Spatial-Dynamic Externalities and Coordination in Invasive Species Control

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Abstract

This study investigates a spatial externality common in invasive species control decisions made by multiple, spatially-connected decision makers (i.e., individual landowners, state and federal agencies, etc.). The externality arises due to the different spatial considerations of decision-makers which drives a wedge between individual and social damages, and results in a suboptimal level of individual control. The number of decision makers, the size of individual parcels, and the spatial configuration of small and large parcels influence the severity of the externality and consequently the insufficiency of privately supplied invasive species control. To internalize the externality, this paper provides a corrective mechanism in which individuals compensate invaded individuals for control actions that preserve uninvaded areas. These results shed light on the coordination problem in transboundary invasive species issues.

1. Introduction

Invasive species are causing tremendous losses in the US with several billions of dollars spent on decreasing their spread (Pimentel et al., 2005). Altering the spread of an established invader is a long-term trade-off between the flow of damages and relative control costs. This trade-off critically depends on the ecological and economic factors that dictate the evolution of the invasion and subsequent damages (Regev et al. 1976; Olson and Roy, 2008). Invasive species also impact large spatial areas requiring control actions by multiple affected individuals, e.g., land owners, regional governments, countries (Wilen, 2007). While invasive species control suffers from well-known problems of public good provision (Perrings et al. 2002), the problem is novel in that the public good is determined by spatial-dynamic processes unique to each species and the spatial configuration of decision makers. This paper integrates ecological and economic processes to study a spatial externality common in invasive species control decisions made by spatially-connected individuals.

Previous economic research has characterized the degree of invasion in two ways. One is based on population density, in which an invasive species population grows within a fixed area and effectiveness of control is measured as a reduction in invasive species numbers (Bhat et al., 1993; Bhat et al., 1996; Bhat and Huffaker, 2007; Bicknell et al., 1999; Horan and Wolf, 2005; Kim et al., 2006; Burnett et al., 2008; Olson and Roy, 2010; Finnoff et al., 2011; Homans and Horie, 2011). Another way to describe the impact of invasive species is spatial spread represented by an expansion of invaded area (Sharov and Liebhod, 1998; Rich et al., 2005b; Wilen, 2007; Olson and Roy, 2010; Epanchin-

Niell and Wilen, 2012; Sims and Finnoff, 2013). In this case, control is manifested as a reduction in the spread rate.

Spatial externalities are likely when control efforts impact the movement of an invasive species across the landscape (Grimsrud et al. 2008; Rich et al. 2005a). A negative externality is caused by the pattern of spread, such as the emigration of pests from high-density to low-density. Bhat et al. (1993, 1996) show that multiple landowners necessitate a centralized control strategy which incorporates the effect of species diffusion on control. Rich et al. (2005b) find that regional control of foot and mouth disease (FMD) spread is diminished by spatial spillovers from neighboring regions which perform less. A positive externality arises when benefits from individualistic control spillover to neighbors in the form of a decrease or delay of damages (Brown et al. 2002; Wilen, 2007). In each case, the fundamental reason for the externality is that individual participants base control decisions on a subset of the total area at risk of invasion. Variable transfer payment agreements (Bhat and Huffaker 2007) and chained bilateral negotiation (Wilen 2007) have been suggested as ways to internalize these types of externalities.

Another positive externality centers on the deficiency of private efforts to ameliorate impacts to regional commodity markets. When an individual producer becomes invaded, that producer experiences a decline in profit due to 1) physical damage to the commodity which reduces yields and 2) impacts to the regional commodity market in the form of reduced demand for commodities from this region or the imposition of costly phytosanitary standards (Acquaye et al, 2005). While initially invaded producers fully internalize their physical damage when making control decisions, they only incur a subset of the total economic impact to the regional market.[1] This creates a tendency for individual producers to have control incentives that are not aligned with their neighbors (Cook et al. 2010). The degree to which an individual producer will internalize the full market impact is determined by the size of the producer with larger producers experiencing a larger share of the market damages.

For example, citrus canker was detected near Miami in 1995 (Gottwald et al. 2001). The USDA now believes that long-distance spread of the disease by hurricanes in 2004 and 2005 makes eradication infeasible and a new citrus canker management plan is being developed (Olson 2006). Restrictions against the importation of citrus fruit have already had serious impacts on regional producers in Florida (Acquaye et al, 2005). If the bacterial agent that causes citrus canker becomes endemic to Florida, it will effectively result in prohibition of interstate commerce of fresh citrus fruit, which comprises approximately 20% of the State's $8 billion commercial citrus industry (Muraro 1986). Other highly susceptible cultivar such as grapefruit will become less economically viable due to requirements for multiple bacterial sprays (Gottwald et al. 2001). Individual orchard owners would only incur a portion of these market-level impacts and would not consider how their control decisions would alter the spread of citrus canker and lessen market impacts on other orchard owners. On a more broad scale, countries may prohibit

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[1] For example, Horan and Wolf (2005) show that foot and mouth disease, bovine TB (tuberculosis), and other diseases may cause (1) losses due to the death of livestock and the reduction of meat from infected livestock, etc.; (2) loss associated with the imposition of trade sanctions on the disease outbreak regions; (3) threats to human health; and (4) threats to wildlife. The social planner considers these losses as the social damages caused by TB, while an individual only includes his/her own individual economic losses, i.e. reduction of production, decrease in market value and other losses. Bicknell et al. (1999) show similar factors prevent individual livestock producers from eradicating diseases in their own herd.

importation of citrus fruit from the United States due to the risk on invasion spreading to other states. The state of Florida would only incur a portion of the impacts from a federal quarantine and would not consider how their control decisions would lessen market impacts on other citrus-producing states such as California.[2]

This paper utilizes optimal control to investigate market externalities that arises as an invasive species spreads across a landscape that is a source of supply for a regional commodity market. Increases in invaded area result in two types of damage. The first is physical damage to the commodity. The second is a market response where damage arises from 1) reduced demand for commodities from this region due to trade restrictions and quarantines or consumer perceptions of lowered commodity quality or 2) from the introduction of costly phytosanitary standards for exports from the region. Costly control efforts may slow, stop, or reverse the spread of the invasion in the region. Individual producers make control decisions based only on a subset of the area at risk (i.e., their private property or jurisdictional considerations) and thus do not consider the full impact of the invasion on the local commodity market. In short, a mismatch between the spatial extent of individual producers and the potential invaded area leads to a divergence between the control incentives of individual producers and the market as a whole.

The paper provides theoretical and numerical results on the impact of spatial configuration and size of individual producer on the deficiency of privately supplied

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[2] Other examples include the impact of emerald ash borer (EAB) and gypsy moth on forestry markets and foot and mouth disease (FMD) on regional beef markets. All logs and firewood from EAB and gypsy moth quarantine areas are required to undergo costly heat treatments (USDA, 2010). The price of meat in FMD-endemic markets is about 50% lower than in FMD-free markets (Ekboir et al., 2002). Prohibitions against importing meat from FMD-endemic countries results in losses to all producers in those countries (Ekboir et al., 2002; Jarvis et al. 2008).

invasive species control. Because individual producers are relatively spatially myopic, their shadow cost of an additional unit of invaded area is smaller than it would be for the market as a whole. This implies that commodity markets which rely on a large number of small producers will support larger invasions. If the market is comprised of producers of various sizes, invaded area will be larger if the species is introduced on a smaller producer and then spreads to larger producers. In an effort to internalize this externality, we also identify the timing and sequence of side-payments in which uninvaded and fully invaded individuals compensate individuals currently engaged in control for actions that preserve uninvaded areas and lessen impacts to the regional market as a whole. Such side-payments, organized by industry trade groups or cooperatives, become an attractive approach for coordinating invasive species control efforts in light of increasingly limited federal funds designated for such efforts.

2. Modeling a Species Invasion

For convenience, the definition of each variable in the model is summarized in Table 1. A regional commodity market is supplied by a rectangular strip whose width is normalized to one.[3] This rectangular strip is divided into I individually owned parcels of land, labeled as 1 to I from west to east (see Figure 1). Hereafter let "parcel" refer to the single piece of land owned by each individual producer and "region" the total area of I parcels. Let A_i represent the size of the parcel owned by producer $i = 1, 2, \ldots, I$ and $A = \sum_{i=1}^{I} A_i$ reflect the size of the region. The species is introduced on parcel 1 and

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[3] The assumption of a rectangular potentially invaded area with invasion spreading from one end to the other fits observations of some invasions better than other but was chosen for exposition. It is an assumption that could be relaxed for any spatial configuration or initial invaded area.

spreads from west to east along the length of the rectangle. Cumulative distance spread up to time *t* is given by $x(t)$ with x_0 representing the distance spread when the invasion was first detected at $t_0 = 0$. Each parcel is either "not invaded" (the invasion front has not yet reached the western border of the parcel), "being invaded" (the invasion front has reached and is spreading within the parcel), or "fully invaded" (the invasive species has fully spread across the parcel).

The spread of an invasive species is "a process by which the species expands its range from a habitat in which it currently occupies to one in which it does not" and there are two processes (Liebhold and Tobin, 2008). Local or short-range dispersal due to the growth of the population is characterized by a constant spread rate. In contrast, long-distance dispersal such as human-mediated or wind dispersal results in isolated colonies, which grow and eventually merge into the main population of invasive species. The combination of the two processes (known as stratified dispersal) causes spread to accelerate over time (Liebhold and Tobin, 2008). Let the spread of the invasion be given by $\dot{x} = gx(t)$ where $g > 0$ is the intrinsic constant spread rate of the invasive species. Here we assume no new introductions and an exponentially increasing invaded area which is consistent with a combination of short and long range dispersal.

Economic damages due to invasion in the region have a physical and market component. Physical damages refer to the pecuniary losses due to crop death or product decline. For example, Rice Water Weevil causes an average yield loss of 7% ($64.05 /acre) in the US (Hummel, 2009) and about 10%-20% yield loss in the north of China (Yu et al., 2008). Market damage results from price effects, restricted markets, and the imposition of costly phytosanitary standards. Consumers may consider commodities from an invasion region as damaged goods resulting in lower demand. Demand may also be reduced due to quarantines or trade restrictions intended to prevent new introductions outside the regional market. These market damages occur throughout the region since consumers and regulators will rarely be able to distinguish between commodities from invaded and non-invaded parcels.

The physical and market damages are shown in Figure 2. Once the region becomes invaded, physical commodity damage on invaded parcels causes the supply curve to shift from S_0 to S_1 and quantity supplied falls from Q_0 to Q_1. We make the small regional market assumption such that only producers are affected by this reduction in supply. If phytosanitary measures are required in the regional market, the marginal cost of production for both invaded and non-invaded producers will increase and the quantity supplied will be reduced further to Q_2. If consumers perceive commodities from the regional market as inferior or if access to export markets are restricted, regional demand will shift from D to D' leading to a lower price for commodities from the invaded region.

