Reputation and Multiproduct-firm Behavior: Product line and Price Rivalry Among Retailers

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1. Introduction

The current consumer demand for healthful and safe foods has become a driving force in the food retail market; for example the continued resistance to genetically modified or genetically engineered foods and the growth of organic foods. A considerable number of studies using stated preference method have shown that consumers have a high willingness-to-pay (WTP) to healthful and high-quality food (i.e. Wang et al., 1997). In this situation, when making policy, retailers should also adjust their strategies to meet the changing nature of consumer preference. However, consumers' stated demands are not always an accurate reflection of their actual demands. Several studies on consumer behavior have found that consumers' stated preferences do not always match their revealed preferences (Olsen and Smith, 2001; Homburg et al., 2005; Laroche et al., 2001). An example from the fluid milk sector shows that although many studies find that consumers have a high stated WTP for milk that is not from cows treated with recombinant bovine somatropin (henceforth referred to as rBST-free milk for short), actual sales data show that the market share of rBST-free milk declined during the period 1997 to 2002 (Dhar, 2005). In these cases, it may be in the best interest of the retailers to follow what consumers actually do – as opposed to what they say – to determine what to provide.

However, there are good reasons to listen to what consumers say, and not necessarily to what they actually do. This is especially true in the case of products that receive high levels of public scrutiny, such as genetically modified goods. Another example from fluid milk is grocery retailers such as Safeway, Walmart and Dean Foods implemented a ban on selling milk from cows treated with rBST in 2007/2008. What is the incentive for them to specialize in a niche market? A similar question is why organic stores, such as whole foods, choose to restrict their product line rather than to proliferate it. Our motivation is to investigate the issue of restriction or specialization of firms' product line relative to proliferation of their product line. In our view, retailers that decide not to sell any controversial products can generate a lot of positive publicity. It can send a positive signal to the public if a firm willingly bans a product because it is concerned with the public's health. In addition, this type of action can serve to differentiate the retailer from its competitors and further establish their own reputations as a purveyor of high quality products to attract consumers who favor more healthful foods.

Our study examines competition among multiple-product retailers in a vertically and horizontally differentiated industry where each retailer can choose a niche or full product line strategy. We model a firm's decision with respect to product line offerings in the presence of reputation effects. The objective of our work is to explore the role of reputation in product line competition and the conditions under which retailers adjust their product offering. We ask and answer these questions: (1) Can a retailer benefit from a niche product (high quality) strategy relative to a full product line when there are reputational benefits to be gained? (3) Can the reputational benefits gained due to product line choice affect competition among multiple-product retailers?

When modeling competition among firms, most studies impose the assumption that each firm produces a single product due to the analytical convenience. However, multiproduct firms are ubiquitous in many industries. Studies are also developed to address issues associated with competition among multiproduct firms. Among the literature modeling multiproduct firm competition, some analyzed competition in term of product line choice. Katz (1984) analyzed multiproduct firms' competition in markets where the products produced by each firm have interaction on demand side. He claimed that the degree of competition of one market can be affected by that of a related market. The consequence is that firms may choose not to provide full product lines. Gilbert and Matutes (1993) used a two-dimensional model to address the problem that commitment can affect a multiproduct firm's choice of product line. Their conclusion is that in the presence of commitment the smaller the degree of brand-specific differentiation, the more likely the firm specializes. In the absence of commitment, firms produce a full product line. Canoy and Peitz (1997) modeled a differentiation triangle where the low quality good is not subject to horizontal differentiation. They also addressed the questions concerning when and why firms produce a full product line or follow a niche strategy. They modeled competition among firms in the differentiation triangle with sequential entry. Their main finding is that incumbents have different motivation to develop a niche or full line strategy. Incumbent 1 who has a first-mover advantage choose products for the purpose of profit. Incumbent 2 has strategic incentive to determine product line, either for the purpose of deterring entry, or accommodating entry. Different from their studies, our work is the first to endogenize reputation explicitly into a duopoly model where goods are differentiated in two dimensions. Katz (1984) introduced firm-specific reputation into his model, yet he treated it as one of the two dimensions which distinguish products manufactured by various multiproduct firms.

Many studies have examined the issue of product line proliferation. Cheng, Peng and Tubuchi (2010) investigates a two-stage competition in a vertically differentiated market and show that each firm has an incentive to produce a range of qualities under the assumption of an increasing and quadratic marginal cost of quality. To the best of our knowledge, very few studies have attempted to address why firms choose to restrict their product line or specialize in a niche market. Our study is the first to examine this issue in the context of both vertically and horizontally differentiation.

The structure of this paper is as follows. Section 2 presents the theoretical model of competition among multiproduct retailers. We describe the simultaneous game is introduced and solve for the Nash equilibrium. Conclusions are discussed in Section 3.

2. The model

We present a two-dimensional product differentiation model to depict firm behavior under a duopolistic market structure. We assume that the products are differentiated both vertically (quality) and horizontally (location). Consumers have preferences for quality that are a function of their income (or something similar) and we assume that the distribution is uniform. Retailers are located at two ends of a linear city and can offer the same product line or not. Our model is based on Gilbert and Matutes' (1993) setup, and we build on their model by introducing reputation as an endogenous attribute. The key assumption that drives our results is that consumers "care" about reputation; that is, firm reputation enters into their utility function.

General assumptions and notations

Retailers

Two retailers A and B, indexed by j, compete in a duopolistic market of a generic good. The good is differentiated according to vertical characteristics (quality) and horizontal characteristics (location). We use i to index the vertically-differentiated variants of the good, where $i = H$ (high-quality variant) and L (low-quality variant). The two retailers can choose a full product line by providing both variants, or they produce only one variant and focus on a specialized product line. We assume the high quality variant generates higher reputation for those who provide it. The good is also differentiated by location, for the two retailers lie at two opposite ends of a linear city of length 1 or $l^A = 0$ and $l^B = 1$.

The retailers' optimization problem is

$$(1) \qquad Max \ \pi_i^j = \sum_{i=1,2} \left(P_i^j - c_i\right)\varphi_i^j,$$

where π is profit, P denotes the price, c is the marginal cost, and φ is the market share of the respective good by the respective retailer.

Consumers

On the consumer side, we assume the following: (a) consumers are uniformly distributed along a linear city of length 1; (b) each consumer located at δ has a preference for quality denoted by θ which is uniformly distributed over the interval $[0,1]$; (d) each consumer purchases one and only one good[1].

The utility of a representative consumer buying product i from firm j is $U_i^j = \theta \mathrm{s}_i - t(1 - r)\left|\delta - l^j\right| - P_i^j$. In this utility function, s_i denotes the quality of product i and $\mathrm{s}_H \geq \mathrm{s}_L$; t is transportation cost; l represents the location of the two retail stores. Our contribution to the literature is to incorporate reputation into the utility function. Specializing in high-quality variants can generate higher reputation for retialers. If a retailer has high reputation, i.e. $r = 1$, the utility function becomes $U_i^j = \theta \mathrm{s}_i - P_i^j$. If a retailer purchases from a retailer with low reputation, i.e. $r = 0$, the consumers' utility is $U_i^j = \theta \mathrm{s}_i - t\left|\delta - l^j\right| - P_i^j$.

In our model, reputation enters into the utility function by effectively offsetting the disutility caused by transportation cost, it is essentially, though not actually, a reimbursement of travel expenses for any consumer who purchases from a retailer with higher reputation. We can easily see that the greater t is, the greater the "gain" a consumer receives purchasing from a store with high reputation.

