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Authors:

Flavio Menezes, John Quiggin

Working Paper: R13_1

2013

RSMG Working Paper Series

Risk and Uncertainty Program

Schools of Economics and Political Science

The University of Queensland

St Lucia

Brisbane

Australia 4072

Web: www.uq.edu.au

Inferring the strategy space from market outcomes

Flavio Menezes John Quiggin

17 January 2013

Abstract

In this paper we show that, if demand varies stochastically, and firms compete after the realization of demand shocks, the strategy space may be inferred from market evidence. The key idea is that, in equilibrium, each firm acts as a monopolist, choosing the optimal price-quantity combination from a residual demand curve determined by a given observation of market demand and the (equilibrium) strategies of the other firms.

JEL Code L11

Keywords: competition, supply schedules, equilibrium locus

1 Introduction

A central issue in the analysis of oligopoly is the identification of the strategy space. Most of the literature has focused on the polar cases where the strategic variable is either quantity (Cournot 1838) or price (Bertrand 1883). However, from the 1980s onwards, a substantial literature has developed dealing with various forms of competition in supply schedules (Grossman 1981, Robson 1981, Turnbull 1983, Klemperer and Meyer 1986; 1989, Grant and Quiggin 1996; 1997, Vives 1986, Menezes and Quiggin 2007, 2011).

In the general case where any supply schedule is admissible, there is little definite that can be said. Klemperer and Meyer (1989) show that, in this case, any market-clearing outcome where no firm makes negative profits can be supported as a Nash equilibrium.

One way of reducing the multiplicity of equilibria is to introduce demand uncertainty. Typically, if demand is subject to additive shocks, and the supply curve is required to define an equilibrium for every possible value of the shock, a unique equilibrium will emerge. Robson proved the first result of this kind showing that, if the supply curve is required to be linear, with slope and intercept as strategic variables, and marginal costs are constant, the unique equilibrium is Bertrand. Turnbull showed that, for the case of quadratic costs (linearly increasing marginal costs) Robson's supply function equilibrium coincides with the consistent conjectures equilibrium of Bresnahan (1981). Klemperer and Meyer generalized these results to the case when the strategy space admitted arbitrary one-dimensional manifolds in price-quantity space as supply schedules. As in Robson (1981), the case of constant marginal costs yields Bertrand as the unique equilibrium.

An alternative approach is to restrict the strategy space, for example by assuming that the slope of the supply curve (that is, effectively, the competitiveness of the market) is fixed, perhaps in an earlier stage of a multi-stage game (Kao, Menezes and Quiggin 2012a). In this case, varying β, the slope of the supply curves in the strategy space yields a family of equilibria, ranging from Cournot ($\beta = 0$) to Bertrand ($\beta = \infty$). Negatively sloped supply curves, corresponding to 'meet the competition' pricing strategies, may also be considered. (Kao, Menezes and Quiggin 2012b)

By contrast with the approach of Robson and KM, this approach is appropriate in the case where firms can make strategic choices after observing shocks to demand. In this context, the assumptions that the strategy space consists of linear supply schedules, and that the demand curve is linear, do not entail any loss of generality. The relevant aspects of the problem are completely specified by the slope and location of the supply and demand schedules at the equilibrium allocation.

It remains to be considered how the competitiveness of the market, that is, the value of β may be determined. In this paper we show that, if demand varies stochastically, and firms compete after the realization of demand shocks, the strategy space may be inferred from market evidence.

The key idea is that, in equilibrium, each firm acts as a monopolist, choosing the optimal price-quantity combination from a residual demand curve determined by a given observation of market demand and the (equilibrium) strategies of the other firms. Aggregating across firms, we can determine an equilibrium relationship between price and quantity for any particular realization of the demand shock.[1] We can then back out the value of β or, more precisely, the value of β imputed by firms to their competitors.

The paper is organized as follows. We begin by formulating the firm's problem in the general case, and solving for the optimal price-quantity pair. We then derive the equilibrium relationship between market price and aggregate quantity for the cases of monopoly and symmetric oligopoly, allowing both additive shocks to demand and shocks to the slope of the demand curve. We consider some implications for empirical work and directions for future research.

2 Model

We begin by examining a standard oligopoly problem with linear demand, and N firms, producing output at zero cost. We write the inverse demand function as:

$$p = a - b[q_1 + ... + q_N], \tag{1}$$

The strategy space for each firm consists of all linear supply schedules with a given slope β. More precisely, we specify the strategic choice for firm i as a choice of supply schedules, parametrized by the strategic variable α_i as follows:

$$q_i = \alpha_i + \beta p \tag{2}$$

where the strategic variable α_i is a scalar variable representing upward or downward shifts in supply and $\beta \geq 0$ is an exogenous parameter reflecting the intensity of competition. The slope of the residual demand curve facing any given firm is determined by the slopes of the demand schedule and of

[1] Busse (2012) considers the comparative static properties of the equilibrium locus for the special cases of monopoly and Cournot oligopoly.

the supply schedules of other firms. The parameter β may, therefore, be interpreted as representing the aggressiveness of competition in the market.

The assumption of linear demand and supply schedules simplifies the analysis without any substantive loss of generality. The crucial assumption is that the strategy space for each firm consists of a family of smooth non-intersecting concave supply schedules, including all potentially optimal price-quantity pairs.[2] Given this assumption, non-linear demand and supply curves can always be replaced with the linear approximation applicable at the unique equilibrium.

