Heteroscedasticity and the Estimation of the Risk Balancing Model

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1. Introduction

The purpose of this research is to examine the the effect of increases in risk on agricultural debt.

2. Risk Balancing Theory

- The basic concept follows from Collins [1]

$$\delta^*(t) = 1 - \frac{\rho\sigma_A^2(t)}{[\mu_A(t) - K(t)]} \quad (1)$$

where $\delta^*(t)$ is the optimal debt to asset ratio, ρ is the producer's absolute risk aversion, $\mu_A(t)$ is the expected return on agricultural assets, $K(t)$ is the cost of debt, and $\sigma_A^2(t)$ is the variance of the rate of return on agricultural assets.

- In this study, we multiply the equation through by the level of agricultural assets to obtain

$$D^*(t) = A(t) - \frac{\rho\sigma_A^2(t)}{[R_A(t) - K(t)A(t)]} \quad (2)$$

where $D^*(t)$ is the level of agricultural debt, $R_A(t)$ is the level of agricultural returns, and $K(t)A(t)$ is the opportunity cost of return on agricultural assets (valued in terms of the cost of debt).

- In order to scale the problem, we then divide through by the number of acres and take the first-order Taylor series expansion to yield

$$\tilde{D}^*(t) = \alpha_0+\alpha_1\tilde{R}_A(t)+\alpha_3\sigma_A^2(t)\,\alpha_2\left(K(t)\tilde{A}(t)\right)+\epsilon(t) \quad (3)$$

where $\tilde{D}^*(t) = D^*(t)/L(t)$ (given that $L(t)$ is the number of acres) and $\tilde{A}(t) = A(t)/L(t)$.
  - In general we expect $\alpha_1 \gg 0$ – or that increases in the expected return increases the optimal debt level.
  - $\alpha_2 \ll 0$ – increases in the level of risk decreases the optimal debt level.
  - $\alpha_3 \ll 0$ – increase in the opportunity cost of capital decreases the optimal debt level.

3. Econometric Specification

3.1 Expected Profit

- Several approaches have been used to model expected profit. For example, Moss, Shonkwiler and Ford [2] used a time series (autoregressive) formulation to model expected returns on agricultural assets.
- In this study we use a linear profit function based on input and output prices

$$\tilde{\pi}(t) = \beta_0+\beta_1p_1(t)+\beta_2p.2(t)+\beta_3w_1(t)+\beta_4w_2(t)+\beta_5w_3(t)+\nu(t) \quad (4)$$

where $\tilde{\pi}(t)$ is the profit per acre, $p_1(t)$ is the price index for crops sold, $p_2(t)$ is the price index for livestock sold, $w_1(t)$ is the price index for seeds, $w_2(t)$ is the price index for fertilizer, and $w_3(t)$ is the price index for fuel.

- The estimated coefficients for this simple model for this formulation are presented in Table 1.
  - In general, we would expect that increases in the output prices would increase the profit per acre. However, the coefficient for crop production is negative in three states (California, Florida, and Georgia). The reason for this anomaly may be the composition of crops in each state. For example, both California and Florida produce a significant quantity of fruits and vegetables which may not be well represented in the general crop price index. The results for the livestock output are closer to our expectations with the only negative estimate in Georgia.
  - The results for the input variables are less satisfactory. In general, the seed index only conforms to our expectations in the corn/soybean states – again, this may be due to the prevalence of fruit and vegetable production in California and Florida. The Fertilizer parameters are generally positive, but insigificant. However, the effect of fuel prices conforms to our expectations.

Table 1: *Profit Function Estimates*

	California	Florida	Georgia	Illinios	Indiana	Iowa
Const	-48.1752	-28.1476	-10.7902	-7.4273	-9.8602	-15.9632
	(13.8874)	(17.1540)	(15.5898)	(10.7975)	(11.0974)	(12.1308)
Crop	-0.3126	-1.1701	-1.2654	0.5253	0.5372	0.4792
	(0.5254)	(0.6490)	(0.5898)	(0.4085)	(0.4199)	(0.4589)
Live	1.6445	1.6666	-0.0583	0.7685	0.5501	0.8470
	(0.5420)	(0.6695)	(0.6085)	(0.4214)	(0.4331)	(0.4735)
Seed	1.9461	3.2760	2.7844	-0.2065	-0.0921	-0.2148
	(0.3717)	(0.4591)	(0.4172)	(0.2890)	(0.2970)	(0.3246)
Fertilizer	0.0448	-0.5641	0.8615	0.0880	0.1454	0.1463
	(0.5924)	(0.7317)	(0.6650)	(0.4606)	(0.4734)	(0.5174)
Fuel	-0.6511	-0.5079	-0.8065	-0.0863	-0.0284	0.1395
	(0.3482)	(0.4301)	(0.3908)	(0.2707)	(0.2782)	(0.3041)

