STARs with the GIDDS: Smooth Transition Functions and Structural Change

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1 Introduction

Understanding and documenting the cause and nature of structural change is at the center of most applied work in consumer demand analysis. At its core, structural change in demand analysis implies change in parameters of a specific model. There could be a number of reasons why an otherwise stable demand system might experience changes over time. Attitude of consumers towards a product might change following, say, health concerns (Brown and Schrader, 1990).

However, detecting structural change in the data has proved to be an elusive quest. The general approach in the parametric world of demand analysis is such that demand functions are clearly specified and their parameters estimated and tested for consistency with demand theory. The initial applications to food demand used functional forms that are flexible and tried to see if parameter estimates were consistent with negativity, homogeneity, and symmetry conditions (Blanciforti, Green, and King, 1986). A rejection of homogeneity, symmetry, or negativity restrictions is usually taken as an indicator of changes in preferences. An alternative approach is the test for parameter stability. The Chow test has been applied to demand models to determine if structural change has occurred. Using these tests, several studies detected some parameter instability in models applied to US meat demand (Chavas (1983); Eales and Unnevehr (1988)).

Structural change can also be detected by explicitly modeling it. Models that use time-series data usually include a time trend to represent gradual changes over time. It is also a standard approach to use dummy variables to capture seasonality for high frequency time series data (quarterly or monthly data, for example). The coefficients on the dummy variables can be interpreted as intercept shifts. A dummy variable can also be used to measure changes at a particular point in the time series data. However, Moschini and Moro (1996) argue that such use of dummy or time trend variables to search for structural change is rather ad hoc and does not tell us much about either the timing or nature of structural change.

Holt and Balagtas (2009) propose a new approach to remedy some of the problems associated with hitherto time series testing of structural change. Their new method allows structural change to be smooth over time and that the change is non-monotonic. For this they use the now popular smooth transition auto regressive models (STAR). They used the Inverse Almost Ideal Demand System (IAIDS) for functional form and the US meat demand data for an application. Their overall finding is that augmenting the standard IAIDS with smooth transition improves model fit and produces much fewer or no non-negativity violation of the Antonelli matrix. It is argued that any particular findings of structural change in demand may reflect model specification error, such as a poor choice of functional form, rather than true structural change (Chalfant and Alston (1988); Alston and Chalfant (1991); Davis (1997)). Consequently, it often pays to examine the robustness of findings in terms of their sensitivity to changes in model specification or other aspects of the analysis.

This paper evaluates the validity of smooth transition function as a tool for testing structural change by using Bartern's generalized inverse differential demand system (GIDDS). As we will show below, the GIDDS model nests the Inverse AIDS among others. STAR models hold a great promise for allowing a non-linear transition between two regimes if there is structural change. But we still know little about nature and behavior of these models as applied in demand analysis. It is important to test the validity and usefulness of the approach by using a general demand model specification.

2 Model, Method and Data

2.1 Model

Barten (1993) showed that the Rotterdam, the differential AIDS and other two demand systems (CBS and NBR) can be nested within a generalized ordinary demand system. Eales, Durham, and Wessells (1997) further developed this generalized ordinary differential demand system(GODDS) and used it to compare inverse demands with ordinary demand systems. Using demand for fish in Japan, Eales, Durham, and Wessells (1997) report that the inverse demand system dominates the ordinary demand system in forecasting performance and in non-nested tests. This paper will use the inverse demand specification of GIDDS for the test of structural change. In what follows the estimable model is given without necessarily deriving it. Following Eales, Durham, and Wessells (1997), GIDDS model is:

$$dw_i = (\phi_i + \theta_1 w_i)dlnQ + \sum_{j=1}^{N}(\phi_{ij} + \theta_2 w_i(\delta_{ij} - w_j))dlnq_i \tag{1}$$

