FURTHER EVIDENCE ON THE STRUCTURE OF CONSUMER DEMAND IN THE U. S.: AN APPLICATION OF THE SEPARABILITY HYPOTHESIS

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I. INTRODUCTION

The applied economic literature abounds with reports on the estimation of parameters of consumer demand functions. Early attempts were hindered by an inadequate understanding of the theory and inappropriate statistical methods. In following the developments both from the point of view of the evolution of methods and the applied results the patterns are rather clear. Until rather recently demand parameters were estimated by single equation least squares methods. However, as the theory and econometric methods developed, it became clear that these early results were only under special circumstances correct. That is, it became clear that specific demand equations should be estimated within the complete systems explaining market prices and quantities. This observation spawned a number of empirical studies incorporating simultaneous equations methods in estimating systems of demand functions. Results obtained by this more general approach were again however of rather limited generality. The limitations in this case were due to the *ad hockery* involved in specifying the systems. More specifically, since demand equations contain all prices and income, the estimation could not proceed without the addition of information in the form of exclusion restrictions. The lack of a general theoretical basis for providing this information led to the use of conditions involving highly specialized behavioral assumptions.

Recent studies in the theory of consumer behavior have provided a number of useful hypotheses concerning the incorporation of prior information in demand function estimation. In particular, the advent of the various separability concepts has provided a more substantive behavioral underpinning for exclusion restrictions on sets of parameters in systems of demand equations. Although these hypotheses have been applied in estimating consumer demand functions from the aggregate data in the U. S. and other economies, the results are far from conclusive. In fact, as we shall subsequently show, the currently available empirical results for the U. S. economy contain many conflicting and intuitively unacceptable parameter values.

Our objective in this paper is to add to the current stock knowledge regarding these important policy parameters. Specifically, we examine the structure of consumer demand in the U. S. for the period 1929–1969. The estimation method incorporates an additive utility function and is based on a procedure

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proposed by Barten and Theil [1, 11]. We selected the Barten-Theil demand model because of the natural manner in which the parameter restrictions implied by the additive hypothesis are taken into account, the comparative ease with which the expenditure data required for estimating the parameters of the model could be obtained, and finally, the availability of a systematically developed error structure which is adaptable for empirical applications.

The discussion proceeds as follows. Section two begins with a brief description of the Barten-Theil demand model. This is followed by a development of the stochastic structure and estimation method. Section three contains a discussion of the data and underlying variables employed in applying the model. The main empirical results are presented in section four; the results are evaluated in section five and compared to those of previous studies in section six. Lastly, some concluding observations in regard to both the empirical results and the methods employed are offered in section seven.

II. THE EMPIRICAL DEMAND MODEL

In this section we review the procedure to be applied in estimating the demand parameters from the U. S. data. As the methods to be employed are essentially the same as those developed by Barten [1], most of the derivations will be omitted. The purpose is to set out the notation and statistical assumptions and to indicate the importance of the additivity condition in reducing the parameters required for estimating the full system of demand equations.

The Demand Equation

Let $q = [q_i]$ denote an n-element column vector of quantities of commodities, $p = [p_i]$ denote an n-element vector of the corresponding commodity prices and m denote money income. It is well known that under suitable mathematical restrictions the first order conditions associated with the problem of maximizing a utility function[1] $u = u(q)$ subject to the linear budget constraint $p'q = m$ yield demand functions of the form,

$$q_i = q_i(p, m); \qquad (i = 1, \cdots, n). \quad (1)$$

The expenditure proportion differentials related to this system of demand functions can be written as

$$\overset{*}{w}_{it} Dq_{it} = \sum_j b_{ij}[Dp_{jt} - \sum_k \mu_k Dp_{kt}] + \mu_i Dq_t; \qquad (i = 1, \cdots, n). \quad (2)$$

where $w_i (= p_i q_i/m)$ is the expenditure proportion for the ith commodity; $\overset{*}{w}_{it} (= 1/2 \{w_{it} + w_{it-1}\})$ is the average expenditure proportion between two successive periods for the ith commodity; $b_{ij} (= \lambda p_i p_j u^{ij}/m)$, with λ denoting the marginal utility of income and u^{ij} denoting the (i, j)th element of the inverse of the Hessian associated with the consumer utility maximization problem, is the coefficient of the jth relative price;[2] $\mu_i (= p_i \partial q_i/\partial m)$ is the marginal propensity to spend for the ith commodity; and finally, D is the logarithmic difference operator.

Despite the rather cumbersome notation, the terms of which equation (2) is composed are rather easily interpreted. The term $[Dp_{jt} - \sum_k \mu_k Dp_{kt}]$ is the log change in the relative price of the jth commodity. It is obtained by subtracting a weighted average of prices (the weights being marginal propensities to spend) from the log change in the absolute price of the jth commodity. The term Dq_t $(= \sum_i \overset{*}{w}_{it} Dq_{it})$ is the log change in real income summed over the i left hand variables.

From standard neoclassical demand theory we know that as a result of the structure of

[1] It is assumed that the utility function is twice differentiable. The first-order derivatives $u_i = \partial u/\partial q_i$ are assumed to be positive, and the matrix of the second-order derivatives, i.e., the Hessian matrix $U = [\partial^2 u/\partial q_i \partial q_j]$ is assumed to be symmetric and negative definite. The negative definiteness of the Hessian matrix guarantees that the utility function has a maximum.

