Spatial externalities in aquifers with varying thickness: Theory and numerical results for the Ogallala aquifer*

Jeffrey M. Peterson
Department of Agricultural Economics
Kansas State University
216 Waters Hall
Manhattan, KS 66506-4011, U.S.A.
Phone: 785 477-0924
Fax: 785 532-6925
E-mail: jpeters@ksu.edu

Alexander E. Saak
Markets, Trade, and Institutions Division
International Food Policy Research Institute
2033 K Street, NW
Washington, DC 20006-1002 USA
Phone: 202 862-4543
Fax: 202-467-4439
E-mail: asaak@cgiar.org

June 3, 2013

Abstract

This paper studies the divergence in the planning and equilibrium solutions for a multicell aquifer with heterogeneity in cell depths. A spatial model is developed that explicitly accounts for the lateral movement of water between cells. The optimal planning problem maximizes the discounted stream of rents earned from irrigation over an infinite horizon. The optimal steady state of this problem is derived and compared to the competitive equilibrium steady state, which results from myopic rent maximization among users. Studying the steady-state conditions in the two outcomes allows for the nature the spatial externalities to be characterized and reveals the effects of varying cell depths. In a two-cell specification of the model, closed-form expressions are derived for the difference in optimal steady state water table elevations between the two cells. The gap in optimal heights is shown to depend on an interaction between the speed of lateral flows in the aquifer, the asymmetry in cell depths, and the curvature properties of the irrigation benefits function. The 2-cell model is then applied numerically to quantify the spatial externalities and asymmetry effects in Sheridan County, Kansas, which overlies the Ogallala aquifer. Simulated welfare losses in this model are relatively large and are sensitive to the asymmetry in cell depths.

Introduction

Based on the seminal model of Gisser and Sanchez (1980), groundwater use has traditionally been analyzed under the assumptions that (a) there are a large number of identical resource users and (b) the resource itself is a bathtub aquifer – a basin with parallel sides and a flat bottom in which changes in the groundwater level are transmitted instantaneously to all users. Under these assumptions, the location of users in the area overlying the aquifer is immaterial, and a representative user exists.

*This research was funded in part by United States Department of Agriculture (USDA-NIFA Multistate Research Project W-2190) and in part by the National Science Foundation (grant GEO0909515).

These models capture the common pool externalities in groundwater use. When there are no policy restrictions on extraction rates, individual users have an incentive to extract more quickly than the socially efficient rate, because most of the future costs of current extraction are transferred to others. Although the competitive and planning solutions differ analytically in these models, empirical studies based on this framework typically find that the gap between them is small (Koundouri, 2004). One implication discussed in the literature is that the welfare gains from policies to conserve groundwater for future uses are limited, so that policy attention should be focused on allocating water to the highest valued uses in each period (Gisser, 1983).

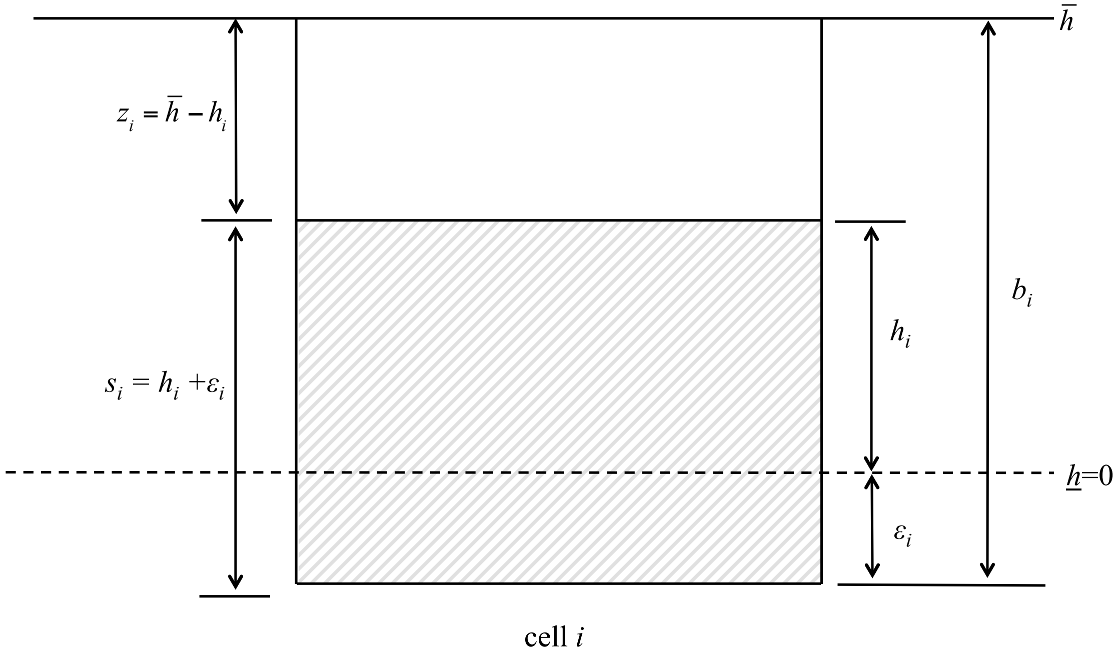
An emerging literature, however, is beginning to reconcile the intertemporal and cross-sectional allocation issues in a cohesive framework. A number of recent studies have relaxed the assumptions of the traditional models, considering settings where lateral flows are not instantaneous and/or where resource conditions are spatially heterogeneous. Brozović et al. (2010), Pfeiffer and Lin (2012), and Saak and Peterson (2007) develop models based on hydrological principles where the speeds of lateral flow are finite. Wang and Segarra (2011)and Saak and Peterson (2012) consider user heterogeneity in land productivity and farm size, respectively. Athanassoglou et al. (2012) consider both finite transmissivity and user heterogeneity, where users may differ in land productivity and in the transmissivity to neighboring aquifer cells.