3.2 Estimating the Risk

- Again, several different approaches have been used to estimate the risk. Moss, Shonkwiler and Ford [2] used an Autoregressive Conditional Heteroscedasticity (ARCH).
- In this study, we use a Loess estimator based on the squared residuals from Equation 4.
- Specifically, we begin by setting $\tilde{V}(t) = \nu(t)^2$.
- Next, we model this variance using a locally linear least squares estimator

$$\min_{\gamma_0,\gamma_1} \sum_{s=1}^{T} k(t,s)\left(\tilde{V}(s) - \gamma_0(t) - \gamma_1(t)\,s\right)^2 \quad (5)$$

where $k(t,s)$ is a kernel which decreases as t and s diverge.

- Using the estimated coefficients from Equation 5, we can compute an variance estimate for each point in time $\hat{V}(t)$

$$\hat{V}(t) = \sum_{s=1}^{T} k(t,s)\left[\gamma_0(t)+\gamma_1(t)\,s\right] \quad (6)$$

- The implied variances presented in Figure 1 indicate that the relative risk in agriculture was relatively small throughout the 1960s through about 1975 for all states. Figure 2 presents the variance and Loess estimator for Florida. In general, the risk increased in the 1970s (probably due to the citrus freezes) to a maximum in 1985 and then declined throughout the rest of the sample. These results contrast somewhat with the results from Illinois presented in Figure 3. Figure 4 presents the estimated variance for the sample of states.

4. Risk Balancing Results

- The results of the risk-balancing model are presented in Table 2.
  - Consistent with our expectations, increases in expected income (or profit per acre) lead to increases in the debt per acre in every state.
  - However, the coefficient for variance is only negative in three states (Florida, Illinois, and Iowa).
  - Further, the estimated coefficient of the opportunity cost of capital is positive in all states.

Table 2: *Effect of Estimated Variance on Debt*

	California	Florida	Georgia	Illinios	Indiana	Iowa
Constant	1.8380	10.5687	17.1144	-44.9582	-5.8688	-24.7761
	(1.5419)	(5.1661)	(5.0170)	(20.5422)	(24.9811)	(28.6365)
Pred Var	0.0877	-0.0465	0.0145	-0.0825	0.0545	-0.0552
	(0.0063)	(0.0084)	(0.0287)	(0.0411)	(0.0719)	(0.1313)
Exp Income	0.1189	1.1797	0.3288	2.4847	1.4572	2.5084
	(0.0254)	(0.0537)	(0.1015)	(0.4329)	(0.6894)	(0.7906)
Int Paid	0.2196	0.5183	2.2163	0.7435	1.2761	0.7732
	(0.0205)	(0.0887)	(0.1462)	(0.1331)	(0.1966)	(0.1899)