It is crucial to observe that the importance of β is not as a description of the way each firm regards its own supply decisions, but as a description of how it perceives the strategic choices of others. For any given firm i, the vector α_{-i} representing the strategic choices of the other firms determines a residual demand curve. Given any perceived strategy space rich enough to allow the selection of any point on the residual demand curve, the firm's best response will yield the same equilibrium price and quantity. This point was first made by Klemperer and Meyer (1989) considering the duopoly problem when the possible strategic variables are prices and quantities:

> *The equilibria are supported by each firm's choosing the strategic variable that its rival expects; although each firm sees its own choice between price and quantity as irrelevant,its choice is not irrelevant to its rival because the choice determines the residual demand that its rival faces.*

This point may be applied to our model, to say that each firm's perception of the slope of its own supply curve is irrelevant, but the perception of its slope is relevant to other firms in the market. Given the market demand and the equilibrium strategies of other players, each firm picks a pair (p, q_i) from the residual (inverse) demand curve determined by market demand and the decisions of other players. The firm may conceive of itself as choosing a price, quantity, markup or any well behaved function of the market price and its own output quantity. In doing so, the firm seeks to maximise profit, and therefore acts exactly like a monopolist faced with the same demand curve (in fact, monopoly may be treated as a special case, as shown below).

For the linear case, the inverse residual demand curve for firm i will take the form

$$p = \theta_i - \gamma_i q_i$$

[2] Busse (2012) also considers the equilibrium locus, for the special cases of monopoly and Cournot oligopoly, and observes the link to conjectural variations. We became aware of this work after completing the present paper.

where

$$\begin{aligned}\theta_i &= \frac{a - b\sum\limits_{j\neq i}\alpha_j}{1 + b\left(\sum\limits_{j\neq i}\beta_j\right)}\\ \gamma_i &= \frac{b}{1 + b\left(\sum\limits_{j\neq i}\beta_j\right)}\end{aligned}$$

If the firm's return is

$$pq_i - c\left(q_i\right) = \theta q_i - \gamma q_i^2 - c\left(q_i\right)$$

then the foc on q_i is

$$\begin{aligned}p - \gamma_i q_i - c'\left(q_i\right) &= 0\\ \theta_i - 2\gamma q_i - c'\left(q_i\right) &= 0\\ q_i &= \frac{\theta_i - c'\left(q_i\right)}{2\gamma}\\ p &= \frac{\theta_i + c'\left(q_i\right)}{2}\end{aligned}$$

2.1 Monopoly

For the monopoly case, we have $\theta = a, \gamma = b$ so

$$\begin{aligned}q &= \frac{a - c'\left(q\right)}{2b}\\ p &= \frac{a + c'\left(q\right)}{2}\end{aligned}$$

So, if b is constant, we get the locus

$$q = \frac{p - c'\left(q\right)}{b}$$

while if a is constant, we get the locus

$$p = \frac{a + c'\left(q\right)}{2}$$

3 Symmetric oligopoly

Now consider the symmetric oligopoly case. If γ is constant, we get the locus

$$\begin{aligned} q &= \frac{p - c'(q)}{\gamma} \\ p &= c'(q) + \gamma q \end{aligned}$$

while if θ is constant

$$p = \frac{\theta_i + c_i'(q_i)}{2}$$

In a symmetric equilibrium

$$\begin{aligned} \theta &= \frac{a - (n-1)\alpha}{1 + (n-1) b\beta} \\ \gamma &= \frac{b}{1 + (n-1) b\beta} \\ Q &= nq \end{aligned}$$

so the case where β is the strategic variable, costs are constant and shocks are additive we have:

$$\begin{aligned} p &= c + \gamma \frac{Q}{n} \\ &= c + \frac{b}{n + n(n-1) b\beta} Q \end{aligned}$$

For Cournot with constant marginal costs, we get $\gamma = b, \theta = a - (n-1)\alpha$, so the case of additive shocks reduces to

$$Q = n\left(\frac{p - c}{b}\right)$$

4 Electricity pool prices

A possible area for empirical application is that of electricity pool markets. A typical setup is that firms submit their supply schedules on a daily basis, so that they may be assumed to have a good estimate of demand shocks, relative to the average demand level over the course of the year.[3] Since

[3] This may be contrasted with the Klemperer-Meyer equilibrium solution, which is appropriate in the case when firms must commit to the same supply schedule for an extended period of time, encompassing a range of demand shocks (examples ..)>

consumers are typically on long-term contracts, they can be assumed not to be price-responsive in the short term, so that shocks may be taken as additive. Furthermore, since most consumers are small, the demand curve can be estimated from micro-data.

In this case, assuming firms behave strategically, they will adjust their submitted demand curve in response to changes in the residual demand they face. The equilibrium locus, in combination with the estimated market demand curve, provides the information necessary to infer the perceived competitiveness of supply.

5 Concluding comments

The problem of determining the strategy space is not unique to oligopoly theory. Similar issues arise, for example, in the theory of contests (Menezes and Quiggin 2010). The comparative static approach used here may be applied more generally to infer the strategy space from observed outcomes.

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