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Output Price Subsidies in a Stochastic World

Robert G. Chambers
Professor and Adjunct Professor, respectively, University of Maryland and University of Western Australia

and

John Quiggin
Australian Research Council Federation Fellow, University of Queensland

Schools of Economics and Political Science
University of Queensland
Brisbane, 4072
rsmg@uq.edu.au
http://www.uq.edu.au/economics/rsmg

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

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Robert G. Chambers[2] and John Quiggin[3]

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[1]We thank Bruce Gardner for comments on an earlier version of this paper.

[2]Chambers is Professor of Agricultural and Resource Economics at the University of Maryland, College Park and Adjunct Professor, School of Agricultural and Resource Economics, University of Western Australia.

[3]Quiggin is an Australian Research Council Professorial Fellow at the University of Queensland.

Output Price Subsidies in a Stochastic World

This paper develops an analytically tractable approach to the comparative statics of output subsidies for firms, with monotonic preferences over costs and returns, that face price and production uncertainty. In what follows, we first present the basic model. The modelling of deficiency payments, support-price schemes, and stochastic supply shifts in a state-space framework is then discussed. We then show how these notions can be used, via a simple application of Shephard's lemma, to analyze input-demand shifts once comparative-static results for supply are available. With this foundation in place, an analysis of supply comparative statics proper is presented.

The Producer Model

We model a price-taking firm facing a stochastic environment in a two-period setting. The current period, 0, is certain. Uncertainty in period 1 is represented by a state space $\Omega = \{1, 2, ..., S\}$. Period 0 prices of inputs are denoted by $\mathbf{w} \in \Re^N_{++}$ and are non-stochastic. In the absence of government intervention, the output price in period 1 is stochastic, and we denote by $\mathbf{p} \in \Re^S_{++}$ the vector of state-contingent (spot) output prices corresponding to the vector of state-contingent outputs. Producers take these state-contingent output prices and the prices of all inputs as given. Let $\mathbf{1} \in \Re^S_{++}$ denote the S vector of ones.

The firm's stochastic production technology is represented by a strictly convex and closed single-product, state-contingent input set

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re^N_+ : \mathbf{x} \text{ can produce } \mathbf{z}\}.$$

where $\mathbf{x} \in \Re^N_+$ is a vector of inputs committed in period 0, and $\mathbf{z} \in \Re^S_+$ is a vector of state-contingent outputs (Chambers and Quiggin, 2000). Define the cost function by

$$c(\mathbf{w}, \mathbf{z}) = \min\{\mathbf{wx} : \mathbf{x} \in X(\mathbf{z})\}$$

and the cost minimizing input vector by

$$\mathbf{x}(\mathbf{w}, \mathbf{z}) = \arg\min\{\mathbf{wx} : \mathbf{x} \in X(\mathbf{z})\}.$$

For simplicity, we shall assume that c is at least twice smoothly differentiable in all arguments.As Chambers and Quiggin (2000) demonstrate, the assumption of differentiability in $\mathbf{z}$ rules out a number of stochastic production technologies.. By the strict convexity of $X(\mathbf{z})$, $\mathbf{x}(\mathbf{w},\mathbf{z})$ is unique, and thus, by Shephard's Lemma,

$$\mathbf{x}(\mathbf{w},\mathbf{z}) = \nabla_{\mathbf{w}} c(\mathbf{w},\mathbf{z}), \tag{1}$$

where ∇ denotes the gradient.

Producer preferences over current period expenditures, c, and period 1 stochastic income, $\mathbf{y}$, are represented by a cardinal certainty equivalent $e(c,\mathbf{y})$ that is strictly decreasing in c and increasing in $\mathbf{y}$. Familiar examples are: the expected-utility of net-returns certainty equivalent

$$e(c,\mathbf{y}) = u^{-1}\left(\sum_s \pi_s u(y_s - c)\right),$$

where u is a strictly increasing *ex post* utility function, and $\boldsymbol{\pi}$ denotes a vector of subjective probabilities; the maximin net returns specification, which corresponds to maximal risk aversion over net returns,

$$\begin{aligned} e(c,\mathbf{y}) &= \min\{y_1 - c, ..., y_S - c\} \\ &= \min\{y_1, ..., y_S\} - c; \end{aligned} \tag{2}$$

the additively separable effort specification, popularised by Newbery and Stiglitz

