Vertical Integration or Contract Farming on Biofuel Feedstock Production: A Technology Innovation Perspective

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Abstract Both the goal of energy independence and the desire to lower greenhouse gas emission have triggered the search for alternate energy sources. For second generation biofuel production, a key question is which form of industrial organization should be adopted in order to stimulate stable feedstock production. Using a two-stage optimal control framework, we analyze the optimal form of industrial organization should be adopted where technology innovation is endogenous and biorefinery faces credit constraint. Our results show that, under certain assumptions, it is optimal to adopt vertical integration in the beginning and move to contract farming later. Moreover, the tighter credit constraint that a biorefinery faces, the sooner the biorefinery would adopt contract farming.

Keywords: Contract Farming; Vertical Integration; Biofuel Feedstock; Technology Innovation

JEL Classification Numbers: Q16, Q42.

1 Introduction

Both the goal of energy independence and the desire to lower greenhouse gas emission have triggered the search for alternate energy sources. First generation biofuel production which uses corn or sugarcane etc as feedstock, albeit a business success, causes the "food vs. fuel" competition. As a consequence, the biorefinery industry faces strong need in adopting new technology that uses feedstock such as switchgrass for biofuel production. The inadequate spot market supply of the feedstock and high uncertainty in production technology indicates that manufacturers in the sector design a suitable business model that will allow for stable feedstock production. In this paper, we aim at providing a two-stage optimization framework to analyze the optimal form of industrial organization for biorefineries to adopt to produce the feedstock.

Spot Market, vertical integration[1], and agricultural contracts are the three ways for agricultural commodities to be transferred from farm to their next step of processing (MacDonald and Korb, 2011). As Acemoglu et al. (2009) suggested, vertical integration is more likely to occur in capital-intensive industries and when the contracting cost is higher. Therefore, vertical integration is less common in agriculture (Hayenga et al., 2000) while contract farming constitutes 39 percent value of U.S. agricultural production (MacDonald and Korb, 2011) and is widely accepted worldwide (Angeles et al., 2008).

Based on the involvement of contractor in production activities, the form of contracts in contract farming can be divided in two categories: namely, marketing contracts and production contracts (Farm Business Economics Branch, Rural Economy Division, ERS, 1996). In marketing contracts, agreement has to be made between growers and buyers on 'what to be made' and 'what are the commitment for future sale' (da Silva, 2005). i.e., market contracts specify the quantity and quality of the designated crop in transaction and set either a predetermined price for the crop or a formula for pricing based on market price at the time of

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[1] Here, we apply Bijman (2008)'s definition and distinguish the concepts of vertical integration and vertical coordination where the former implies unified ownership and the latter describes the coordination among activities in the supply chain.

transferring. Consequently, contractors share price risks with contractees.

Many forms of marketing contracts exist in the current U.S. agriculture market. In terms of delivery and payment date, a marketing contract can be categorized as spot contract or forward contract. A spot contract set an agreement on delivering a commodity and payment by the date of contract while a forward contract execute the delivery and payment at a later time. The most standard forward contracting system is the futures and derivatives market where major agricultural products are traded. A futures contract specifies the quantity, precisely measured quality, payment method and date of delivery of the contracting commodity. When the trade volume of a commodity is relatively small (energy crops, for instance) or the quality of a commodity is hard to be standardized, there is no futures market available for such commodities. Yet, both growers and processors may find it helpful to stabilize price uncertainty through contracting. In fact, for those commodities not on the futures market, growers and processors typically sign a cash forward contract. A cash forward contract is a forward contract other than futures contract (Paul et al., 1976). In such a contract, the specifications can be customized upon agreement between the two parties. Moreover, as there is no third-party involved, a credit premium may be added to the contract to reduce credit risk.

In the case of production contracts, arrangements will be made on 'how to produce' certain products (da Silva, 2005). Buyers are more involved in the production process under production contracts. They may specify inputs being used in production and share risks in both production and sale price with growers. Personal service contract and bailment are some frequently used production contracts (Kunkel et al. (2009)). Under a personal service contract, growers do not possess the ownership of the crop nor do they assume responsibilities of losses of crops. All the contractee provide is their services under given requirements from the contractor on the variety of seed, production technology, etc,. Meanwhile, under a bailment contract, the contractee stores the crop for the contractor, but they do not possess the crop.

Bijman (2008) mentioned another way to categorize different types of contracts, namely formal and informal contracts. He explains the some reasons for most agricultural contracts being informal: a. it is often hard for a third party to precisely measure if the desired characteristics of the commodity are meet and b. the involvement of a third party will incur higher transaction cost. A consequence of such a categorization is that Bijman (2008) explicitly reveals the importance of reputation in informal contractual relations. This is especially relevant to perennial crops since, by the end of first contract term, the producers' productivity and ability to meet certain crop criteria and the contractors' fulfillment of payment are observable to both parties. Therefore, in such a repeated game environment, the trustworthiness of both party in early stage often determines the possibility of contract renewal in later stages.