[2] Since it is assumed that the Hessian matrix U is symmetric and negative definite, it follows that the matrix $B = [b_{ij}]$ is symmetric and negative definite.

the consumer optimization problem the parameters μ_i and b_{ij} must satisfy the following conditions:

$$\sum_i \mu_i = 1, \tag{3.a}$$

$$b_{ij} = b_{ji} \tag{3.b}$$

and

$$\sum_j b_{ij} = \phi\mu_i, \tag{3.c}$$

i.e., the so called Engel aggregation, the symmetry relation and the homogeneity condition, respectively. The new term introduced in (3.c) is the income flexibility parameter $\phi(= \partial m/\partial\lambda \cdot \lambda/m)$, or the reciprocal of Frisch's [5, 183] money flexibility. An additional restriction is obtained by summing over i in equation (3.c) and using (3.a) to show that the income flexibility parameter is equal to the sum of the price coefficients which, since they are negative (by the second order conditions for the consumer optimization problem), imply that ϕ is negative.

In comtemplating the estimation of (2), i.e., the b_{ij}'s and μ_i's, it is apparent that the unconstrained system contains $n^2 + n$ unknown parameters. By imposing the restrictions mentioned in (3.a), (3.b) and (3.c), the number of unknown parameters is reduced to $\frac{1}{2}(n^2 + n) - 1$. As mentioned earlier, this number is quite large in view of the usual commodity definitions and the length of economic time series. Hence, estimation of (2), or equivalently the structure of the consumer demand, requires the introduction of additional information. The modern consumption theory suggests restrictions on the utility function as an alternative to eliminating parameters from the equations composing (2) on an *ad hoc* basis. One such restriction is direct additivity. In the special case when the utility function is directly additive, the marginal utility of the ith commodity depends only upon the quantity of the ith good alone. Such an assumption is obviously overly restrictive if commodities are considered separately. However, if commodities are grouped and the additivity is between groups, then the condition is more plausible. Under the latter condition the behavioral assumption is that consumers make budgeting decisions by commodity groups—considering only group price indices—and that within groups decisions are made under the unrestricted condition. This condition is called "block additivity" and will be employed throughout the remainder of this paper.

When the additivity condition is imposed, we can replace the condition (3.c) with

$$b_{ii} = \phi\mu_i \tag{3.c$'$}$$

and the condition $\sum_i \sum_j b_{ij} = \phi$ by $\sum_i b_{ii} = \phi$. The power of these restrictions is immediately seen. The expenditure proportion differentials (2) can be written as

$$\overset{*}{w}_{it} Dq_{it} = \phi\mu_i[Dp_{it} - \sum_k \mu_k Dp_{kt}] + \mu_i Dq_t; \quad (i = 1, \cdots, n). \tag{4}$$

There are just n parameters to be estimated: the $n - 1$ independent values of μ_i and ϕ. This does of course represent a drastic departure from the previous specification (2), and as importantly, is suggested by an easily interpretable behavioral assumption.

Stochastic Specification

Thus far we have assumed that the quantity demanded of a particular commodity q_i is a function of only prices and income. Apart from prices and income there are, however, other non-economic elements which influence consumer demand. Because these factors are not explicitly introduced in the demand equations, it is proper to assume that the demand system (4) is incomplete. We complete the model by the introduction of an error term. This error term is denoted V_{it} and is added to each of the i equations in the system defined by (4).

In specifying the characteristics of the error term we have two choices. It can be specified on an arbitrary basis or on the basis of the structure of the consumer optimization model. In following Barten and Theil we choose the latter model. Specifically, the characteristics of the error terms V_{it} are deduced through the inclusion of a random shock vector in the utility maximization process. By introducing the shock vector in the linear part of a simplified quadratic utility function, Barten and Theil obtain an expression for V_{it} which takes the form

$$V_{it} = -\sum_j (b_{ij} - \phi\mu_i\mu_j)[\Delta a_{jt}/\bar{E}(\lambda)], \qquad (6)$$

where Δa_j is the random shock element of the jth marginal utility [11, 230].

On the condition that the random shock vector Δa is composed of elements which are independently and normally distributed with zero means and a homoscedastic variance σ^2, the disturbance terms V_{it} can be shown to have the following properties:

$$\bar{E}(V_t) = 0, \qquad (7.a)$$

$$\bar{E}(V_t V_{t'}') = 0 \quad \text{for} \quad t \neq t' \qquad (7.b)$$

and

$$\bar{E}(V_t V_t') = \sigma^2[(1/\phi)B - \mu\mu'] = \Omega. \qquad (7.c)$$

That is, the V_t have zero means, are temporally uncorrelated and have a contemporaneous variance covariance matrix Ω.

It may be observed that the contemporaneous covariance matrix Ω is positive semi-definite with rank $n - 1$. Summing (6) over i gives the restriction $\sum_i V_{it} = 0$, from which it follows that $\Omega l = 0$, where $l' = [1, \cdots, 1]$. In the case of direct-additivity where $b_{ij} = 0$ for all $i \neq j$ and $b_{ii} = \phi\mu'$, the covariance matrix Ω takes the form

$$\bar{E}(V_t V_t') = \sigma^2(M - \mu\mu'), \qquad (8)$$

where M is an $n \times n$ diagonal matrix whose diagonal is the vector μ. This implies that (a) var $(V_{it}) = \sigma^2\mu_i(1 - \mu_i)$ and (b) cov $(V_{it}V_{jt}) = -\sigma^2\mu_i\mu_j$. In short, under the assumption of direct additivity, the random shock model gives variances and covariances of disturbances which are proportional (in term of marginal propensities) to deviates from multivariate normal distribution [11, 233].