The duopoly game

The competition between the two retailers in terms of product line choice can be described by a simultaneous game. In the simultaneous game, retailer A and B choose product line and prices simultaneously. While deciding the product line, retailer A faces the strategy set $\{(\emptyset),\ (AH),\ (AL) \text{and}\ (AH, AL)\}$, where AH represents retailer A offering the high quality product, AL represents retailer A offering the low quality product, and ∅ represents the null set (or no products). Similarly, retailer B's strategy set is $\{(\emptyset),\ (BH),\ (BL) \text{ and } (BH, BL)\}$. Combining these strategies yields four scenarios: $\{(AH, \emptyset), (BH, \emptyset)\}$, $\{(AH, \emptyset), (BH, BL)\}$, $\{(AH, AL), (BH, BL)\}$ and $\{(AH, AL), (BH, \emptyset)\}$. Since our hypothesis is that specializing in high quality products can lead to high reputation, retailers have no incentives to specialize in low quality and we do not consider the situation where only the low quality good is provided[2]. We also ignore the case where one or both retailers offer no products, but it is easy to verify that these are never optimal strategies. Therefore, there are three scenarios to consider in the duopoly game. The first scenario is $\{(AH, AL);\ (BH, BL)\}$. In this scenario, both retailers engage in symmetric competition in which both of them choose to provide a full line. The second scenario is $\{(AH);\ (BH)\}$. This is also a symmetric competition, but both of them choose to specialize in high quality products. The third one is $\{(AH);\ (BH, BL)\}$, which is an asymmetric competition in which retailer A provides only high quality and retailer B chooses a full line. We can prove that $\{(AH, AL);\ (BH, BL)\}$ and $\{(AH);\ (BH)\}$ have the same results. Thus we only look at two scenarios in our study: $\{(AH, AL),\ (BH, BL)\}$ and $\{(AH);\ (BH, BL)\}$. We treat $\{(AH, AL),\ (BH, BL)\}$ as the baseline scenario where both retailers have the same reputation, in other words, reputation does influence retailers' decision of product line strategy. In scenario 2, retailer A focuses on the high quality goods and gains higher reputation than the retailer who offers a full line. To find the Nash Equilibrium of the duopoly game, we need to know their payoff which is simply their profit from each scenario. This is to be accomplished in the following subsections.

Baseline scenario: a symmetric competition

The baseline duopoly model draws on Gilbert and Matutes' (1993). In their model, each retailer chooses to provide a full product line. They do not consider the effect of retailer reputation. We first take a look at the consumers' decision under horizontal and vertical differentiation, respectively.

Horizontal differentiation

We consider the model under horizontal and vertical differentiation separately. In the setting of horizontal differentiation consumers are indifferent between buying good i from retailer A or B if $\theta s_i - P_i^A - t|\delta - l^A| = \theta s_i - P_i^B - t|\delta - l^B|$ .This yields

$$(2) \qquad \hat{\delta}_i = \frac{P_i^B - P_i^A + t}{2t}.$$

If a consumer is located in the interval [0, $\hat{\delta}_i$), she buys product i from retailer A. If she is located between the range of ($\hat{\delta}_i$, 1], she prefers to purchase from retailer B. At $\hat{\delta}_i$ the consumer is indifferent between buying from retailer A or B. $\left|P_i^B - P_i^A\right| \leq t$ which indicates the price differential between the two retailers cannot exceed transportation cost. A necessary condition for this to be true is that if $\left|P_i^B - P_i^A\right| \geq t$, retailer B has no demand for good i, while retailer A captures the entire market of good i. If $P_i^B = P_i^A$, $\hat{\delta}_i = \frac{1}{2}$, retailer A and B split the market

Vertical differentiation

We next look at the consumer's decision between choosing a high quality or low quality good. The consumer is indifferent between buying good H and good L from a given retailer j if $\theta s_L - P_L^j - t\left|\delta - l^j\right| = \theta s_H - P_H^j - t\left|\delta - l^j\right|$. It yields

$$(3) \qquad \hat{\theta}^j = \frac{P_H^j - P_L^j}{s_H - s_L}.$$

If a consumer's preferences lie between [0, $\hat{\theta}^j$), she buys a low-quality good from a retailer j. If her preferences are located between($\hat{\theta}^j, 1$], she will choose a high-quality good from the retailer. At $\hat{\theta}^j$ she is indifferent as long as $P_H^j - P_L^j \geq 0$ and $P_H^j - P_L^j \leq s_H - s_L$. These two conditions ensure $\hat{\theta}^j$ between 0 and 1. If $P_H^j - P_L^j > s_H - s_L$, there is no demand for the high quality products. In this case, we need to restrict the price differential to be equal to or lower than the quality differential to ensure both products are purchased. At $P_H^j = P_L^j$, retailer j charges the same price for both goods and drives the demand for the low- quality goods to zero.

Horizontal and vertical differentiation

In the baseline case, we want to look at how retailer A and B compete with each other when they both choose to product a symmetric full product line. The products they provide are both horizontally and vertically differentiated.

We can think of the space that depicts consumers and firms when there is both vertical and horizontal differentiation as being a unit square. Let $ij \succ \overline{ij}$ denotes that a consumer prefers product i from retailer j to product $\bar{\imath}$ from retailer $\bar{\jmath}$. A consumer is indifferent between buying good i from retailer j and buying good $\bar{\imath}$ from retailer $\bar{\jmath}$ if $\theta s_i - P_i^j - t\left|\delta - l^j\right| = \theta s_{\bar{\imath}} - P_{\bar{\imath}}^{\bar{\jmath}} - t|\delta - l^{\bar{\jmath}}|$. For example, for a consumer who is indifferent with buying AL and BH, her physical location and preference for quality have the following relationship:

(4) $$\hat{\theta} = \frac{2t(\hat{\delta}_l - \hat{\delta})}{s_H - s_L} + \hat{\theta}^B.$$

Then we plug equation (1) and (2) into (3) yielding

(5) $$\hat{\theta} = \frac{P_L^B - P_L^A + P_H^B - P_L^B + t - 2t\hat{\delta}}{s_H - s_L}.$$

From equation (5), we can see that the consumer's choice between AL and BH is determined by the following factors: (1) price differentials; (2) transportation cost t; and (3) quality differential $s_H - s_L$. The effect of price differential occurs at two levels: within-retailer but across products ($P_H^B - P_L^B$) and across retailers but within products ($P_L^B - P_L^A$). Therefore, the consumer's choice between AL and BH can be describe by a two-step process, she first chooses between BL and BH, which is the quality decision in a given retailer, if $P_H^B - P_L^B$ exceeds $s_H - s_L$, she would rather to choose BL. Next she compares BL and AL, If $P_L^B - P_L^A$ is greater than t, she will purchase AL. Thus a consumer purchases AL rather than BH if her preference ranking is $AL \succ BL \succ BH$.

A general form of equation (4) can be written as

(6) $$\hat{\theta} = \frac{2t(\hat{\delta}_i - \hat{\delta})}{s_{\bar{\imath}} - s_i} + \hat{\theta}^{\bar{\jmath}}.$$

Equation (5) describes the relationship between θ and δ when $ij \succ \overline{\imath\jmath}$, which is the consumer prefer good i from retailer j over good $\bar{\imath}$ from retailer$\bar{\jmath}$. $\hat{\delta}$ is where the consumer does not care about which retailer to go given a good i . $\hat{\theta}^{\bar{\jmath}}$ is where the consumer feels indifferent about the quality given a retailer $\bar{\jmath}$.

So far, we have described how consumers choose between AL and BL (see equation (2)), AL and AH (see equation (3)) and AL and BH (see equation (4)). Next we would like to see under what circumstances AL can be chosen by a consumer located in $\hat{\delta}$ with quality taste $\hat{\theta}$.

To summarize the conditions we talked previously, a consumer located at $\hat{\delta}$ with taste parameter for quality $\hat{\theta}$ will purchase AL if and only if

(i) $AL \succ BL$ requires $\hat{\delta} \leq \hat{\delta}_L$

(ii) $AL \succ BH$ requires $\hat{\theta} \leq \frac{2t(\hat{\delta}_L - \hat{\delta})}{s_H - s_L} + \hat{\theta}^B$

(iii) $AL \succ AH$ requires $\hat{\theta} \leq \hat{\theta}^A$,

where $\hat{\delta}_L$ is where consumers are indifferent between AL and BL, $\hat{\theta}^A$ and $\hat{\theta}^B$ are where consumers are indifferent between AL and AH, and BL and BH, respectively.