$$e(c,\mathbf{y}) = \hat{e}(\mathbf{y}) - c, \tag{3}$$

where $\hat{e}$ is a nondecreasing function;The additively separable effort model does not correspond to the separable effort model of Newbery and Stiglitz. If $\hat{e}$ is consistent with an expected-utility certainty equivalent then the additively separable effort model is

$$u^{-1}\left(\sum_s \pi_s u(y_s)\right) - c,$$

whereas the Newbery-Stiglitz specification corresponds to

$$\sum_s \pi_s u(y_s) - c.$$

and the Sandmovian net-returns specification

$$e(c,\mathbf{y}) = \tilde{e}(\mathbf{y} - c\mathbf{1}), \tag{4}$$

where $\tilde{e}$ is a nondecreasing function (Chambers and Quiggin, 2000).

Deficiency Payments, Support Prices, and Supply Shifts in a State-space Setting

In the usual stylized representation of a deficiency-payment scheme, the government requires producers to sell their output in the market for the market price. If the market price falls below the predetermined target price, p_T, the government rebates the difference between the market price and the target price to the producer. From an *ex ante* perspective, the introduction of a deficiency-payment scheme modifies the producer's state-contingent price vector from $\mathbf{p}$ to $\mathbf{p}^T$ with typical element

$$p_s^T = \max\{p_s, p_T\}. \tag{5}$$

Price-support schemes, although they operate through a different mechanism, have a similar effect on the producer's *ex ante* price perceptions.

The introduction of a deficiency-payment or support-price scheme truncates the price distribution so that the probability of facing a price lower than p_T is zero. Thus, the support of the price distribution faced by producers is 'narrowed' in the sense that the lowest state-contingent price faced by producers moves to the right, but the highest price faced by the producer remains unchanged. Both deficiency-payment and support price schemes thus have two important effects: one raises state-contingent prices without lowering any of the same, and the other stabilizes prices faced by decreasing the range of prices faced. The stabilizing effect has been examined in Chambers and Quiggin (2000). Thus, we focus on the first characteristic. Borrowing from the literature on vector dominance we say that $\mathbf{a}' \in \Re^S$ is a *statewise-dominant shift* of the random variable $\mathbf{a} \in \Re^S$ if $\mathbf{a}' \geq \mathbf{a}$.

Ultimately our interest is in determining under what conditions the introduction of a deficiency-payment or support-price scheme increases supply and increases or decreases factor demands. In a nonstochastic world, the effect of introducing a deficiency-payment or support-price scheme on supply and factor demand is well understood. For example, as demonstrated by Gardner, the introduction of a target-price makes supply perfectly inelastic (at the supply rationally produced in the presence of the target price) for all prices less than the target price. For prices above the target price, the supply schedule corresponds to marginal cost.

One goal of this paper is to demonstrate that a virtually identical analysis can apply in a stochastic world. But to do that, we must first pose and answer the question of what it means for supply to shift outward in a stochastic world? Various notions have appeared in the literature (discussed critically in Chambers and Quiggin (2003)), including increases in expected supply and rightward shifts of an output distribution with fixed support. The most pertinent definition for our purposes is one that characterizes what happens to *ex post* supply in each state of Nature. As we show below, the introduction of a deficiency payment or price-support scheme can cause supply in some states of Nature to fall. Therefore, we shall be particularly concerned with statewise dominant shifts in optimal $\mathbf{z}$.

Input Comparative Statics

By (1), any input adjustment resulting from the introduction of a support-price or deficiency-payment scheme can be modelled as a change in the cost minimizing demands induced by the resulting optimal adjustment of the state-contingent outputs. Thus, in considering such input adjustment, it is logical first to ascertain how inputs adjust to changes in state-contingent outputs. Once comparative-static results are obtained for supply shifts below, they can be combined with these results to obtain comparative static results for the firm using multiple inputs via a simple application of Shephard's lemma.

This, however, is a simple matter of determining whether inputs are regressive or non-regressive in state-contingent outputs. So for example, if $\mathbf{z}' \geq \mathbf{z}$ (state-contingent supply is unambiguously higher), then a sufficient condition for input i to increase as a result of the induced change is that input i be nonregressive in all state-contingent outputs. Conversely, a sufficient condition for input i to decrease is that it be regressive in all state-contingent outputs.