As Rehber (1998) pointed out, a major motivation for contract farming is that contractors could gain greater control over specific characteristics of the commodity they demand. Moreover, under contract farming, it is more likely to guarantee a stable supply of raw material. This feature is especially crucial for processing firms. Processing firms, refineries for instance, generally face high fixed cost to setup the processing facilities. In order to get sufficient raw material to match the processing capacity, processors may find it necessary to sign contracts to prevent unforeseenable inadequate input by the time of harvest. Another motivation of contract farming, as introduced in Bogetoft and Olesen (2004), is risk sharing. This is certainly the case when an unfamiliar crop is to be produced. Under such a circumstance, contractors typically provide the necessary technical assistance for growers so that farmers are less uncertain about the production process and contractors face less risk on not enough supply on the market. Note that the risk of not fulfilling the contract remains for the two parties. Both Minot (2007) and Rehber (1998) realize that the technology used in production is crucial for the success of a contract. At the same time, the two articles also find that the nature of the crop is a crucial determinant for the success of a contractual relation. It is easy to imagine that when a crop is more perishable or requires more quality control, contract farming is more welcomed.

Farm Business Economics Branch, Rural Economy Division, ERS (1996) summarizes some of the advantages of contract farming over traditional cash market. The first advantage they highlighted is that contract farming brings stable income to growers. Contracts in agricultural production can be viewed as a tool that allows risks to be transferred from less risk bearable farmers to the risk neutral end of the industry. For instance, in biofuel production, new energy crops may have to be introduced for greater biomass to meet the need of biofuel production. A production contract may assuage farmers' willingness to attempt the new varieties. Bijman (2008) noticed that on the social level, government subsidized contract farming may lead to better technological adoption for the society, which is referred to as the 'donor's ambition'. Despite of those advantages, Rehber (1998) also listed some of the disadvantages of contract farming. He mentioned that under contract farming, the growers are more often in a weaker position comparing to their counterpart. Also, production contracts may lead to loss of independence of farmers.

Besides the question of why contracts exist, it is very natural for economists to ask why different forms of contracts exist. Cheung (1969) is one of the early literature that dedicated to give an answer. His argument was though fixed wage or land lease contracts could bring about lower transaction costs, sharecropping (or profit-sharing) contracts leads to risk sharing. Consequently, trade-off between risk-sharing and transaction costs becomes one of the determinants for which type (or mixture) of contracts will be chosen. However, Rao (1971) used sharecropping data in India to empirically test this statement and the results were not in favor of Cheung's arguments. Stiglitz (1974) explained the existence of multiple types of contracts by a model of screening and claimed that the different forms of contracts help contractors to sort the farms' abilities which is unobservable for the contractors. But, Eswaran and Kotwal (1985) argued in their article, there is no reason to believe that in most rural areas, the assumption of ignorance of farmers' ability is inappropriate. Yet, an important contribution of Stiglitz (1974) is he proved that fixed wage or land lease contract could exist if and only if farmers or producers have to be risk neutral.

The early works on this topic considered merely production uncertainty. Later on, scholars have included more aspects to this issue. Both Eswaran and Kotwal (1985) and Bhattacharyya and Lafontaine (1995) use a double-sided moral hazard model to explain the existence of sharecropping. Hueth and Ligon (1999) considered unobserved or unmeasurable product quality. Recent literature developments have been focused on optimal contract form selection. Larson et al. (2005) analyzed four alternatives, namely spot, standard, acreage and revenue contracts, that producers could offer farmers. Using quadratic programming model, the authors found that acreage and revenue contracts are more effective than the other two types. Yang et al. (2012) take into account risk aversion of farmers and land quality issues. They show that higher risk aversion of farmers leads to higher sought for fixed term contracts while refinery could get higher profit in a region where there is concentrated lower quality lands and lower risk aversion farmers.

Another issue is the interplay between technology change and contractual structure change. In fact, the existing models cannot bring satisfying explanation to the stylized fact introduced in Rao (1971) and Day (1967). In the former, Rao found a switch from land lease contracts to fixed wage contract in India in the 1960s because of technology improvement and the latter realized that after advancement in technology, there is trend of moving from profit-contracts to fixed wage contracts in the US.

2 Model

Consider a two-stage cost minimization problem. In the first stage, the refinery chooses the optimal size of processing facility and in the second stage, the firm selects an optimal production plan of feedstock supply.