Under condition (i), retailer A wins out in a competition with retailer B for sales of the low-quality good. In (ii), the rivalry is between different retailers producing different goods, AL wins. In case (iii), the competition is between different goods from the same retailers and the low quality goods win. Therefore the three conditions are both necessary and sufficient conditions of $AL \succ BH$. In other words, a consumer located at δ with taste parameter θ will purchase AL if and only if

(7) $$\hat{\delta} \leq \hat{\delta}_L \, ; \hat{\theta} \leq min\left[\hat{\theta}^A, \frac{2t(\hat{\delta}_L - \hat{\delta})}{s_H - s_L} + \hat{\theta}^B\right].$$

Given $\hat{\theta}^A > \hat{\theta}^B$ and $\hat{\delta}_L > \hat{\delta}_H$, the sales region of AL is demonstrated in Figure 1

Similarly, the consumer will purchase the AH if and only if

(i) $AH \succ BH$ requires $\hat{\delta} \leq \hat{\delta}_H$

(ii) $AH \succ BL$ requires $\hat{\theta} \geq -\frac{2t(\hat{\delta}_H - \hat{\delta})}{s_H - s_L} + \hat{\theta}^B$

$AH \succ AL$ requires $\hat{\theta} \geq \hat{\theta}^A$

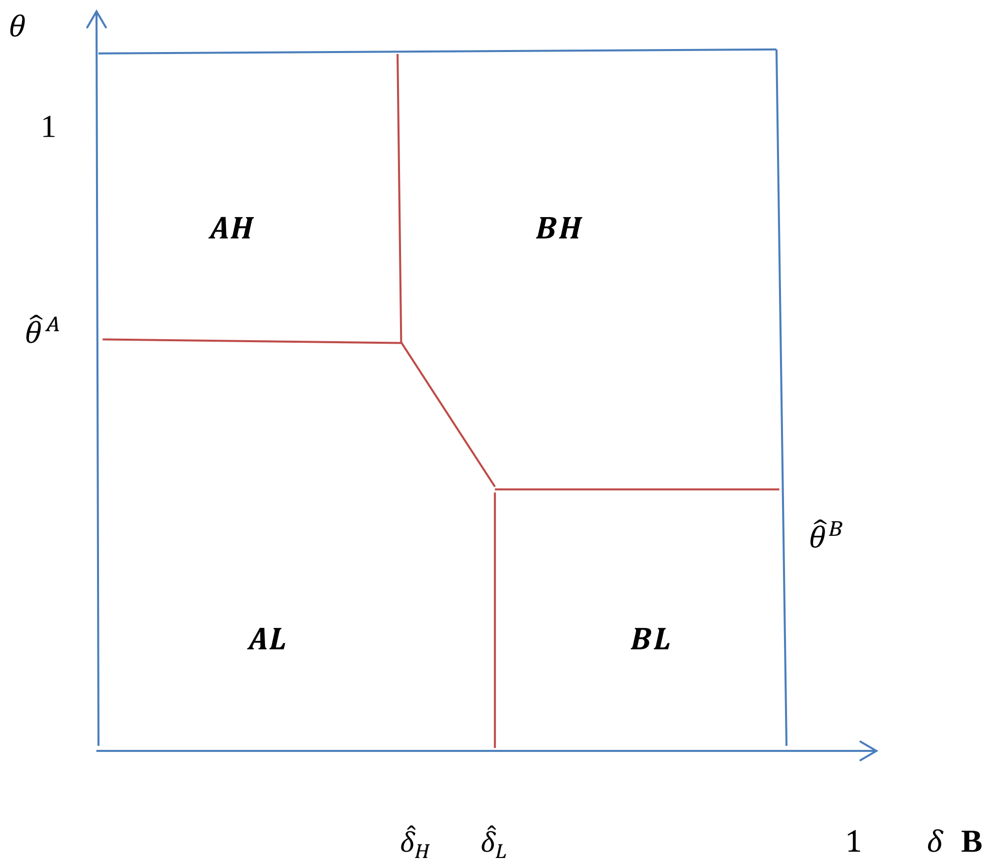
(iii) Which can also be written as

(8) $$\hat{\delta} \leq \hat{\delta}_H ; \hat{\theta} \geq \max\left[\hat{\theta}^A, \hat{\theta}^B - \frac{2t(\hat{\delta}_H - \hat{\delta})}{s_H - s_L}\right].$$

Given $\hat{\theta}^A > \hat{\theta}^B$ and $\hat{\delta}_L > \hat{\delta}_H$, the sales region of AH is demonstrated in Figure 1.

Likewise, we can draw sales regions for retailer B in the same unit square. Then we obtain the sales regions for all the products, which is illustrated in figure1.

Figure 1. Sales region of all products



Market shares

We now can calculate the market shares for each of the goods based on the market segmentation above. They are the areas of different regions in figure 1. We have

$$\varphi_L^A = \hat{\delta}_L \hat{\theta}^A, \tag{9}$$

$$\varphi_H^A = \hat{\delta}_H (1 - \hat{\theta}^A) - \frac{1}{2}(\hat{\delta}_H - \hat{\delta}_L)(\hat{\theta}^B - \hat{\theta}^A), \tag{10}$$

$$\varphi_L^B = (1 - \hat{\delta}_L)\hat{\theta}^B, \tag{11}$$

and

(12) $$\varphi_H^B = \left(1-\hat{\delta}_H\right)\left(1-\hat{\theta}^B\right) - \frac{1}{2}\left(\hat{\delta}_H-\hat{\delta}_L\right)\left(\hat{\theta}^B-\hat{\theta}^A\right),$$

where φ_L^A, φ_H^A, φ_L^B and φ_H^B are market shares for good AL, AH, BL and BH.

Profit-maximization problem and solutions

We assume that the two retailers engage in Bertrand –Nash competition in each submarket. Here we assume the retailers have the same cost of providing a good i and $c_H > c_L$. We also assume unit transportation cost[3].

Retailer A's profit-maximizing problem is written as

(13) $$\max_{P_L^A, P_H^A} \pi^A = \pi_L^A + \pi_H^A$$
$$= (P_L^A - c_L)(1-\hat{\delta}_L)\hat{\theta}^B + (P_H^A - c_H)(\hat{\delta}_H\left(1-\hat{\theta}^A\right)$$
$$-\frac{1}{2}\left(\hat{\delta}_H-\hat{\delta}_L\right)\left(\hat{\theta}^B-\hat{\theta}^A\right)).$$

Retailer B's problem is

(14) $$\max_{P_L^A, P_H^A} \pi^B = \pi^{AL} + \pi^{AH}$$
$$= (P_L^A - c_L)(1-\hat{\delta}_L)\hat{\theta}^B + (P_H^A - c_H)(\hat{\delta}_H\left(1-\hat{\theta}^A\right)$$
$$-\frac{1}{2}\left(\hat{\delta}_H-\hat{\delta}_L\right)\left(\hat{\theta}^B-\hat{\theta}^A\right)).$$

The equilibrium prices we calculated are $P_H^A = P_H^B = c_H + 1$ and $P_L^A = P_L^B = c_L + 1$. The market shares are $\varphi_L^A = \varphi_L^B = \varphi_H^A = \varphi_H^B = \frac{1}{2}$ and the profit of each firm is $\frac{1}{2}$, given $c_H - c_L = (s_H - s_L)/2$, where $c_H - c_L$ is a measure of difference between marginal cost of offering high quality and low quality products and we use k to represent it in the rest of the paper. $(s_H - s_L)$

is consumers' perception of quality differential between high and low quality. We use k and s to denote marginal cost differential and quality differential in the remainder of the paper. k and s play essential roles in the scenario of an asymmetric competition. We will give more details about these two parameters in the following description of the asymmetric scenario. The computational details are shown in Appendix 1.

From the equilibrium we can see that the price- cost markup is the same for each good. This is the same as in the single-product completion where each retailer only provides one quality. Our conclusion is the same as Gilbert and Matutes (1993): price- cost markups are independent of the number of goods provided and thus profit is independent of the number of variants provided, which means a multi-product strategy cannot bring more profit for retailers relative to a single- product one. Furthermore, we can also reach the conclusion that the margin for a given quality is also independent of consumers' tastes for quality attributes. The Nash equilibrium tells us that both retailers offer the full product line, although the retailers are not any better off by offering the product line vis-à-vis a single product. This is because if one firm is producing a single variant, it could always generate additional sales without losing profit by introducing a second variant with the same price-cost margin. The additional profit made by the single-product retailer has several sources: (1) from the consumers that switch from the old variant to the new one without switching retailers; and (2) from the consumers who used to buy the new variant from the other retailer and now switches.

Scenario 2: an asymmetric competition

In scenario 2, retailer A specializes in the high quality submarket. Here she gains higher reputation through the specialization. In contrast, retailer B retains his full product line and has a low reputation.

Market shares

A consumer who purchases from retailer A gains extra utility due to the store reputation, and the utility can offset her travel cost. The utility function of this consumer is $U_i^j = \theta s_i$, as $r = 1$. If she shops from retailer B, her utility function is $U_i^j = \theta s_i - t|\delta - 1| - p_i^j$, as $r = 0$.

