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Markets for Influence

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Abstract

We specify an oligopoly game, where firms choose quantity in order to maximise profits, that is strategically equivalent to a standard Tullock rent-seeking game. We then show that the Tullock game may be interpreted as an oligopsonistic market for influence.Alternative specifications of the strategic variable give rise to a range of Nash equilibria with varying levels of rent dissipation.

1 Introduction

Tullock's (1980) seminal contribution marks the beginning of a large research effort aimed at understanding some economic interactions, typically referred to as contests, where agents spend resources in order to get ahead of their rivals and win a prize. Economic interactions that have been studied in this literature include elections, litigation, internal labour market tournaments, sales contests, R&D races, and rent seeking.[1]

This note shows that the standard formulation of the Tullock rent-seeking game, where individuals choose effort or resources to win a prize, is strategically equivalent to a Cournot oligopoly game where the elasticity of demand is unitary and firms choose quantity to maximise their profits. We derive the associated isomorphism and show how alternative specifications of the success function in the Tullock contest correspond to specifications of the cost function for oligopolists. Conversely, oligopolistic markets may be regarded as a generalized class of contests.

This result has an important implication; it suggests that the exclusive focus of the Tullock contest literature on effort or resources as the strategic variable might be misleading. There is an obvious contrast with oligopoly models, where both prices and quantities (Bertrand or Cournot models) were considered as possible strategic variables even before the game theory revolution that has dominated the field of industrial organisation over the last three decades. More recently, a number of papers have proposed alternative strategic variables, such as supply curves (Klemperer and Meyer, 1989) and markups (Grant and Quiggin, 1994). In addition, there have been numerous attempts to motivate the choice of particular strategic variables, for example as outcomes of a multistage game (Kreps and Scheinkman, 1983). These issues have received comparatively less systematic attention in the contests literature.

The contests literature does examine games for which the equilibria resemble Bertrand. For example, when the contest success function is discontinuous, complete dissipation of rent (equivalent to zero profits in the oligopoly case) is an equilibrium outcome. However, our result shows that even confining attention to the standard success function with probability proportional to relative effort, alternative choices of the strategic variable can yield the full gamut of equilibrium outcomes from Cournot to Bertrand.

This finding leads us to explore the relationship between contests and markets more carefully. We argue that a natural economic interpretation of contests may be presented in terms of oligopsonistic markets for influence. The influence variable may be interpreted as electoral support, legal expertise, connections within labour markets and so on. As markets for influence become more competitive, the implicit price of influence increases and the net rent shared by purchasers of influence decreases. Once again, this interpretation leads to an isomorphism with a model of imperfect competition, in this case that of firms acting as oligopsonists in a factor market.

[1] For a recent survey of the literature see, for example, Konrad (2007).

This paper is organised as follows. Section 2 formally defines the strategic equivalence between the two classes of games. Section 3 then discusses the implications of such equivalence by recasting contests as markets for influence. Section 4 concludes.

2 Tullock Contests and Cournot Competition

Our starting point is the most well-known model of contests, namely, the Tullock rent-seeking game. This class of games can be represented by a set of n players, who choose effort levels $e_1, e_2, ..., e_n$ in order to win a prize of fixed value V, and a parameter $R > 0$. The winner of the contest is the player who spends the most effort. Player i's payoff in this class of games is given by:

$$\pi_i(e_1, e_2, ..., e_n) = V \frac{e_i^R}{\sum_{j=1}^{n} e_j^R} - e_i. \tag{1}$$

The equilibria for this family of games (both symmetric and asymmetric, pure and mixed-strategy) are well-known.[2]

To develop the isomorphism with oligopoly games, consider now a unit elasticity demand curve with normalisation such that:

$$p = \frac{V}{\sum_{j=1}^{n} q_j}$$

and assume that i's cost function is given by $c_i(q_i) = q_i^{\frac{1}{R}}$. Thus, the revenue for player i is

$$r_i = \frac{V q_i}{\sum_{j=1}^{n} q_j} = \frac{c_i^\gamma}{\sum_j c_j^\gamma}.$$

Therefore, we can write i's payoff as follows :

$$\pi_i(c_1, c_2, ..., c_n) = V \frac{c_i^R}{\sum_{j=1}^{n} c_j^R} - c_i. \tag{2}$$

Thus, incentives in the oligopoly game, with c_i as the strategic variable, are strategically equivalent to those in a standard Tullock contest with e_i substituted for c_i. That is, we have established the following result:

[2] See, for example, Baye, Kovenock and de Vries (1994). Importantly, Baye and Hoppe (2003) show that this family of games is isomorphic to certain innovation and patent-race games. It follows then that our main result also applies to these other classes of games. That is, there are isomorphisms between oligopoly games and specific innovation and patent-race games.