In order for farmers to be willing to participate in the contractual relationship, the refinery has to provide a contract that makes farmers at least earn the same amount before

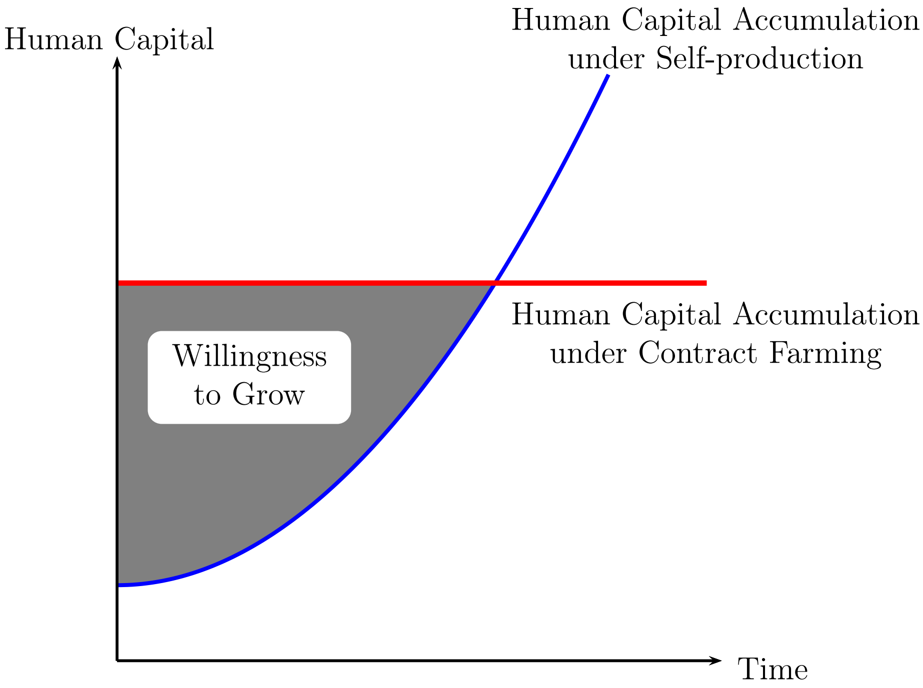

Figure 1: Illustration of Human Capital Accumulation over Time

contracting. Let w be the unit price of feedstock under contract farming. Participation constraint implies that there exists a $\underline{w}$ such that farmers will accept the offer if $w \geq \underline{w}$. We then postulate that farmers also are interested in human capital accumulation over time. Then contract farming may bring about higher gain in human capital by learning the production technology. Then, let A denote the production technology, we have $w = w(A)$. We assume that $w(A) \leq 0$. See figure 1 for illustration.

The argument induced the function $w = w(A)$ and note that w is decreasing in A.

Using backward induction, we first consider the second stage of the problem where the firm can either produce on its own with production cost function $C(t) = C(w_1, q_1(t), A(t))$ where w_1 is a vector of input prices, q_1 is the production quantity and A is the stock of knowledge of production technology. At the same time, the firm could also sign contract with farmers' to produce level of $q_2(t)$ at a fixed cost of w_2. Then the firm's cost minimization problem can be written as:

$$\min \int_0^T e^{-rt}[C(w_1, q_1(t), A(t)) + w(A)q_2(t)]dt. \tag{1}$$

The state equation is the dynamics of knowledge accumulation, we assume that the growth rate of A is a function of self-production level and existing knowledge stock:

$$\dot{A} = g(q_1, A), \tag{2}$$

where we assume that $g_{q_1} > 0, g_A < 0$. Moreover, the total production is limited to the firm's processing capacity $\bar{Q}$:

$$q_1(t) + q_2(t) = \bar{Q}, \ \ \forall t \in [0, t]. \tag{3}$$

Theorem 1 *(Bang-Bang Solution for linear case). Suppose $C(\cdot)$ and $g(\cdot)$ are linear in q_1, then there exists switch time $\tau \in [0, T]$ if the following equality holds at τ:*

$$e^{-rt}[w(A) - C(w_1, A)] = \lambda g(A)$$

Proof. In the case that $C(\cdot)$ and $g(\cdot)$ are linear in q_1, we let $C(w_1, A, q_1) = C(w_1, A)q_1$ and $g(A, q_1) = g(A)q_1$.

Notice that the capacity constraint must be met with equality, we may rewrite the Hamiltonian as:

$$H = e^{-rt}[C(w_1, A)q_1 + w(A)(\bar{Q} - q_1)] + \lambda g(A)q_1. \tag{4}$$

where $0 \leq q_1 \leq \bar{Q}$.

Notice that the Hamiltonian is linear in q_1: $H = [e^{-rt}C(w_1, A) - e^{-rt}w(A) + \lambda g(A)]q_1 + w(A)\bar{Q}$.

Then the minimization problem has a Bang-Bang Solution:

$$\begin{aligned} q_1^* = \bar{Q} \text{ if } e^{-rt}C(w_1, A) - e^{-rt}w(A) + \lambda g(A) < 0. \\ q_1^* = 0 \text{ if } e^{-rt}C(w_1, A) - e^{-rt}w(A) + \lambda g(A) > 0. \end{aligned} \tag{5}$$

Therefore, if there is any switching happened, it must be the case that $e^{-rt}C(w_1, A)+\lambda g(A) = e^{-rt}w(A)$. ■

The theorem is essentially saying that if we could observe a switch from one way of getting the feedstock to another, it must be the case that, at the switching point, the marginal cost from self-production subtracting the marginal benefit of knowledge accumulation is exactly the marginal cost from contract farming.