An individual producer’s damage function captures these physical and market damages at each stage of the invasion. When a parcel is uninvaded, that producer only experiences market damages. When a parcel is currently being invaded or is fully invaded, that producer experiences both physical and market damages. This implies that an individual’s damage is a function of both the size of the invasion and the size of the individual producer’s parcel: $D_i[x(t), A_i] \geq 0$. As the invasion grows, market impacts felt by the individual producer increase due to more restricted market access or more stringent phytosanitary standards ($\partial D_i/\partial x > 0$); however, the majority of these damages are felt early in the invasion ($\partial^2 D_i/\partial x^2 \leq 0$).[4] The total damage to all producers in the regional market is defined as $D[x(t), A] = \sum_{i=1}^{I} D_i[x(t), A_i]$.[5]

A control action is available to producers which will reduce the rate of spread across their parcel from g to $[g - u_i(t)]$ where $u_i \geq 0$ is the reduction in the spread rate due to control by producer *i* at time t. Control efforts slow spread if $u_i(t) < g$, reverse spread if $u_i > g$, and stop spread if $u_i = g$.[6] The cost of spread control is determined by the control rate c$[u_i(t)]$ with $\partial c/\partial u_i > 0$ and $\partial^2 c/\partial {u_i}^2 > 0$.[7] Efforts to control spread are focused in a barrier zone along the edge of the expanding population front (Sharov and Liebhold, 1998), so producers can take control actions only when their parcel is being invaded. Once the invasion spreads from parcel *i* to a neighboring parcel, producer *i* stops control and the neighboring producer starts spread control.[8] In this way, the individual spread control process is akin to a relay.

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[4] For example, the majority of quarantines and phytosanitary standards will be enacted when the invasion is initially detected. Eventually the region will become so invaded that any increase in invaded area will trigger a minimal market response.

[5] Invasive species spread may also cause significant environmental damage such as reduction in biodiversity and native species extinction. Due to the paper's focus on spatial externalities in commodity markets, we abstract from these impacts. As a result, *D* should not be interpreted as a measure of social damages from invasion.

[6] Sharov and Liebhold (1998) investigate the conditions which determine the best spread rate for managing an invasive species along the invasion front. Due to the concavity of control cost function and linear damage function in their model, it is never optimal to stop spread. The model presented herein considers alternative specifications of the control cost and damage functions which allows for slowing, reversing (eradication), stopping of spread.

[7] Control costs may also be a function of A_i which would change the degree of control chosen on a particular parcel. However, this would not change the nature of the externality between neighbors since collective and individual control decisions would take place on the same parcel and be subjected to the same costs.

[8] This assumption only applies to control actions that alter the spread of the invasion and does not preclude the possibility that producers may continue population-related control actions once fully invaded. Due to the paper's focus on externalities between neighbors, we have chosen to simplify the model by focusing only on control actions that alter the spread of the invasion.

In what follows we first characterize the collective control decision from the perspective of all producers. We then illustrate the spatial externality by contrasting the collective decision with that of the private control decisions of each individual producer.

3. Collective Control by all Regional Producers

The collective control strategy represents the decisions of a coordinated control effort by all producers. Such coordinated efforts would reflect the control decisions of an industry trade group or agricultural cooperative. For example, a cooperative of ranchers was created to share costs and coordinate efforts to control knapweed in Montana (Fiege 2005) and yellow starthistle in California (Epanchin-Niell et al. 2010). The objective of a collective control strategy is to choose $u(t)$ to minimize the total damages and control costs across the region

$$(1) \qquad \max_{u(t)} \int_0^\infty \{R(A) - D[x(t), A] - c[u(t)]\} e^{-rt} dt$$

subject to $\dot{x} = [g - u(t)]x(t)$, $x(0) = x_o > 0$, and $u(t) \geq 0$ where $R(A)$ is the agricultural revenue earned in the entire region before invasion, and *r* is the discount rate. There are three cases for the terminal condition. The first case is when it is optimal to preserve a portion of the region leading to a steady-state invaded area that is smaller than the region. The second case finds the region being fully invaded such that the terminal condition is equal to the size of the region. The third case corresponds to control decisions that optimally reverse the spread leading to eventual eradication.[9]

The current value Hamiltonian for this problem is

$$(2) \quad H[u(t), x(t), \omega(t)] = \{R(A) - D[x(t), A] - c[u(t)]\} + \omega(t)[g - u(t)]x(t)$$

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[9] Mathematically, invaded area will only approach eradication at the limit. For exposition, we consider the invasion eradicated when it reaches a very small value.

where ω is the current value costate variable for invaded area.[10] The costate variable represents the shadow cost of an incremental increase in invaded area (or the marginal value of uninvaded land). The corresponding necessary conditions for an interior solution are

$$(3) \quad -\omega^*(t)x^*(t) - \frac{\partial c(\cdot^*)}{\partial u} = 0$$

$$(4) \quad \dot{\omega}^* = \left[r - \left(g - u^*(t)\right)\right]\omega^*(t) + \frac{\partial D(\cdot^*)}{\partial x}$$

$$(5) \quad \dot{x}^* = [g - u^*(t)]x^*(t)$$

From (3), the optimal control rate is adjusted to ensure the marginal control cost equals the incremental damage of invasion area: $\partial c(\cdot^*)/\partial u = -\omega^*(t)x^*(t)$. No control is optimal if the incremental damage is lower than the marginal control cost: $-\omega^*(t)x^*(t) < \partial c(\cdot^*)/\partial u$. If this condition holds at every point of invasion, the damage caused by the invasion is ignored and the invasion spreads at its natural rate across the region. If $-\omega^*(t)x^*(t) > \partial c(\cdot^*)/\partial u$ control is implemented to the maximum degree.

Solving equation (3) for $\omega^*(t)$, taking the time derivative and using (5) yields

$$(6) \quad \dot{\omega}^* = -\frac{\frac{\partial^2 c(\cdot^*)}{\partial u^2}\frac{du(t)}{dt} - [g - u^*(t)]\frac{\partial c(\cdot^*)}{\partial u}}{x^*(t)}$$

Substituting (6) into (4), and using (3) and (5)

$$(7) \quad \dot{u}^* = \frac{r\frac{\partial c(\cdot^*)}{\partial u} - \frac{\partial D(\cdot^*)}{\partial x}x^*(t)}{\frac{\partial^2 c(\cdot^*)}{\partial u^2}} = \frac{r\frac{\partial c(\cdot^*)}{\partial u} - \left\{\sum_{i=1}^{I}\frac{\partial D\left[x^*(t), A_i\right]}{\partial x}\right\}x^*(t)}{\frac{\partial^2 c(\cdot^*)}{\partial u^2}}$$

The optimized dynamic system is described by the coupled nonlinear system of differential equations in (5) and (7).

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[10] See Appendix for a more detailed explanation of the costate variable in invasive species control problems.

Figure 3 presents a phase diagram of this system when it is optimal to maintain a portion of the region as uninvaded (case 1).[11] The slope of the $\dot{u} = 0$ isocline (solid line) is positive reflecting the desire for more control in response to larger damages. The $\dot{x} = 0$ isocline (dashed line) is realized by equating $u^*(t)$ and the invasive species natural spread rate. The steady state invaded area $x^{ss} = r\,\frac{\partial c/\partial u}{\partial D/\partial x}$ is solved directly from equations (5) and (7). As expected, the area that should be preserved from invasion is negatively related to the discount rate and marginal control cost but positively related to marginal damage. Off- equilibrium conditions suggest a saddle-point stable trajectory, indicated by the dotted line. From isosector I, control is increased as the invasion approaches eradication and from isosector IV, decision makers give up on control as the area becomes fully invaded.

4. Dynamic Control of Invasive Species by Individual Producers

Invasive species control becomes the responsibility of individual producers in the absence of an organization or institution that represents the interest of the regional market. Individual control falls short of collective control because control actions involve private costs but partially public benefits. Individual producers will first suffer market damage before their parcel is invaded since market damages impact all producers in an invaded region. When the invasion spreads onto a new parcel, that producer also begins to

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[11] The other two cases can be derived directly from the first case. In case 2, the whole region is fully invaded regardless of the externality, and only the time when the region becomes fully invaded is different between the individual control and the social control. In this case, the social planner is able to delay the inevitable (McIntosh at al. 2010). In case 3, if it is also optimal for the individual control relay to eradicate the invasion, there is just a time difference between the individual and social control to reverse the invasion back to the west border of the region. But if it is not optimal for the individual control to eradicate the invasion, the difference will be no invasion versus partial invasion.

experience physical damage (which reaches a maximum when fully invaded) and chooses a level of control at each point in time to minimize the present value of her own individual damage and control cost flow into perpetuity. Once the invasion spreads to other parcels, she will stop control since her control efforts no longer alter the spread of the invasion. However, she continues to experience market damage (which increases as the invasion spreads to other parcels) and a constant amount of physical damage.

Each producer's control strategy is one turn of an individual spatial control relay. The invasive species is first discovered on parcel 1 at τ_0 which signifies the start of the individualistic control relay. For i > 1, τ_{i-1} is the time at which parcel i-1 becomes fully invaded. This also represents the time the invasion initially occurs on parcel i signifying a transfer in the individual control relay.

The nature of the control relay limits the degree of strategic behavior on the part of individual producers. Control decisions made by previously invaded producers determine the time subsequent producers become invaded but do not influence subsequent control decisions. In contrast, a currently invaded producer must anticipate the control decisions of subsequently invaded individuals. This allows the private optimum control strategy to be treated as a chain of individual optimal control problems solved through backward induction. Links between individual control problems are handled through initial conditions and terminal salvage values. An individual's initial condition reflects the control decisions of all previously invaded individuals. The salvage value represents an individual's anticipation of future damages given all subsequent individuals behave optimally. The optimal control path of the last individual in the region or the individual that stops spread (the steady state individual) determines the terminal salvage value of the preceding individual's control problem. This procedure is repeated to find all the control decisions of all producers in the region.