Estimation Procedure

The estimation of b_{ii} and μ_i from (4) would be a straight forward application of generalized least-squares method if: a) the right hand side were linear in the unknown parameters, and b) the covariance matrix for the disturbances was non-singular. To solve the first problem, a Taylor series expansion is used to derive a linear version of demand equations (4).

$$\begin{aligned} \overset{*}{w}_{it}Dq_{it} - \mu_i^0 Dq_t &= b_{ii}[Dp_{it} + (1/\phi^0)Dq_t - \sum_k \mu_k^0 Dp_{kt}] \\ &\quad -\mu_i^0 \sum_j b_{jj}[Dp_{jt} + (1/\phi^0)Dq_t \\ &\quad - \sum_k \mu_k^0 Dp_{kt}] + V_{it}; \\ &\qquad (i = 1, \cdots, n), \end{aligned} \qquad (8)$$

where μ_i^0 and ϕ^0 represent initial values for μ_i and ϕ. The logarithmic change in λ due to logarithmic income and price change is $D\lambda_t = (1/\phi)Dq_t - \sum_k \mu_k Dp_{kt}$. From this it follows that $D\lambda_t^0 = (1/\phi^0)Dq_t - \sum_k \mu_k Dp_{kt}$ is the initial value of the log-change in the marginal utility of income. Hence, $D\overset{0}{\mu}_{it} = Dp_{it} + D\lambda_t^0$ is written as the value of the log-change in the marginal utility of the ith commodity. Equation (8) can now be written in the following form:

$$\begin{aligned} \overset{*}{w}_{it}Dq_{it} - \mu_i^0 Dq_t &= b_{ii}D\overset{0}{\mu}_{it} \\ &\quad - \mu_i^0 \sum_k b_{kk}D\overset{0}{\mu}_{kt} + V_{it}; \\ &\qquad (i = 1, \cdots, n). \end{aligned} \qquad (9)$$

In matrix notation we can thus write the system as

$$\begin{aligned} Y_t &= Z_t B - \mu^0 Z_t' B + V_t \\ &= (I - \mu^0 l')Z_t B + V_t = X_t B + V_t.^3 \end{aligned} \qquad (10)$$

[3] Where

$$Y_t = \begin{bmatrix} \overset{*}{w}_{1t} & Dq_{1t} - \mu_1^0 & Dq_t \\ & \vdots & \\ \overset{*}{w}_{nt} & Dq_{nt} - \mu_n^0 & Dq_t \end{bmatrix}, \qquad Z_t = \begin{bmatrix} D\overset{0}{\mu}_{1t} \\ \vdots \\ D\overset{0}{\mu}_{nt} \end{bmatrix},$$

$$\mu^0 = [\mu_1^0 \cdots \mu_n^0]', \qquad B = [B_{11} \cdots B_{nn}]$$

Y_t is the $n \times 1$ vector of observations on the dependent variables, The system of which (10) refers to only one time period may be written as

$$Y = XB + V \tag{11}$$

where $Y = [Y_1, \cdots, Y_T]'$ is the $nT \times 1$ vector of observations on the dependent variables, $X = [X_1, \cdots, X_T]'$ is the $nT \times n$ matrix of observations on the independent variables, $B = [B_1, \cdots, B_n]'$ is the $n \times 1$ vector of coefficients and $V = [V_1, \cdots, V_T]'$ is the $nT \times 1$ disturbance vector.

The system (11) may be characterized by the following set of assumptions:

$$Y = XB + V \tag{12.a}$$

$$\bar{E}(V) = 0 \tag{12.b}$$

$$\bar{E}(VV') = \sum = I \otimes \Omega, \tag{12.c}$$

where I is the $T \times T$ identity matrix, $\otimes$ denotes the knonecker product, and Ω is the $n \times n$ covariance matrix of the contemporaneous disturbance terms. Under the assumption of additivity $\Omega = \sigma^2(M - \mu\mu')$, M is a diagonal $n \times n$ matrix, which has elements of the vector μ for diagonal elements. (12.d)

X is a $Tn \times n$ matrix which is fixed. The elements which make up matrix X are Dq_t, Dp_{it}, μ_i^0, and ϕ^0. It is assumed that the disturbance terms are independent of Dq_t and Dp_{it} and that the initial values for μ_i and ϕ are selected independently of the disturbance terms.

$$\text{Rank of } X = n. \tag{12.e}$$

Finally, in order to obtain estimates of the elements of the vector B, one should account for the problem of the singularity of the covariance matrix Ω. This problem can be eliminated by using the following estimator suggested by Theil [10]:

$$b = (X'HX)^{-1}X'HY, \tag{13}$$

where H is an $nT \times nT$ diagonal matrix defined by $H = I \otimes M_0^{-1}$. Estimator (13) is a best linear unbiased estimator if (a) $\Omega = \sigma^2(M - \mu\mu')$ and (b) $\mu = \mu_0$ [1, 233]. The covariance matrix of b is

$$\sum_{bb} = (X'HX)^{-1} \cdot X'H(I \otimes \Omega)HX(X'HX)^{-1}.\text{[4]} \tag{14}$$