When the consumer is indifferent between buying high-quality good from retailer A (AH) and B (BH), the consumer obtains equal utility from AH and BH, which is

$$\theta \mathrm{s}_H - P_H^A = \theta \mathrm{s}_H - P_H^B - t(1-\delta). \tag{15}$$

Thus consumers are physically located at

$$\tilde{\delta}_H = \frac{P_H^B - P_H^A + t}{t}. \tag{16}$$

When the consumer is indifferent between BH and BL, she obtains the same utility from BH and BL, which is

$$\theta \mathrm{s}_L - P_L^B - t(1-\delta) = \theta \mathrm{s}_H - P_H^B - t(1-\delta). \tag{17}$$

Consumers' preferences for quality are located at

$$\tilde{\theta}^B = \frac{P_H^B - P_L^B}{s_H - s_L}. \tag{18}$$

When the consumer is indifferent between AH and BL, we have

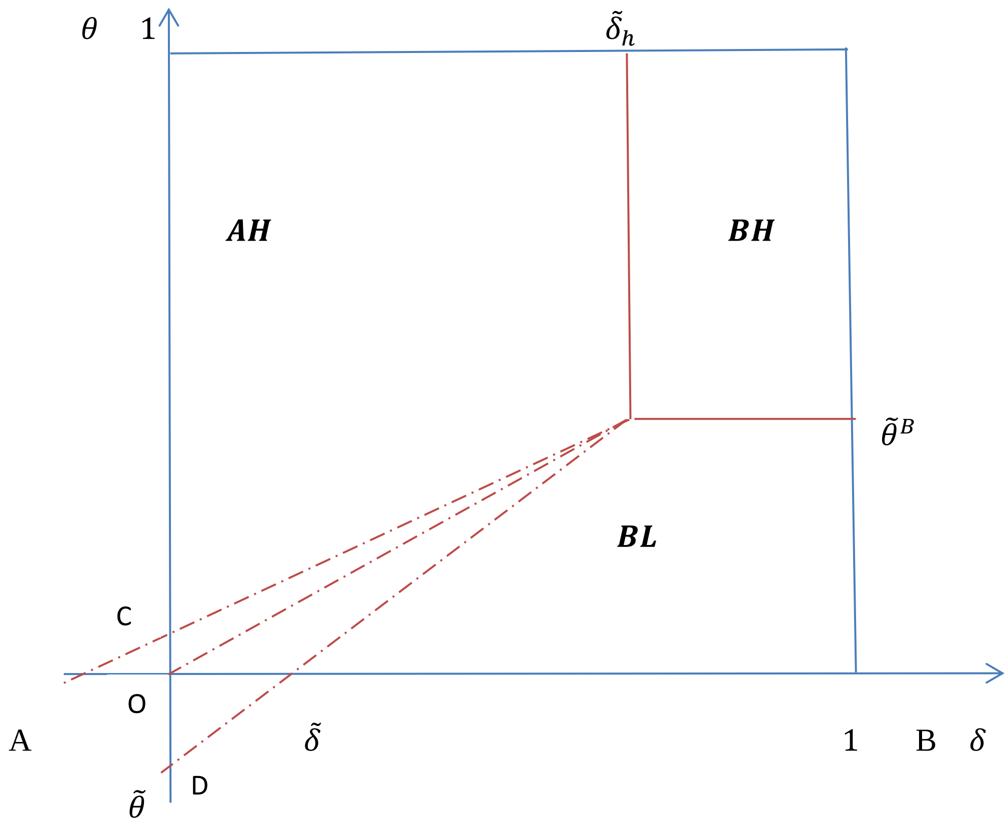
(19) $$\theta s_H - P_H^A = \theta s_L - P_L^B - t(1-\delta).$$

The relationship between consumers' locations and their preferences for quality can be represented by

(20) $$\tilde{\theta} = \frac{P_H^A - P_L^B - t(1-\tilde{\delta})}{s_H - s_L}.$$

In figure 8, we can see how the entire market is segmented by AH, BH and BL. We need to note that the locations of the difference curve between AH and BL depend on the relationship between price differential $P_H^A - P_L^B$ and transportation cost t. When $P_H^A - P_L^B = t$, we have $\tilde{\theta} = 0$, and AO is the indifference curve between AH and BL. Similarly, when $P_H^A - P_L^B > t$, AC is the indifference curve; and when $P_H^A - P_L^B < t$, AD is the indifference curve. Therefore, there are three cases of market segmentation. We can also prove that the consumer is indifferent among AH, BH and BL at *($\tilde{\delta}_H, \tilde{\theta}^B$)*.

Figure 2. Market segmentation if retailer A specializes in high quality



Market shares under each case can be computed as follows:

The market shares in case 1 are

(21) $$\varphi_1^A = \varphi_{1H}^A = \tilde{\delta}_H * 1 - \frac{1}{2}\tilde{\delta}_H \tilde{\theta}^B$$

and

(22) $$\varphi_1^B = \varphi_{1H}^B + \varphi_{1L}^B = \left(1 - \tilde{\delta}_H\right)\left(1 - \tilde{\theta}^B\right) + \frac{1}{2}\tilde{\delta}_H \tilde{\theta}^B + \left(1 - \tilde{\delta}_H\right)\tilde{\theta}^B,$$

Where φ^{AH}, $\varphi^{BH}$and φ^{BL} are the shares of AH, BH and BL. φ^{A} and φ^{B} are the total market shares captured by retailer A and B. Subscript 1 denote case 1.

The market shares in case 2 are

(23) $$\varphi_2^A = \varphi_{2H}^A = \tilde{\delta}_H * 1 - \frac{1}{2}\tilde{\delta}_H\tilde{\theta}^B$$

and

(24) $$\varphi_2^B = \varphi_{2H}^B + \varphi_{2L}^B = \left(1-\tilde{\delta}_H\right)\left(1-\tilde{\theta}^B\right) + \frac{1}{2}\tilde{\delta}_H\tilde{\theta}^B + \left(1-\tilde{\delta}_H\right)\tilde{\theta}^B.$$

Lastly, in case 3, we have the following market shares:

(25) $$\varphi_3^A = \varphi_{3H}^A = \tilde{\delta}_H * 1 - \frac{1}{2}\tilde{\delta}_H\tilde{\theta}^B + \frac{1}{2}(-\tilde{\delta})\tilde{\theta}$$

and

(26) $$\varphi_3^B = \varphi_{3H}^B + \varphi_{3L}^B = \left(1-\tilde{\delta}_H\right)\left(1-\tilde{\theta}^B\right) + \frac{1}{2}\tilde{\delta}_H\tilde{\theta}^B + \left(1-\tilde{\delta}_H\right)\tilde{\theta}^B - \frac{1}{2}(-\tilde{\delta})\tilde{\theta}.$$

Retailers’ problems and solutions

After having each product’s market share under various cases, we next need to solve retailers’ profit-maximization problems and compute price equilibrium. Their problems are summarized in table 1.

Table 1. Retailers' profit-maximization problems under various cases

	Retailer A	Retailer B
Case 1	$\max_{P_1^{AH}} \pi_1^A = \pi_{1H}^A = (P_{1H}^A - c_H)\varphi_{1H}^A$	$\max_{P_1^{BL},P_1^{BH}} \pi_1^B = \pi_{1L}^B + \pi_{1H}^B$ $= (P_{1L}^B - c_L)\varphi_{1L}^B + (P_{1H}^B - c_H)\varphi_{1H}^B$
Case 2	$\max_{P_2^{AH}} \pi_2^A = \pi_{2H}^A = (P_{2H}^A - c_H)\varphi_{2H}^A$	$\max_{P_2^{BL},P_2^{BH}} \pi_2^B = \pi_{2L}^B + \pi_{2H}^B$ $= (P_{2L}^B - c_L)\varphi_{2L}^B + (P_{2H}^B - c_H)\varphi_{2H}^B$
Case3	$\max_{P_3^{AH}} \pi_3^A = \pi_{3H}^A = (P_{3H}^A - c_H)\varphi_{3H}^A$	$\max_{P_3^{BL},P_3^{BH}} \pi_3^B = \pi_{3L}^B + \pi_{3H}^B$ $= (P_{3L}^B - c_L)\varphi_{3L}^B + (P_{3H}^B - c_H)\varphi_{3H}^B$

Note: The profit-maximization problems are subject to several constraints which are presented in Appendix 2.

Here we only present the results of case 2. We also tried the other two cases. Case 1 does not make sense when imposing constraints. Case 3 has similar results as case 2. Solutions to the profit-maximizing problem under case 2 can be found in table 2. Appendix 2 outlines details of how the problem is solved.