For exposition, assume the individual control relay by producers results in a positive steady-state (x_m^{ss}) being reached within parcel $m \in [1, I]$ at time τ_m^{ss}. Producer *m* is facing an infinite horizon optimal control problem since control must be exerted indefinitely to keep the invasion at steady state. When the invasion reaches the west border of her parcel, producer *m* solves

$$(8) \quad \max_{u_m(t)} \int_{\tau_{m-1}}^{\infty} \{R(A_m) - D_m[x(t), A_m] - c[u_m(t)]\} e^{-rt} dt$$

subject to $\dot{x} = [g - u_m(t)]x(t)$, $u_m(t) \geq 0$, and initial condition

$$x(\tau_{m-1}) = \begin{cases} x_o & m = 1 \\ \sum_{j=1}^{m-1} A_j & m > 1 \end{cases}.$$

The current value Hamiltonian for the control path on parcel *m* is

$$(9) \quad H_m = \{R(A_m) - D_m[x(t), A_m] - c[u_m(t)]\} + \omega_m(t)[g - u_m(t)]x(t)$$

where $\omega_m(t)$ is the current value costate variable associated with invaded area on parcel *m*. The corresponding current value necessary conditions can be written as

$$(10) \quad -\frac{\partial c(\cdot^*)}{\partial u_m} - \omega_m^*(t)x^*(t) = 0$$

$$(11) \quad \dot{\omega}_m^* = \left[r - \left(g - u_m^*(t)\right)\right]\omega_m^*(t) + \frac{\partial D_m(\cdot^*)}{\partial x}$$

$$(12) \quad \dot{x}^* = [g - u_m^*(t)]x^*(t)$$

Following the series of substitutions outlined in section 3, producer m's optimized dynamic system is described by equation (12) and

$$(13) \quad \dot{u}_m^* = \frac{r\frac{\partial c(\cdot^*)}{\partial u_m} - \frac{\partial D_m(\cdot^*)}{\partial x} x^*(t)}{\frac{\partial^2 c(\cdot^*)}{\partial {u_m}^2}}$$

Now let's turn our attention to the control decisions of all previously invaded producers denoted as $q \in [1, m-1]$. Unlike parcel *m*, the control problem on parcel *q* is characterized by a fixed terminal state $x(\tau_q) = \sum_{j=1}^{q} A_j$ but a free terminal time since control actions alter the rate of spread across the parcel. Individual *q* solves

$$(14) \quad \max_{u_q(t)} \int_{\tau_{q-1}}^{\tau_q} \{R(A_q) - D_q[x(t), A_q] - c[u_q(t)]\} e^{-rt} dt + e^{-r\tau_q} s_q[x(\tau_q)]$$

subject to $\dot{x} = [g - u_q(t)]x(t)$, $u_q(t) \geq 0$, initial condition

$$x(\tau_{q-1}) = \begin{cases} x_o & q = 1 \\ \sum_{j=1}^{q-1} A_j & q > 1 \end{cases}$$

with τ_{q-1} given, and terminal condition $x(\tau_q) = \sum_{j=1}^{q} A_j$ with $\tau_q < \tau_m^{ss}$. $s_q[x(\tau_q)]$ is a salvage value and represents the present value damages suffered by producer *q* after becoming fully invaded (τ_q to ∞)

$$(15) \quad s_q\left(x(\tau_q)\right) = \int_0^{\tau_q^{ss}} -D_q(x(t), A_q)\, e^{-rt} dt + \frac{-D_q(x_m^{ss}, A_q) e^{-r\tau_q^{ss}}}{r}$$

where $\tau_q^{ss} = \tau_m^{ss} - \tau_q$ represent the time between individual *q* becoming fully invaded and the individual control relay reaching the steady state. The first term in (15) captures the present value of increasing market damages (and constant production damages) that accrue to producer *q* after being fully invaded but before the individual control steady state is reached. The second term captures the constant flow of physical and market damages that accrue to producer *q* after the steady state invaded area is reached. Producer

q takes the control decisions of all subsequently invaded producers as given when evaluating s_q.

From (14), the current value necessary conditions can be written as

$$(16)\quad -\frac{\partial c(\cdot^*)}{\partial u_q} - \omega_q^*(t)x^*(t) = 0$$

$$(17)\quad \dot{\omega}_q^* = \left[r - \left(g - u_q^*(t)\right)\right]\omega_q^*(t) + \frac{\partial D_q(\cdot^*)}{\partial x}$$

$$(18)\quad \dot{x}^* = \left[g - u_q^*(t)\right]x^*(t)$$

The current value transversality condition is

$$(19)\quad R(A_q) - D_q\left[x^*(\tau_q), A_q\right] - c\left[u_q^*(\tau_q)\right] + \omega_q^*(\tau_q)\dot{x}^*(\tau_q) = rs_q\left[x(\tau_q)\right]$$

which states that the optimal choice for τ_q should equate the marginal cost of a longer time horizon (left-hand side) with the marginal benefit of delaying the salvage value (right-hand side). The optimal dynamic system for producer q is composed of (18) and

$$(20)\quad \dot{u}_q^* = \frac{r\frac{\partial c(\cdot^*)}{\partial u_q} - \frac{\partial D_q(\cdot^*)}{\partial x}x^*(t)}{\frac{\partial^2 c(\cdot^*)}{\partial u_q{}^2}}$$

The optimal dynamic system for producer m and producer q is the same as the collective system except for $\partial D_m(\cdot^*)/\partial x < \partial D(\cdot^*)/\partial x$ and $\partial D_q(\cdot^*)/\partial x < \partial D(\cdot^*)/\partial x$. This implies two things about the individual spread control relay.[12] First, the individual producer's optimal control path on each parcel will be lower than the collective control path. From (13) and (20), the individual isocline $\dot{u}_i = 0\,(i = 1, \cdots, I)$ is underneath the social isocline $\dot{u} = 0$. Second, the steady state invaded area under individual control $x_m^{ss} = r\frac{\partial c/\partial u_m}{\partial D_m/\partial x}$ is larger than under the collective control process $x^{ss} = r\frac{\partial c/\partial u}{\partial D/\partial x}$.

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[12] See the appendix for a detailed proof of these results.

To illustrate these findings, consider the hypothetical scenario in Figure 4 where an initial invaded area of size x_0 is discovered at $t_0 = 0$. The individual isoclines $\dot{u}_1 = 0$ and $\dot{u}_2 = 0$ are always under the collective isocline $\dot{u} = 0$ and the individual control rate at a given invasion area is smaller than the collective one, $u_1^*(x) \leq u^*(x)$ and $u_2^*(x) \leq u^*(x)$. Because the individual's optimal control rate is lower than the collective one for a given invaded area before the steady-state, the invasion is spreading faster under individual control than under collective control. At time τ_1, parcel 1 is fully invaded, then the owner of parcel 2 starts her control and reaches the steady state x_2^{ss} at time τ_2^{ss}. The steady state invaded area under collective optimal control is smaller and reached sooner at time t^{ss}.

5. Subsidy Scheme

The deficiency of individual control is due to the limited spatial interest of the individual. Specifically, the parcel owner is only concerned with her private economic losses, thus excluding losses on neighboring land parcels' when making her optimal control decision. A central management authority representing the regional market has an incentive to encourage producers who are currently being invaded to enact more control.

Side payments between individual producers could be organized to motivate individual producers to take more control. Here we present a series of side payments that allow uninvaded or fully invaded producers to compensate other producers for control actions that preserve uninvaded land. When a producer is currently being invaded and making control decisions, she receives a side payment; and when a producer is fully invaded or uninvaded, she makes side payments to the producer currently engaged in control. These side payments may be organized and carried out by regional agricultural cooperatives or market interest groups. Unlike past work on pollution control, our side-payments are time-varying due to the dynamic nature of the invasion. Also unlike the variable transfer payment of Bhat and Huffaker (2007), the evolution of the invasion does not change the bargaining strengths of participants. As a result our ex ante schedule of side payments is sufficient to ensure continued compliance as the invasion unfolds.

The payment is provided as a rate $h(t)$ per uninvaded area of the region, $[A - x(t)]$. Only producers who are currently being invaded and engaging in control receive a side payment. The rate per uninvaded area that internalizes the externality when producer *i* is controlling is the other producers' marginal market damages at $x(t)$

$$(21) \quad h(t) = \sum_{\substack{k=1 \\ k \neq i}}^{I} \frac{\partial D_k[x^*(t), A_k]}{\partial x}$$

This payment rate ensures the costate variable for the individual producer is identical to the costate variable for the collective control decision. As the invasion area grows, the total subsidy offered declines. This reflects the non-increasing subsidy rate due to non-increasing marginal damage from market impacts accruing to other producers and the decline in uninvaded areas $[A - x(t)]$.

The total payment $h(t)[A - x(t)]$ is funded by all other producers who benefit from producer *i*'s control efforts. The total spillover from control by producer *i* to each of the other producers *k* is

$$(22) \quad B_k = \int_{\tau_{i-1}}^{\tau_i} D_k\left[e^{gt} \sum_{j=1}^{i-1} A_j\right] dt - \int_{\tau_{i-1}}^{\tau_i} D_k\left[e^{(g-u_i)t} \sum_{j=1}^{i-1} A_j\right] dt \;\; k = 1, \cdots, I; k \neq i.$$

The first term represents the accumulation of market damage experienced by all other producers if no control is exerted on parcel i. The second term represents the slower accumulation of damage experienced by all other producers if control on parcel *i* reduces the spread rate to $g - u_i$.

Due to market damages, both fully invaded and uninvaded producers are willing to pay an amount equal to their market damage at *x*(*t*). Let

$$(23) \quad F_{ki}(t) = \frac{\partial D_k[x^*(t), A_k]}{\partial x}\left(A - x(t)\right) \qquad k = 1, \cdots, I, k \neq i$$

be the side payment paid by producer *k* to producer *i* when the invasion is of size *x*(*t*).

Assume the invasion has yet to reach the steady-state parcel *m*. Let $u_q^s(t)$ represent individual q's reduction in the invasive species spread rate in response to this series of side payments from all other producers *k*. The producer on parcel q solves

$$(24) \max_{u_q^s(t)} \int_{\tau_{q-1}}^{\tau_q} \left\{R(A_q) - D_q\left[x, A_q\right] - c\left[u_q^s\right] + h(t)[A - x]\right\} e^{-rt} dt + e^{-r\tau_q} s_q\left[x\left(\tau_q\right)\right]$$

subject to dx/dt, all relevant initial and terminal conditions and

$$s_q\left(x\left(\tau_q\right)\right) = \int_0^{\tau_q^{SS}} \left\{-D_q\left[x(t), A_q\right] - F_{qi}(t)\right\} e^{-rt} dt + \frac{\left[-D_q(x_m^{SS}, A_q) - F_{qm}(\tau_m^{SS})\right] e^{-r\tau_q^{SS}}}{r}$$

where F_{qi} and F_{qm} are the side payment provided by producer *q* to subsequently invaded producers *i* and *m*. Producer *q* recognizes that while she is currently receiving a side payment, she will also be making side payments to other producers in the future. Because she will still be experiencing market damages after she becomes fully invaded, payments from producer *q* to other individuals remain incentive compatible after becoming fully invaded. The producer on steady-state parcel *m* makes side payments to all other producers before becoming fully invaded. Eventually parcel *m* becomes invaded and producer *m* solves

$$(25) \quad \max_{u_m^s(t)} \int_{\tau_{m-1}}^{\infty} \{R(A_m) - D_m[x(t), A_m] - c[u_m^s(t)] + h(t)[A - x(t)]\} e^{-rt} dt$$

Because she must engage in perpetual control to keep the invasion at steady state, producer *m* receives a continuous stream of side payments from other producers to preserve the remaining un-invaded area.