Where δ_{ij} is one when i=j and zero otherwise, w_i is the budget share of good i. Q is the Stone qunantity index and q_j is quantity of good j. The ϕ's represent coefficients and the θ's are the nesting parameters for the inverse system. The inverse Rotterdam is derived by setting both nesting parameters to one. Setting both nesting parameters to zero will give us the inverse AIDS. Inverse CBS is derived if θ_1 is zero and θ_2 is one. The inverse NBR results if we set θ_1 to one and θ_2 to zero. Adding up, homogeneity, and symmetry imply the following restrictions, respectively.

$$\sum_i \phi_i = -\theta_1, \sum_i \phi_{ij} = 0, \sum_j \phi_{ij} = 0, \phi_{ij} = \phi_{ji} \tag{2}$$

(Eales, Durham, and Wessells, 1997) also derive the flexibilities associated with the above inverse system. The compensated price flexibility associated with inverse GIDDS is given by;

$$f^*_{ij} = \frac{\phi_{ij}}{w_i} + (\theta_2 - 1)(\delta_{ij} - w_j) \tag{3}$$

The uncompensated price flexibility is given as;

$$f_{ij} = \frac{\phi_{ij} + \phi_i w_j}{w_i} + (\theta_2 - 1)\delta_{ij} + (\theta_1 - \theta_2)w_j \tag{4}$$

Finally, the scale flexibility is computed as follows;

$$f_i = \frac{\phi_{ij}}{w_i} + \theta_1 - 1 \tag{5}$$

2.2 Method and Data

This paper follows Holt and Balagtas (2009) and outlines the structural change model below. We drop the i subscript to save space.

$$dw_t = f(X_t, \theta^*) + \varepsilon_t \tag{6}$$

where

$$\theta^* = \theta_1 + \theta_2 D(t) \tag{7}$$

where D(t) is a variable that indicates structural change. If structural change is thought to be discrete, one-time event at time t^*, then D(t) can be specified to equal one if $t > t^*$ and zero otherwise. Structural change is is depicted as follows:

$$dw_t = f(X_t, \theta_1)(1 - G(t^*; \gamma, c)) + f(X_t, \theta_2)G(t^*; \gamma, c) \tag{8}$$

where, t^*=$\frac{t}{T}$ is a time index, and G(.), is the transition function, which is continuous and smooth function of t^*, and bounded on the unit interval. γ and c are parameters that define characteristics of the transition function. Central to this analysis the choice of the transition function. A common specification of the transition function is the first-order logistic function given by:

$$G(t^*;\gamma,c) = [1 + e^{\frac{-\gamma(t^*-c)}{\sigma_{t^*}}}]^{-1}, \gamma > 0 \tag{9}$$

Here γ determines the speed with which the model shifts from one regime to another , c is the centrality parameter that determines what point in the sample the structural change is fifty percent complete (Holt and Balagtas, 2009), σ_{t^*} is the standard deviation of the normalized trend variable, and dividing by it makes the transition function unit free. As γ approaches zero, the transition function becomes effectively linear in t. Values between zero and infinity correspond to a transition that is s-shaped, indicating varying degrees of smooth transitioning between one regime to another. Models that use this function are called the Logistic Smooth Transition Autoregressive (LSTAR) models.

Another approach is to model the transition using an exponential function :

$$G(t^*;\gamma,c) = 1 + e^{(\frac{-\gamma(t^*-c)^2}{\sigma^2_{t^*}})}, \gamma > 0 \tag{10}$$

In this case, the structural change implied by the transition function is non-monotonic and is symmetric around c. As t^* approaches zero or one, *G()* goes to 1. As γ approaches zero, the exponential function approaches zero while if approaches infinity, the exponential function approaches one. Models that use this transition function are called Exponential Smooth Transition Autoregressive (ESTAR).

We use quarterly data on consumption and retail prices for beef, pork, and poultry. The data starts at the first quarter of 1960 and ends at the fourth quarter of 2009, giving us a total of 200 sample observations. This analysis uses the maximum likelihood estimator(MLE) to calculate the parameters. Under normality, SUR iterated to convergence is equivalent to MLE (Greene, 2011). The basic GIDDS model is estimated first by adding quarterly dummy variables to the model. Then the structural change models that rely on the transition function are estimated.