Corollary 2 *If $C(A) < w(A)$ at some $\tau \in [0, T]$ and $|C_A| > |w_A|$, then contract farming is not optimal for all $t \in [\tau, A]$.*

Proof. By theorem 1, we know that contract farming would happen if and only if

$$e^{-rt}[w(A) - C(w_1, A)] < \lambda g(A)$$

Since λ is the shadow price for knowledge accumulation, we know that $\lambda(t) \leq 0$ for all $t \in [0, T]$. Given that $C(A) < w(A)$ at τ, also notice that

$$\frac{d(C - w)}{dt} = (C_A - w_A)\dot{A} = (C_A - w_A)gq_1$$

Since $|C_A| > |w_A|$, we know that $\frac{d(C-w)}{dt} < 0$. Therefore, $C(A) < w(A)$ for all $t \in [\tau, A]$. we have $e^{-rt}[w(A) - C(w_1, A)] > 0$ for all $t \in [\tau, T]$. Thus, the inequality

$$e^{-rt}[w(A) - C(w_1, A)] < \lambda g(A)$$

will never hold after $t = \tau$. ■

The intuition for the corollary is simple: if self-production incurs lower cost at some time, then only reason for refinery to switch after that time is that the contracting cost has to reduce more than self-production cost. But $|C(A)| > w(A)|$ eliminates this possibility. Then the question becomes: if $C(A) > w(A)$, will switching occur. The next two theorems will answer this question.

Theorem 3 *In the linear case introduced above, if production technology improvement does not affect contracting cost, i.e.* $w_A = 0$ *for all* $t \in [0, T]$*, then switching cannot occur.*

Proof. Suppose that contract farming is optimal for the refinery at some time $\tau \in [0, T]$, then it must be the case that $C(w_1, A) - w(A) > 0$ at $t = \tau$. If, on the contrary, $C(w_1, A) - w(A) < 0$, note that $\lambda \leq 0$ and $g(A) \geq 0$, then, we must have

$$e^{-rt}[C(w_1, A) - w(A)] + \lambda g(A) < 0$$

which means self-production should be optimal.

We will then show that the expression

$$e^{-rt}(C(w_1, A) - w(A)) + \lambda g(A)$$

is monotonically non-increasing in t if $C(\cdot) > w$.

Notice that

$$\frac{d[e^{-rt}C(w_1, A) + \lambda g(A)]}{dt} = e^{-rt}C_A\dot{A} - re^{-rt}(C - w) + \lambda g_A\dot{A} + \dot{\lambda}g$$

Moreover, combining the results that $\dot{A} = g(A)q_1$ and

$$\dot{\lambda} = -e^{-rt}C_Aq_1 + \lambda g_Aq_1.$$

we then have

$$\frac{d[e^{-rt}C(w_1, A) + \lambda g(A)]}{dt} = -re^{-rt}(C - w).$$

Since $C(w_1, A) > 0, e^{-rt} > 0$, the expression must be non-increasing in t.

Since $C(\cdot)$ is decreasing in t, then $C > w$ at $t = \tau$ implies $C > w$ for all $t \leq \tau$. Therefore, combined with the previous result, we must have $e^{-rt}(C(w_1, A) - w(A)) + \lambda g(A)$ is decreasing in t for all $t \leq \tau$. But we are given that the expression is positive at $t = \tau$, then we have:

$$e^{-rt}[C(w_1, A) - w(A)] + \lambda g(A) > 0, \forall t \in [0, \tau].$$

For $t \in [\tau, T]$, we first notice that since $q_1^* = 0$ for all $t \leq \tau$, we have $A(t)$ is a constant for all $t \leq \tau$ and consequently $C(A)$ is a constant. This leads to $\lambda = 0$ because the available technology is never being used. Therefore, the expression $e^{-rt}[C(w_1, A) - w(A)] + \lambda g(A)$ remains to be positive for all $t \geq \tau$.

So far, we have shown that if refinery finds contract farming to be attractive at any time τ, then it is to be adopted for all time periods, which means it's not possible to switch from or to contract farming. Therefore, switching cannot happen for all $t \in [0, T]$. ■

Theorem 3 is essentially saying that if there is no return to scale in self-production of feedstock (by assuming cost function is linear in q_1) and production technology does not provide the extra benefit in contract cost reduction, then switching cannot occur. Here, the pure effect of technology improvement on self-production determines whether a refinery should choose self-production or not. Then we relax the assumption of $w_A = 0$, the following result will hold

Theorem 4 *If $C > w$ for all $t \in [0, T]$, then the only switching scenario is switching from self-production to contract farming.*