6. Numerical Simulation

To illustrate the model's main findings, consider the following hypothetical example. A single commodity is produced in the region and sold in a perfectly competitive market. Before being invaded, parcel i produces $Q(A_i)$ and after being invaded output decreases by $Q(A_i)\kappa_i[x(t)]$ where

$$\kappa_i[x(t)] = \begin{cases} 0 & x(t) < \sum_{j=1}^{i-1} A_j \\ \alpha \frac{x(t) - \sum_{j=1}^{i-1} A_j}{A_i} & \sum_{j=1}^{i-1} A_j \leq x(t) < \sum_{j=1}^{i} A_j \\ \alpha & x(t) \geq \sum_{j=1}^{i} A_j \end{cases} \quad (26)$$

is the percent decrease in output on parcel i for an invasion of size $x(t)$. As the invasion expands, κ_i increases to α which represent the percent reduction in output from full invasion. The quantity of output is normalized such that one km^2 of area yields one unit of output, $Q(A_i) = A_i$. Individual i suffers production damage $PA_i\kappa_i$ where P is the given price of the commodity before the invasion. The region is a small player in the commodity market such that the reduction of output in the region due to the invasion does not affect the competitive market price.

Let $D^c[x(t)] = zx(t)^a$ be the total market effects of invasion on the whole region with $a \leq 1$ and $z > 0$. Therefore, the total damage to the regional market from invasion is

$$D[x(t), A] = P\alpha x(t) + zx(t)^a. \quad (27)$$

D_i^c is the market damage on parcel i, which may be proportional to the total market damage weighted by some index, such as land shares or output levels. For exposition we allocate market damage among producers according to land share. Therefore, parcel i's total damage from invasion is

$$(28) \quad D_i[x(t), A_i] = \begin{cases} zx(t)^a \frac{A_i}{\sum_{i=1}^{I} A_i} & \kappa_i[x(t)] = 0 \\ PA_i\kappa^i[x(t)] + zx(t)^a \frac{A_i}{\sum_{i=1}^{I} A_i} & 0 < \kappa_i[x(t)] < 1 \\ PA_i\alpha + zx(t)^a \frac{A_i}{\sum_{i=1}^{I} A_i} & x(t) \geq \sum_{i=1}^{q} A_i \end{cases} .$$

The collective and individual control costs are identical cubic functions of the reduction invasive species spread:

$$(28) \quad c(u) = \eta[u(t)]^3$$

For convenience, the definition and values of each parameter in the simulations are summarized in Table 2.

The production damage and the market damage of the land parcel i for invasion size $x(t)$ are shown in Figure 5. The perpendicular line implies inelastic supply which would be consistent with a region where producers do not adopt invasive-resistant crops or convert the parcel to other uses over the course of the invasion.[13] Once the invasion has been detected, the price received for products from the region drops from P to P_t. This decline is captured by the market damage function $D^c[x(t)]$. If parcel i is not yet invaded, only market damage occurs and the damage is area I+II. If parcel i is currently being invaded, the production level decreases from $Q(A_i)$ to $\{1 - \kappa_i[x(t)]\}Q(A_i)$ causing both

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[13] Adaptation, such as planting more resistant crop varieties or removing land from cultivation in response to the establishment of pests can be a constructive means in decreasing the damage caused by the spread of the invasion. While the general model does not rule out the possibility of adaptation, the numerical example assumes land owners are excluded from using any adaptation. In short, the numerical simulation considers intensive margin but not extensive margin responses to invasion.

market and production damages (areas I+II +III). Once fully invaded, the production damage reaches its apex, but the market damage continues to increase as the invaded area expands beyond the parcel.

Assume the total area is 18 km^2 and there are three equally sized parcels in this region, i.e., $A_1 = A_2 = A_3 = 6$ km^2. Later the number, order, and sizes of parcels are varied. Two initial invaded areas (2 km^2 and 12 km^2) are considered to illustrate the effect of species detection on the control decision. The control path is simulated in GAMS (23.9.2) using the PATH and CONOPT solvers.

The results of optimal control under collective decision making with a small and large initial invaded area are shown in Figure 6. In the small initial invasion case, the control rate is increasing from 8% to 10%, and the invasion spreads from 2 km^2 and eventually stops at the steady-state, 4.3 km^2. In the case of large initial invasion, the control rate declines from 14% to 10%, and the invasion is reversed from 12 km^2 to 4.3 km^2. In both small and large initial invasion cases, the same robust steady-state is reached.

The simulation exercise of individuals before steady-state is applied by a backward algorithm design.[14] A comparison of the individualistic and collective optimal path for invasion area and reduction rate with a small initial invasion is shown in Figure 7. The limited interest of the individual parcel owner results in a lower control path. The absolute value of individualistic control relay's costate variable is smaller than the social one, revealing the insufficient individualistic interest in controlling invasive species spread. As a result, the individual control relay steady-state invaded area is 6.9 km^2; larger than the social one (4.3 km^2).

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[14] See the Appendix for a detailed backward algorithm.

The insufficiency of privately supplied invasive species control will fluctuate with changes in the number of individual producers, the size of individual parcels, and the order that parcels are invaded. Table 4 summarizes three alternative ownership scenarios which hold the total area constant at 18 km^2. The first alternative scenario A(4.5) divides the region among more producers which decreases the size of individual parcels from 6 to 4.5 km^2. Scenarios 2 and 3 allow for heterogeneity in ownership size and differ in regards to the order in which parcels are invaded. The evolution of the invasion in each scenario is presented in Figure 8. Given a fixed size of suitable habitat, increasing the number of individual owners (A4.5) causes the individualistic control relay to deviate farther from the collective control process resulting in a larger steady state invaded area. When landowners are heterogeneous in size, the size of producer reaching the steady-state plays a big role in the invasion result - the larger her parcel, the smaller the steady-state invaded area. For example, both steady states are reached within the second parcel, but the configuration of case A(4,4,4,6) has a larger steady state invasion than the case of A(6,5,4,3) due to the smaller second parcel size.

The side payment rate and optimal sequence of side payments needed to internalize the externality in the benchmark scenario (three individual owners with 6 km^2 parcel each) is presented in Figure 9. As the invasion grows, the side payment rate does not change in response to the constant marginal market damage. But the non-invaded land diminishes as the invasion spreads, compelling the individual to adopt more control efforts. In fact, the payment from a participant is determined by his/her benefits from the control behavior. The non-controlling producers have an incentive to encourage the controlling producer to choose a higher level of control to reduce their market damage. This multiple-source side payment scheme alleviates the tight budget burden, and at the same time internalizes the externality in a way that encourages coordination among participants.

7. Conclusion

This paper has developed a dynamic control model to synthesize the biological and economic properties of invasive species, such as the spread rule, damage and control cost, and the participants' control behaviors. It is found that a spatial externality results from individual control by multiple spatially-connected producers in a regional commodity market being invaded. This outcome is due to the presence of market-level impacts from invasions such as trade restrictions, reduced demand for regional commodities, or costly phytosanitary requirements. These impacts are treated as public benefits by individual producers making invasive species control decisions. Although individual producers consider physical commodity damage from an invasion, their limited spatial consideration causes them to ignore the market-level impacts on other regional producers. A series of side payments is suggested to internalize the externality and coordinate individual control efforts in the regional market. The numerical simulation suggests three conclusions.

First, commodity markets which rely on a large number of small producers will support larger invasions. Relying on individual producers to control the spread of an invasion will be problematic in a highly competitive regional commodity market. This suggests that the trend from small family farms to larger industrial farms may actually result in smaller invasions in the future.[15]

Second, if the market is comprised of producers of various sizes, the order of invasion becomes an important factor in determining the ultimate size of the invasion. Specifically, the invaded area will be larger if the species is introduced on a smaller producer and then spreads to larger producers. This suggests that the location of the initial invasion may be just as important as the size of the initial invasion.

Third, the series of side payments needed to internalize the externality is nonincreasing function of invaded area and critically depends on the curvature of the market damage function. In contrast to much of the pollution literature, a constant rate payment will only be optimal if market damages are linear. This suggests that highly nonlinear market damages may be more difficult to internalize due to the need to adjust the payment rate based on the state of the invasion. Regardless, initiating the series of side payments early in the invasion will be critical in internalizing the externality.

As Cook et al. (2010) point out, "In terms of post-border measures, the emergence of producer biosecurity cooperatives to better cope with the heterogeneity of (potentially) affected parties may yield benefits in terms of both incentive alignment and burden sharing in response effort it self-learning decision-support systems can be developed". Our results represent an initial step in this direction. However, the spatial control of

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[15] This is consistent with Hansen and Libecap's (2004) study of the Dust Bowl which reveals that the abundance of small farms in the 1930s compromised the control of wind erosion. The limited scale of small farmers encouraged less erosion control than larger farmers. Small farms with intensive cultivation and less erosion control cause increased blowing of sand to the leeward farms and reduce their benefits of control. The collective control necessitated the establishment of soil conservation districts and improved the coordination of farmer's erosion control. In the same way, the number and size of participants will also influence invasive species control.

invasions is complex and there are several avenues for future research. Our model of one-dimensional spread may be well suited for certain invasions (aquatic invasions in a river system) but may be overly simplistic for invasions where two-dimensional spread is a major component. Recent research focused on overcoming the “curse of dimensionality” hurdle that arises in optimal control problems in two-dimensional state space appears promising (Epanchin-Niell and Wilen 2012; Brock and Xepapadeas 2010). Consideration of long-distance invasive species spread is implicit in our model but explicit consideration of the spatial-dynamic processes that lead to outlier populations and the detection of those outliers is an important area for future reaserch (Homans and Horie 2011). More experimental case studies are also important but often difficult to perform due to a lack of data.

Appendix

The Costate Variable in Invasive Species Control

The costate variable and the state variable play an important role in the optimal control path. Understanding the relation between them reveals the tradeoff associated with control and highlights the fundamental reasons for different invasive species control results. In this section, the relation between the costate variable and the state variable is discussed. Then, the components of the costate variable are analyzed.

From Equation (3), $-c_u(\cdot^*) - \omega^*(t)x^*(t) = 0$ for $u^*(t) > 0$. Using the implicit function theorem, get

$$\text{(A.1)} \qquad \frac{\partial \omega^*(t)}{\partial x} = -\frac{\omega^*(t)}{x^*(t)} > 0.$$

In Zone III of Figure 3, the initial invasion area is small and the steady state is reached within the region, therefore the invasion area grows with time t, $\omega^*(t)$ increases as the invasion increases, i.e., $\frac{d\omega^*(t)}{dt} > 0$ in Zone III. While in Zone II, the invasion area diminishes due to a high rate of control. Here $\omega^*(t)$ decreases as the invasion decreases, i.e., $\frac{d\omega^*(t)}{dt} < 0$. Based on the equation of motion for the costate variable, the optimal control process is interpreted in section 3 of the paper.