More generally, using (1), and assuming that $\delta \mathbf{z} = (\mathbf{z}' - \mathbf{z})$ is differentiably small, then a necessary and sufficient condition for input i to increase as a result of the induced state-contingent output change is that

$$\sum_s \frac{\partial^2 c(\mathbf{w}, \mathbf{z})}{\partial w_i \partial z_s} \delta z_s \geq 0.$$

Supply Comparative Statics

Our focus now turns to sufficient conditions for the introduction of a support-price or deficiency-payment scheme to induce a statewise-dominant shift in the output distribution. Formally, we seek sufficient conditions for which $\mathbf{p}^T \geq \mathbf{p} \Rightarrow \mathbf{z}^T \geq \mathbf{z}$, where $\mathbf{z}^T$ denotes the vector of state-contingent supplies in the presence of output-price subsidies. This is a problem of monotone comparative statics (Topkis; Milgrom and Shannon).

Consider first the risk-neutral case. In the absence of the output price subsidy, the risk-neutral farmer solves

$$Max_{\mathbf{z}} \left\{ \sum_s \pi_s p_s z_s - c(\mathbf{w}, \mathbf{z}) \right\}.$$

In the presence, for example, of a deficiency payment scheme, the same risk-neutral farmer solves

$$Max_{\mathbf{z}} \left\{ \sum_s \pi_s \max\{p_s, p_T\} z_s - c(\mathbf{w}, \mathbf{z}) \right\}.$$

The problem is written in this form to emphasize its formal equivalence to the maximization problem faced by a multi-product, profit maximizing firm in a nonstochastic world. Viewed from this context, increases in particular state-contingent prices are analogous to increases in the prices of particular products. From the theory of the multi-product firm, *it is well-known that an increase in any one product price is not sufficient to ensure that the supply of all products increase.* Such is true here as well. However, studies of stochastic supply response frequently impose production structures that do not permit supply to rise in some states and to fall in others. Innes, for example, uses a multiplicative cost structure that requires that either all state-contingent outputs rise or all state-contingent outputs fall in response to the institution of deficiency payments.

Intuition suggests that supply rising in some states but falling in others may be the normal response to such price changes and not the exception. Certainly, in the absence of any input adjustment, one expects an increase in the price of one product to be associated with a fall in production of other products as the producer substitutes along the product transformation curve. If the input adjustment associated with the price change does not overwhelm that substitution effect, the production of other products, whose prices do not rise, falls. The familiar notion of *gross output substitutability* captures this tendency to substitute away from the production of less profitable commodities to more profitable commodities. It follows by simple analogy, therefore, that a statewise-dominant shift in the output price distribution does not guarantee a statewise-dominant shift in the output distribution.

For example, suppose a deficiency payment scheme is introduced. Farmers now see higher prices in a subset of Ω (those states where the deficiency payment is effective) than before the introduction of the scheme. Prices in the complement of that subset, however, remain unchanged. The resulting relative price changes may entice farmers to divert resources formerly devoted to production in states where the deficiency payment is not effective towards production in states where it is now effective.

The cost function exhibits *output cost complementarities* between outputs s and k if

$$\frac{\partial^2 c(\mathbf{w}, \mathbf{z})}{\partial z_s \partial z_k} \leq 0, \quad s \neq k.$$

It exhibits *output substitutability* if the inequality is reversed. It is well-known that if a technology exhibits cost complementarities between all outputs, then an increase in any or all output prices never leads to a decrease in any profit maximizing supply (Topkis). Translated to the present case, the presence of cost complementarities is sufficient to guarantee that a statewise-dominant shift in the price distribution leads to a statewise-dominant shift in the output distribution for a risk-neutral producer. (This is formalized below.) However, in the absence of such strong complementarity in the underlying cost structure, statewise-dominant shifts in the price distribution may not lead to statewise-dominant shifts in the output distribution.