Proposition 1 *A standard Tullock contest characterised by the payoff function given in (1) is strategically equivalent to a Cournot oligopoly game with demand function* $p = \frac{V}{\sum_{j=1}^{n} q_j}$ *and cost function* $c_i(q_i) = q_i^{\frac{1}{R}}$.

2.1 Imperfectly competitive markets as contests

The interpretation of contests as taking places in markets, which is afforded by the proposition above, may be turned around. Participants in oligopolistic markets may be considered as taking part in a contest for market share. In the case where the elasticity of demand is unitary, this interpretation is represented by the isomorphism given above. More generally, oligopolistic markets may be considered as analogous to contests where the strategic choices of the players determine both the value of the prize (total revenue) and the probability of winning (market share).

One important implication of this literature, which has received only limited attention in the industrial organization literature, is that, in determining the rent accruing to participants, the cost function is just as important as the choice of strategic variable. Depending on the cost function, any outcome in the range from perfect competition to joint monopoly pricing may be sustained as a Cournot equilibrium.

3 Tullock Contests as Markets for Influence

The results derived above suggest that individual behavior in Tullock contests may usefully be related to the behavior of firms in imperfectly competitive markets. To pursue this idea further, it seems natural to consider more carefully the idea, familiar from public-choice theoretic discussions of political processes, that contests represent a particular kind of market, namely a market for influence. If this analogy is taken seriously, the participants in contests may be regarded as buyers in oligopsonistic markets. To formalize the idea, we need to define concepts analogous to prices, quantities, and supply schedules.

To address this task, we introduce the idea of a price of influence which is given by the inverse demand function

$$p(\theta_1, \theta_2, ..., \theta_n) = \sum_i \theta_i$$

where θ_i is the influence acquired by player i and p is the unit price of influence. In the electoral case, for example, we might adopt the interpretation that p is the price paid by the candidates for each vote and θ_i the total number of voters induced to vote for candidate i. Accordingly, the expenditure for player i is

$$e_i = p\theta_i, i = 1, 2...n.$$

As before, we assume that the success probabilities are given by

$$\frac{e_i}{\sum_{j=1}^{n} e_j} = \frac{\theta_i}{\sum_{j=1}^{n} \theta_j} \tag{3}$$

where we assume, for simplicity, that $R = 1$ and the prize is normalized to one so that i's payoff is given by $\pi_i - e_i$. One can immediately see that such context is essentially isomorphic to a oligopsony game as described below.

Proposition 2 *A standard Tullock contest characterised by payoff function* $\frac{e_i}{\sum_{j=1}^{n} e_j} - e_i$, $i = 1, ..., n$, *is strategically equivalent to a oligopsony game where:*

(i) the strategic variable for firm i is the quantity purchased of an input $x_i > 0$;

(ii) output is given by the production function $f(x_i) = x_i$;

(iii) the (constant) output priceis $p = A - 1$

(iv) A is sufficiently large that the input supply price $w = A - \frac{1}{\sum_{i=1}^{n} x_i}$ is always positive.

Proof: Each firm i chooses x_i to maximise profits, which can be written as:

$$\pi_i = pf(x_i) - wx_i = (A-1)x_i - \left(A - \frac{1}{\sum_{i=1}^{n} x_i}\right) x_i = -x_i + \frac{x_i}{\sum_{i=1}^{n} x_i}. \tag{4}$$

Then replace x_i with e_i. □

Proposition 2 above suggests that a Tullock contest will deliver outcomes that are as competitive as those where the strategic variable is the quantity of influenced purchased. This result is the oligopsony equivalent of Proposition 1. This raises the following question: Would outcomes be more competitive if the strategic variable were total expenditure?