Accounting for the components of the costate variable and following Lyon (1999), start with Equation (4) with $\omega^*(t^{ss})$ given by the steady state condition that $\omega^*(t^{ss}) = \frac{-D_x(x^{ss})}{r}$. Equation (4) can be written

(A.2) $$\frac{d\omega^*(t)}{dt} - \left[r - \left(g - u^*(t)\right)\right]\omega^*(t) = D_x(\cdot^*).$$

Adding $-t\frac{du^*(t)}{dt}\omega^*(t)$ to both sides of Equation (A.2), the general solution for this differential equation is

(A.3) $$\omega^*(t) = e^{[r-(g-u^*(t))]t}\left\{\int e^{-[r-(g-u^*(t))]t}\left[D_x(\cdot^*) - t\frac{du^*(t)}{dt}\omega^*(t)\right]dt + K\right\},$$

$$= e^{[r-(g-u^*(t))]t}\left\{\int e^{-[r-(g-u^*(t))]t}\left[D_x(\cdot^*) + t\frac{du^*(t)}{dt}\frac{c_u(\cdot^*)}{x^*(t)}\right]dt + K\right\}.$$

where K is a yet to be determined constant of integration. The effective discount rate is a combination of the normal discount rate, the natural spread rate of the invasive species, and the control rate, i.e., $r - [g - u^*(t)]$. The effective discount rate can be positive, $r > [g - u^*(t)]$, negative, $r < [g - u^*(t)]$, and zero, $r = [g - u^*(t)]$. Let

(A.4) $$F(\mathrm{t}) = \int e^{-[r-(g-u^*(t))]t}\left[D_x(\cdot^*) + t\frac{du^*(t)}{dt}\frac{c_u(\cdot^*)}{x^*(t)}\right]dt$$

such that Equation (A.3) can be simplified to

(A.5) $\omega^*(t) = e^{[r-(g-u^*(t))]t}[F(t) + K]$, and

(A.6) $\omega^*(t^{ss}) = e^{\left[r-\left(g-u^*((t^{ss}))\right)\right](t^{ss})}\,[F(t^{ss}) + K]$

Since at steady state $u^*(t^{ss}) = g$, then

(A.7) $K = e^{-\mathrm{r}t^{ss}}\,\omega^*(t^{ss}) - F(t^{ss})$.

Therefore,

$$\omega^*(t) = e^{[r-(g_n-u^*(t))]t}\left[e^{-\mathrm{r}t^{ss}}\omega^*(t^{ss}) + F(\mathrm{t}) - F(t_{ss})\right] \tag{A.8}$$

$$= \overbrace{e^{-r(t^{ss}-t)-\left(g-u^*(t)\right)t}\omega^*(t^{ss})}^{\text{The instant effect}}$$

$$\underbrace{-e^{[r-(g-u^*(t))]t}\int_t^{t^{ss}} e^{-[r-(g-u^*(t))]s}\left\{D_x(\cdot^*) + s\left[\frac{rc_u(\cdot^*)}{x^*(t)} - D_x(\cdot^*)\right]\frac{c_u(\cdot^*)}{c_{uu}(\cdot^*)}\right\} ds}_{\text{The cumulative effect}}.$$

The value of the costate variable at t, $\omega^*(t)$, is composed of two components. The first is the present value of uninvaded land after the invasion has been stopped (costate variable at the steady state) which we refer to as an instant effect of invasion. This component, $e^{-r(t^{ss}-t)-\left(g-u^*(t)\right)t}\omega^*(t^{ss})$, integrates the normal discounting of time, $e^{-r(t^{ss}-t)}$, and the biological discount rate $e^{-\left(g-u^*(t)\right)t}$. Similar to “the Scarcity Effect” in the optimal control problem of exhaustible resources (Lyon, 1999), this first component captures the scarcity value of uninvaded land - an exhaustible resource. The instant effect of invasion only includes the current time’s damage, no future effect, which therefore is called an instant damage effect. Because $\omega^*(t) < 0$, a decreasing costate variable implies an increasing $\omega^*(t)$ in absolute value.The absolute value of the instant effect component increases with time and it approaches the steady-state value of $\frac{-D_x(x^{ss})}{r}$ as t approaches t^{ss}. The diminishing instant effect induces less control in early periods.

The second component is the discounted value of damage and control cost flow for an incremental increase in invaded area from t to t^{ss} which we refer to as the cumulative effect of damage and control cost. In other words, this term represents the cumulative effect of a current incremental invasion at t on damage and control cost from t to t^{ss}, i.e., $-e^{[r-(g-u^*(t))]t}\int_t^{t^{ss}} e^{-[r-(g-u^*(t))]s}\left\{D_x(\cdot^*) + s\left[\frac{rc_u(\cdot^*)}{x^*(t)} - D_x(\cdot^*)\right]\frac{c_u(\cdot^*)}{c_{uu}(\cdot^*)}\right\}ds$. It corresponds to “the Cost Effect” in the natural resource optimal control problem (Lyon, 1999). This cumulative effect increases as t approaches t^{ss}, which, due to the negativity of this component, implies the value of this component is increasing to zero and the absolute value is decreasing.

The cumulative effect gives more weight to damages early in the invasion and induces more control. The instant effect provides more weight on later control, while the cumulative effect causes more attention on earlier control. These two opposing flows interact together to decide the optimal control path.

Noting $\omega^*(t) \leq 0$, $x^*(t) \geq 0$, and $c_{uu}(\cdot) > 0$, and applying the implicit function theorem to equation (4)

(A.9) $\frac{\partial u^*(t)}{\partial \omega^*(t)} = -\frac{x^*(t)}{c_{uu}(\cdot)} < 0$, and

(A.10) $\frac{\partial u^*(t)}{\partial x^*(t)} = -\frac{\omega^*(t)}{c_{uu}(\cdot)} > 0.$

Equation (A.9) implies if at a given invasion point the decision maker values the future damages of the incremental invasion less, the control be less, i.e., as the costate variable increases, the control rate decreases ceteris paribus. But the invasion’s spread has an increasing effect on the control rate according to equation (A.10).

Also from Equation (3) when $u^*(t) > 0$,

(A.11) $u^*(t) = c_u^{-1}\big(-\omega^*(t)x^*(t)\big)$.

Totally differentiate Equation (A.11) to get

(A.12) $du^*(t) = -c_{uu}^{-1}(\cdot)x^*(t)d\omega^* - c_{uu}^{-1}(\cdot)\omega^*(t)dx^*$.

As discussed above, the optimal costate variable may increase as the invasion expands. However, the control rate increases, when $-c_{uu}^{-1}(\cdot)x^*(t)d\omega^* - c_{uu}^{-1}(\cdot)\omega^*(t)dx^* > 0$. This implies

(A.13) $d\omega^* < -\frac{\omega^*(t)}{x^*(t)}dx^*$.

In case one (the steady state reached within the region), when the initial invasion area is small ($x_0 < x_{ss}$), the optimal control process implies $dx^* > 0$, then $-\frac{\omega^*(t)}{x^*(t)}dx^* > 0$. As long as $d\omega^* < -\frac{\omega^*(t)}{x^*(t)}dx^*$, the control rate is increasing. When the initial invasion area is above the steady state, the optimal control process implies $dx^* < 0$, then $-\frac{\omega^*(t)}{x^*(t)}dx^* < 0$. As long as $d\omega^* > -\frac{\omega^*(t)}{x^*(t)}dx^*$, the control rate is decreasing.

From Equation (3) and (7)

(A.14) $\frac{du(t)}{dt} = -\frac{\frac{d\omega^*(t)}{dt}x^*(t) - [g-u^*(t)]c_u(\cdot^*)}{c_{uu}(\cdot^*)} = -\frac{x^*(t)\left\{\frac{d\omega^*(t)}{dt} + [g-u^*(t)]\omega^*(t)\right\}}{c_{uu}(\cdot^*)}$.

If the control rate is increasing over time, i.e., $\frac{du(t)}{dt} > 0$, it must be true that $\frac{d\omega^*(t)}{dt} + [g - u^*(t)]\omega^*(t) < 0$. Substituting equation (5) yields the following condition for the costate variable

(A.15) $\omega^*(t) < \frac{-D_x(\cdot^*)}{r}$.

In contrast, if the control rate is decreasing over time, i.e., $\frac{du(t)}{dt} < 0$:

(A.16) $\omega^*(t) > \frac{-D_x(\cdot^*)}{r}$.

Figure A.1 shows the control of invasive species spread as two different converging paths; one originating from a small initial invaded area ($x_0 < x^{ss}$) and the other from a large initial invaded area ($x_0 > x^{ss}$). The changing control rate and the costate variable in Figure A.1a corresponds to the optimal control process in Zone III of Figure 3. In that isosector, $\frac{du^*(t)}{dt} > 0$ only when $\omega^*(t) < \frac{-D_x(\cdot^*)}{r}$ and $d\omega^* < -\frac{\omega^*(t)}{x^*(t)}dx^*$ so the control path follows the trajectory (the dotted line) which converges to (x^{ss}, u^{ss}). Once $\omega^*(t) > \frac{-D_x(\cdot^*)}{r}$, the control deviates from Zone III to Zone IV.

If the initial invasion area is above the steady state, the optimal control process is in Zone II of Figure 3, which corresponds to Figure A.1b. Following the same logic, if $\omega^*(t) > \frac{-D_x(\cdot^*)}{r}$ the control rate is decreasing over time, and the steady-state is reached. If $\omega^*(t) < \frac{-D_x(\cdot^*)}{r}$, the control deviates from Zone II to Zone I.

Proof of Individual Control Deficiency

For the parcel $q \in [1, m-1]$, the individuals not reaching the steady-state, the nonlinear system of differential equations is

(A.17) $\frac{du^{q*}(t)}{dt} = \frac{rc_{u^q}(\cdot^*) - D_x^q(\cdot^*)x^*(t)}{c_{u^q u^q}(\cdot^*)}$,

(A.18) $\frac{dx^*(t)}{dt} = [g - u^{q*}(t)]x^*(t)$.

The nonlinear system of differential equations under the social optimal control is

(A.19) $\frac{du^*(t)}{dt} = \frac{rc_u(\cdot^*) - D_x(\cdot^*)x^*(t)}{c_{uu}(\cdot^*)}$

$$= \frac{rc_u(\cdot^*) - \{\sum_{i=1}^{I} D_x^i[x^*(t), A_i] + D_x^e(x^*(t))\}x^*(t)}{c_{uu}(\cdot^*)} ,$$

(A.20) $\frac{dx^*(t)}{dt} = [g - u^*(t)]x^*(t)$.