The results are summarized in table 1. The results are based on different values of following parameters: marginal cost differential (k), quality differential (s) and transportation cost (t). k is a measure of the difference between the marginal cost of producing the high and low quality goods. The higher k is, the more costly it is to produce a high quality product relative to a low quality product. The factors that affects k could be size of the retailers and technology-related factors which become obstacles to economies of scope, etc. In our study, we let it vary between 0 and 10[4], where 0 means it makes no extra cost to improve quality and 10 means it is expensive for retailers to offer high quality. s measures consumers' perception of quality difference between high quality and low quality. It varies between 0 and 10[5] in our study, which means consumers either perceive no difference between the two goods (0) or a very large difference between high quality and low quality (10). The magnitude of t affects how likely consumers are willing to commute to a store. In our example, we choose values of 2, 5 and 10[6] represent low, moderate and high transportation costs. We can infer the following results from table 2.

1) Transportation cost plays an important role in affecting consumer and retailer behavior. Both retailer A and B choose product line subject to the magnitude of transportation cost. For retailer A, she can be better off by specializing in the high quality goods when transportation cost reaches a certain threshold (which is conditional on the other parameters). This is because the higher transportation cost is, the more consumers "gain" by purchasing from the high reputation retailer. We can easily see that retailer A benefits from an increase in transportation costs through higher profit, a larger market share and greater market power (i.e., markup). For retailer B, in most cases, she also gains in the asymmetric competition in terms of greater profit and markups. However, her market share of each product is declining with an increase in transportation cost.

To summarize, we can say both retailers benefit from asymmetric competition, but retailer A is a winner relative to retailer B, given increasing transportation costs.

2) The cost differential k and quality differential s has a significant influence on the retailer's choice of product line. We can see from the table, retailer A's market share and markup for AH are greater at the point when consumers perceive a big difference between the high quality good and low quality good and when providing high quality good is cheap than the point when consumers perceive a small difference between the high quality good and low quality good and providing high quality good is costly. For retailer B, If consumers perceive the high quality good to be far superior to the low quality good and difference in marginal cost between the two is small, the retailer B would provide more high quality products relative to low quality products. We can see that the market share of BH is greater than BL. If consumers perceive the high quality good to be far superior to the low quality good, but the marginal cost of producing the high quality good is also much higher than that of the low quality good, then retailer B will capture almost none of the high quality good market.

Table 2. Results of the asymmetric competition given certain parameters

k	s	t	Markups(AH, BH, BL)			Mkt Shares (AH, BH, BL)			Profits(A,B)	
8	8	2	0.14	0.10	2.71	0.07	0.02	0.91	0.01	2.49
		5	2.08	1.39	3.43	0.42	0.03	0.55	0.87	1.92
		10	5.36	3.57	4.83	0.54	0.02	0.55	2.88	2.20
2	8	2	1.26	0.65	1.14	0.63	0.25	0.12	0.79	0.30
		5	3.26	1.67	2.03	0.65	0.25	0.10	2.12	0.62
		10	6.59	3.36	3.60	0.66	0.25	0.09	4.34	1.17
2	2	2	1.02	0.68	1.06	0.51	0.03	0.46	0.52	0.50
		5	3.00	2.00	2.00	0.60	0	0.40	1.81	0.79
		10	6.34	3.66	3.66	0.63	0	0.37	4.02	1.34

Equilibrium of the game

Based on the payoffs of the symmetric and asymmetric scenarios, we can find the Nash equilibrium of the simultaneous game. The game is presented in tables 2 and 3 where we let $k = 8$, $s = 8$, and consider both low and high transportation costs, respectively.

Table 3. Payoff matrix with low t

	Retailer B		
Retailer A		H	$H\&L$
	H	0.5, 0.5	0.01, 2.49
	$H\&L$	2.49, 0.01	<u>0.5</u>, <u>0.5</u>

Table 4. Payoff matrix with high t

	Retailer B		
Retailer A		H	$H\&L$
	H	0.5, 0.5	<u>2.88</u>, <u>2.20</u>
	$H\&L$	<u>2.20</u>, <u>2.88</u>	0.5, 0.5

From table 3, we can see that the Nash equilibrium is $\{(AH, AL);\ (BH, BL)\}$, when t is low. When t is high, as table 4 shows, there are two Nash equilibria $\{(AH);\ (BH, BL)\}$ and $\{(AH, AL);\ (BH)\}$. This indicates that the game ends up with a symmetric competition in which both retailers provide a full product line when transportation cost is low. When the transportation cost is high, the game ends up with an asymmetric competition in which one retailer specializes in high quality and the other one offers a full product line.

3. Conclusions

In this study we analyzed competition among multiproduct firms, specifically retailers, in terms of product line choice and prices. Our objective is to investigate the conditions under which retailers would choose to restrict their product line. In contrast to firms that offer a very wide product line to price discrimination, we show that that if reputation is a function of product line, concentrating on a high quality niche market can be an optimal strategy for retailers.

We modeled a simultaneous game in a vertically differentiated market where two retailers decide their product offerings prices simultaneously. We examined a case of symmetric competition in which both retailers provide a full product line and an asymmetric case in which one retailer decides to specialize in a high quality product. We found that retailers can be better off from specialization if there is a reputation effect. The way we have modeled how reputation takes effect is to assume that it reimburses consumers by offsetting the travel cost they incur from shopping. In our model, travel cost is the only factor which generates disutility other than price. Therefore, based on the assumptions of our model, we also found transportation cost to be an important factor, the higher the transportation cost, the greater the reimbursement consumers can gain, and the more likely they will choose to purchase from a retailer with higher reputation. In this case, the retailer's incentive to specialize in high quality goods can be increased.

Judd (1985) and Gilbert and Matutes (1993) both concluded that specialization depends on the extent of brand-specific differentiation. It occurs if the degree of brand-specific differentiation is small. In contrast with their findings, our study further investigates beyond brand-specific differentiation of products and considers reputation as a differentiating feature among firms. We explore the question what may affect specialization even though the brand-specific differentiation is constant. Our model sheds light on endogenous reputation formation and its effect on multiproduct firms' choice of product line.

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[1] This assumption imposed on consumers is for the purpose of simplifying the analysis.

[2] In this study, we are not interest in the case where the retailers would choose specialize in low quality. Specialization in low quality does not have anything to do with "reputation effect" which is our main interest. We also checked and it is not an optimal strategy.

[3] We have checked that in the baseline symmetric scenario, the magnitude of transportation costs does not affect the results.

[4] We use number 0 and 10 to denote, respectively, low and high difference in consumers' perception of high quality and low quality. k can be an arbitrary number. We choose these two numbers because we want to see the difference from a wide range of k.

[5] s is also arbitrary, just as k. The reason we make it vary between 0 and 10 is the same as k. We want want to see the difference in a wide range.

[6] We have tried various numbers between 0.01 and 20. Note that if transportation costs are so high that consumers would not patronize either retailer. So we think t should be restricted to ensure consumers have a chance to switch between the two retailers. Ideally, we want to describe t in terms of percent of price of each good, however, we have no further information of the magnitude of marginal cost of each good.

Appendix 1: solving retailers' problems in Baseline scenario

In this appendix, we solve retailer A and B's problems and derive the equalibrium prices, market shares and profits. We assume unit transportation cost. According to the market share segmentation in figure 1

$$\begin{aligned}
\widehat{\delta}_h &= \frac{p_{Bh} - p_{Ah} + 1}{2}, \\
\widehat{\delta}_l &= \frac{p_{Bl} - p_{Al} + 1}{2}, \\
\widehat{\theta}^A &= \frac{p_{Ah} - p_{Al}}{s},
\end{aligned}$$

and

$$\widehat{\theta}^B = \frac{p_{Bh} - p_{Bl}}{s},$$

where A and B represents retailer A and B. h and l refers to high quality and low quality. p is price. So p_{Ah} means price of high quality manufactured by retailer A.