Some definitions

Define the pointwise maximum of $\mathbf{x}$ and $\mathbf{y}$ as the *join* denoted by

$$\mathbf{x} \vee \mathbf{y} = (\max\{x_1, y_1\}, ..., \max\{x_S, y_S\}),$$

and denote by $\mathbf{x} \wedge \mathbf{y}$ pointwise minimum of $\mathbf{x}$ and $\mathbf{y}$ in $\Re^S$. A mapping $f : \Re^S \to \Re$ is *supermodular* if for all $\mathbf{y}, \mathbf{y}'$

$$f(\mathbf{y} \vee \mathbf{y}') + f(\mathbf{y} \wedge \mathbf{y}') \geq f(\mathbf{y}) + f(\mathbf{y}').$$

If $-f$ is supermodular, then f is *submodular.* Twice differentiable supermodular functions satisfy (Topkis)

$$\frac{\partial^2 f(\mathbf{y})}{\partial y_s \partial y_k} \geq 0, \quad s \neq k.$$

Thus, for example, the cost function is supermodular in $\mathbf{z}$ only if outputs are substitutes. Conversely, *the cost function is submodular in z only if it exhibits output complementarities.*

A function h has *increasing differences in* $(\mathbf{n}, \mathbf{v})$ if $\mathbf{v}'' \geq \mathbf{v}'$ and $\mathbf{n}'' \geq \mathbf{n}'$ implies that

$$h(\mathbf{n}'', \mathbf{v}'') - h(\mathbf{n}', \mathbf{v}'') \geq h(\mathbf{n}'', \mathbf{v}') - h(\mathbf{n}', \mathbf{v}').$$

If one sets $\mathbf{n} = \mathbf{w}$, and $\mathbf{v} = \mathbf{z}$, then for c increasing differences requires for $\mathbf{w}'' \geq \mathbf{w}'$ and $\mathbf{z}'' \geq \mathbf{z}'$ that

$$c(\mathbf{w}'', \mathbf{z}'') - c(\mathbf{w}', \mathbf{z}'') \geq c(\mathbf{w}'', \mathbf{z}') - c(\mathbf{w}', \mathbf{z}').$$

Taking $w''_k = w'_k + \delta$ and $w''_j = w'_j$ $(j \neq k)$ and letting $\delta \to 0$ then gives

$$\frac{\partial c(\mathbf{w}', \mathbf{z}'')}{\partial w_k} \geq \frac{\partial c(\mathbf{w}', \mathbf{z}')}{\partial w_k}.$$

By (1), this last expression requires that the cost minimizing demand for input k be nonregressive in all state-contingent outputs.

The importance of these definitions to our analysis is demonstrated by considering the generic optimization problem

$$\max_{\mathbf{q}} \{h(\mathbf{q},\alpha)\},$$

where $h : \Re^S \times \Re \to \Re$. Assume that there exists an unique optimal solution to this problem, which we denote as $\mathbf{q}(\alpha)$. Topkis (Theorem 2.8.1, p.76) has shownThese conditions are only sufficient. Topkis and Milgrom and Shannon identify weaker sufficient conditions involving quasi-supermodularity and varying versions of the increasing difference or single crossing property. However, supermodularity of e, which is cardinal, ensures that $W(c, \mathbf{y})$ is quasi-supermodular (Topkis). Hence, the results that follow apply to quasi-supermodular W.

Lemma 1 *(Topkis)* $\alpha' \geq \alpha \Rightarrow \mathbf{q}(\alpha') \geq \mathbf{q}(\alpha)$ *if* h *is supermodular in* $\mathbf{q}$ *for all* α *and* h *satisfies increasing differences in* $(\mathbf{q},\alpha)$.

Define

$$\mathbf{p}(\alpha) = (1-\alpha)\,\mathbf{p}{+}\alpha\mathbf{p}^{T}.$$

The farmer's problem in the absence of and in the presence of a deficiency-payment or support-price scheme can be written as polar cases ($\alpha = 0$ and $\alpha = 1$, respectively) of the maximization problem:

$$\max_{\mathbf{z}} \left\{ e\left(c\left(\mathbf{w},\mathbf{z}\right), \mathbf{p}\left(\alpha\right)\cdot\mathbf{z}\right)\right\}.$$

Thus, the monotone-comparative static results of Topkis and Milgrom and Shannon apply. In what follows, assume that unique solutions to the producer's problem always exist and denote them by

$$\mathbf{z}(\alpha) = \arg\max\left\{ e\left(c\left(\mathbf{w},\mathbf{z}\right), \mathbf{p}\left(\alpha\right)\cdot\mathbf{z}\right)\right\}.$$