Proof. Since $\dot{\lambda} = -H_A$, we have the differential equation for the shadow price:

$$\dot{\lambda} = -e^{-rt}(C_A - w_A)q_1 - w_A\bar{Q} - \lambda g_A q_1$$

Since the problem is free terminal state with fixed termination time, we have $\lambda(T) = 0$. Let $f(t)$ defined to be

$$f(t) = \int [re^{-rt}(C - w) - gw_A\bar{Q}]dt.$$

We will show that

$$\lambda = g^{-1}[-e^{-rt}(C - w) + c_0 + f(t)]$$

where the constant c_0 is: $c_0 = e^{-rT}(C - W) - f(T)$. First note that the homogeneous equation for the problem is:

$$\dot{\lambda} = \lambda g_A q_1$$

The solution is given by:

$$\lambda = g^{-1}$$

By the method of constant variation, the solution to the inhomogeneous problem is:

$$\lambda = g^{-1}[c_0 - \int [e^{-rt}(C_A - w_A)q_1 + w_A\bar{Q}]gdt$$

By integration by parts, we have:

$$\int e^{-rt}(C_A - w_A)gq_1 dt = \int e^{-rt}d(C - w)$$

$$= e^{-rt}(C - w) - \int (C - w)de^{-rt} = e^{-rt}(C - w) + r\int e^{-rt}(C - w)dt$$

Now, since $\lambda(T) = 0$, we have $c_0 = e^{-rT}(C - W) - f(T)$.

Then, we look at the expression $e^{-rt}(C - w) + \lambda g$, and observe:

$$e^{-rt}(C - w) + \lambda g = c_0 + f(t)$$

Note that $f'(t) = re^{-rt}(C - w) - gw_A\bar{Q}$. Given that $C > w$ and $w_A < 0$, we have $f'(t) > 0$. Thus, $f(t)$ is increasing in t. Suppose contract farming occurs at $t = \tau$ then $c_0 + f(\tau) > 0$, but $f(t)$ increases over time. Thus, $c_0 + f(t) > 0$ for all $t \in [\tau, T]$. Therefore, switching could happen only in the case of switching from self-production to contract farming. ■

Figure 2 demonstrates the scenario introduced in theorem 4. When self-production quantity switch from $\bar{Q}$ to 0 at time τ, we first notice that the production technology stops to accumulate. Therefore, the optimal production technology path has a non-differentiable point at τ. Since cost function $C(\cdot)$ and contracting cost function $w(A)$ are decreasing in A,