In view of the conditions $b_{ii} = \phi\mu_i$ and $\sum_i b_{ii} = \phi$, one has estimates for μ and ϕ defined as

$$\hat{\mu} = (1/\hat{\phi})\hat{b} \quad \text{and} \quad \hat{\phi} = l'\hat{b}. \tag{15}$$

Finally, in addition to providing a more formal framework for the estimation and interpretation of the consumer demand structure, the Theil-Barten approach has some useful implications for the sampling properties of the parameter estimates. These implications result from possibilities for improved sampling variances which exist as a result of the estimation of the system as a set of seemingly unrelated regressions and the iteration on the initial value of μ, μ_0. First, it is clear from equation (10) that X_t is a diagonal matrix. This suggests that we can consider the estimation problem posed in equations (12.a), (12.b) and (12.c) within a seemingly unrelated regressions framework [14]. That is, by reordering the elements of y so as to collect the T observations on each of the elements of the parameter vector B and rearranging the non-zero elements of X in a block diagonal form so as to preserve the equality in equation (12.a), the estimation

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Z_t is the $n \times 1$ vector of the initial value of log-change in the marginal utility of the (ith) commodity,
μ^0 is the $n \times 1$ vector of the initial values of the marginal propensities to spend on various commodities,
B is the $n \times 1$ vector of coefficients,
$\mathcal{Z}_t$ is the $n \times n$ diagonal matrix, which has the elements of the vector $\mathcal{Z}_t$ for diagonal elements,
I is the $n \times n$ identity matrix,
X_t is the $n \times n$ matrix $(\mathcal{Z}_t - \mu^0\mathcal{Z}_t')$, i.e., matrix of observations on the independent variables, and
V_t is the $n \times 1$ vector of residuals

[4] Where Ω is estimated by $(1/T'') \sum_t \hat{V}_t\hat{V}_t'$, here

$$T'' = T - \frac{\text{tr}\,[\sum_t M_0^{-1}X_t(X'HX)^{-1}X_t'M_0^{-1}][\sum_t \hat{V}_tV_t']}{\text{tr}\,[M_0^{-1}(\sum_t \hat{V}_t\hat{V}_t')]}.$$

problem can be viewed in the seemingly unrelated regressions framework. Since reordering of V corresponding to the reordering of Y does not result in a diagonal variance covariance matrix and the elements of X_t identified with the various elements of B are different, it follows from the seemingly unrelated regressions results that asymptotic sampling variances are equal to or smaller than if the equations had been estimated separately [14].

Potential advantages of the iteration on successive values of the elements of μ are discussed in Theil [10]. In the example given by Theil, however, there was only slight improvement in the estimated sampling variances of the estimators. Our results were somewhat more encouraging on this count in that the sampling variances (reported later in Table II) were smaller than those obtained with the initial value of μ, μ_0. Having provided a statistical model for estimating a complete set of consumer demand functions, we turn now to analyze the empirical implications of such a model by means of time series data.

III. THE DATA AND THE VARIABLES[5]

The demand model presented in the preceding section is estimated using a time series of annual observations on prices and personal consumption expenditures in the United States for the period 1929–1969. These data are based on official estimates of the U. S. Department of Commerce [13]. To obtain the observations of the dependent variables, the following procedure was used. First, the logarithms of the total quantities demanded were calculated, using the identity: $\ln q_{it} = \ln (p_{it}\, q_{it}) - \ln p_{it}$. Next, the logarithm in mid-year population was subtracted from the logarithm quantity for each year to obtain per capita quantity demanded. Then, first differences were calculated, omitting years 1942–1946. Finally, the observations of the dependent variables of the demand equations were obtained by multiplying the average expenditure proportions by the corresponding first differences.

The price data for the nine groups of commodities and services are retail price indices (1958 = 100). These price indices are called "implicit deflators," which means that the weights of the various components of each commodity or service shift over time. By use of these data the first differences of the logarithmic prices were obtained. The income (total expenditure) variable in the demand equations is expressed as $\sum_k \overset{*}{w}_{kt}\, Dq_{kt} = Dq_t$. Thus, when all observations on the dependent variables are summed, income per capita values are obtained.

[5] In this study all consumer goods have been aggregated into nine separate groups of commodities and services: durable goods, food, clothing and shoes, gasoline and oil, other non-durable goods, housing, household operation services, transportation services and other services. This grouping of consumer goods follows the same principles of classification as employed by the U. S. Department of Commerce. It must be emphasized that this classification is in no sense ideal and is not in strict accordance with the concept of additive preferences model. It should also be observed that with one exception the names of the commodities and services are the same as those used in the Department of Commerce publications. The exception is that alcoholic beverages was added to other non-durable goods.

IV. EMPIRICAL RESULTS

Estimation of the Parameters

To estimate the parameters of the system (11), it is necessary first to obtain initial values for the marginal propensities to spend, μ_i's and for the income flexibility parameter ϕ. The initial values for μ_i's were obtained by applying the least-squares estimator to each equation separately.[6] These estimated values for the initial estimates of the marginal propensities to spend are listed in Table I.