This research has revealed that spatial heterogeneity adds a new dimension to the underlying common pool externalities. When users are heterogeneous, the socially optimal solution will involve different pumping rates and water table heights across locations. In general, the planner will assign higher pumping rates to cells where water has higher marginal benefits, partly because of the inherent productivity advantage, but also because higher pumping temporarily lowers the water table height in that cell and induces a lateral flow of groundwater toward the locations where it can be used most beneficially. The speed of lateral flows then determines both the magnitude of the optimal gap in water table heights as well as the spatial extent over which externalities are transmitted.

In this paper, we address an unexplored source of variation by studying a multicell aquifer with varying cell depths. This source of variation is important in many aquifers. For example, in contrast to the relatively flat land surface and homogenous soils overlying large portions of the High Plains aquifer in the central United States, the bedrock floor of the aquifer is very uneven; the measured saturated thickness of the aquifer in 1980 ranged from less than one foot to over 1000 feet (Miller and Appel 1997). We formulate a spatial model that explicitly accounts for the lateral movement of water between cells, which results from spatial differences in water table heights. We then develop the optimal planning problem, which maximizes the discounted stream of rents earned from irrigation over an infinite horizon. The optimal steady state of this problem is derived and compared to the competitive equilibrium steady state, which results from myopic rent maximization among users. By studying the steady-state conditions in the two outcomes, we can characterize the nature of the spatial externalities and the way they are affected by variation in cell depths.

To sharpen the analysis, we also develop a two-cell specification of the model, from which we can derive closed-form expressions for competitive steady state water table heights and for the difference in optimal steady state heights. The gap in optimal heights is shown to depend on an interaction between the speed of lateral flows in the aquifer, the asymmetry in cell depths, and the curvature properties of the irrigation benefits function. The 2-cell model is then applied numerically to quantify the spatial externalities and asymmetry effects in Sheridan County, Kansas, which overlies the Ogallala aquifer. Simulated welfare losses in this model are relatively large and are sensitive to

Figure 1: Physical characteristics of an aquifer cell.



the asymmetry in cell depths.

Model

Consider a region with a flat land surface underlain by an aquifer with an uneven bottom. The land surface is divided into $i = 1, \ldots, n$ equally sized cells. Figure1 shows the relevant physical measures on a cross section of the i^{th} cell of the aquifer. The mean elevation of the bottom of the aquifer across all cells is normalized to $\underline{h} = 0$, while $\bar{h} > 0$ is the constant elevation of the land surface. This implies the mean cell has a depth from the land surface to aquifer base of $\bar{h}$. Cell i's depth is $b_i = \bar{h} + \epsilon_i$, where ϵ_i is the deviation from mean depth ($\sum_{i=1}^{n} \epsilon_i = 0$), and the elevation of the cell base is $\bar{h} - b_i = -\epsilon_i$.