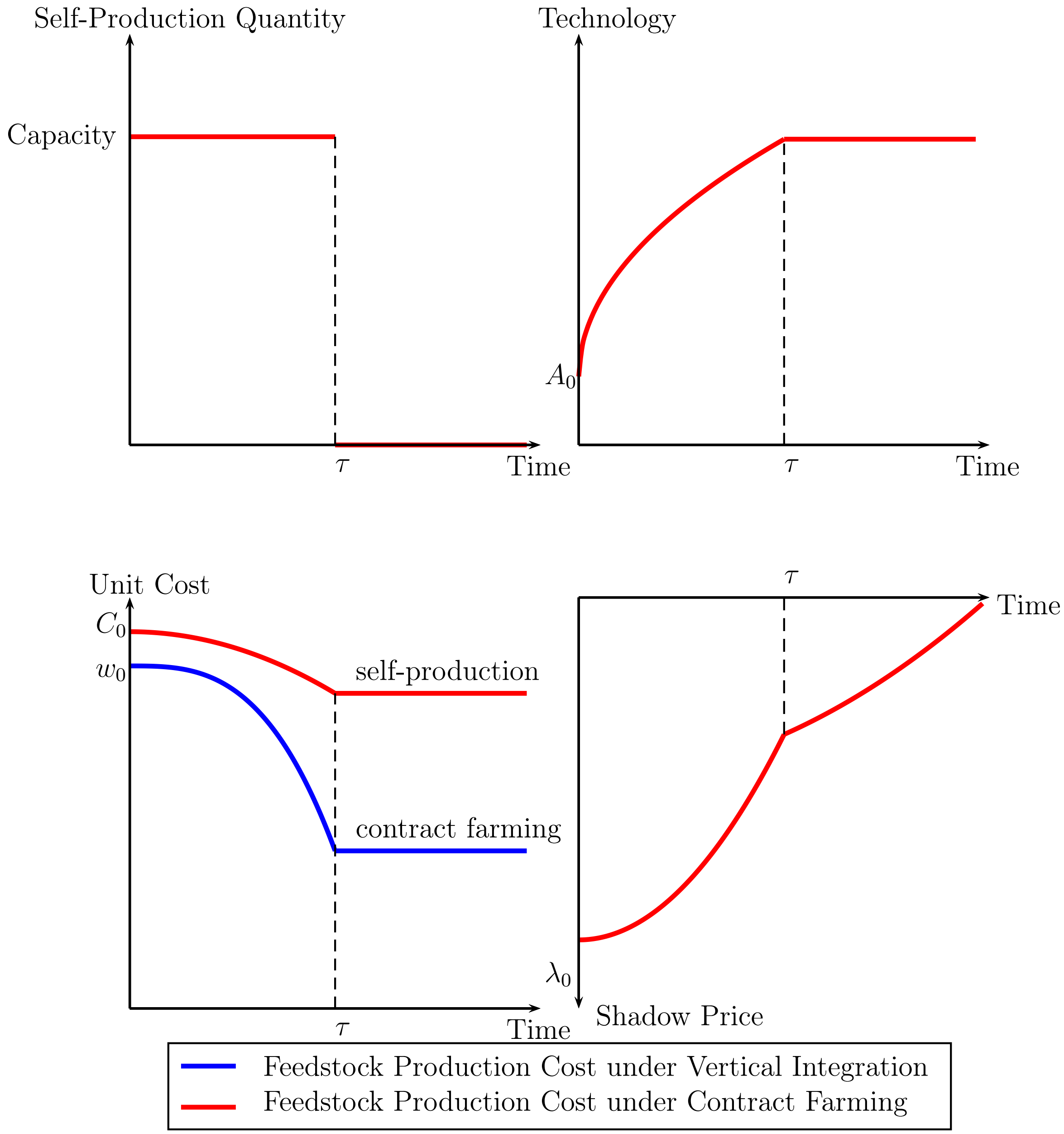

Figure 2: Illustration of optimal switching time

we see the decreasing pattern over time for the two functions. However, as technology stops to grow, the value of the two functions remains constant for all $t \in [\tau, T]$.

It should be noted that the optimal path of shadow price follow different patterns before and after τ. Recall that

$$\dot{\lambda} = -[e^{-rt}(C_A - w_A) + \lambda g_A]q_1 - w_A \bar{Q}$$

When $q_1^* = \bar{Q}$, the shadow price captures two components, the first part $-[e^{-rt}(C_A - w_A) + \lambda g_A]q_1$ is the benefit resulted from reduced self-production cost over contracting cost. The second part $w_A \bar{Q}$ is the benefit from lowering contracting cost. Notice that, as $q_1^* = 0$ after $t = \tau$, the dynamics of λ follows:

$$\dot{\lambda} = -w_A \bar{Q}$$

which means that after the production technology growth stops, the shadow price of technology improvement reflects only the benefit from lowered contracting costs.

Now we go back to the first stage and find the optimal processing capacity. In the first stage, the refinery faces the decision of choosing size of processing facility. We use $F(\bar{Q})$ to denote the cost of facility.

Moreover, the firm faces a credit constraint: the firm has total disposable capital I. Let $V^*(\bar{Q})$ denote the discounted total cost of feedstock production, then we have:

$$V^*(\bar{Q}) + F(\bar{Q}) \leq I.$$

Lemma 5 *On the optimal feedstock production path, the following identity must hold:*

$$V_{\bar{Q}}^* = \int_0^T e^{-rt} w(A) dt. \tag{6}$$

where

$$V^*(\bar{Q}) = \int_0^T e^{-rt}[C(w_1, q_1^*(t), A(t)) + w(A)q_2^*(t)]dt. \tag{7}$$

Proof. Using LaFrance and Barney (1991), notice that:

$$V_{\bar{Q}}^* = \int_0^T H_{\bar{Q}}|_{q_1 = q_1^*, q_2 = q_2^*} dt.$$

where $H_{\bar{Q}} = w(A)$ by equation (4). Thus, we have the desired equality. ■

Suppose the credit constraint binds, then $V^*(\bar{Q}) + F(\bar{Q}) = I$. By implicit function theorem, we get:

$$\frac{d\bar{Q}}{dI} = \frac{1}{V^*_{\bar{Q}} + F_{\bar{Q}}}$$

By Lemma 1, we know that $V^*_{\bar{Q}} > 0$, assume that $F_{\bar{Q}} > 0$, then $\frac{d\bar{Q}}{dI} > 0$.

Theorem 6 *If credit constraint is relaxed, then the switching time will delay. i.e.* $\frac{d\tau}{dI} > 0$.

Proof. Notice that at the switching time,

$$e^{-rT}(C - w) = \int_{\tau}^{T} [re^{-rt}(C - w) - gw_A\bar{Q}]dt$$

Using implicit function theorem on the equation above, we have:

$$\frac{d\tau}{d\bar{Q}} = \frac{\int_{\tau}^{T} -gw_A\bar{Q}dt}{re^{-r\tau}(C - w) - w_A g\bar{Q}}$$

In the expression above, we utilize the Leboniz Theorem and realize that

$$\frac{d(\int_{\tau}^{T} [re^{-rt}(C - w) - gw_A\bar{Q}]dt)}{d\tau} = re^{-r\tau}(C - w) - w_A g\bar{Q}$$

Note that $C - w > 0$ and $w_A < 0$. Then the expression is positive. and

$$\frac{d(\int_{\tau}^{T} [re^{-rt}(C - w) - gw_A\bar{Q}]dt)}{d\bar{Q}} = \int_{\tau}^{T} -gw_A\bar{Q}dt$$