3 Results

The results for the basic GIDDS model are reported in Table 1. Most of the parameters are statistically significant at the 95 percent level of confidence. Table 2 presents the compensated and uncompensated flexibilities calculated at the mean value of the budget shares. Compensated own-price flexibilities are negative for all commodities, while scale flexibilities are close to negative one. All the uncompensated price flexibilities are negative.

The coefficient estimates for LSTAR and ESTAR are reported in Tables 3 and 4. Most of the coefficients are statistically significant. The gamma parameter for LSTAR model is 2.59 and the location parameter c is 0.399. This implies that there was a break and smooth transition to regime two of the distribution around 1979. The ESTAR estimates also tell similar story and with a gamma and c-value of 3.25 and 0.28, respectively, implying a non-monotonic smooth transition to the second regime occurred around 1974. This finding is comparable to earlier studies that employed STAR models. For example, Holt and Balagtas (2009) who also report that structural change in US meat demand occurred sometime in the 1970s. Rodriguez (2011) tested a host of models and finds that the timing of structural change varies depending on the type of model and transition variable used with the models. As a result our results are not directly comparable. In one of his models an LSTAR model with LA/IAIDS - Rodriguez (2011) finds structural change occurring in the early 1970s.

Scale and price flexibility results for the STAR models are reported in Table 5 and Table 6. These tables report the flexibilities computed at the mean values of the budget shares. We make the following observations from these. First, scale and own-price flexibilities are negative in all regimes for both models, and for all meat types. Similarities between the two models, however, end there. The LSTAR model tells the story where the own price and scale flexibility for pork declines in absolute terms as we move to regime two. The own price flexibility of beef declines in absolute terms while its scale flexibility increase in absolute value. The LSTAR model tells the story where the own price and scale flexibility for pork declines in absolute terms as we move to regime two. The own price flexibility of beef declines in absolute terms while its scale flexibility increase in absolute value. Own price flexibility of poultry rises substantially (in absolute terms) as we move to regime two. The ESTAR model indicates that pork and poultry become luxury goods in moving from regime 1 to regime 2 with their scale flexibilities as well as own price flexibilities declining in absolute value. In going from regime one to regime two, beefs own-price and scale flexibilities rise in absolute terms becoming a necessity in regime two. Figures 1 and 2 show the transition function for the two models where we have used time as the transition variable.

Table 7 presents the overall statistical comparison of the basic GIDDS model with the structural change models. We make note of the following issues. First, the structural change models outperform the basic GIDDS model in terms of log-likelihood, the Akaike information criterion (AIC), and the Bayesian information criterion (BIC) values. Second, the basic GIDDS fits the data better (higher R-squared) than the LSTAR model and compares favorably with the ESTAR model. This mixed performance of the STAR models stands in contrast to Holt and Balagtas (2009) who report that the STAR models outperform the basic Inverse AIDS model

4 Concluding Remarks

Documenting when and how structural change affects parameters of consumer demands is an important exercise. We followed the methodology outlined in Holt and Balagtas (2009) and put their approach to the test by using a general demand system that nests the Inverse

AIDS system they used. We employed the Barten's generalized inverse differential demand system (GIDDS).We also find that structural change might have occurred sometime in the early 1970s. Earlier models that used STAR models also report structural break during the same time frame. We find some evidence that the structural change models improve the fit, but it is not very strong.

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Variable definition:

a1/b1 are intercepts. p1 and p2 correspond to the ϕ's in equation (1) above. t1 and t2 are the nesting parameters i.e. θ_1 and θ_2 in equation 1. Parameters that start with "b' are reserved for the dummy variables. p11, p12, are own and cross price flexibility for beef. p21, p22, are own and cross price flexibility for pork. There are two regimes in the structural change estimation. The nomencluture of results in Tables 3 and 4 remains the same excep that all parameters ending with "b" are regime two parameters.

