Integrating Spatial Dimension into Jointly Dynamic Groundwater Extraction

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Abstract:

One of the most important groundwater problems in Oregon, Washington and Idaho is the long-term decline of the groundwater surface level, which has been intensified by wells through discharging water from aquifers. Groundwater contamination from agriculture aggravates the depletion problem in irrigated regions since quality has a decisive role in ways of water use. We developed a spatial agricultural groundwater extraction model by coupling a hydrological model and a contamination migration model. We find that the optimal groundwater extraction is reduced if spatial interactions are incorporated, and that spatial heterogeneities such as crop varieties and soil types affect individual extraction. The socially optimal paths of shadow prices of groundwater quantity and quality depend on time preference, stock effect and dilution effect.

Keywords: Groundwater, Dynamic, Spatial, Groundwater quantity, Groundwater Quality, Agricultural Water Use

1 Introduction

One of the most important groundwater problems in Idaho, Oregon and Washington is the long-term decline of the groundwater surface level, which has been intensified by wells through discharging water from aquifers. Groundwater contamination aggravates the depletion problem since quality has a decisive role in ways of water use. In the Pacific Northwest, groundwater underlying irrigated regions can contain higher concentrations of chloride, nitrate, sulfate, and residue from fertilizers and pesticides. Nitrates are the most widespread pollutants of groundwater now due to human activity, particularly the intensification of agriculture (Goldberg 1989).

Researchers modeled dynamic agricultural groundwater extraction considering both quantity and quality (Hellegers et al. 2011, Roseta-Palma 2003). They investigated the role of groundwater contamination in individual extraction decision and the socially optimal pumping path. They found the optimal shadow price of groundwater quantity was higher due to stock effect and dilution effect, if there was a negative externality due to groundwater quality degradation.

Although it is an improvement to simultaneously determine the optimal quantity and quality paths, however, there is no spatial consideration in the previous studies, which may be important to the efficiency of groundwater extraction. Most studies used a representative farmer in their models to simplify the analysis ignoring spatial variations of farms' attributes such as crop varieties, management practices and soil quality. Also, the framework of a representative farm ignores interactions between farms: farmers would pay more for extraction due to the decreasing surface level of groundwater, if their neighbors extracted the groundwater from the same aquifer; the more pollutants emitted from the upstream farms, the higher contamination level of groundwater received by the farms located at the end of the groundwater flow in the same aquifer.

This paper thus develops a theoretical dynamic groundwater extraction model by coupling a hydrological model and a contamination migration model and incorporating the spatial dimension. We modeled both the individual and social planner's decisions in groundwater extraction.

The remainder of this paper is divided into five sections. Section 2 presents a hydrological model of groundwater, a groundwater contamination model, and an economic model of individual groundwater extraction. Section 3 discusses the individual and social planner's decisions in groundwater extraction. The last section concludes the paper.

2 Models

Hydrological Groundwater Model

In this paper we assume one single aquifer possessing large storage reserves and providing high well yields.

Notations:

t	Time
i	Farms, i=1,2, farm 1 is located upstream, farm 2 downstream
j	Irrigation technology, j=1,2
S_t	Stock size of groundwater
A_{it}	Extraction water for farm i

h_{ij} Share of groundwater used by crops, (1- h_{ij}) being returned to the groundwater stock

R_t Natural groundwater recharge

Following Hellegers et al. (2001), the groundwater stock change is

(1) $\dot{S}_t = R_t - \sum_i h_{ij} A_{it}$

$S_t \geq 0$, and given an intial condition S_0

Groundwater Contaminant Migration Model

The movement of groundwater contaminant is a complex 3-D process (Sun 1989). The factors affecting contaminant migration through porous media include contaminant concentration, chemical properties of the contaminant and the environment, geologic setting and site development (Egboka et al. 1989).

Notations:

n Contaminant species, n=1,…, M

C_n Concentration of contaminants, n=1, ….,M

θ Water content

$B(C_n)$ Absorption/desorption of contaminant n

D Hydrodynamic diffusion/dispersion tensor including the effects of molecular diffusion and longitudinal and transverse dispersion

u Darcy velocity

R_n Chemical reactions between contaminants

q Flow rates at injection and production wells

$\tilde{C}_n$ Concentration at wells, $\tilde{C}_n$ specified at injection wells and $\tilde{C}_n = C_n$ at production wells

The 3-D spatial transport equation which captures all above factors is (Hamed 1996):