Compare Equation (A.17) with (A.19), at any specific *x*(*t*). If $du^{q*}(t)/dt = 0$, $u^{q*}(t)$ satisfies $rc_{u^q}(\cdot^*) = D_x^q(\cdot^*)x^*(t)$, and if $du(t)/dt = 0$, $u(t)$ ensures $rc_u(\cdot^*) = \{\sum_{i=1}^{I} D_x^i[x^*(t), A_i]\}x^*(t)$.By the properties of the individual damage function and the social damage function, $\sum_{i=1}^{I} D_x^i[x^*(t), A_i] > D_x^q(\cdot^*)$for any $q \in [1, m-1]$, implying that at specific *x*(*t*), $rc_{u^q}(\cdot^*) < rc_u(\cdot^*)$. With non-decreasing marginal control cost functions $u^q|_{\frac{\mathrm{du}^{q*}(\mathrm{t})}{\mathrm{dt}}=0} < u|_{\frac{du}{dt}=0}$ for the same *x*(*t*).This implies the $du^{q*}(t)/dt = 0$ isocline is below the $du(t)/dt = 0$ isocline for any $q \in [1, m-1]$.

For the parcel *m*, the individual reaching the steady-state, the nonlinear system of differential equations under the individual control is

(A.21) $\frac{du^{m*}(t)}{dt} = \frac{rc_{u^m}(\cdot^*) - D_x^m(\cdot^*)x^*(t)}{c_{u^m u^m}(\cdot^*)}$,

(A.22) $\frac{dx^*(t)}{dt} = [g - u^{m*}(t)]x^*(t)$.

Compare Equation (A.21) with (A.19), at a specific *x*(*t*), if $du^{m*}(t)/dt = 0$, $u^{m*}(t)$ satisfies $rc_{u^m}(\cdot^*) = D_x^m(\cdot^*)x^*(t)$, and if $du(t)/dt = 0$, $u(t)$ ensures $rc_u(\cdot^*) = \{\sum_{i=1}^{I} D_x^i[x^*(t), A_i]\}x^*(t)$. By the properties of the individual damage function and the social damage function, $\sum_{i=1}^{I} D_x^i[x^*(t), A_i] > D_x^m(\cdot^*)$. Just as the proof of *q* parcel, $u(t) > u^{m*}(t)$ for any *x*(*t*) implying

(A.23) $u(t)|_{\frac{du^*(t)}{dt}=0, x(t)} > u^{m*}(t)|_{\frac{du^{m*}(t)}{dt}=0, x(t)}$

which implies the $du^{m*}/dt = 0$ isocline is below the $du/dt = 0$ isocline.

At the social steady state, $u(t) = g$; and at the individual steady-state, $u^{m*}(t) = g$. The individual and social steady-states are characterized by $rc_{u^m}(g) = rc_u(g)$ and

(A.24) $\left\{\sum_{i=1}^{I} D_x^i[x^{ss}(t^{ss}), A_i]\right\} x^{ss}(t^{ss}) = D_x^m\left(x_m^{ss}(\tau_m^{ss})\right) x_m^{ss}(\tau_m^{ss})$.

But $\sum_{i=1}^{I} D_x^i[x^*(t), A_i] > D_x^m(\cdot^*)$ at any $x(t)$, and $D_{xx}(\cdot) > 0$ and $D_{xx}^m > 0$ for all $m \in [1, I]$. This implies that equation (A.24) can only hold when

(A.25) $x^{ss}(t^{ss}) < x_m^{ss}(\tau_m^{ss})$,

for any $m \in [1, I]$.

Q.E.D

The Backward Algorithm

As in the theoretical analysis, an individual control relay reaches the steady-state at parcel m. The simulation exercise of all individuals before steady-state is applied by a backward algorithm design. The backward algorithm is performed through these steps:

1. Calculate the control path of individual m and the steady-state of x_{ss}^m.

   Calculate individual m's costate variable at τ_{m-1}, the initial costate variable's value under individual m's control. Using Equation (19) get

   $$R(A_{m-1}) - D_{m-1}[x^*(\tau_{m-1}), A_{m-1}] - c[u_{m-1}^*(\tau_{m-1})] \\ + \omega_{m-1}^*(\tau_{m-1})\dot{x}^*(\tau_{m-1}) = rs_{m-1}[x(\tau_{m-1})]$$

   and $x(\tau_{m-1}) = \sum_{i=1}^{m-1} A_i$.

2. Calculate individual $m-1$'s control rate, state variable, and costate variable from time $\tau_{m-1} - 1$ to τ_{m-2} by using Equations (16), (17), (18),) and $\omega^{m-1*}(\tau_{m-1})$, and $x(\tau_{m-1})$.

$$\text{(A.26)} \quad \begin{cases} u^{m-1}(\tau_{m-1}-1) = \sqrt[2]{\frac{-\omega^{(m-1)*}(\tau_{m-1}-1)x^{*}(\tau_{m-1}-1)}{3\eta}} \\ \omega^{m-1*}(\tau_{m-1}-1) = \frac{\omega^{m-1*}(\tau_{m-1})-P\alpha-azx(\tau_{m-1}-1)^{a-1}\frac{A_{m-1}}{\sum_{i=1}^{I}A_i}}{1+r-\left(g-u^{m-1*}(\tau_{m-1}-1)\right)} \\ x(\tau_{m-1}-1) = \frac{x(\tau_{m-1})}{1+g-u^{m-1*}(\tau_{m-1}-1)} \end{cases}$$

3. Reiterate steps 2 and 3 from individual $m-2$ to the initial invaded owner.

$$\text{(A.27)} \quad R(A_{m-2}) - D_{m-2}[x^{*}(\tau_{m-2}), A_{m-2}] - c[u_{m-2}^{*}(\tau_{m-2})] + \omega_{m-2}^{*}(\tau_{m-2})\dot{x}^{*}(\tau_{m-2}) = rs_{m-2}[x(\tau_{m-2})]$$

$$x(\tau_{m-2}) = \sum_{i=1}^{m-2} A_i$$

$$\text{(A.28)} \quad \begin{cases} u^{m-2}(\tau_{m-2}-1) = \sqrt[2]{\frac{-\omega^{(m-2)*}(\tau_{m-2}-1)x^{*}(\tau_{m-2}-1)}{3\eta}} \\ \omega^{m-2*}(\tau_{m-2}-1) = \frac{\omega^{m-2*}(\tau_{m-2})-P\alpha-azx(\tau_{m-2}-1)^{a-1}\frac{A_{m-2}}{\sum_{i=1}^{I}A_i}}{1+r-\left(g-u^{m-2*}(\tau_{m-2}-1)\right)} \\ x(\tau_{m-2}-1) = \frac{x(\tau_{m-2})}{1+g-u^{m-2*}(\tau_{m-2}-1)} \end{cases}$$

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Table 1 Variables in theoretical model.

Variable	Definition
$x(t)$	Invasion area at time t.
$\kappa_i[x(t)]$	The percentage of physical production loss in parcel i at time t.
$u(t)$	Collective control rate at time t.
$u_i(t)$	Individual i's control rate at time t.
$\omega(t)$	Collective costate variable- the social shadow cost of an incremental increase in invasion at time t.
$\omega_i(t)$	A costate variable of individualistic control- individual i's shadow cost of an incremental increase in invasion at time t.
τ_{i-1}	The time of invasion reaching the west border of parcel i.
τ_i	The time of invasion reaching the east border of parcel i.
τ_m^{ss}	The time of reaching steady-state under the individualistic control relay.
t^{ss}	The time of reaching steady-state under collective control.
x^{ss}	The steady state of invaded area under collective control.
x_m^{ss}	The steady state of invaded area under the individualistic control relay.

Table 2. Parameter values for numeric simulation

parameter	Definition	value
r	The discount rate	5%
g	The natural spreading rate of invasive species, a constant parameter.	10%
A_i	A parameter- parcel i's area.	6km^2
P	The price of commodity before invasion.	\$20 thousands
α	A parameter- a constant percentage of reducing production due to the invasion.	15%
z	A parameter which adjust the severity of market damage function.	$d_1 = 0.2 \times P = 4$
a	A parameter which adjusts the concavity of market damage function.	1
η	A parameter of the variable control cost.	\$20,000 thousands

Table 3. Alternative land ownership scenarios

Type	Notation	number of owners	Individual parcel	Total area
Benchmark	A(6,6,6)	3	A_1=6 A_2=6 A_3=6	18
Scenario 1	A(4.5×4)	4	A_1=4.5 A_2=4.5 A_3=4.5 A_4=4.5	18
Scenario 2	A(6,5,4,3)	4	A_1=6 A_2=5 A_3=4 A_4=3	18
Scenario 3	A(4,4,4,6)	4	A_1=4 A_2=4 A_3=4 A_4=6	18

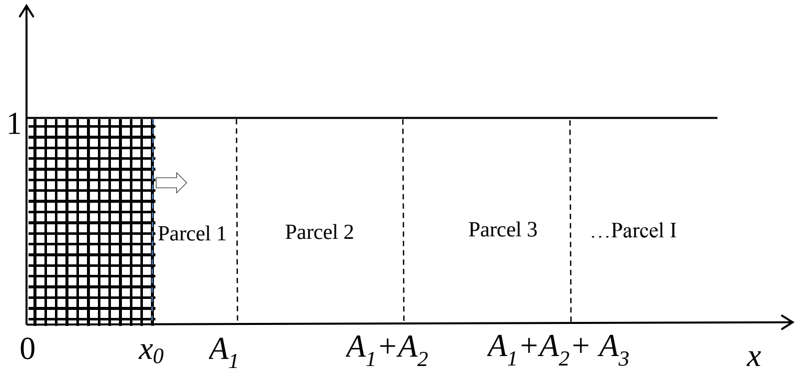


Figure 1. Species invasion across multiple management jurisdictions.

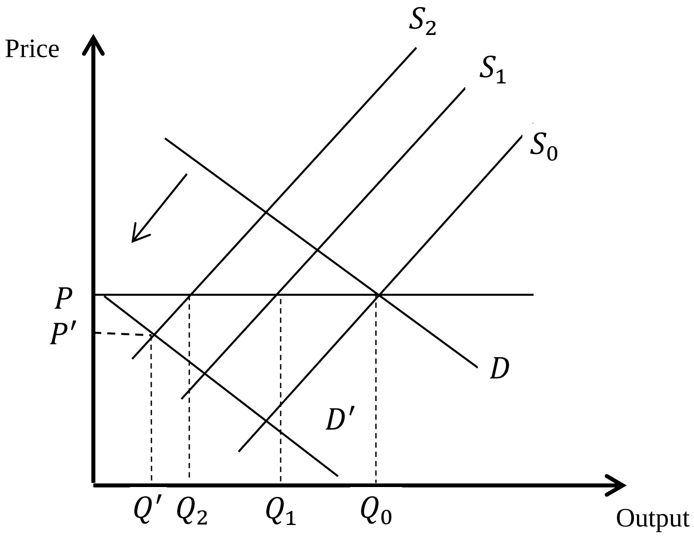


Figure 2. Physical damages and market response to invasion in a regional commodity market.