Additively-separable effort

We first address the case where

$$e\left(c\left(\mathbf{w},\mathbf{z}\right), \mathbf{p}\left(\alpha\right)\cdot\mathbf{z}\right) = \hat{e}\left(\mathbf{p}\left(\alpha\right)\cdot\mathbf{z}\right) - c\left(\mathbf{w},\mathbf{z}\right).$$

Because this contains as special cases risk-neutral preferences, maximally risk-averse net-returns preferences, and constant absolute risk averse net-return preferences (Chambers and Quiggin, 2000), it encompasses three of the most closely examined preference specifications. If the cost structure is submodular in $\mathbf{z}$, $\hat{e}$ is supermodular in $\mathbf{y}$, and $\hat{e}$ satisfies increasing differences in ($\mathbf{z}$,α) then Lemma 1 implies that the introduction of output-price subsidies evokes a statewise-dominant shift in the output distribution.

Submodularity of the cost structure is equivalent to requiring that state-contingent outputs be complementary. This guarantees in the multi-product case that increasing any output price leads no product output to decrease. Thus, we intuitively expect a similar result to emerge in the risk-neutral case, and it does. A risk-neutral case $\hat{e}$ is trivially supermodular in $\mathbf{y}$. It also satisfies increasing differences since differentiating first by z_k and then by α obtains

$$\pi_k \max\left\{0, p_T - p_k\right\} \geq 0.$$

Hence,

Theorem 2 *(Topkis) If c is submodular in* $\mathbf{z}$, *the introduction of a price-subsidy scheme leads to a statewise-dominant shift in the output distribution for risk-neutral producers.*

Now consider the maximally risk-averse case. It is well-known that $\hat{e}$ is supermodular in $\mathbf{y}$ (e.g. Topkis, p. 48). Thus, in principle, one could use monotone comparative static methods to determine the effect of introducing price subsidies on state-contingent supply. However, a more direct analysis is perhaps both more intuitive and revealing. If cost is strictly increasing in all state-contingent outputs and producer preferences are given by

$$\min\left\{p_1(\alpha)z_1,...,p_S(\alpha)z_S\right\}-c(\mathbf{w},\mathbf{z}),$$

then the optimal solution requires for all s (Chambers and Quiggin, 2000)

$$z_s(\alpha)=\frac{p_1(\alpha)}{p_s(\alpha)}z_1(\alpha). \tag{6}$$

Choose indexes so that $p_1(0)=\max\{p_1,...,p_S\}$ and assume that $p_T\leq p_1$.In words, we are simply assuming that p_T is not placed higher than the upper support of the original discrete price distribution. Using (6) to concentrate the objective function then yields:

$$L(z_1,\alpha)=p_1z_1-c\left(\mathbf{w},z_1\left(1,\frac{p_1}{p_2(\alpha)},...,\frac{p_1}{p_S(\alpha)}\right)\right),$$

which is concave in the single choice variable z_1 so long as c is convex in $\mathbf{z}$. To simplify, maintain that assumption. Hence, the first-order condition

$$1-\sum_{k=1}^{S}\frac{c_k\left(\mathbf{w},z_1\left(1,\frac{p_1}{p_2(\alpha)},...,\frac{p_1}{p_S(\alpha)}\right)\right)}{p_k(\alpha)}=0 \tag{7}$$

is both necessary and sufficient. Thus, $z_1(1)\geq z_1(0)$ if and only if

$$\sum_{k=1}^{S}\frac{c_k\left(\mathbf{w},z_1(0)\left(1,\frac{p_1}{\max\{p_2,p_T\}},...,\frac{p_1}{\max\{p_S,p_T\}}\right)\right)}{\max\{p_k,p_T\}}\leq\sum_{k=1}^{S}\frac{c_k\left(\mathbf{w},z_1(0)\left(1,\frac{p_1}{p_2},...,\frac{p_1}{p_S}\right)\right)}{p_k}.$$

This last inequality is satisfied if $\sum_{k=1}^{S}\frac{c_k\left(\mathbf{w},z_1\left(1,\frac{p_1}{p_2(\alpha)},...,\frac{p_1}{p_S(\alpha)}\right)\right)}{p_k(\alpha)}$, which represents the marginal cost of a nonstochastic increase in period 1 income (Chambers and Quiggin, 2000), is decreasing in α.

