2013

RSMG Working Paper Series

Risk and Uncertainty Program

TITLE: The value of information and the value of awareness

Author:

John Quiggin

Working Paper: R13_2

Schools of Economics and Political Science

The University of Queensland

St Lucia

Brisbane

Australia 4072

Web: www.uq.edu.au

Abstract

Recent literature has examined the problem facing decisionmakers with bounded awareness, who may be unaware of some states of nature. A question that naturally arises here is whether a value of awareness (VOA), analogous to VOI, can be attributed to changes in awareness.In this note it is shown, in a sense that will be made precise, that the sum VOA+VOI is constant and independent of the choice set. It follows that, the greater is VOA, the less is VOI. This point is illustrated for a simple two-state case, then proved for general classes of choice sets. The analysis is then extended to cover altenative concepts of choice under unawareness.
Keywords:value of information, awareness, JEL code: D8

The value of information and the value of awareness

John Quiggin
School of Economics
University of Queensland, Brisbane 4072 Australia
+61 7 400747165 (ph), + 61 7 33556473(fax)
j,quiggin@uq.edu.au

18 January 2013

The value of information and the value of awareness

In a modern economy, economic agents routinely make important decisions under conditions of uncertainty. Such problems have been addressed by the literature on the economics of information, beginning with the classic work of Blackwell (1951). In the standard Bayesian framework developed by Blackwell, individuals are endowed with a prior probability distribution over a set of possible states of the world. Before making a decision, they may observe information that enables them to update their prior beliefs, yielding a posterior distribution. The difference between the expected return to the optimal decision based on prior beliefs and the expected return based on posterior beliefs is the value of information (VOI).

Recent analysis of choice under uncertainty has challenged the assumption that individuals are aware of all possible states of the world. A growing literature has dealt with the analysis of beliefs and preferences in situations where individuals may be unaware of some possible states of the world (Grant and Quiggin 2013, Halpern and Rêgo 2008, Karni and Viero 2012).[1] This raises the possibility that individuals will become aware of new possibilities and raises the question of how they should adjust their beliefs in response. A question that naturally arises here is whether a value of awareness (VOA), analogous to VOI, can be attributed to changes in awareness.

In this paper, I examine VOA, represented as the improvement in expected return when a decisionmaker takes all possible states of nature into account in selecting a state-contingent income vector from a choice set. I show, in a sense that will be made precise, that the sum VOA+VOI is constant and independent of the choice set. It follows that, the greater is VOA, the less is VOI. This point is illustrated for a simple two-state case, then proved for general classes of choice sets.

An alternative approach to unawareness is to suppose that individuals have access to a partial description of the world, with the result that their representation of the state space is coarser than the full state space that would be considered by an unboundedly rational decision maker (Heifetz, Meier, and Schipper 2006). The implications of this representation for VOA and VOI are also considered.

1 Illustrative example

Consider a risk-neutral decisionmaker choosing over a set of state-contingent income vectors $Y \sqsubseteq \mathbb{R}^2_+$. Assume Y is strictly convex, satisfies free out-

[1] In the case where state-contingent outcomes are bundles of commodities or characteristics, rather than monetary values, individuals might be unaware of some possible consequences, a case considered by Karni and Viero (2012). In common with most of the literature on the value of information, however, this paper focuses on the case where outcomes are expressed in monetary terms.

put disposal and is symmetric, in the sense that if $(y_1, y_2) \in Y$ then also $(y_2, y_1) \in Y$. Further assume that the two states of nature are equiprobable, and measure the decisionmaker's expected welfare as $\frac{1}{2}y_1 + \frac{1}{2}y_2$.

Define

$$\begin{aligned} y^* &= \max\{y : (y, 0) \in Y\} = \max\{y : (0, y) \in Y\} \\ \bar{y} &= \max\{y : (y, y) \in Y\} \end{aligned}$$

We now consider three cases

(a) Unawareness. The decisionmaker is only aware of one state of nature. By symmetry we can assume that this is state 1, in which case the decisionmaker will chooses the output $(y^*, 0)$ and receives the expected return $\frac{y^*}{2}$.