(2) $$\frac{\partial}{\partial t}[\theta C_n + B(C_n)] - \nabla \cdot (D\nabla C_n - uC_n) = q\tilde{C}_n + R_n(C_1, \ldots, C_M).$$

In equation (2), pumping wells are treated as point sources and sinks. The instant contaminant concentration at one specific location is dependent on physical, chemical and biological attributes of location including water velocity, porosity, source or sink, distance, time, hydraulic conductivity and so on. The process of contaminant transportation is a combination of advection effect, diffusion effect, dispersion effect, absorption effect, retard effect and chemical reactions between pollutants (Hamed 1996). To capture all these effect, it is impossible to get analytical solutions by solving partial differential equations. Researchers now turn to develop numerical solutions like the finite difference methods (FDM), the finite element methods (FEM) and other alternatives (Sun 1989).

We assume that only a fixed part of extraction water will return to the groundwater, which simplifies the contaminant transportation process through the soil indexed by one parameter, h_{ij}. Also, we only model nitrate transport through groundwater between two farms, one of which is an emission source and the other a sink using one dimension advection-dispersion equation. Within the source-sink system, the nitrate release of one farm upstream will affect another downstream but not vice versa.

Notations:

D_x Horizontal dimension of hydrodynamic dispersion coefficient tensor;

u_x Average velocity of fluid at horizontal dimension

x Distance variable

C_{it}^R Nitrate concentration in recharge flows at farm i

C_{it}^S Nitrate concentration in the groundwater stock at farm i's well

The simplified 1-D groundwater contaminant transportation model is (Bear 1972):

(3) $\dot{C}_t = D_x \frac{\partial^2 C}{\partial x^2} - u_x \frac{\partial C}{\partial x}$,

s.t. $C(x, 0) = 0$, $C(0, t) = C_0$, $C(\infty, t) = 0$.

The analytical solution is given by Fried and Combernous (1971):

(4) $C(x, t) = \frac{C_0}{2}\left\{\operatorname{erfc}\left(\frac{x-u_x t}{2\sqrt{D_x t}}\right) + \exp\left(\frac{u_x x}{D_x}\right)\operatorname{erfc}\left(\frac{x+u_x t}{2\sqrt{D_x t}}\right)\right\}$

where $\operatorname{erf}(x) = \frac{2}{\pi}\int_0^x e^{-y^2} dy$, $\operatorname{erf} c(x) = 1 - \operatorname{erf}(x)$.

The change of nitrate concentration at farm i is (Rauscher 2007):

(5) $\dot{C}_{it}^S = \left(C_{it}^R - C_{it}^S\right)\frac{R_t - h_{ij}A_{it} + A_{it}}{S_t}$,

given an initial condition C_0^S, where $C_{1t}^S \geq 0$, $C_{2t}^S \geq 0$,

(6) $C_{2t}^S = \frac{C_{1t}^S}{2}\left\{\operatorname{erfc}\left(\frac{x-u_x t}{2\sqrt{D_x t}}\right) + \exp\left(\frac{u_x x}{D_x}\right)\operatorname{erfc}\left(\frac{x+u_x t}{2\sqrt{D_x t}}\right)\right\}$.

Economic Model

Notations:

p Exogenous price of agricultural output

$CR_i(A_{it})$ Crop varieties of farm i

$SO_i(A_{it})$ Soil type of farm i

$PR_i(A_{it})$ Precipitation of farm i

$f(\cdot)$ an increasing and strictly concave agronomic function

w water price

K_j cost of technology per hectare.

We assume farmers are risk-neutral and maximize their profits. The objective of each farmer is:

(7) $$\pi_{ij}(A_i) = \max\{pf([h_{ij}A_i, CR_i(A_i), SO_i(A_i), PR_i(A_i)] - wA_i - k_i\}.$$

The first order necessary condition is:

(8) $$pf_1 = \frac{w}{h_{ij}} - \frac{p(f_2 CR_i' + f_3 SO_i' + f_4 PR_i')}{h_{ij}}$$

3 Results and Discussion

Open access

Notation:

$c_i(S_t, A_{-it})$ Unit cost of groundwater extraction, decreasing and strictly convex in the groundwater stock, increasing and convex in the neighbor's extraction; -i indexes other farms

The marginal cost of extracting groundwater is $c_i(S_t, A_{-it})$ rather than constant water price w in equation (8). The standard open access model with marginal extraction benefits equal to marginal extraction cost gives:

(9) $$pf_1 = \frac{c_i(S_t, A_{-it})}{h_{ij}} - \frac{p(f_2 CR_i' + f_3 SO_i' + f_4 PR_i')}{h_{ij}}.$$

Since farmers' marginal extraction cost is increasing with neighbors' extraction, $\frac{\partial c_i}{\partial A_{-it}} > 0$, groundwater extraction needs to be reduced to keep a large size of groundwater stock (stock effect).