Hence, the ultimate determinant of whether z_1 increases or decreases as a result of an increase in α is whether the marginal cost of a nonstochastic income increase is increasing or decreasing in α. If it is decreasing, then marginal cost falls. This opens a gap between marginal cost and marginal return, and z_1 optimally rises to close the gap. If it is increasing, then the opposite intuition applies. Differentiating $\sum_{k=1}^{S} \frac{c_k\left(\mathbf{w}, z_1\left(1, \frac{p_1}{p_2(\alpha)}, \ldots, \frac{p_1}{p_S(\alpha)}\right)\right)}{p_k(\alpha)}$ with respect to α obtains:

$$-\sum_{k=2}^{S} \frac{c_k}{p_k(\alpha)^2} \max\{0, p_T - p_k\} - \sum_{k=1}^{S} \sum_{j=2}^{S} \frac{c_{kj}}{p_k(\alpha)} \frac{p_1}{p_j(\alpha)^2} \max\{0, p_T - p_j\}. \tag{8}$$

At the margin, subsidizing the stochastic output price allows, via (6), the producer to reduce output in states of nature where the subsidy is effective without affecting revenue earned in those states of nature. This permits a cost saving, and hence the marginal cost of nonstochastic income falls by this effect. This effect is captured by the first term above, which is negative. Unless the cost structure is additive across states of nature, that is $c_{kj} = 0$, $k \neq j$, this output reduction in the states where the subsidy is effective also affects marginal cost of the other state-contingent outputs. This is the effect measured by the second term in (8). If the cost structure is supermodular, $c_{kj} \geq 0$, these secondary effects reinforce the impact effect. In that case, the marginal cost of producing a sure income falls as a result of a move toward a price subsidy. However, if the cost structure is submodular (exhibits output complementarities, $c_{kj} \leq 0$), these secondary state-contingent output adjustments tend to increase marginal costs in all other states. Whether the secondary effect overwhelms the primary effect is then an empirical issue.

From (6), it follows that for $s \neq 1$

$$\begin{aligned}
\frac{\partial z_s}{\partial \alpha} &= \frac{p_1}{p_s}\frac{dz_1}{d\alpha} - z_1\left(\frac{p_1}{p_s^2}\right)\max\{0, p_T - p_s\} \\
&= \frac{p_1}{p_s}\left[\frac{dz_1}{d\alpha} - z_1\frac{\max\{0, p_T - p_s\}}{p_s}\right].
\end{aligned}$$

Thus, in any state where the price subsidy is not effective, production rises if costs are supermodular. But even if the cost structure is supermodular in state-contingent outputs, the possibility exists that some outputs in the states where the subsidy is effective can fall as a result of the introduction of output subsidies. In particular, for those states with the lowest *ex ante* spot prices, this seems a clear possibility. An intuitive explanation is that in the very low price states, keeping income at the optimal sure level associated with (6) requires a large level of state-contingent production. Therefore in the absence of output-price subsidies, the farmer fully self insures his income risk by matching low price states with high production levels. Introducing output-price subsidies allows the farmer to safely divert productive resources from this self-insurance effort towards production in higher price states without sacrificing income.

Summarizing:

Theorem 3 *For a completely risk-averse producer with a supermodular cost structure, output-price subsidies evoke a supply increase for all states where $p_T \leq p_s$, and for states where $p_T > p_s$ supply increases if*

$$\frac{dz_1}{d\alpha}\frac{1}{z_1} \geq \frac{p_T - p_s}{p_s}.$$

Theorem 2 and Theorem 3 demonstrate that the polar cases of risk neutrality and complete aversion to risk lead to polar sufficient conditions (respectively, submodularity and supermodularity) on the cost function for the introduction of a price-support scheme to lead to supply increases in high-price states. This is illuminating. It indicates the difficulties that one can encounter difficulties in transferring familiar monotone comparative static results for the risk-neutral firm to firms that care about risk. As their risk aversion becomes unboundedly large, even the strongest submodularity (complementarity) restrictions on the cost function, and thus on the technology, are no longer sufficient to ensure even the simplest comparative static results.