One way of interpreting this is that the decisionmaker perceives the technology as the projection of Y onto the axis corresponding to the state of which he is aware, that is, the set

$$\begin{aligned} Y_1 &= \{y_1 : \exists y_2 \geq 0 : (y_1, y_2) \in Y\} \\ &= [0, y^*] \end{aligned}$$

(b) Full awareness with no information. The decisionmaker is aware of both possible states and has prior probabilities (p_1, p_2). As noted, we assume that she judges them as equiprobable, so $p_1 = p_2 = \frac{1}{2}$. He chooses

$$\hat{\mathbf{y}} = \arg\max_{\mathbf{y} \in Y} \{p_1 y_1 + p_2 y_2\}$$

Given the stated assumptions the maximizer is $(\bar{y}, \bar{y})$ and the decisionmaker's expected welfare is $\bar{y}$

(c) Full awareness and full information. Before choosing (y_1, y_2) the decisionmaker observes the state of nature $s \in \{1, 2\}$. She then chooses to produce y^* in state s and 0 in state $3 - s$, deriving expected welfare y^*

For this simple model, we may derive

$$\begin{aligned} VOA &= \bar{y} - \frac{y^*}{2} \\ VOI &= y^* - \bar{y} \\ VOA + VOI &= \frac{y^*}{2} \end{aligned}$$

We can normalize by setting set $y^* = 2$, so that $VOA + VOI = 1$. Subject to this normalization, the total gain $VOA + VOI$ is independent of the choice set and in particular of the degree of substitutability between the state-contingent claims y_1 and y_2. With this normalization, VOA represents the proportion of the total gain due to full awareness and VOI the proportion due to full information.

The crucial observation is that the value of awareness is perfectly negatively correlated with the value of information. The difference between the return with minimal awareness and the return with full awareness and full information is independent of the choice set (up to the arbitrary normalization described above).

Both VOA and VOI must lie in the interval $[0,1]$. We now show that these bounds are sharp.

First, consider the set

$$Y = \{(y_1, y_2) : y_1 + y_2 = 2\}$$

bounded above by the dashed diagonal line in Figure 1. This is the choice set that would arise in the presence of a complete market for state-contingent claims with actuarially fair pricing. In this case, $y^* = 2$, $y = 1$, $VOA = 0$, $VOI = 1$.

Conversely, consider the set $\{(y_1, y_2) : y_1, y_2 \leq 2\}$ bounded by the dotted lines in Figure 1. Given any sequence of strictly convex sets Y that converge to the set $\{(y_1, y_2) : y_1, y_2 \leq 2\}$, we must have $y \to y^* = 2$ and hence $VOA \to 1$, $VOI \to 0$.

2 Main result:

For any S, and $y^* = (y_1^*, ...y_S^*)$ define $\Theta(\mathbf{y}^*)$ to be the class of convex choice sets Y such that, for each s

$$y_s^* = \max\{(y : \exists \mathbf{y} \in Y, y_s = y)\}$$

Then let π be a probability distribution over S. Assume, that a minimally aware decision maker will be aware of state s with probability π_s and also that state s will be realized with probability π_s.

Then we have:

Proposition 1 *For any $Y \in \Theta(\mathbf{y}^*)$ and probability distribution π,*

$$VOA(Y) + VOI(Y) = \sum_s \pi_s (1 - \pi_s) y_s^*$$

Observe that, for the two-state example given above, $\pi_1 = \pi_2 = \frac{1}{2}$ and $y_1^* = y_2^* = 1$, so $VOA + VOI = 1$ as derived above.

For comparative statics, we have

Proposition 2 *Let $Y, Y' \in \Theta(\mathbf{y}^*)$. Then the following are equivalent.*
(i) $Y \subseteq Y'$
(ii) $VOA(Y) \leq VOA(Y'), VOI(Y) \geq VOI(Y')$

The results derived above may usefully be considered in the light of state-contingent production theory (Chambers and Quiggin 2000). Considering Y as the set of state-contingent outputs that may be produced with a given ex ante input, the value of information about the true state of nature is greatest when the state-contingent outputs are substitutes in production. In this case, information about the true state of nature enables the producer to divert resources towards production of the state-contingent output for that state. The polar case is that of a state-allocable technology, giving rise to a linear choice set. By contrast, the value of awareness about possible states of nature is greatest when the state-contingent outputs are complements. In this case, if the decisionmaker becomes aware of a previously unconsidered state of nature, the marginal cost of generating the associated state-contingent output is small. The polar case is that of a stochastic production function, where the choice set is 'output-cubical' having a kink at the unit vector with all outputs equal.