5 Tables

Table 1: Basic GIDDS parameter estimates

Parameter	Coeff	SE	tratio	p value
a1	0.01	0.00	3.62	0.00
p1	-0.34	0.14	-2.48	0.01
t1	0.70	0.26	2.65	0.01
p11	0.04	0.06	0.71	0.48
p12	-0.01	0.03	-0.29	0.77
t2	0.36	0.23	1.53	0.13
b1	0.00	0.00	0.38	0.70
b2	-0.01	0.00	-5.13	0.00
b3	-0.01	0.00	-4.77	0.00
a2	-0.01	0.00	-5.61	0.00
p2	-0.17	0.07	-2.49	0.01
p22	0.03	0.05	0.74	0.46
b12	-0.00	0.00	-0.99	0.32
b22	0.01	0.00	7.12	0.00
b32	0.02	0.00	8.70	0.00

Table 2: Basic GIDDS Flexibilities

Compensated Flexibilities			
	Beef	Pork	Poultry
Beef	-0.226	0.148	0.117
Pork	0.294	-0.346	0.092
Poultry	0.205	0.081	-0.268
Scale	-0.970	-0.943	-0.953

Uncompensated Flexibilities			
	Beef	Pork	Poultry
Beef	-0.722	-0.101	-0.167
Pork	-0.187	-0.590	-0.112
Poultry	-0.194	-0.0104	-0.418

Table 3: LSTAR parameters

Parameter	Coeff	SE	tratio	p value
a1	0.03	0.01	2.70	0.01
p1	-0.88	0.43	-2.07	0.04
t1	1.80	0.81	2.23	0.03
p11	-0.02	0.12	-0.21	0.84
p12	0.01	0.06	0.21	0.84
t2	0.39	0.43	0.90	0.37
b1	-0.03	0.02	-2.14	0.03
b2	-0.04	0.01	-2.98	0.00
b3	-0.05	0.02	-2.95	0.00
a2	-0.00	0.00	-0.97	0.33
p2	-0.46	0.21	-2.18	0.03
p22	-0.00	0.08	-0.01	0.99
b12	-0.01	0.01	-1.56	0.12
b22	0.01	0.00	1.27	0.21
b32	0.02	0.01	3.12	0.00
a1b	-0.00	0.00	-0.28	0.78
p1b	-0.84	0.36	-2.34	0.02
t1b	1.63	0.76	2.15	0.03
p11b	-0.14	0.16	-0.84	0.40
p12b	0.06	0.08	0.78	0.44
t2b	1.11	0.63	1.77	0.08
b1b	0.01	0.00	2.38	0.02

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Table3 – Continued

Parameter	Coeff	SE	tratio	p value
b2b	-0.00	0.00	-1.25	0.21
b3b	-0.00	0.01	-0.38	0.70
a2b	-0.00	0.00	-2.51	0.01
p2b	-0.39	0.20	-1.94	0.05
p22b	-0.09	0.12	-0.76	0.45
b12b	-0.00	0.00	-0.83	0.41
b22b	0.01	0.00	3.79	0.00
b32b	0.01	0.00	3.09	0.00
γ	2.59	1.26	2.06	0.04
c	0.40	0.10	4.09	0.00

Table 4: ESTAR parameters

Parameter	Coeff	SE	tratio	p value
a1	0.03	0.01	3.80	0.00
p1	1.12	0.77	1.46	0.15
t1	-1.68	1.33	-1.26	0.21
p11	-0.18	0.22	-0.81	0.42
p12	0.09	0.10	0.87	0.38
t2	0.91	0.77	1.18	0.24
b1	-0.03	0.01	-2.70	0.01
b2	-0.03	0.01	-3.97	0.00
b3	-0.05	0.01	-4.55	0.00
a2	-0.00	0.00	-0.15	0.88
p2	0.43	0.33	1.31	0.19
p22	-0.12	0.15	-0.81	0.42
b12	-0.01	0.01	-1.48	0.14
b22	-0.00	0.01	-0.33	0.74
b32	0.01	0.01	2.08	0.04
a1b	-0.00	0.00	-0.12	0.91
p1b	-0.70	0.22	-3.15	0.00
t1b	1.29	0.44	2.91	0.00
p11b	0.06	0.09	0.66	0.51
p12b	-0.02	0.04	-0.48	0.63
t2b	0.38	0.35	1.08	0.28
b1b	0.01	0.00	2.50	0.01