[6] The estimates were obtained from the following equation:

$$\overset{*}{w}_{it}\Delta(\ln q_{it}) = a_i + \sum_j S_{ij}\Delta(\ln p_{jt}) + \mu_i Dq_t + V_{it}$$

where $S_{ij} = w_i(e_{ij} + \eta_i w_j)$, and $\mu_i = w_i \eta_i$. The coefficients μ_i and S_{ij}, satisfy the constraint:

$$\sum_i \mu_i = 1, \qquad \sum_j S_{ij} = 0, \qquad S_{ij} = S_{ji}$$

i.e., Engel aggregation, homogeneity condition, and symmetry relation respectively.

TABLE I

INITIAL VALUES FOR THE MARGINAL PROPENSITIES TO SPEND: MAJOR COMMODITY GROUPS

Commodity Group	Marginal Propensity To Spend μ°
1 - Durable Goods	0.4143
2 - Food	0.1398
3 - Clothing and Shoes	0.1199
4 - Gasoline and Oil	0.0142
5 - Other Non-Durable Goods	0.1626
6 - Housing	0.0190
7 - Household Operation Services	0.0329
8 - Transportation Services	0.0330
9 - Other Services	0.0639

Values for the income flexibility parameter are also initially unknown. Therefore, an iterative procedure was used to obtain an estimate of ϕ. For the first iteration an initial value for ϕ, say ϕ^0, was put equal to -0.50.[7] It took six iterations to achieve convergence, and the resulting estimate for money flexibility was $\hat{\phi} = -0.3268$. With these initial values of ϕ and the μ_i, we can apply the estimator (13) to the system (11) to obtain estimates for the price coefficient vector b, the marginal propensities to spend vector μ and the income flexibility parameter ϕ. These estimates are presented along with their asymptotic standard errors in Table II. Note that all the estimates are more than twice the corresponding standard errors. In addition, all the estimates are theoretically admissible. That is, all the coefficients of the price vector b have the correct (negative) sign, and all the marginal propensities to spend, μ_i's, are positive, presumably the correct sign. The negative sign of b_{ii} confirms the assumption that the Hessian matrix U is negative definite; also, the negative sign of b_{ii} and ϕ implies that all the marginal propensities to spend are positive under the assumption of additive preference specification.

[7] This value was suggested for the consumers in the median income bracket by Frisch [5, 189].

Elasticities

Additional insight is obtained when the results are examined in terms of the elasticity estimates they imply. These elasticities are particularly interesting as they evolve from this method because they vary with expenditure proportions and are thus different from year to year.

From the definitions of μ_i and w_i it should be clear that the income elasticity of the ith commodity is

$$\eta_i = \mu_i/w_i. \tag{16}$$

Since all μ_i are positive, it follows that the income elasticities are positive. Furthermore, it is easily verified that $\eta_i \gtrless 1$, according to $\mu_i \gtrless w_i$. If $\mu_i > w_i$, then $\eta_i > 1$, and the commodity is a luxury. If the reverse is true, the commodity is of course a necessity.

Specification of the direct and cross price elasticities is somewhat more involved. On differentiation of equation (2), with respect to p_i and p_j, we have, respectively,

$$e_{ii} = (b_{ii} - b_{ii}\mu_i - \mu_i w_i)/w_i \tag{17}$$

and

$$e_{ij} = (b_{ii}\mu_j - w_j\mu_j)/w_i. \tag{18}$$

TABLE II

ESTIMATES OF PARAMETERS FOR THE ADDITIVE PREFERENCE MODEL: MAJOR COMMODITY GROUPS

Commodity Group	Price Coefficient $\hat{b}_{ii}$	Marginal Propensity to Spend $\hat{u}_i$
1 - Durable Goods	-.1009(.0455)	.3089(.0430)
2 - Food	-.0482(.0180)	.1475(.0196)
3 - Clothing and Shoes	-.0361(.0146)	.1106(.0140)
4 - Gasoline and Oil	-.0085(.0019)	.0260(.0063)
5 - Other Non-Durable Goods	-.0490(.0188)	.1501(.0234)
6 - Housing	-.0207(.0048)	.0634(.0199)
7 - Household Operation Services	-.0163(.0035)	.0501(.0087)
8 - Transportation Services	-.0107(.0034)	.0327(.0035)
9 - Other Services	-.0360(.0083)	.1103(.0214)
Income Flexibility $\hat{\phi}$	-.3268(.1096)	

TABLE III

ELASTICITY MATRIX FOR NINE COMMODITY GROUPS*: PERCENTAGE CHANGES IN QUANTITIES DEMANDED RESULTING FROM ONE PERCENT CHANGES IN PRICES OR INCOME