This tension between risk aversion and supermodularity or submodularity of the cost structure is further illustrated by the general additively separable effort case (3). Suppose that the cost structure is submodular in $\mathbf{z}$ and that $\hat{e}$ is supermodular in $\mathbf{y}$ in (3), what further restrictions are sufficient to ensure that output-price subsidies lead to a statewise-dominant shift in supply? If $\hat{e}$ is twice differentiable, increasing differences requires for all s that

$$p_s(\alpha)\,\hat{e}_s(\mathbf{p}(\alpha)\cdot\mathbf{z})$$

be increasing in α. Differentiating establishes (dropping function arguments) the following requirement

$$\left(\hat{e}_s + p_s(\alpha) z_s \hat{e}_{ss}\right) \max\{0, p_T - p_s\} + p_s(\alpha) \sum_{k \neq s}^{S} \hat{e}_{sk} z_k \max\{0, p_T - p_k\} \geq 0,$$

for all s. In states where the price subsidy is not effective, $p_s \geq p_T$, this condition is guaranteed by supermodularity of $\hat{e}$. But in states where the price subsidy is effective, another condition is involved. We summarize in the following theorem:

Theorem 4 *For a producer with a submodular cost structure and an additively separable-effort preference structure of the form (3) with $\hat{e}$ supermodular in $\mathbf{y}$ and $\left(\hat{e}_s + p_s(\alpha) z_s \hat{e}_{ss}\right) \geq 0$ for all states where the price subsidy is effective, output-price subsidies evoke a statewise-dominant shift in $\mathbf{z}$.*

Consider the familiar special case of $\hat{e}$ as an expected-utility certainty equivalent. If $\hat{e}(\mathbf{p} \cdot \mathbf{z}) = u^{-1}\left(\sum_s \pi_s u(p_s z_s)\right)$, then supermodularity in $\mathbf{z}$ requires

$$\left(u^{-1}\right)'' \pi_s p_s u'(p_s z_s) \pi_k p_k u'(p_k z_k) \geq 0,$$

where $(u^{-1})''$ denotes the second derivative of u^{-1}. Thus, the expected-utility representation of $\hat{e}$ is supermodular if and only if the individual producer is risk averse (Quiggin and Chambers, 2003). Similarly, the condition that $\left(\hat{e}_s + p_s(\alpha) z_s \hat{e}_{ss}\right)$ be positive is equivalent to

$$1 + p_s(\alpha) z_s \left[\frac{(u^{-1})''}{(u^{-1})'} \pi_s u'(p_s(\alpha) z_s) + \frac{u''(p_s(\alpha) z_s)}{u'(p_s(\alpha) z_s)}\right] \geq 0.$$

Because $\frac{(u^{-1})''}{(u^{-1})'} \geq 0$ for risk-averse expected utility preferences, a sufficient condition is that the Arrow-Pratt coefficient of relative risk aversion for the states where the price subsidy is effective be less than or equal to one.

General preferences

For the preference structure $e(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z})$, first-order conditions for an interior solution require

(9) $\quad e_0(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z}) c_s(\mathbf{w}, \mathbf{z}) + p_s(\alpha) e_s(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z}) = 0, \quad s = 1, 2, ..., S,$

where e_0 denotes the derivative of e with respect to c. Expression (9) requires that the marginal cost of raising income in state s by one unit, $c_s(\mathbf{w}, \mathbf{z}) / p_s(\alpha)$, equal $-e_s/e_0$. This latter term measures the firm's marginal rate of substitution between period 0 income and period-1, state s income, and hence the firm's present-value (in period 0 terms) of a dollar of income in state s.

Summing over states of Nature gives

$$\sum_{s=1}^{S} \frac{c_s(\mathbf{w}, \mathbf{z})}{p_s(\alpha)} = -\frac{\sum_{s=1}^{S} e_s(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z})}{e_0(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z})}. \tag{10}$$

The left-hand side of (10) is the marginal cost to the producer of a nonstochastic increase in income. The right-hand side of (10) is the producer's marginal rate of substitution between one dollar of income in period 0 and a nonstochastic increase in period 1 income of a dollar. It gives the producer's present value of a nonstochastic increase in period 1 income by one dollar and thus measures the rate of time preference.

Note that Sandmovian net-returns specification that forms the core of most agricultural-economic analysis of producer decisionmaking under risk parametrically requires either that

$$1 = -\frac{\sum_{s=1}^{S} e_s(c, \mathbf{y})}{e_0(c, \mathbf{y})}, \tag{11}$$

or that the right-hand side is set equal to some constant measuring the rate of time preference.