Since $w_A \leq 0$ for all $t \in [\tau, T]$, the integral above is also positive. Then we know that $\frac{d\tau}{d\bar{Q}} > 0$. By the argument above, $\frac{d\bar{Q}}{dI} > 0$. Then, by chain rule, we get:

$$\frac{d\tau}{dI} > 0.$$

■

When the production does not show a pattern of constant return to scale and growth function A is not linear in q_1, interior solution is available. Consider the Cobb-Douglas self-production function:

$$q_1 = A \prod_{i=1}^{N} x_i^{\alpha_i}$$

where unit cost of factor x_i is r_i, this will lead to the cost function:

$$C(r_1, \ldots, r_N, q_1, A) = \phi(r_1, \ldots, r_N)(\frac{q_1}{A})^{1-\sum\limits_{i=1}^{N} \alpha_i}.$$

Let $w_1 = \phi(r_1, \ldots, r_N), \alpha = 1 - \sum\limits_{i=1}^{N} \alpha_i$. Then the cost function becomes:

$$C(w_1, A, q_1) = w_1(\frac{q_1}{A})^{\alpha}.$$

Assume that the technology growth follows:

$$g(A, q_1) = (\frac{q_1}{A})^{\beta}.$$

Then the Hamiltonian can be written as:

$$H = e^{-rt}[w_1(\frac{q_1}{A})^{\alpha} + w(\bar{Q} - q_1)] + \lambda(\frac{q_1}{A})^{\beta}.$$

First order condition gives:

$$e^{-rt}(w_1 \alpha q_1^{\alpha-1} A^{-\alpha} - w) + \lambda \beta q_1^{\beta-1} A^{-\beta} = 0$$

Thus,

$$\lambda = -\frac{e^{-rt}(w_1 \alpha q_1^{\alpha-1} A^{-\alpha} - w)}{\beta q_1^{\beta-1} A^{-\beta}}.$$

The adjoin equation gives:

$$\dot{\lambda} = e^{-rt} w_1 q_1^{\alpha} \alpha A^{-\alpha-1} + \lambda \beta q_1^{\beta-1} A^{-\beta-1}$$

Putting the λ formula into the adjoin equation, we get:

$$\dot{\lambda} = e^{-rt} w q_1 A^{-1}$$

Now, we differentiate the λ equation with respect to t, first note that:

$$\lambda = e^{-rt} \frac{w}{\beta} q_1^{1-\beta} A^{\beta} - e^{-rt} \frac{\alpha}{\beta} q_1^{\alpha-\beta} A^{\beta-\alpha} w_1.$$

Then

$$\dot{\lambda} = -r\lambda + e^{-rt} w q_1 A^{-1} - e^{-rt} \frac{\alpha}{\beta} q_1^{\alpha-\beta} (\beta - \alpha) A^{\beta-\alpha-1} w_1.$$

Recall that we have shown $\dot{\lambda} = e^{-rt} w q_1 A^{-1}$. Therefore,

$$-r[e^{-rt} \frac{w}{\beta} q_1^{1-\beta} A^{\beta} - e^{-rt} \frac{\alpha}{\beta} q_1^{\alpha-\beta} A^{\beta-\alpha} w_1] = e^{-rt} \frac{\alpha}{\beta} q_1^{\alpha-\beta} (\beta - \alpha) A^{\beta-\alpha-1} w_1.$$

Further calculation will give us the q_1^* formula:

$$q_1^* = (\frac{rw\alpha}{w_1})^{\frac{1}{\alpha-1}} A^{\frac{\alpha}{\alpha-1}} (r - \frac{\beta - \alpha}{A})^{\frac{1}{1-\alpha}}.$$

Theorem 7 *Under the Cobb-Douglas setting, given that $\beta < \alpha$, the share of contract farming production decreases over time if $\alpha < 1$ and increases over time if $\alpha > 1$. When $\beta > \alpha$, there may not exist monotonic pattern.*

Proof. Notice that

$$\dot{q_1^*} = \frac{dq_1^*}{dA} \dot{A}$$

where

$$\frac{dq_1^*}{dA} = \frac{q_1^*}{(\alpha - 1)A} [\alpha + \frac{\beta - \alpha}{(\beta - \alpha) - rA}]$$

If $\beta < \alpha$, then the term $\frac{\beta-\alpha}{(\beta-\alpha)-rA} > 0$. Thus, $\frac{dq_1^*}{dA} > 0$ if $\alpha > 1$; $\frac{dq_1^*}{dA} < 0$ if $\alpha < 1$.

Since $\dot{A} \geq 0$, we have that $\dot{q_1^*}$ is of the same sign as $\frac{dq_1^*}{dA}$. $\blacksquare$

Figure 3 illustrates the optimal production path under Cobb-Douglas production function specification. It is clear from the figure that the Cobb-Douglas production function specification yields a solution path that is a smooth version of the linear case. However, the shape of the paths remains.