Commodity Group	Durable Goods	Food	Clothing and Shoes	Gasoline and Oil	Other Non-Durable Goods	Housing	Household Operation	Trans-portation	Other Services	Income Elasticity
Durable Goods	-.8586	-.4440	-.1506	-.0587	-.1993	-.2860	-.0971	-.0502	-.2896	2.4343
Food	-.0165	-.3257	-.0395	-.0154	-.0523	-.0751	-.0255	-.0132	-.0761	.6396
Clothing and Shoes	-.0292	-.2057	-.4384	-.0272	-.0923	-.1325	-.0450	-.0232	-.1342	1.1279
Gasoline and Oil	-.0206	-.1454	-.0493	-.2797	-.0652	-.0936	-.0318	-.0164	-.0948	.7972
Other Non-Durable Goods	-.0297	-.2090	-.0709	-.0276	-.4684	-.1347	-.0457	-.0236	-.1364	1.1463
Housing	-.0118	-.0836	-.0283	-.0110	-.0375	-.2037	-.0183	-.0094	-.0545	.4586
Household Operation	-.0230	-.1623	-.0551	-.0214	-.0728	-.1046	-.3264	-.0183	-.1059	.8901
Transportation	-.0270	-.1903	-.0646	-.0251	-.0854	-.1226	-.0416	-.3626	-.1242	1.0436
Other Services	-.0184	-.1297	-.0440	-.0171	-.0582	-.0836	-.0283	-.0146	-.3171	.7114
Expenditure Proportions	.1268	,2305	.0980	.0326	.1309	.1382	.0562	.0313	.1550	

* Evaluated at mean expenditure proportions 1929–1969.

These expressions can be simplified to $e_{ii} = \phi\eta_i - \eta_i w_i\ (1 + \phi\eta_i)$ and $e_{ij} = -\eta_i w_j\ (1 + \phi\eta_j)$, in case of additive preferences (in view of the condition $b_{ii} = \phi\mu_i$). In matrix notation, price elasticities can be expressed as

$$E = \phi\bar{\eta} - \eta w'\ (I + \phi\bar{\eta}).^{8} \qquad (19)$$

It is easily seen that $e_{ij} \gtrless 0$ according to $|\eta_i w_j \phi\eta_j| \gtrless |\eta_i w_j|$. If $|\eta_i w_j| > |\eta_j w_j \phi\eta_i|$, then $e_{ij} < 0$, and the cross price elasticity reflects the income effect rather than the substitution effect of price change.

Table III contains estimates of price and income elasticities for nine commodity groups evaluated at mean expenditure proportions for the sample period. These are obtained from the parameter estimates (Table II) by evaluating the elasticity formulas (16) and (19) at mean expenditure proportions.

One of the interesting uses of cross elasticities is the study of relationships of substitutability and complementarity between commodity groups. Hence it is of interest to estimate the Slutsky compensated price elasticities (Table IV) in order to classify commodities as substitutes, complements or independent goods. This approach to commodity classification provides a more rigorous definition of substitutability and complementarity than classification by cross-elasticities since the latter approach neglects the income effect. The Slutsky price elasticities are defined by

$$E^{*} = E + \eta w'. \qquad (20)$$

In scalar terms these are: $e^{*}_{ii} = \phi\eta_i\ (1 - w_i\eta_i)$ and $e^{*}_{ij} = -\phi\eta_i\eta_j w_j$. Since all η_i, η_j are positive and ϕ is negative, all e^{*}_{ij} are positive, so that all goods are substitutes.

Finally, estimates of demand elasticities obtained can be put into perspective by comparing them with some previous and related studies. We make these comparisons because our analysis is somewhat more comprehensive than that of other studies, and as earlier mentioned, because of the amount of uncertainty which exists in regard to the values for these important policy parameters which

[8] Here $\bar{\eta}$ stands for the $n \times n$ diagonal matrix, whose diagonal is the vector η.

TABLE IV

ESTIMATES OF SLUTSKY PRICE ELASTICITIES*

Commodity Group	Durable Goods	Food	Clothing and Shoes	Gasoline and Oil	Other Non-Durable Goods	Housing	Household Operation	Trans-portation	Other Services
Durable Goods	−.5497	.1173	.0879	.0206	.1194	.0504	.0398	.0260	.0877
Food	.0645	−.1782	.0231	.0054	.0313	.0132	.0104	.0068	.0230
Clothing and Shoes	.1138	.0543	−.3278	.0095	.0553	.0233	.0184	.0120	.0406
Gasoline and Oil	.0804	.0384	.0288	−.2537	.0391	.0165	.0130	.0085	.0287
Other Non-Durable Goods	.1157	.0552	.0414	.0097	−.3183	.0237	.0187	.0122	.0413
Housing	.0462	.0221	.0165	.0038	.0224	−.1403	.0075	.0049	.0165
Household Operation	.0898	.0429	.0321	.0075	.0436	.0184	−.2763	.0095	.0320
Transportation	.1053	.0503	.0377	.0088	.0511	.0216	.0170	−.3299	.0376
Other Services	.0718	.0342	.0257	.0060	.0348	.0147	.0116	.0076	−.2068

* Evaluated at mean expenditure proportions (1929–1969).

are suggested by the data. In Tables V and VI our results are arrayed with comparable results of selected related empirical studies.

V. DISCUSSION OF THE RESULTS

Income Flexibility

The numerical value of the income flexibility parameter was found to be -0.3268. This value gives an estimate of Frisch's money flexibility $1/\hat{\phi}$ of -3.05, indicating that a 1 percent increase in real income decreases the marginal utility of income by about 3 percent. This value agrees closely with -3.125, the value of $1/\hat{\phi}$ which is reported by Barten [1, 233]. Furthermore, it is interesting to note that our value for $1/\hat{\phi}$ lies between Frisch's money flexibility of -4 for the slightly better off but still poor segment of the population and -2 for the median part of the population [5, 189]. The persistence of this relationship between studies (as well as the accuracy of our estimate) would suggest that the estimate obtained is a reliable policy parameter.