Also, spatial heterogeneities indexed by crop varieties, soil types and precipitation affect individual extraction. Higher precipitation will reduce groundwater use. If we rank crop varieties and soil types with yields and water demand, and yields monotonically increasing in crop varieties and soil types $(f_2 > 0, f_3 > 0)$, water demand monotonically decreasing in crop varieties and soil types $(CR_i^{'} > 0, SO_i^{'} > 0)$, groundwater extraction is lowered.

Comparing equation (9) with equation (8), farmers extract more groundwater than socially optimal allocations due to common property of groundwater. Even if farmers extract less groundwater and save it to use next time, their neighbors would pump the groundwater up, and therefore discourage farmers to reduce groundwater extraction, given the fixed storage of the aquifer.

Socially Optimal Allocation

Notations:

$g(A_{it}, C_{it}^{S})$ Environmental-damage function, increasing in both arguments and having positive second derivatives

V and ϕ Parameters transforming environmental damage into dollars

The social planner's decision is given by:

$$(10) \quad \max \int_0^{\infty} \{\sum_{i=1}^{2} \left[pf\left(h_{ij}A_{it}, CR_i(A_{it}), SO_i(A_{it}), PR_i(A_{it})\right) - c(S_t, A_{-it})A_{it}\right] - \Phi V g\left(A_{it}, C_{it}^{S}\right)\} e^{-\rho t} dt$$

The Hamiltonian function is:

(11) $$H=\sum_{i=1}^{2}\left[pf\left(h_{ij}A_{it},CR_i(A_{it}),SO_i(A_{it}),PR_i(A_{it})\right)-c(S_t,A_{-it})A_{it}\right]-\Phi Vg\left(A_{it},C_{it}^S\right)$$

$$+\lambda\left(R_t-\sum_i h_{ij}A_{it}\right)+\mu_1\left(C_{1t}^R-C_{1t}^S\right)\frac{R_t-h_{1j}A_{1t}+A_{1t}}{S_t}+\mu_2\left(C_{2t}^R-C_{2t}^S\right)\frac{R_t-h_{2j}A_{2t}+A_{2t}}{S_t}$$

where $C_{2t}^S=\frac{C_{1t}^S}{2}\left\{\operatorname{erfc}\left(\frac{x-u_xt}{2\sqrt{D_xt}}\right)+\exp\left(\frac{u_xx}{D_x}\right)\operatorname{erfc}\left(\frac{x+u_xt}{2\sqrt{D_xt}}\right)\right\}$.

The Optimal extraction rules are given by:

(12) $$\frac{\partial H}{\partial A_{it}}=p\left(f_1h_{ij}+f_2CR_i'+f_3SO_i'+f_4PR_i'\right)-c(S_t,A_{-it})-\frac{\partial c(S_t,A_{it})}{\partial A_{it}}A_{-it}$$

$$-\Phi V\frac{\partial g\left(A_{it},C_{it}^S\right)}{A_{it}}-\lambda h_{ij}+\mu_i\left(C_{it}^R-C_{it}^S\right)\frac{1-h_{ij}}{S_t}=0,$$

(13) $$\dot{\lambda}_t=\rho\lambda_t+\sum_{i=1}^{2}\left[\frac{\partial c(S_t,A_{it})}{\partial S_t}A_{it}-\mu_i\left(C_{it}^R-C_{it}^S\right)\frac{R_t-h_{ij}A_{it}+A_{it}}{S_t^2}\right],$$

(14) $$\dot{\mu}_{1t}=\rho\mu_{1t}+\Phi V\frac{\partial g\left(A_{1t},C_{1t}^S\right)}{C_{1t}^S}-\frac{\mu_1C_{1t}^R(R_t-h_{1j}A_{1t}+A_{1t})}{S_t}-\frac{\mu_2C_{2t}^R(R_t-h_{2j}A_{2t}+A_{2t})}{S_t}\frac{C_{2t}^S}{C_{1t}^S},$$

(15) $$\dot{\mu}_{1t}=\rho\mu_{1t}+\Phi V\frac{\partial g\left(A_{1t},C_{1t}^S\right)}{C_{1t}^S}-\frac{\mu_1C_{1t}^R(R_t-h_{1j}A_{1t}+A_{1t})}{S_t}.$$

Rewrite equations (12) - (13) and yield:

(16) $$pf_1=\frac{c_i(S_t,A_{-it})}{h_{ij}}-\frac{p\left(f_2CR_i'+f_3SO_i'+f_4PR_i'\right)}{h_{ij}}+\frac{\partial c(S_t,A_{it})}{\partial A_{it}}\frac{A_{-it}}{h_{ij}}$$

$$+\frac{\Phi V}{h_{ij}}\frac{\partial g\left(A_{it},C_{it}^S\right)}{A_{it}}+\lambda+\frac{\mu_i\left(C_{it}^R-C_{it}^S\right)(1-h_{ij})}{S_th_{ij}},$$

(17) $$\frac{\dot{\lambda}_t}{\lambda_t}=\rho+\sum_{i=1}^{2}\left[\frac{\partial c(S_t,A_{it})}{\partial S_t}\frac{A_{it}}{\lambda_t}-\frac{\mu_i\left(C_{it}^R-C_{it}^S\right)(R_t-h_{ij}A_{it}+A_{it})}{S_t^2\lambda_t}\right],$$

(18) $$\frac{\dot{\mu}_{1t}}{\mu_{1t}}=\rho+\frac{\Phi V}{\mu_{1t}}\frac{\partial g\left(A_{1t},C_{1t}^S\right)}{C_{1t}^S}-\frac{\mu_1C_{1t}^R(R_t-h_{1j}A_{1t}+A_{1t})}{S_t\mu_{1t}}-\frac{\mu_2C_{2t}^R(R_t-h_{2j}A_{2t}+A_{2t})}{S_t\mu_{1t}}\frac{C_{2t}^S}{C_{1t}^S},$$

(19) $$\frac{\dot{\mu}_{2t}}{\mu_{2t}}=\rho+\frac{\Phi V}{\mu_{2t}}\frac{\partial g\left(A_{2t},C_{2t}^S\right)}{C_{2t}^S}-\frac{\mu_2C_{2t}^R(R_t-h_{2j}A_{2t}+A_{2t})}{S_t\mu_{2t}}.$$

The spatial heterogeneity, indicated by the second term in the right hand side of equation (16), has effects on the socially optimal groundwater extraction. Two farms therefore have different groundwater allocations. The farm with crop varieties and soil types producing high yields and demanding less water extracts less groundwater. Also, the third term in equation (16) captures spatial quantity interactions between farms by changing marginal their extraction cost.

The optimal path of shadow price of groundwater quantity depends on time preference, stock effect and dilution effect. The stock size is large as total extraction is less in equation (16). If stock size of groundwater is relatively small, there are larger stock effect and dilution effect, and the shadow price could decline over time. If there exists a positive externality by implementing groundwater contaminant abatement technologies, dilution effect will lower the shadow price of groundwater quantity.

I assume there is a threshold effect in environmental benefits indicated by convexity of environmental damage function $g(A_{ijt}, C_{ijt}^{S})$, which increases shallow price of groundwater quality and reduces extraction for both farms.

The optimal extraction of groundwater quantity for the farm upstream will be less than the farm downstream due to its negative externality along groundwater flow, holding others constant. The difference between two farms' optimal extraction amount is dependent on emissions of and distance from the farm upstream. The upstream farm should extract less groundwater and thus reduce pollution effect on the downstream farm based on equation (18) and (19).

Thus, the socially optimal groundwater charge should be:

$$(20) \qquad T = \frac{\partial c(S_t, A_{it})}{\partial A_{it}} \frac{A_{-it}}{h_{ij}} + \frac{\Phi V}{h_{ij}} \frac{\partial g(A_{it}, C_{it}^{S})}{A_{it}} + \lambda + \frac{\mu_i (C_{it}^{R} - C_{it}^{S})(1 - h_{ij})}{S_t h_{ij}}.$$

4 Conclusion

In this paper, we found that that spatial dimension changes the individual and socially optimal allocation of groundwater. We added two spatial components to the dynamic groundwater extraction model. Quantity interaction is reflected in the change of marginal extraction cost while quality issue is solved by a contaminant migration equation, which captures all factors influencing the spatial distribution of contaminants.

The uniform water charges and water permits are inefficient without consideration of spatial quantity and quality issues. The optimal groundwater management should take into consideration of quantity and quality interactions as well as location characteristics. We need to empirically estimate the efficiency losses due to ignoring spatial dimension in optimal groundwater extraction. The future research can also extend the existing model to consider the spatial configuration within the same aquifer from multiple farms. The numerical methods may need to solve the full migration function.

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