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Table4 – Continued

Parameter	Coeff	SE	tratio	p value
b2b	-0.00	0.00	-1.28	0.20
b3b	-0.00	0.00	-0.59	0.55
a2b	-0.01	0.00	-3.12	0.00
p2b	-0.30	0.11	-2.64	0.01
p22b	0.04	0.07	0.63	0.53
b12b	-0.00	0.00	-1.46	0.15
b22b	0.01	0.00	4.95	0.00
b32b	0.01	0.00	4.22	0.00
γ	3.26	1.54	2.12	0.04
c	0.28	0.02	15.32	0.00

Table 5: LSTAR Flexibilities

Regime One			
	Beef	Pork	Poultry
Beef	-0.82	-0.06	-0.07
Pork	-0.15	-0.71	-0.12
Poultry	-0.06	-0.05	-0.66
Scale	-0.93	-0.99	-0.76
Regime Two			
	Beef	Pork	Poultry
Beef	-0.73	-0.17	-0.18
Pork	-0.27	-0.50	-0.25
Poultry	-0.37	-0.19	-1.05
Scale	-1.01	-0.86	-0.75

Table 6: ESTAR Flexibilities

Regime One			
	Beef	Pork	Poultry
Beef	-0.63	0.07	0.06
Pork	-0.12	-0.79	-0.23
Poultry	-1.03	-0.53	-1.14
Scale	-0.48	-1.00	-2.27
Regime Two			
	Beef	Pork	Poultry
Beef	-0.74	-0.16	-0.21
Pork	-0.21	-0.52	-0.09
Poultry	-0.08	-0.04	-0.21
Scale	-1.08	-0.87	-0.70

Table 7: **Some measures of model fitness**

	Basic Inverse GODDS	LSTAR	ESTAR
No.Parameters estimated	15	32	32
Log Likelihood	1522.82	1827.14	1832.46
System R-sqr	0.93	0.92	0.94
AIC	-26.08	-26.27	-26.33
BIC	-25.78	-25.91	-25.97
γ	-	3.26	0.28
c	-	2.59	0.40

6 Figures

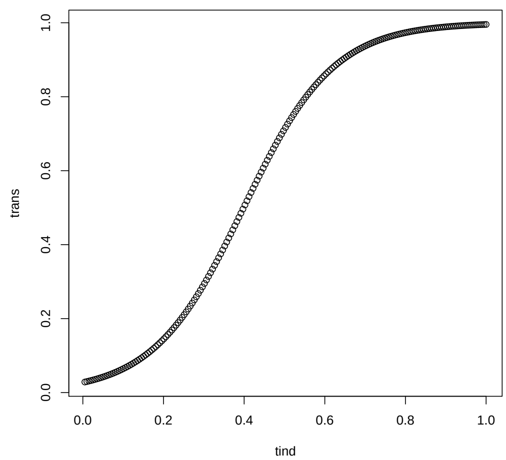

Figure 1: LSTAR Transition Function

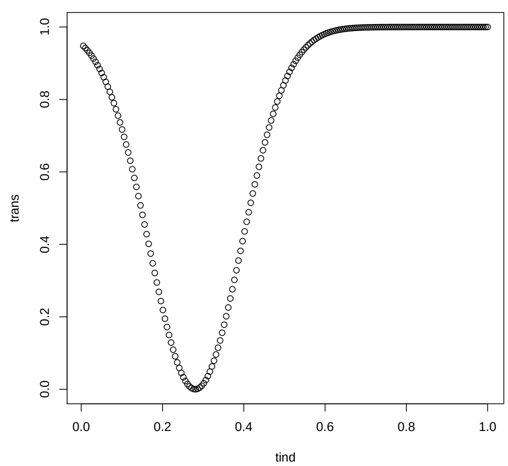

Figure 2: ESTAR Transition function