Marginal Propensities to Spend

As previously indicated, Table II reports estimates of the parameters of the additive preference model. In examining the table it is of interest to note that the estimated marginal propensities to spend, μ_i's, are all positive, i.e., all theoretically admissible, significantly different from zero, and sum to 1. Also worth noting is that the estimated marginal propensities to spend do not differ considerably from their initial values in Table I. This suggests that the generalized least squares estimator $b = (X'HY)^{-1}X'HY$ is numerically not very different from the mean of true best linear unbiased estimator.

Although the marginal propensities to spend may be alternatively viewed (as they subsequently are) in terms of income elasticities, some comments concerning the implications of the results appear in order at this point. Durable goods, the commodity group which contains automobiles and auto parts and furniture and household equipment, has the highest marginal propensity (0.31). The second highest marginal propen-

TABLE V

COMPARISON OF INCOME ELASTICITY ESTIMATES FOR MAJOR COMMODITY GROUPS WITH OTHER STUDIES

Commodity Group	Source and Period					
	Hassan et al. (1929-1969)	Boutwell (1935-1962)	Burk (1947-1969)	Hein[a] (1949-1965)	Houthakker & Taylor[b] (1929-1964)	Powell et al. (1949-1963)
Durable Goods	2.4343	.6662				
Food	.6396		.7	.079	.71[c]	
Clothing and Shoes	1.1279				1.24	.888
Gasoline and Oil	.7972			1.209	.41	
Other Non-Durable Goods	1.1463			1.302	.79	.413
Housing	.4586				.06	-.048
Household Operation Services	.8901				.88	1.027
Transportation Services	1.0436				.76	.399
Other Services	.7114			.417	.78	.874

[a] Short run estimates.
[b] Short run estimates.
[c] Including beverages.

TABLE VI

COMPARISON OF DIRECT PRICE ELASTICITY ESTIMATES FOR MAJOR COMMODITY GROUPS WITH OTHER STUDIES

Commodity Group	Source and Period					
	Hassan et al. (1929-1969)	Boutwell (1935-1962)	Brandow (1923-1956)	Hein[a] (1946-1965)	Houthakker & Taylor[b] (1929-1964)	Powell et al. (1949-1963)
Durable Goods	-.8586	-1.8775				
Food	-.3257		-.3413	-.521	-.47[c]	
Clothing and Shoes	-.4384				-.57	-.622
Gasoline and Oil	-.2797			-.307	-.16	
Other Non-Durable Goods	-.4684			-.351	-.38	-.303
Housing	-.2037				-.03	.038
Household Operation Services	-.3264				-.37	-.699
Transportation Services	-.3626				-.32	-.274
Other Services	-.3171			-.800	-.41	-.641

[a] Short run estimates.
[b] Short run estimates.
[c] Including beverages.

sity (0.15) is associated with the commodity class called other non-durable goods. This commodity group includes, among other things, alcoholic beverages and drug preparations and sundries. The marginal propensity to spend for food is also about (0.15). The size of this value is somewhat surprising and may be reflecting shifts to higher quality or more highly processed foods. The remainder of the propensities are about as would be anticipated. The possible exception is transportation services, which is (0.03). This rather low value may be explained by the fact that the commodity groups include local transportation services likely to be used by low-income individuals as well as air travel, which is a high-income good. The low marginal propensity may therefore be reflecting the balance between these two items.

Income Elasticities

In examining Table III, our attention is first drawn to the pattern of income elasticities. The main feature of the results obtained is that all elasticities are positive. This implies, of course, that all commodity groups are normal—a result which is not surprising in view of the small number of commodity groups.

Five commodity groups have income elasticities that are less than unity; i.e., as income increases, demand will increase less than proportionally. These commodity groups are, therefore, necessities, the consumption of which tends to remain approximately the same irrespective of income level; they are food, gasoline and oil, housing, household operation services and other services. The results for food, in particular, are worth noting, since Engel's law concerning the decreasing share of expenditure on food is confirmed. Income elasticities for clothing and shoes, other non-durable goods, and transportation services are all greater than 1. Demand for these commodity groups is likely to change approximately proportionally with income. These commodity groups may therefore be thought of as luxuries, although moderate ones. The highest estimated income elasticity was obtained for durable goods. The elasticity is near 2, implying that these goods are chiefly luxuries. As the general level of income rises, demand for durable goods will increase rapidly. As mentioned earlier, this commodity class is highly influenced by automobile purchases.

Price Elasticities

An examination of the direct price elasticities in Table III indicates that all have the negative sign, which *a priori* theory would lead one to expect, and that they have magnitudes which indicate an inelastic price response. These low direct price elasticities are to be expected for the highly aggregated groups of commodities. It is usually argued that direct price elasticity of a commodity tends to be more inelastic than that of the goods which make up the particular commodity group. This is of course due to the possibility for within-group-substitution by consumers.

As might be expected from the previous results regarding expenditure proportions and income elasticities, direct price elasticities for durable goods are high. The direct price elasticities for food, gasoline and oil, housing, household operation services, and transportation services are all comparatively low in value. Clothing and shoes and other non-durable goods show high value for their direct price elasticities, although smaller in absolute values than those of durable goods.