Retailer A's problem is

$$\begin{aligned}
Max\ \pi^A &= (p_{Al} - c_l)\widehat{\delta}_l\widehat{\theta}^A + (p_{Ah} - c_h)\widehat{\delta}_h(1 - \widehat{\theta}^A) \\
&= (p_{Al} - c_l)\frac{p_{Bl} - p_{Al} + 1}{2}\frac{p_{Ah} - p_{Al}}{s} \\
&\quad + (p_{Ah} - c_h)\frac{p_{Bh} - p_{Ah} + 1}{2}(1 - \frac{p_{Ah} - p_{Al}}{s})
\end{aligned}$$

The first order condition with respect to p_{Al} is

$$\begin{bmatrix}
\frac{1}{2s}(p_{Al} - p_{Ah})(p_{Al} - c_l) \\
-\frac{1}{2s}(p_{Al} - c_l)(1 - p_{Al} + p_{Bl}) \\
-\frac{1}{2s}(p_{Al} - p_{Ah})(1 - p_{Al} + p_{Bl}) \\
-\frac{1}{2s}(c_h - p_{Ah})(1 - p_{Ah} + p_{Bh})
\end{bmatrix} = 0.$$

The reaction function is:

$$\begin{aligned}
&p_{Al} = \frac{1}{3} + \frac{1}{3}c_l \pm \\
&\frac{1}{3}\sqrt{\begin{array}{c} 3c_h p_{Bh} - 3c_h p_{Ah} - c_l p_{Ah} - c_l p_{Bl} + 3c_h - c_l + c_l^2 \\ -3p_{Ah}p_{Bh} - p_{Ah}p_{Bl} \\ -4p_{Ah} + 2p_{Bl} + 4p_{Ah}^2 + p_{Bl}^2 + 1 \end{array}} \\
&+\frac{1}{3}p_{Ah} + \frac{1}{3}p_{Bl}
\end{aligned}$$

The first order condition with respect to p_{Ah} is

$$\begin{aligned} &\left[\begin{array}{c} \frac{1}{2}\left(\frac{1}{s}\left(p_{Ah}-p_{Al}\right)-1\right)\left(p_{Ah}-c_h\right) \\ -\frac{1}{2}\left(\frac{1}{s}\left(p_{Ah}-p_{Al}\right)-1\right)\left(1-p_{Ah}+p_{Bh}\right) \\ -\frac{1}{2s}\left(p_{Ah}-c_h\right)\left(1-p_{Ah}+p_{Bh}\right) \\ -\frac{1}{2s}\left(c_l-p_{Al}\right)\left(1-p_{Al}+p_{Bl}\right) \end{array}\right] \\ = \quad & 0, \end{aligned}$$

The reaction function is:

$$\begin{aligned} p_{Ah} \quad = \quad & \frac{1}{3}+\frac{1}{3}s+\frac{1}{3}c_h+\frac{1}{3}p_{Bh}+\frac{1}{3}p_{Al} \\ & \pm\frac{1}{3}\sqrt{\begin{array}{c} 3c_l-c_h-s-c_hp_{Bh}-c_hp_{Al}-3c_lp_{Al}+3c_lp_{Bl} \\ -sc_h+c_h^2+2p_{Bh}-4p_{Al}-p_{Bh}p_{Al} \\ -3p_{Al}p_{Bl}-sp_{Bh}+2sp_{Al} \\ +p_{Bh}^2+4p_{Al}^2+s^2+1 \end{array}}. \end{aligned}$$

Retailer B's problem is

$$\begin{aligned} Max\ \pi^B \quad &= \quad (p_{Bl}-c_l)(1-\widehat{\delta}_l)\widehat{\theta}^B+(p_{Bh}-c_h)(1-\widehat{\delta}_h)(1-\widehat{\theta}^B) \\ &= \quad (p_{Bl}-c_l)(1-\frac{p_{Bl}-p_{Al}+}{2})\frac{p_{Bh}-p_{Bl}}{s_h-s_l} \\ & \quad +(p_{Bh}-c_h)(1-\frac{p_{Bh}-p_{Ah}+}{2})(1-\frac{p_{Bh}-p_{Bl}}{s}). \end{aligned}$$

The first order condition with respect to p_{Bl} is

$$\begin{aligned} &\left[\begin{array}{c} \frac{1}{s}\left(\frac{1}{2}\left(p_{Bl}+1-p_{Al}\right)-1\right)\left(p_{Bl}-c_l\right) \\ +\frac{1}{s}\left(p_{Bl}-p_{Bh}\right)\left(\frac{1}{2}\left(p_{Bl}+1-p_{Al}\right)-1\right) \\ +\frac{1}{s}\left(\frac{1}{2}\left(1-p_{Ah}+p_{Bh}\right)-1\right)\left(c_h-p_{Bh}\right) \\ +\frac{1}{2s}\left(p_{Bl}-p_{Bh}\right)\left(p_{Bl}-c_l\right) \end{array}\right] \\ = \quad & 0. \end{aligned}$$

The reaction function is:

$$\begin{aligned} & p_{Bl}=\frac{1}{3}+\frac{1}{3}c_l \\ & \pm\frac{1}{3}\sqrt{\begin{array}{c} 3c_hp_{Ah}-3c_hp_{Bh}-c_lp_{Bh}-c_lp_{Al}+3c_h \\ -c_l+c_l^2-3p_{Ah}p_{Bh}-p_{Bh}p_{Al}-4p_{Bh} \\ +2p_{Al}+4p_{Bh}^2+p_{Al}^2+1 \end{array}} \\ & +\frac{1}{3}p_{Bh}+\frac{1}{3}p_{Al}. \end{aligned}$$

The first order condition with respect to p_{Bh} is

$$
\begin{aligned}
&\left[\begin{array}{c}
\left(\frac{1}{s}\left(p_{Bh}-p_{Bl}\right)-1\right)\left(\frac{1}{2}\left(p_{Bh}+1-p_{Ah}\right)-1\right) \\
+\frac{1}{2}\left(\frac{1}{s}\left(p_{Bh}-p_{Bl}\right)-1\right)\left(p_{Bh}-c_h\right) \\
+\frac{1}{s}\left(\frac{1}{2}\left(p_{Bh}+1-p_{Ah}\right)-1\right)\left(p_{Bh}-c_h\right) \\
+\frac{1}{s}\left(\frac{1}{2}\left(1-p_{Al}+p_{Bl}\right)-1\right)\left(c_l-p_{Bl}\right)
\end{array}\right] \\
= \quad & 0.
\end{aligned}
$$

The reaction function is:

$$
\begin{aligned}
p_{Bh} \quad = \quad & \frac{1}{3}+\frac{1}{3}c_h+\frac{1}{3}s+\frac{1}{3}p_{Ah}+\frac{1}{3}p_{Bl} \\
& \pm\frac{1}{3}\sqrt{\begin{array}{c}
3c_l-c_h-s-c_hp_{Ah}-c_hp_{Bl}-3c_lp_{Bl} \\
+3c_lp_{Al}-sc_h+c_h^2+2p_{Ah}-4p_{Bl}-p_{Ah}p_{Bl} \\
-3p_{Bl}p_{Al}-sp_{Ah}+2sp_{Bl}+p_{Ah}^2+4p_{Bl}^2++s^2+1
\end{array}}.
\end{aligned}
$$

The equation system of $p_{Al}, p_{Ah}, p_{Bl}, p_{Bh}$ is

$$
\begin{aligned}
p_{Al} \quad = \quad & \frac{1}{3}+\frac{1}{3}c_l+\frac{1}{3}p_{Ah}+\frac{1}{3}p_{Bl} \\
& \pm\frac{1}{3}\sqrt{\begin{array}{c}
3c_hp_{Ah}-3c_hp_{Bh}-c_lp_{Bh}-c_lp_{Al}+3c_h-c_l \\
+c_l^2-3p_{Ah}p_{Bh} \\
-p_{Bh}p_{Al}-4p_{Bh}+2p_{Al}+4p_{Bh}^2+p_{Al}^2+1
\end{array}} \\
p_{Ah} \quad = \quad & \frac{1}{3}+\frac{1}{3}s+\frac{1}{3}c_h+\frac{1}{3}p_{Bh}+\frac{1}{3}p_{Al} \\
& \pm\frac{1}{3}\sqrt{\begin{array}{c}
3c_l-c_h-s-c_hp_{Bh}-c_hp_{Al}-3c_lp_{Al} \\
+3c_lp_{Bl}-sc_h+c_h^2+2p_{Bh}-4p_{Al}-p_{Bh}p_{Al} \\
-3p_{Al}p_{Bl}-sp_{Bh}+2sp_{Al}+p_{Bh}^2+4p_{Al}^2+s^2+1
\end{array}} \\
p_{Bl} \quad = \quad & \frac{1}{3}+\frac{1}{3}c_l+\frac{1}{3}p_{Bh}+\frac{1}{3}p_{Al} \\
& \pm\frac{1}{3}\sqrt{\begin{array}{c}
3c_hp_{Ah}-3c_hp_{Bh}-c_lp_{Bh}-c_lp_{Al}+3c_h-c_l \\
+c_l^2-3p_{Ah}p_{Bh} \\
-p_{Bh}p_{Al}-4p_{Bh}+2p_{Al}+4p_{Bh}^2+p_{Al}^2+1
\end{array}} \\
p_{Bh} \quad = \quad & \frac{1}{3}z+\frac{1}{3}c_h+\frac{1}{3}s_h-\frac{1}{3}s_l+\frac{1}{3}p_{Ah}+\frac{1}{3}p_{Bl} \\
& \pm\frac{1}{3}\sqrt{\begin{array}{c}
3c_l-c_h-s-c_hp_{Ah}-c_hp_{Bl}-3c_lp_{Bl} \\
+3c_lp_{Al}-sc_h+c_h^2+2p_{Ah}-4p_{Bl}-p_{Ah}p_{Bl} \\
-3p_{Bl}p_{Al}-sp_{Ah}+2sp_{Bl}+p_{Ah}^2+4p_{Bl}^2++s^2+1
\end{array}}.
\end{aligned}
$$

