role in the producer's optimal portfolio, although that role may be reduced by the effects of basis risk and yield uncertainty.

Conclusions

This paper has analyzed producer hedging decisions when the effective cash price distribution facing the producer is truncated. Truncation may occur from participation in the current marketing loan program available to farmers as a result of the 1996 FAIR Act, or for other reasons such as engaging in minimum price contracts with elevators. Results suggest that truncation can have a significant impact on hedging decisions and create a role for options in the producer's optimal portfolio.

In particular, cash price truncation provides the producer with an implicit put option with strike price given by the truncation point. Thus, a risk averse producer perceiving unbiased futures and options markets will generally sell puts, as well as futures contracts, in order to reduce risk while still appropriating the option value provided by truncation. In the absence of yield or basis risk, the optimal positions entail a complete hedge ($x = z = y$) which can be replicated by simply selling call options, leaving no role for futures as a hedging instrument. However, yield and basis risk generally reduce the size of the optimal futures and option positions, with the magnitude of the reduction depending on the size of the expected basis, the extent of basis risk, and the degree of price-yield correlation. In the presence of yield and basis risk it is no longer possible to replicate the optimal hedge position with a call option and so both futures and options play a role in generating the producer's optimal hedge. In all cases, however, cash price truncation creates an important role for options in the producer's optimal risk management portfolio.

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