The analogy with oligopoly can help us to answer this question. Grant and Quiggin (1994) show that the equilibrium outcome with revenue as the strategic variable is less competitive (higher price, lower aggregate quantity, higher profit) than the Cournot-Nash equilibrium. This is because (loosely speaking) if one player chooses to deviate by increasing revenue, this entails an increase their own output and a reduction in the market price, and the Nash assumption that other players will hold revenue constant implies that they must increase quantity. Converse reasoning for the oligopsony case suggests that the outcome of a standard Tullock contest with expenditure as a strategic variable will be more competitive (lower price, higher aggregate quantity, more rent dissipation) than the Cournot-Nash equilibrium. This is because an increase

in expenditure by one player raises the market price, and therefore lowers the equilibrium quantity associated with a given expenditure level.

To verify this we first remind the reader that in the standard analysis of Tullock games, player i chooses e_i to maximise (1). The unique (symmetric) Nash equilibrium is well-known and given by $e_i^* = \frac{n-1}{n^2} = e^*$ for $i = 1, ..., n$. To see this, note that $\frac{n-1}{n^2}$ is the solution to $\frac{\partial \pi_1}{\partial e_1} |_{e_2=e_3=...=e_n=e} = \frac{1}{e_1+(n-1)e} - \frac{e_1}{(e_1+(n-1)e)^2} - 1 = 0$. This implies that the total resources spent by players add up to $\sum_{i=1}^{n} e_i^* = \frac{n-1}{n}$.

Second, consider the Cournot-Nash strategic representation where the candidates choose quantity θ_i to maximise:

$$\pi_i = \frac{p\theta_i}{p\sum_j \theta_j} - p\theta_i.$$

It is easy to see that this representation has a unique symmetric equilibrium where

$$\theta_1^C = \theta_2^C = ... = \theta_n^C = \frac{\sqrt{n-1}}{n\sqrt{n+1}},$$

and consequently

$$p^C = \frac{\sqrt{n-1}}{\sqrt{n+1}}$$

and

$$e^C = \frac{(n-1)}{n(n+1)} \text{ and } \sum_{i=1}^{n} e_i^C = ne^C = \frac{(n-1)}{(n+1)}. \tag{5}$$

This impliesless rent dissipation than the standard solution for the Tullock contest as$\frac{(n-1)}{(n+1)} \leq \frac{n-1}{n}$ always holds.

Finally, we consider a strategic representation of markets for influence that is equivalent to a 'Bertrand' model of oligopoly. Under this scenario the candidates compete for voters in the 'prices' space. We impose the standard assumptions in Bertrand competition, where the voters will vote for the candidate who offers the higher price. In the event that both candidates offer the same price, voters are equally split among the two candidates. It is not difficult to see that the Bertrand (auction) logic implies that in equilibrium:

$$p_1^B = ... = p_n^B = 1.$$

That is, any price lower than one leads to 'undercutting'. Under this equilibrium, there is zero profit, that is, full rent dissipation, as

$$\theta_1^B = ... = \theta_2^B = \frac{1}{n} = e_1^B = ... = e_n^B. \tag{6}$$

The discussion suggests that by considering the full range of strategies available to participants in Tullock contests, it is possible to obtain a wide range of symmetric equilibrium outcomes, just as in the case of oligopoly.

4 Concluding comments

In this paper, we have shown that the standard Tullock contest game is strategically isomorphic to a Cournot oligopoly game if effort in the Tullock contest is mapped to outputs in the oligopoly game. Consideration of this isomorphism indicates some differences in the aspects of the problem considered in the two literatures. Analysis of Tullock contests has focused on differences in the success function, while the oligopoly literature has paid more attention to the determination of the strategic variable. In each case, a range of possible outcomes from complete rent dissipation to sharing of the maximum rent may be obtained in appropriate cases.

Understanding of the relationship between contests and imperfectly competitive markets is hampered by the absence of explicit prices and quantities in the standard contest model. When contests are represented as markets for influence, we derive a natural strategic equivalence between the standard Tullock contest and an oligopsonistic market in which expenditure is the strategic variable. Unlike the corresponding case for oligopoly, this outcome turns out to be less competitive (and hence less dissipative of rent) than the Cournot solution.

Representation of Tullock contests as markets for influence raises a wide range of possible future developments. Most obviously, the literature on industrial organization focuses on the extent to which the choice of strategic variable determines whether market outcomes will yield competitive (Bertrand) outcomes, less competitive (Cournot outcomes) or joint monopoly outcomes, not to mention a wide range of intermediate possibilities. Analogies with Tullock contests, including elections, litigation and so on may be fruitful. Beyond this, it would be natural to consider the implications of the literature on mergers to determine conditions under which participants in a Tullock contests, such as political parties, might benefit from the formation of a coalition.

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