From (1) and (3) we can see that the two equations are symmetric, which implies $p_{Al} = p_{Bl}$. Similarly, from (2) and (4), we know that $p_{Ah} = p_{Bh}$. If we implose $p_{Al} = p_{Bl}$ and $p_{Ah} = p_{Bh}$,the equation system becomes

$$
\begin{aligned}
p_{Al} &= \frac{1}{3}+\frac{1}{3}c_l+\frac{1}{3}p_{Ah}+\frac{1}{3}p_{Al} \\
&\pm\frac{1}{3}\sqrt{\begin{array}{c} p_{Ah}^2+p_{Al}^2-p_{Ah}p_{Al}-c_lp_{Ah} \\ -4p_{Ah}-c_lp_{Al}+2p_{Al}+3c_h+c_l^2-c_l+1 \end{array}} \\
p_{Ah} &= \frac{1}{3}+\frac{1}{3}s+\frac{1}{3}c_h+\frac{1}{3}p_{Ah}+\frac{1}{3}p_{Al} \\
&\pm\frac{1}{3}\sqrt{\begin{array}{c} p_{Ah}^2+p_{Al}^2-p_{Ah}p_{Al}-c_hp_{Al}-4p_{Al} \\ -c_hp_{Ah}+2p_{Ah}+3c_l+c_h^2-c_h \\ -sp_{Ah}+2sp_{Al}-sc_h-s+s^2+1 \end{array}}.
\end{aligned}
$$

We can see that equation (1) and (3) are symmetric. Likewise, equation (2) and (4) are symmetric. So we simplify the equation system above by imposing symmetry and yield

$$
\begin{aligned}
2p_{Al} &= 1+c_l+p_{Ah} \\
&\pm\sqrt{\begin{array}{c} p_{Ah}^2+p_{Al}^2-p_{Ah}p_{Al}-c_lp_{Ah} \\ -4p_{Ah}-c_lp_{Al}+2p_{Al}+3c_h+c_l^2-c_l+1 \end{array}} \\
2p_{Ah} &= 1+s+c_h+p_{Al} \\
&\pm\sqrt{\begin{array}{c} p_{Al}^2+p_{Ah}^2-p_{Al}p_{Ah}-c_hp_{Al}-4p_{Al} \\ -c_hp_{Ah}+2p_{Ah}+3c_l+c_h^2-c_h \\ +1-sp_{Ah}+2sp_{Al}-sc_h-s+s^2 \end{array}}.
\end{aligned}
$$

The equation system has a solution if $p_{Al}=c_l+1$ and $p_{Ah}=c_h+1$

Proof:

$$
\begin{aligned}
2p_{Al} &= 1+c_l+p_{Ah} \\
&\pm\sqrt{\begin{array}{c} p_{Ah}^2+p_{Al}^2-p_{Ah}p_{Al}-c_lp_{Ah}-4p_{Ah} \\ -c_lp_{Al}+2p_{Al}+3c_h+c_l^2-c_l+1 \end{array}}.
\end{aligned}
$$

if we rearrange it, it yields

$$
\begin{aligned}
&2p_{Al}-1-c_l-p_{Ah} \\
&= \pm\sqrt{\begin{array}{c} p_{Ah}^2+p_{Al}^2-p_{Ah}p_{Al}-c_lp_{Ah}-4p_{Ah}-c_lp_{Al} \\ +2p_{Al}+3c_h+c_l^2-c_l+1 \end{array}}.
\end{aligned}
$$

Then plug $p_{Al}=c_l+1$ and $p_{Ah}=c_h+1$ into $(1)^{'}$ and yield

$$
\begin{array}{rl}
& 2(c_l+1)-1-c_l-(c_h+1) \\
= & \pm\sqrt{\begin{array}{c}(c_h+1)^2+(c_l+1)^2-(c_h+1)(c_l+1)-c_l(c_h+1) \\ -4(c_h+1-c_l(c_l+1)+2(c_l+1)+3c_h+c_l^2-c_l+1\end{array}}.
\end{array}
$$

For the righ hand side

$$
\begin{array}{rl}
& \sqrt{\begin{array}{c}(c_h+1)^2+(c_l+1)^2-(c_h+1)(c_l+1)-c_l(c_h+1) \\ -4(c_h+1)-c_l(c_l+1)+2(c_l+1)+3c_h+c_l^2-c_l+1\end{array}} \\
= & \sqrt{\begin{array}{c}c_l-c_h-c_l\left(c_h+1\right)-c_l\left(c_l+1\right)+c_l^2 \\ -\left(c_h+1\right)\left(c_l+1\right)+\left(c_h+1\right)^2+\left(c_l+1\right)^2-1\end{array}} \\
= & \sqrt{\left(c_h-c_l\right)^2}.
\end{array}
$$

For the left hand side

$$
2(c_l+1)-1-c_l-(c_h+1)=c_l-c_h.
$$

Then we have

$$
c_l-c_h=\pm\sqrt{\left(c_h-c_l\right)^2}
$$

$$
c_l-c_h=-\sqrt{\left(c_h-c_l\right)^2}
$$

Similarly, we can proove that the second equation also holds at $p_{Al}=c_l+1$ and $p_{Ah}=c_h+1$.

To summary, $p_{Al}=c_l+1$ and $p_{Ah}=c_h+1$ is the solution to the equation system $(1)^{'}$ and $(2)^{'}$. We cannot guarantee it is the unique solution for this equation system. It guarantees only a local maximum. Similarly, we get $p_{Bl}=c_l+1$ and $p_{Bh}=c_h+1$ as the solution to retailer B's problem. Therefore, the solution to the entire equation system is $p_{Al}=p_{Bl}=c_l+1$ and $p_{Ah}=p_{Bh}=c_h+1$.Then we go back the the profit function and plug the solution into the profit functions. We have

$$
\begin{array}{rcl}
\pi^A & = & (p_{Al}-c_l)\widehat{\delta}_l\widehat{\theta}^A+(p_{Ah}-c_h)\widehat{\delta}_h(1-\widehat{\theta}^A) \\
& = & (p_{Al}-c_l)\dfrac{p_{Bl}-p_{Al}+1}{2}\dfrac{p_{Ah}-p_{Al}}{s} \\
& & +(p_{Ah}-c_h)\dfrac{p_{Bh}-p_{Ah}+1}{2}(1-\dfrac{p_{Ah}-p_{Al}}{s}) \\
& = & \dfrac{1}{2},
\end{array}
$$

and

$$
\begin{aligned}
\pi^B &= (p_{Bl} - c_l)(1 - \widehat{\delta}_l)\widehat{\theta}^B + (p_{Bh} - c_h)(1 - \widehat{\delta}_h)(1 - \widehat{\theta}^B) \\
&= (p_{Bl} - c_l)(1 - \frac{p_{Bl} - p_{Al} +}{2})\frac{p_{Bh} - p_{Bl}}{s_h - s_l} \\
&\quad + (p_{Bh} - c_h)(1 - \frac{p_{Bh} - p_{Ah} +}{2})(1 - \frac{p_{Bh} - p_{Bl}}{s}) \\
&= \frac{1}{2}.
\end{aligned}
$$

We can see that the two retailers earn the same profit if they both provide the full product line. The profit they make in this situation is the same as they both reduce to the same single quality and the mark-ups keep unchanged. For example, if retailer A and B both produce high quality,

$$\pi^A = (p_{Ah} - c_h)\widehat{\delta}_h * 1 = (p_{Ah} - c_h) * \frac{1}{2} * 1 = \frac{1}{2},$$

and

$$\pi^B = (p_{Bh} - c_h)(1 - \widehat{\delta}_h) * 1 = (p_{Bh} - c_h) * (p_{Bh} - c_h)(1 - \widehat{\delta}_h) * \frac{1}{2} * 1 = \frac{1}{2}.$$