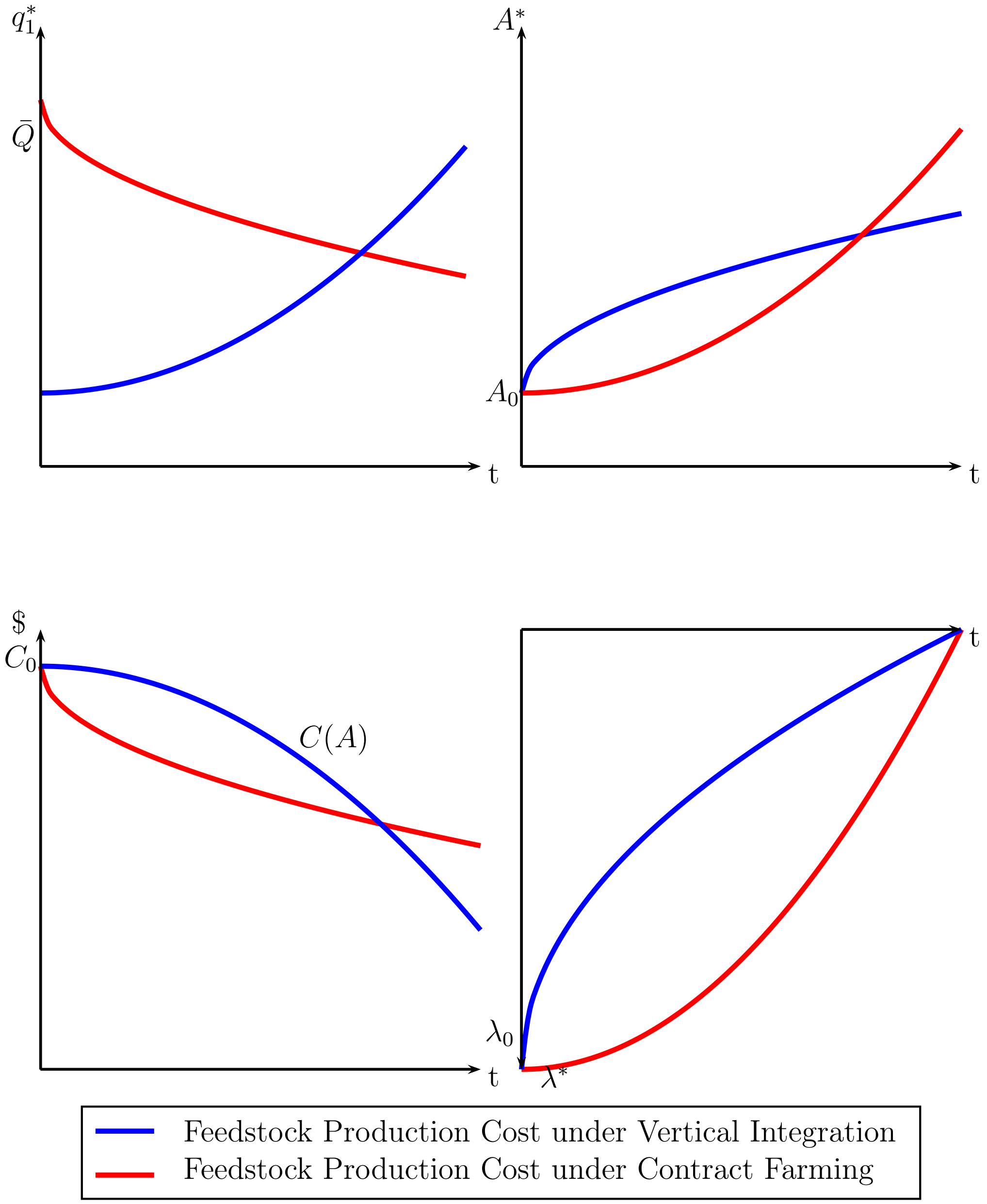

Figure 3: Illustration of Theorem 7

3 Conclusion and Future Directions

In this paper, we provide a two-stage optimal control framework to analyze the optimal form of industrial organization should be adopted where technology innovation is endogenous and biorefinery faces credit constraint. Here, the first stage of the problem is how much resource should a biorefinery allocate to building processing facility and feedstock production. The second stage is to select a form of industrial organization to carry out feedstock production. Using backward induction, we first develop an optimal control model to study the optimal production paths. Our results show that, under certain assumptions, it is optimal to adopt vertical integration in the beginning and move to contract farming later. Moreover, in the first stage, we show that the tighter credit constraint that a biorefinery faces, the sooner the biorefinery would adopt contract farming.

The main limitations of our model are: 1. in our model, the contracting cost is an exogenous variable. However, this variable should be endogenously determined because the biorefinery would take growers' preference into consideration to provide the contract menus. Therefore, one of the future direction of this research is to utilize contract theory to model how the contract is determined. 2. The other limitation of the model is that the results provided by the model rely heavily on the assumption of the production functional forms. As a consequence, another possible improvement of the model is to apply comparative dynamics tools to get rid of the functional form assumptions.

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