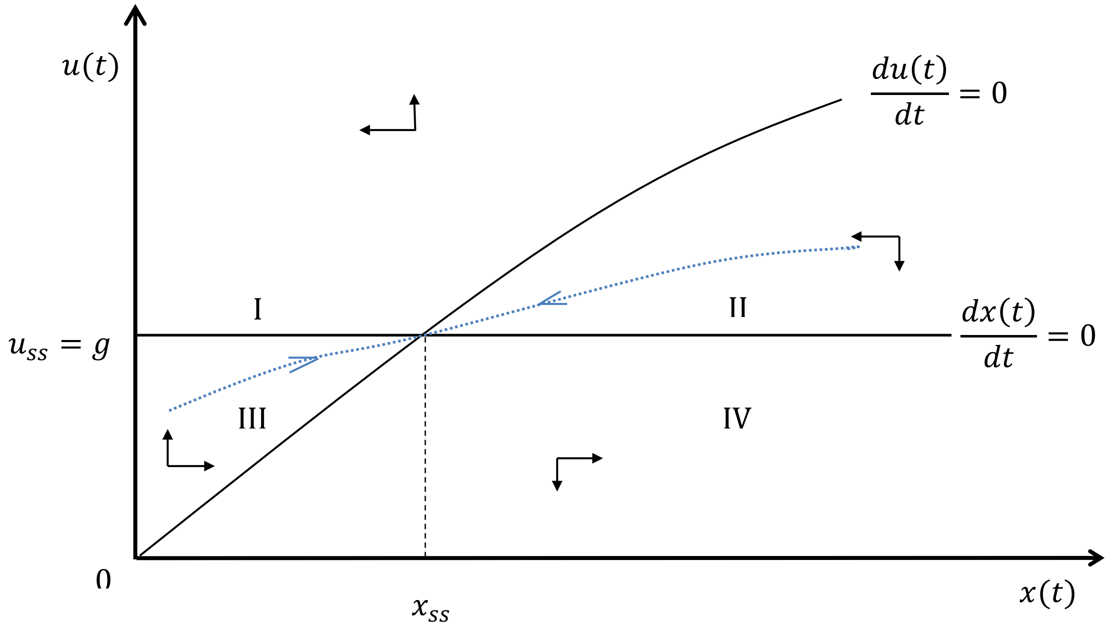


Figure 3. Phase diagram of the system: $\frac{dx(t)}{dt} = 0$ and $\frac{du(t)}{dt} = 0$

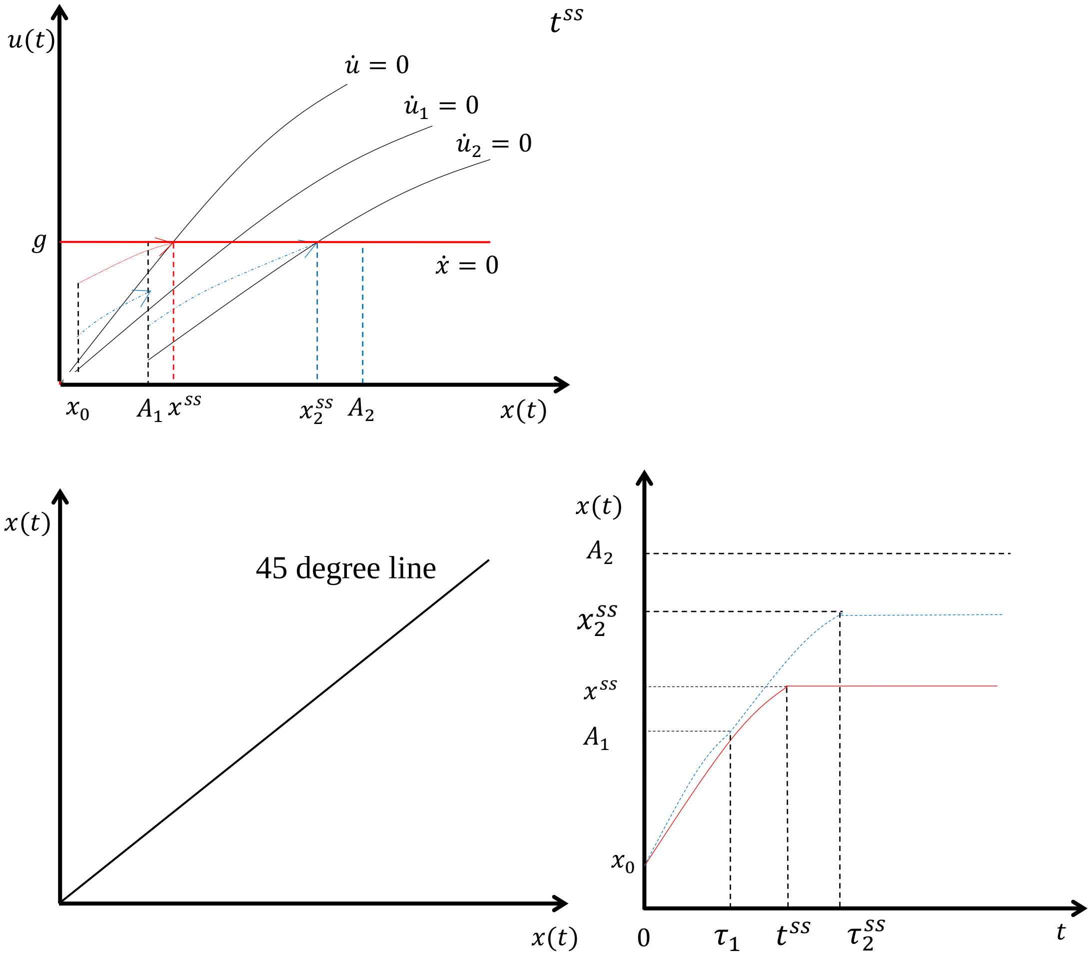


Figure 4. Collective and individual control process for hypothetical invasion

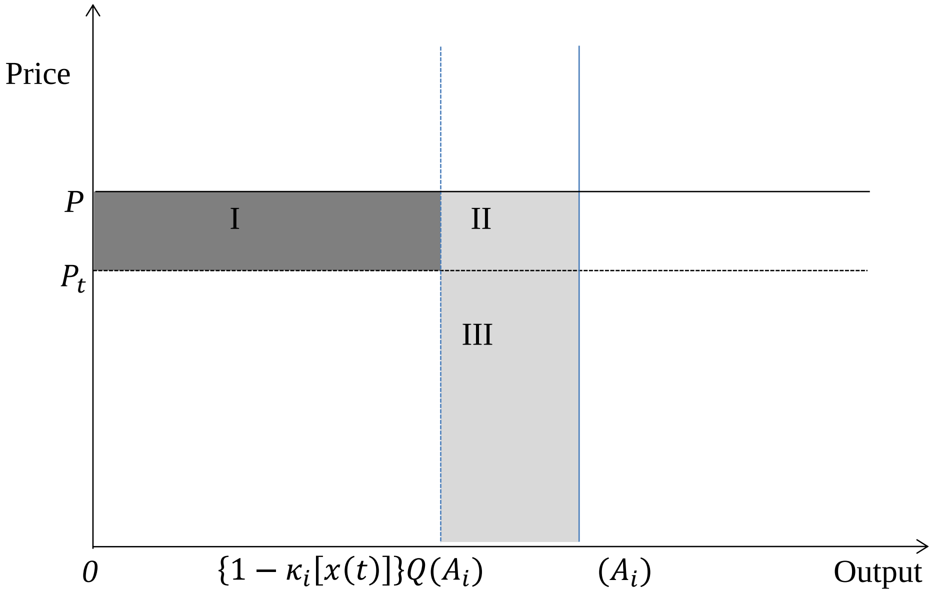
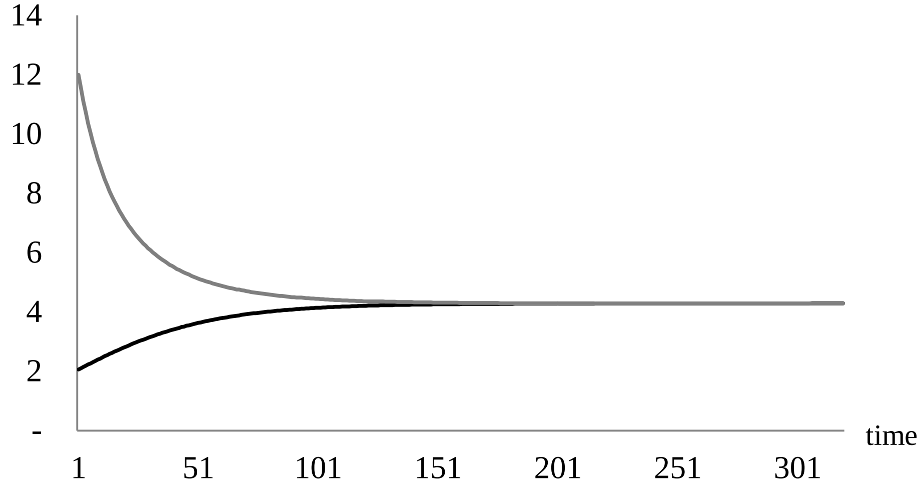
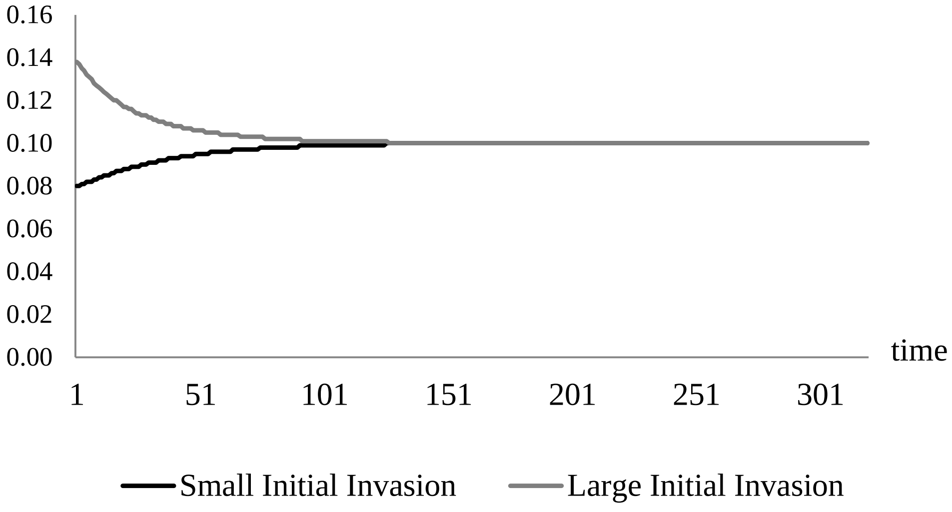


Figure 5. Market and production damage on parcel i.