3 Unawareness as coarsening

There has been little formal analysis of choice in the case when unawareness is represented as a coarsening of the state space. Considering the illustrative example, the question arises as to how an individual who does not distinguish between states 1 and 2 will evaluate elements $(y_1, y_2) \in Y$. One approach is suggested by the informal analysis of choice provided in an example by Heifetz, Meier, and Schipper (2006) who examine willingness to trade between two individuals, each of whom is unaware of a contingency that might affect the value of the asset in question. In the Heifetz, Meier, and Schipper (2006) example, individuals disregard the states of nature in which this contingency changes the value of the asset

With this treatment, the analysis of VOA and VOI is the same as that given above. Unawareness is represented as the projection of the choice set onto the subspace in which unconsidered contingencies take their 'default' values.

An alternative approach would be to suppose that individuals consider only elements of their choice set that do not differ with respect to unconsidered contingencies. Such choices are represented in Figure 1 by the grey diagonal line through the origin satisfying $y_1 = y_2$. In the specific case of the illustrative example, an individual unaware of the distinction between states 1 and 2 would choose $(\bar{y}, \bar{y})$. Given the assumptions of symmetry and equal probabilities, this is the same choice as $\hat{\mathbf{y}} = \arg\max_{\mathbf{y} \in Y} \{p_1 y_1 + p_2 y_2\}$ the optimal decision for a fully aware, but uninformed, decision maker. Thus the value of awareness is zero.

In the terminology of Chambers and Quiggin (2000), the choice set in the example is *not inherently risky.* That is, although risky choices are open to the decisionmaker, she can do no better, in terms of expected return, than to pick the riskless option. If the choice set is not inherently risky, a decisionmaker who is unaware of the existence of risky choices is no

In general, the riskless option will not maximize expected return. Quiggin and Chambers (1998) show that the riskiness of the choice set may be chararacterized by a certainty premium, defined as the reduction in expected return the decision-maker would have to accept in order to secure a riskless outcome. In the case, where the decisionmaker is initially unaware of risky options, the value of awareness is exactly equal to the certainty premium defined by Quiggin and Chambers (1998).

For the case of a symmetric technology with unequal probabilities[2], it is easy to see

$$VOA + VOI = y^* - \bar{y}$$

where, as before y^* is the maximum return available to a fully aware and informed decisionmaker, who knows the true state of nature, and $\bar{y}$ is the maximum riskless return available to an uninformed and unaware decisionmaker. Hence, as in the case considered above, VOI and VOA are perfectly negatively correlated.

4 References

Blackwell, D. (1951), 'Comparison of Experiments', Proceedings of the Second Berkeley Symposium on Mathematical Statistics, 93–102.

Chambers, R.G. and Quiggin, J. (2000) *Uncertainty, Production, Choice and Agency: The State-Contingent Approach*, Cambridge University Press, Cambridge.

Grant, S. and Quiggin, J. (2013), 'Inductive reasoning about unawareness', *Economic Theory*, forthcoming.

Halpern, J. and Rêgo, L. (2008), 'Interactive unawareness revisited', *Games and Economic Behavior*, 62(1), 232–62.

Heifetz, A., Meier, M. and Schipper, B. (2006), 'Interactive Unawareness', *Journal of Economic Theory*, 130(1), 78–94.

Karni, E. and Viero, M.-L. (2012), "'Reverse Bayesianism": A Choice-Based Theory of Growing Awareness', Working Paper, Johns Hopkins.

Quiggin, J. and Chambers, R.G. (1998), 'Risk premiums and benefit measures for generalized expected utility theories', *Journal of Risk and Uncertainty*, 17(2), 121–37.