The estimated cross price elasticities (Table III) are comparatively small. This is not surprising in view of the implications of the additive preference model. Also worth noting is that the estimated cross price elasticities are all negative, i.e., the cross price elasticities are reflecting income effects rather than the substitution effects of price changes.

Slutsky Price Elasticities

Turning now to the Slutsky utility-compensated price elasticities in Table IV, we observe that all compensated direct price elasticities lie between 0 and -1. Also, all compensated cross price elasticities are positive, so that commodity groups appear as substitutes and not as complements in consumption. It is reassuring to find that all these results are in accordance with the utility basis of the additive preference model. The income elasticity vector η, the price elasticity matrix E, and the Slutsky price elasticity matrix E^* have signs which *a priori* theory would lead one to expect.

VI. COMPARISON WITH SOME PREVIOUS STUDIES

It is interesting to compare the results presented in this study with those obtained from other studies [2, 3, 4, 7, 8, 9]. In doing so, we should keep in mind that due to differences in definitions of commodity groups, in time periods used, and in statistical models employed, these results may not be strictly comparable with estimates of other studies. They are, however, comparable in terms of their applied implications. That is, all are designed to be used in economic planning and economic development.

Several inferences can be drawn from the comparisons provided in the compilation of results in Tables V and VI. We begin with the comparison of income elasticities. The most comprehensive studies presented are by Houthakker and Taylor [8] and Powell et al. [9]. The short run estimates of income elasticities reported by Houthakker and Taylor [8] are the more similar to ours. The larger differences occur for gasoline and oil, housing, other durable goods, and transportation services. In each case the estimates we obtained are larger than those given by Houthakker and Taylor. The most important discrepancy is for housing, where we find an income elasticity of .4586 as against the .06 Houthakker and Taylor estimate. In view of the similarity of commodity classifications, the Powell et al. results appear quite erratic. Specifically, the negative income elasticity for housing, the rather small income elasticity for transportation services and the small value for other non-durable goods are counter to our results and somewhat inconsistent with observed patterns of consumer behavior. The results from the Heien [7] study are comparable to ours except for the income elasticity for food. Here the agreement of our results with the other reported estimates would suggest they are the more appropriate for policy purposes. The only comparison available for durable goods is from a study by Boutwell [2]. Here the difference in the two results may be to some extent explained by our more current data base—since incomes and durable goods purchases in the United States increased substantially during the 1960s. Generally, then, it would appear that our results compare favorably with those advanced on the basis of other empirical research using similar behavioral assumptions.

There is considerably more agreement among the price elasticity estimates. Again, the most comprehensive results are from studies by Houthakker and Taylor [8] and Powell et al. [9]. Of these two sets of results, those of Houthakker and Taylor are in closest agreement with ours. The rather substantial difference in income elasticities for housing carries over to the price elasticities. Powell et al. estimate the price elasticity for housing at .038, Houthakker and Taylor estimate it at $-.03$ and our estimate is $-.2037$. As in the case of the income elasticities comparison, there is only one related result for the durable goods classification. Here we find that Boutwell's [2] estimate is in excess of that reported from our analysis. As a general observation, our price elasticities appear to have less variance between commodity groups than do the others. However, as mentioned before, this type of result is to be anticipated in such aggregated commodity groups.

We conclude this section by noting that an explanation of the differences among the elasticities is rather difficult. It is conceivable that variation among the studies is in part attributable to the use of different functional forms by the authors. Also, it is possible that deflating the variables may have influenced the elasticities [12]. The Houthakker and Taylor estimates are the most complete of the alternatives, and in a number of cases similar to those we present. The differences which exist appear, however, to be of sufficient magnitude to suggest that the results we report can add to the fairly limited empirical information price and income parameters for systems of demand equations.

VII. CONCLUSIONS AND LIMITATIONS

Based on the results of our analysis, several inferences may be drawn from this study. First, the empirical demand model employed appears to be a viable approach to the estimation of income and price elasticities. The major advantage of the approach is the computational ease with which demand systems recognizing the interrelationships among commodities can be obtained. Secondly, the empirical results presented in this paper are generally satisfactory. All the demand elasticities have the anticipated signs and have a magnitude which appears to be reasonable. They are also well within the range of the intuitively reasonable results obtained from other studies. Furthermore, the results seem to justify the assumption of direct additivity when a broadly defined classification of commodities such as food, clothing, etc. is adopted.

We close with some precautionary remarks. Although the demand model we have employed is a step forward in terms of the feasibility of estimating a complete set of demand functions, the additivity assumption is rather restrictive. It rules out the possibility of specific substitution effects, the possibility of inferior goods, and the possibility of complementary goods. In evaluating the results obtained, we again need to emphasize that the empirical demand model specified is a static one. Results for such items as durable goods must therefore be interpreted with considerable caution. Also, since the variances of the sampling distributions of the estimated demand elasticities have not been explicitly derived, their statistical reliability cannot be investigated. While the former limitation is to some extent reduced by the favorable comparison with the Houthakker and Taylor results, it remains a problem. The similarity and comparative completeness of the two sets of results would suggest, however, that the added computational problems posed by the dynamic considerations [8, 205] yielded only modest additional applied information. The latter reservation regarding sampling properties is quite important and can only be lessened by the consistency of the results within our analysis and between studies.

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