(a) Invaded area x (km^2)



(b) Control rate u (%)

Figre 6. Simulation results for collective control of invasion

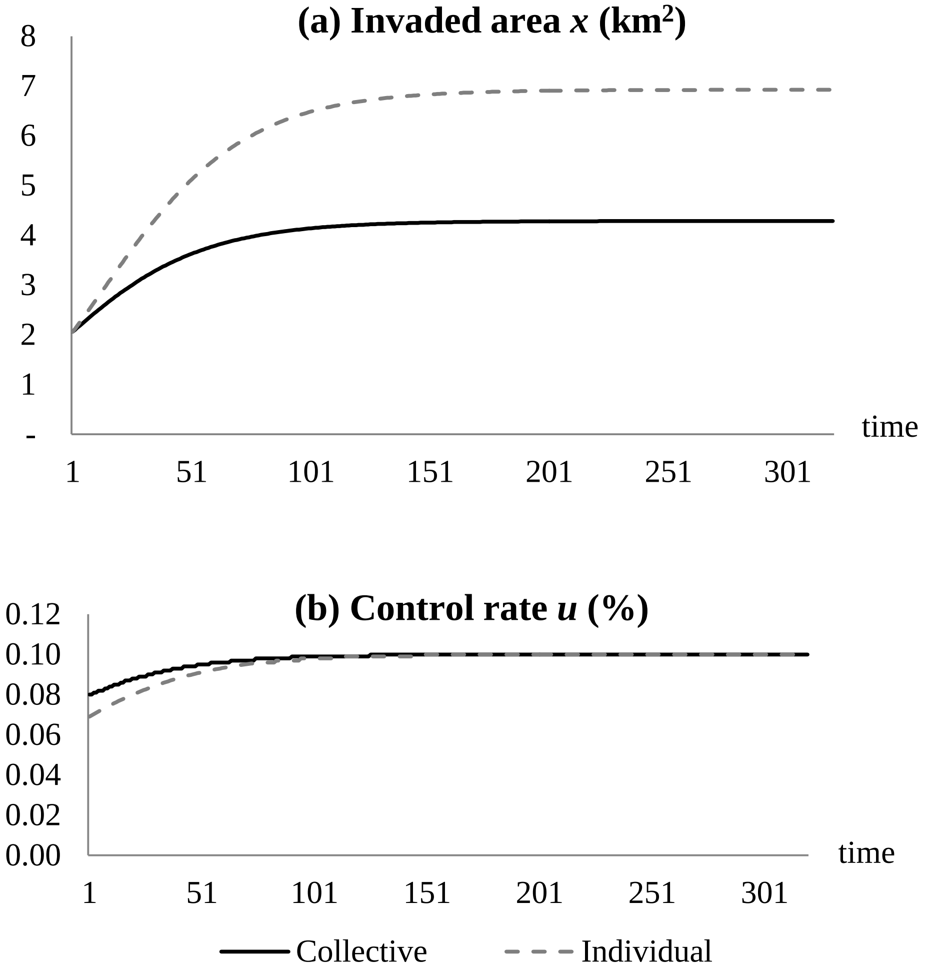


Figure 7. Simulation results for individual control relay

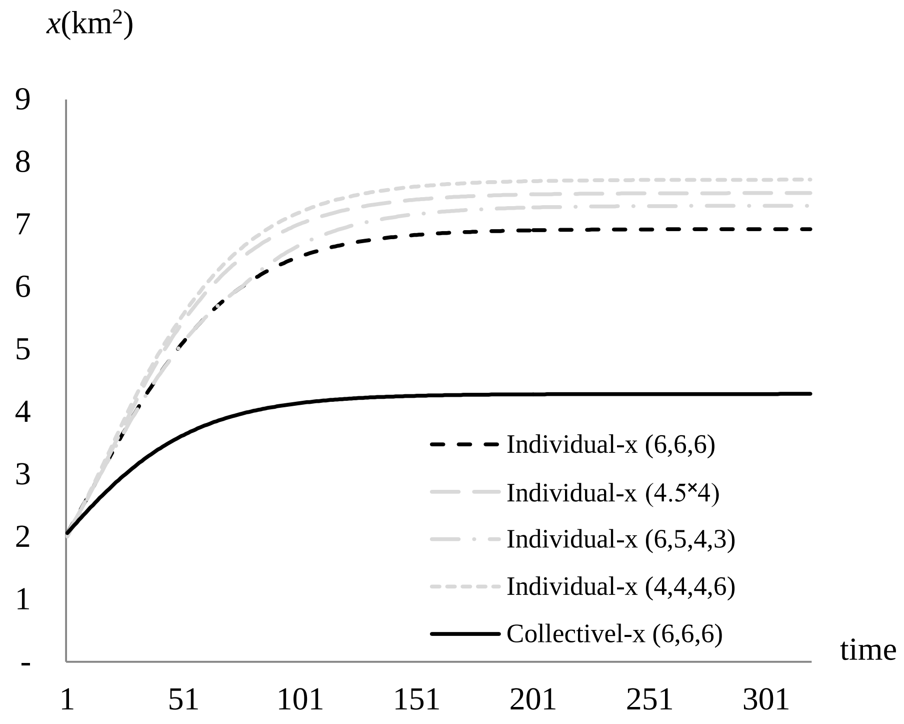
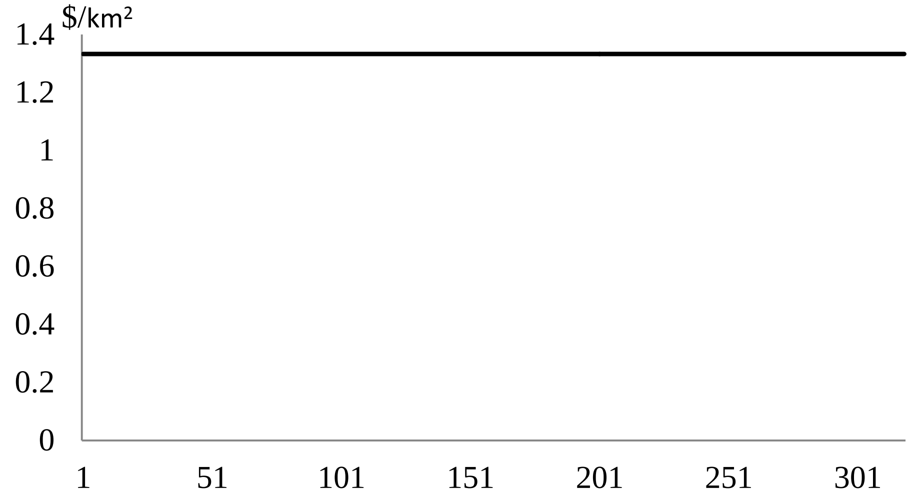


Figure 8. Invasion spread under various ownership scenarios

(a) Side payment rate



(b) Side payment of producer 2 or 3

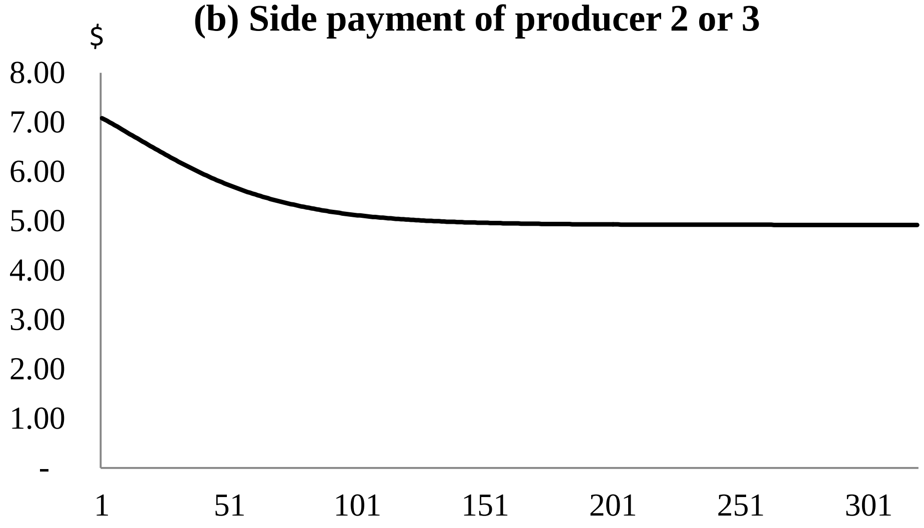
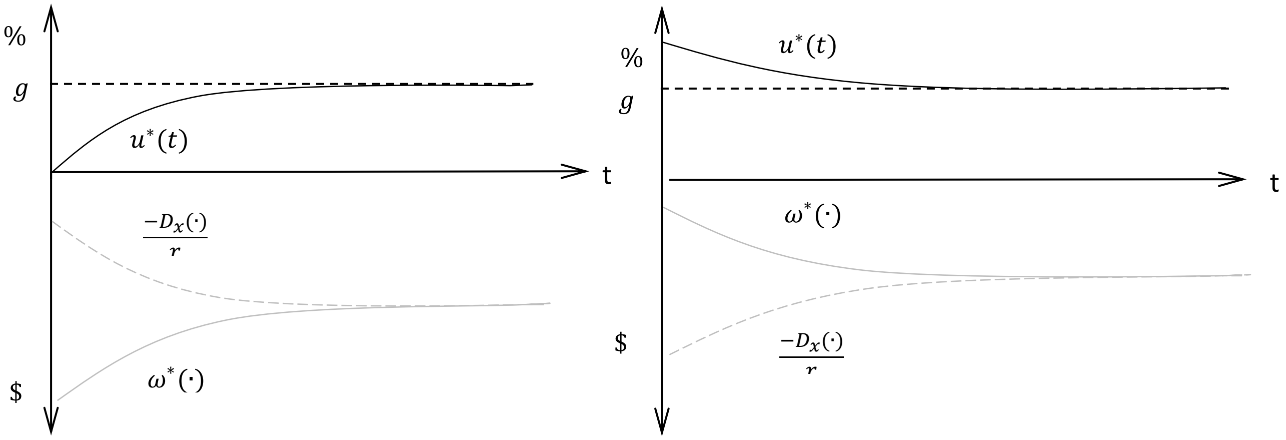


Figure 9. Rate individual producers are compensated for preserving uninvaded area under the optimal sequence of side payments.



(a) A Small initial invasion area. (b) A large initial invasion area.

Figure A.1 The trajectory of optimal control rate and costate variable of case one.