Consider the conditions under which $e(c(\mathbf{w}, \mathbf{z}), \mathbf{p}(\alpha) \cdot \mathbf{z})$ satisfies Lemma 1. Supermodularity of e in $\mathbf{z}$ then requires that the left-hand side of (9) be increasing in z_k for $k \neq s$. Thus, for supermodularity in $\mathbf{z}$, we must have (dropping function subscripts):

$$e_{00} c_s c_k + e_{0k} c_s p_k(\alpha) + e_0 c_{sk} + e_{sk} p_s(\alpha) p_k(\alpha) \geq 0, \quad s \neq k. \tag{12}$$

If e is supermodular in $\mathbf{y}$ and c is submodular in $\mathbf{z}$ (recall e is decreasing in c so that $e_0 < 0$), then by (12) e is supermodular in $\mathbf{z}$ if

$$e_{00} c_k + e_{0k} p_k(\alpha) \geq 0. \tag{13}$$

For $e(c(\mathbf{w},\mathbf{z}),\mathbf{p}(\alpha)\cdot\mathbf{z})$ to satisfy increasing differences in $(\mathbf{z},\boldsymbol{\alpha})$, the left-hand side of (9) must be nondecreasing in α. This is true if for all s

$$c_s\sum_k e_{0k}z_k\max\{0,p_T-p_k\}+(e_s+p_s(\alpha)z_se_{ss})\max\{0,p_T-p_s\}+p_s(\alpha)\sum_{k\neq s}^{S}e_{sk}z_k\max\{0,p_T-p_k\}$$

Thus, we obtain

Theorem 5 *For preferences of the form $e(c,\mathbf{y})$, output-price subsidies evoke a statewise-dominant shift in the output distribution if e is supermodular in $(c,\mathbf{y})$, c is submodular in $\mathbf{z}$, $e_{00}c_k+e_{0k}p_k\geq 0$ for all k, and $e_s+p_s(\alpha)z_se_{ss}\geq 0$ for s such that $p_T>p_s$.*

Supermodularity of e requires that the producer's marginal valuation of each state's income be increasing in income in the other states, so that state-contingent outputs are natural complements, and that his or her marginal valuation of cost also be increasing in each of the state-contingent incomes. For expected-utility preferences, the former is equivalent to risk aversion (Quiggin and Chambers 2003). Submodularity of the cost structure requires that outputs be complementary as discussed. The condition that $e_s+p_s(\alpha)z_se_{ss}\geq 0$, for the states in which the subsidy is effective, repeats the analogous condition found in the case of additively separable preferences. This condition requires that preferences not be "too risk averse".

That leaves the sufficient condition

$$e_{00}c_k+e_{0k}p_k\geq 0,$$

for all k to be explained. Using (9) allows us to rewrite this condition as requiring in equilbrium (recall $e_0<0$) that

$$\frac{e_{00}}{e_0}-\frac{e_{k0}}{e_k}\leq 0. \tag{14}$$

The economic meaning of (14) is perhaps best seen by noting that

$$\frac{\partial}{\partial c}\ln\left(-\frac{e_k}{e_0}\right)=\frac{e_{k0}}{e_k}-\frac{e_{00}}{e_0}.$$

Expression (14), therefore, implies that the present-value of income in state k be increasing in c.

Condition (14) is particularly problematic in some popular specifications of the objective function in agricultural economics. Consider, for example, the Sandmovian net-returns expected-utility objective function. Then, it can be shown that (14) and (11) are globally consistent only if preferences exhibit constant absolute risk aversion (Chambers and Quiggin, 2003) for which the net-returns specification collapses to a special case of the additively-separable effort model (Chambers and Quiggin, 2000).

Concluding comments

This paper has examined the effect of output-price subsidies on a firm with monotonic preferences facing a stochastic output price and a stochastic technology. It has been shown that comparative-static analysis of such subsidies is closely analogous to the comparative-static analysis of price increases for a multiple-output firm. The same basic tools can be used in analyzing both problems, and analysis of comparative-static results under uncertainty does not require a set of specialized analytic tools..

A range of other comparative static results are covered for a number of special cases and for the general preference structure of the paper. Although the effect of output subsidies on input usage are not explicitly treated in our formal analysis, these results can be obtained by combining our results on input comparative statics for $\mathbf{z}$ and our supply comparative statics. This is left to the reader.

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