Appendix 2: solving retailers' problems in scenario 2

According to the market segmentation in figure 2, we have

$$\widetilde{\delta}_h = \widetilde{\delta}_h = \frac{p_{bh} - p_{ah} + t}{t} = (p_{bh} - p_{ah}) * \frac{1}{t} + 1,$$

$$\widetilde{\theta}^B = \frac{p_{bh} - p_{bl}}{s},$$

and

$$\widetilde{\theta} = \frac{p_{ah} - p_{bl} - t(1 - \widetilde{\delta})}{s}$$

when $\widetilde{\theta} = 0$,

$$\widetilde{\delta} = 1 - \frac{p_{ah} - p_{bl}}{t}.$$

when$\widetilde{\delta} = 0$,

$$\widetilde{\theta} = \frac{p_{ah} - p_{bl} - t}{s}.$$

When $p_{ah} - p_{bl} < t = 2$, market shares

$$\varphi_{ah} = \widetilde{\delta}_h * 1 - \frac{1}{2}(\widetilde{\delta}_h - \widetilde{\delta}) * \widetilde{\theta}^B,$$

$$\varphi_{bh} = (1 - \widetilde{\delta}_h)(1 - \widetilde{\theta}^B),$$

and

$$\varphi_{bl} = (\frac{1}{2}(\widetilde{\delta}_h - \widetilde{\delta})\widetilde{\theta}^B + (1 - \widetilde{\delta}_h)\widetilde{\theta}^B).$$

Retailer A's problem is

$$\begin{aligned}
\pi^A &= \pi_h^A = (p_{ah} - c_h)(\widetilde{\delta}_h * 1 - \frac{1}{2}(\widetilde{\delta}_h - \widetilde{\delta})\widetilde{\theta}^B) \\
&= (p_{ah} - c_h)(((p_{bh} - p_{ah}) * \frac{1}{t} + 1) * 1 \\
&\quad -\frac{1}{2}(((p_{bh} - p_{ah}) * \frac{1}{t} + 1) - (1 - \frac{p_{ah} - p_{bl}}{t}))\frac{p_{bh} - p_{bl}}{s}).
\end{aligned}$$

First order condition with respect to p_{ah} is, given transportation cost $t = 2$. We also try other values of t,e.g. $t = 5$ and 10.

$$p_{ah} = \frac{1}{4s}\left(-p_{bh}^2 + 2p_{bh}p_{bl} + 2sp_{bh} - p_{bl}^2 + 4s + 2sc_h\right).$$

Retailer B's problem is:

$$
\begin{aligned}
\pi^B &= \pi_h^B + \pi_l^B = (p_{bh} - c_h)(1 - \widetilde{\delta}_h)(1 - \widetilde{\theta}^B) \\
&\quad +(p_{bl} - c_l)(\frac{1}{2}(\widetilde{\delta}_h - \widetilde{\delta})\widetilde{\theta}^B + (1 - \widetilde{\delta}_h)\widetilde{\theta}^B) \\
&= (p_{bh} - c_h)(1 - ((p_{bh} - p_{ah}) * \frac{1}{t} + 1))(1 - \frac{p_{bh} - p_{bl}}{s}) \\
&\quad +(p_{bl} - c_l)(\frac{1}{2}(((p_{bh} - p_{ah}) * \frac{1}{t} + 1) \\
&\quad -(1 - \frac{p_{ah} - p_{bl}}{t}))\frac{p_{bh} - p_{bl}}{s} \\
&\quad +(1 - ((p_{bh} - p_{ah}) * \frac{1}{t} + 1))\frac{p_{bh} - p_{bl}}{s}).
\end{aligned}
$$

The first order condition with respect to p_{bh} given transportation cost $t = 2$ is

$$
\begin{aligned}
p_{bh} &= \frac{1}{3}s + \frac{1}{3}c_h - \frac{1}{6}c_l + \frac{1}{3}p_{ah} + \frac{1}{2}p_{bl} \\
&\quad -\frac{1}{6}\sqrt{\begin{array}{c} 4s^2 - 4sc_h - 4sc_l - 4sp_{ah} + 12sp_{bl} + 4c_h^2 \\ -4c_hc_l - 4c_hp_{ah} + c_l^2 + 8c_lp_{ah} \\ -6c_lp_{bl} + 4p_{ah}^2 - 12p_{ah}p_{bl} + 9p_{bl}^2 \end{array}}
\end{aligned}
$$

First order condition with respect to p_{bl} is, given different transportation cost $t = 2$,

$$
\begin{aligned}
p_{bl} &= \frac{1}{3}c_l + \frac{2}{3}p_{ah} \\
&\quad +\frac{1}{3}\sqrt{\begin{array}{c} c_l^2 - 2c_lp_{ah} + 4p_{ah}^2 - 12p_{ah}p_{bh} \\ +6c_hp_{ah} + 9p_{bh}^2 - 6c_hp_{bh} \end{array}}.
\end{aligned}
$$

The equation system of the first order conditions is when t=2,

$$
\begin{aligned}
p_{ah} &= \frac{1}{4s}\left(-p_{bh}^2 + 2p_{bh}p_{bl} + 2sp_{bh} - p_{bl}^2 + 4s + 2sc_h\right) \\
p_{bh} &= \frac{1}{3}s + \frac{1}{3}c_h - \frac{1}{6}c_l + \frac{1}{3}p_{ah} + \frac{1}{2}p_{bl} \\
&\quad -\frac{1}{6}\sqrt{\begin{array}{c} 4s^2 - 4sc_h - 4sc_l - 4sp_{ah} + 12sp_{bl} + 4c_h^2 \\ -4c_hc_l - 4c_hp_{ah} + c_l^2 + 8c_lp_{ah} \\ -6c_lp_{bl} + 4p_{ah}^2 - 12p_{ah}p_{bl} + 9p_{bl}^2 \end{array}} \\
p_{bl} &= \frac{1}{3}c_l + \frac{2}{3}p_{ah} + \frac{1}{3}\sqrt{\begin{array}{c} c_l^2 - 2c_lp_{ah} + 4p_{ah}^2 - 12p_{ah}p_{bh} \\ +6c_hp_{ah} + 9p_{bh}^2 - 6c_hp_{bh} \end{array}}
\end{aligned}
$$

Let's make $p_{ah} = c_h + a, p_{bh} = c_h + b$ and $p_{bl} = c_l + c$, where a,b, and c are markups of high-quality product from retailer A, high-quality product from retailer B and low-quality product from retailer B respectively. We also make $k = c_h - c_l$ and $s = s_h - s_l$, the equation system becomes given $t = 2$

$$\begin{aligned}
(b-c)^2 + 4sa + (2k-6s)b - 2kc + k^2 - 4s &= 0 \\
3b^2 - 2ab + 2ac - 3bc + (s-k)a - (2s-2k)b &= 0 \\
3b^2 - 4ab + 4ac - 3c^2 - 2ka + 4kc - k^2 &= 0.
\end{aligned}$$

Similarly, we can derive the equaiton systems given other values of transportation costs. Here we also have when t=5, the equation system is

$$\begin{aligned}
(b-c)^2 + 4sa + (2k-6s)b - 2kc + k^2 - 10s &= 0 \\
3b^2 - 2ab + 2ac - 3bc + (s-k)a - (2s-2k)b &= 0 \\
3b^2 - 4ab + 4ac - 3c^2 - 2ka + 4kc - k^2 &= 0.
\end{aligned}$$

When t=10, the equation system is

$$\begin{aligned}
(b-c)^2 + 4sa + (2k-6s)b - 2kc + k^2 - 20s &= 0 \\
3b^2 - 2ab + 2ac - 3bc + (s-k)a - (2s-2k)b &= 0 \\
3b^2 - 4ab + 4ac - 3c^2 - 2ka + 4kc - k^2 &= 0.
\end{aligned}$$

Here we have a few constrains to impose on the equation system.

Constraint 1 $\quad 0 < \widetilde{\delta}_h = p_{bh} - p_{ah} + 1 < 1$,which means consumers are located along the city between 0 and 1.

From this constraint, we have $0 < b - a + 1 < 1$.

Constraint 2 $\quad 0 < \widetilde{\theta}^B = \frac{p_{bh} - p_{bl}}{s} < 1$,which means consumers' preferences for quality is located between 0 and 1.

From this constraint, we have $0 < k + b - c < s$.

Constraint 3 $\quad 0 < p_{ah} - p_{bl} < t = 2$,which means AD is the indifference curve in figure 2. We present only case 2.

From this constraint, we have $0 < k + a - c < 1$.

Constraint 4 $\quad \widetilde{\delta} * \widetilde{\theta} < 0$,which we can see from figure 2.

There is no analytical solution for these equation systems.We then solve the system through Matlab using numerical methods.