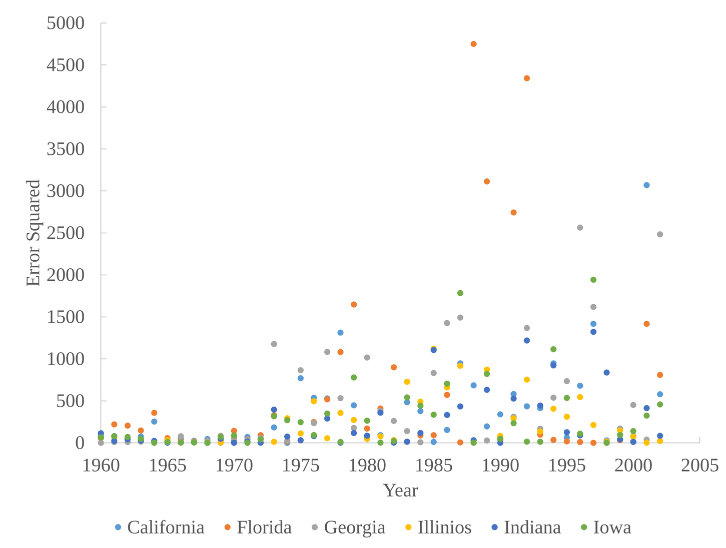

Figure 1: *Estimated Residuals Squared*

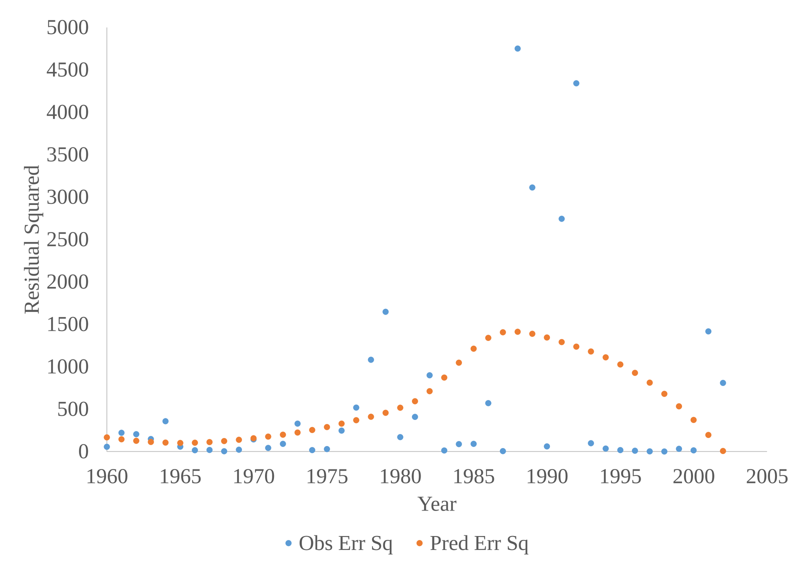

Figure 2: *Loess Estimate of Variance for Florida*

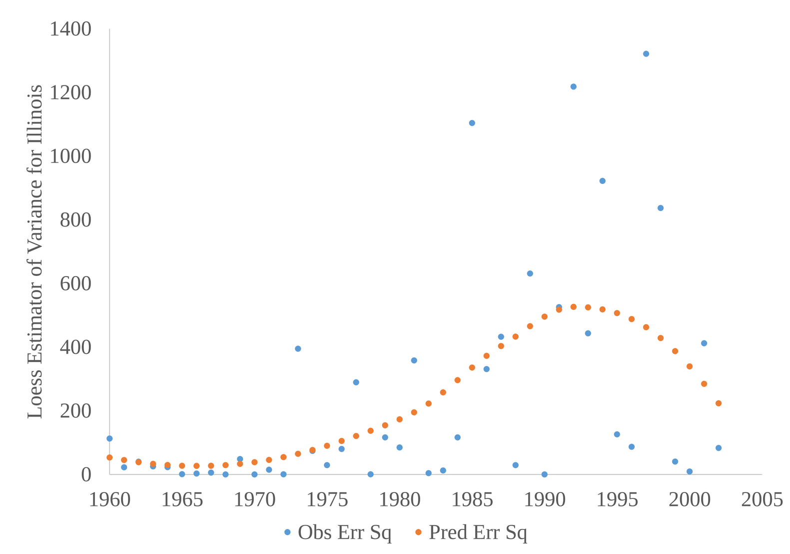

Figure 3: *Loess Estimate of Variance for Illinois*

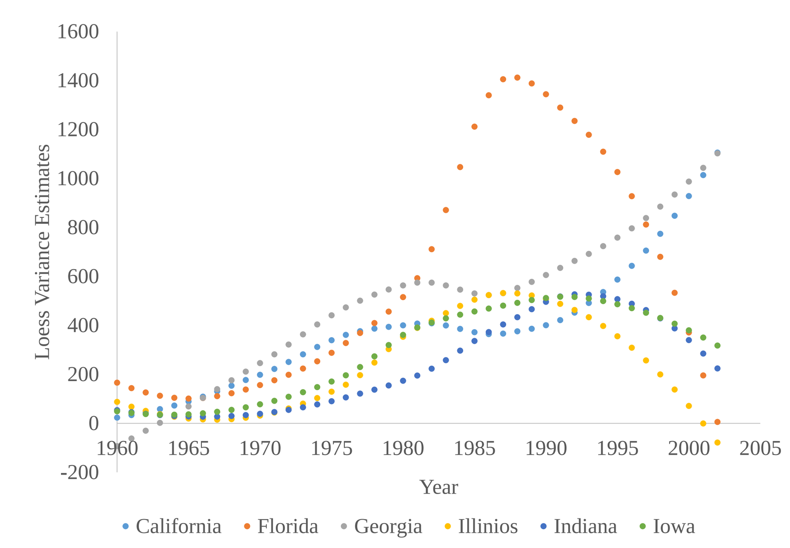

Figure 4: *Loess Estimates of Variance*

References

[1] R. A. Collins. Expected utility, debt-equity structure, and risk balancing. *American Journal of Agricultural Economics*, 65(3):627–629, 1985.

[2] C. B. Moss, J. S. Shonkwiler, and S. A. Ford. A risk endogenous model of aggregate agricultural debt. *Agricultural Finance Review*, 50:73–79, 1990.