[2]With appropriate subdivision of the state space into equally-probable components, consideration of this case involves no loss of generality.

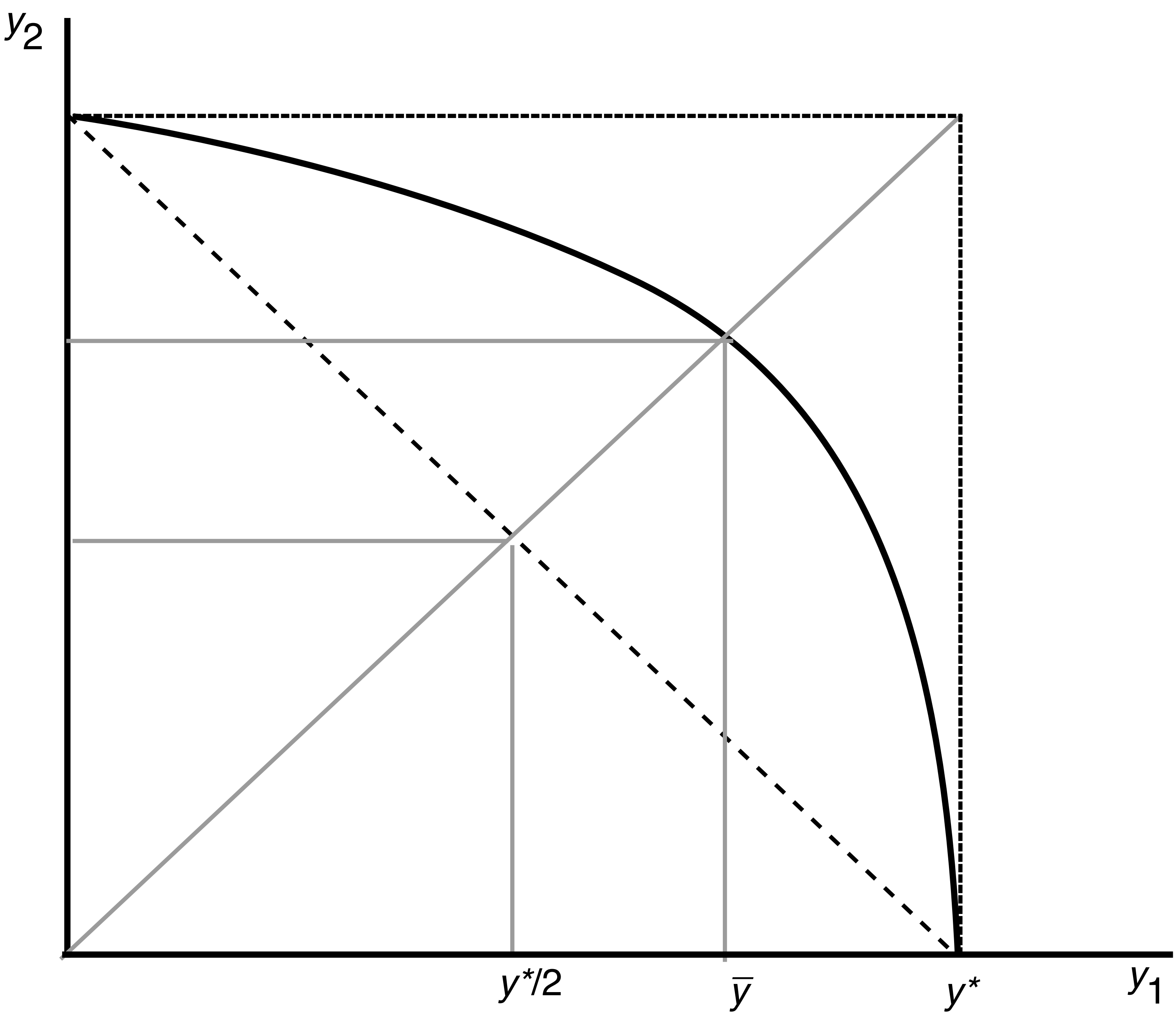

Figure 1: Illustrative example of choice set