The height of the water table in each cell can vary continuously over time, $t \in [0, \infty)$, which affects users' welfare over time due to changing saturated thickness and pumping lifts. Letting $h_i(t)$ denote the elevation of the water table in cell i at instant t, saturated thickness is defined as the difference in water table and base elevations,

$$s_i(t) = h_i(t) + \epsilon_i. \tag{1}$$

Finally, pumping lift is the elevation difference between the land surface and water table, or

$$z_i(t) = \bar{h} - h_i(t). \tag{2}$$

Let $w_i(t)$ be the rate of water withdrawal from cell i at instant t. We normalize our units so that withdrawals are measured as changes in water table height; one unit of w_i is defined as the volume of water stored in a 1-unit thick horizontal slice of aquifer in cell i. This normalization avoids the need to convert between volumes and depths based on the storativity and cell area. The water table height in cell i changes over time according to

$$\dot{h}_i = r - w_i + \sum_{j=1}^{n} \theta_{ji}(h_j - h_i), \quad i = 1, \ldots, n \tag{3}$$

where time arguments have been suppressed to simplify notation, r is the rate of natural recharge (assumed constant across cells), and θ_{ji} is a transfer coefficient measuring the horizontal flow of water from cell j to cell i per unit of gradient (difference in water table elevations). θ_{ji} depends on hydrologic properties as well as the distance between cells. This specification of hydrologic flow is similar to those employed by Athanassoglou et al. (2012), Brozović et al. (2010), Pfeiffer and Lin (2012), among others.

The landholder of cell i (hereafter, user i) earns economic rents from pumping and applying water to irrigate crops. The gross benefits of extracted water for cell i is given by $F(w_i, s_i)$, which depends on water applied as well as saturated thickness. $F(\cdot)$ is assumed to be increasing and (weakly) concave in both its arguments with $F_{ws} \geq 0$; i.e., saturated thickness improves total benefits and also has a nondecreasing effect on marginal benefits of extracted water. While much of the literature on groundwater economics ignores the potential effect of saturated thickness on revenue, a number of studies have found it is potentially important. Greater thicknesses increase the benefits of water use because it increases well yields (extraction rates per pumping hour), thus allowing irrigators to keep up with the peak water demands of water-intensive crops during critical growth stages (O'Brien et al., 2001; Peterson and Ding, 2005).

The cost of pumping is given by $C(w_i, z_i)$, which is assumed to be increasing and weakly convex in both its arguments, with $C_{wz} \geq 0$; increased lifts raise total as well as marginal extraction costs. Substituting (1) and (2) into the benefit and cost functions, the rent earned from cell i can be written

$$\pi_i(w_i, h_i) = F(w_i, h_i + \epsilon_i) - C(w_i, \bar{h} - h_i), \tag{4}$$

with marginal effects of

$$\frac{\partial \pi_i}{\partial w_i} = F_w(w_i, h_i + \epsilon_i) - C_w(w_i, \bar{h} - h_i) \tag{5}$$

$$\frac{\partial \pi_i}{\partial h_i} = F_s(w_i, h_i + \epsilon_i) + C_z(w_i, \bar{h} - h_i). \tag{6}$$

Based on the properties of $B(\cdot)$ and $C(\cdot)$, rents are concave in both w_i and h_i and are globally nondecreasing in h_i ($F_s + C_z \geq 0 \, \forall \, (w_i, h_i)$). The differences in cell depths create heterogeneity in the rent functions and their marginal effects, which can characterized by differentiating (5) and (6) with respect to ϵ_i:

$$\frac{\partial^2 \pi_i}{\partial w_i \partial \epsilon_i} = F_{ws}(w_i, h_i + \epsilon_i) \geq 0 \tag{7}$$

$$\frac{\partial^2 \pi_i}{\partial h_i \partial \epsilon_i} = F_{ss}(w_i, h_i + \epsilon_i) \leq 0. \tag{8}$$

Thus, for two cells $i \neq j$ with i having a greater depth ($\epsilon_i > \epsilon_j$) but with equal water table elevations and pumping rates ($h_i = h_j, w_i = w_j$), we must have

$$\frac{\partial \pi_i}{\partial w_i} \geq \frac{\partial \pi_j}{\partial w_j} \tag{9}$$

$$\frac{\partial \pi_i}{\partial h_i} \leq \frac{\partial \pi_j}{\partial h_j}. \tag{10}$$

The marginal value of extraction is larger in the deeper cell, while the marginal value of water table height is larger in the shallow cell. The first inequality derives from the cross-partial of $F(\cdot)$ (equation (7)). The second inequality arises from the concavity $F(\cdot)$ (equation (8)) and the fact that equal water table elevations implies a smaller saturated thickness in a shallow cell. All else equal, a given increase in the water table height will be of greater value to users in shallow cells who are pumping from thin saturated zones.

Competitive (myopic) pumping

Following a number of studies on common-pool aquifer use (see Koundouri (2004) for a review), we first characterize the competitive equilibrium assuming that all users behave myopically. This assumption is a polar case in which each user believes there is no causal link between current pumping in his cell, $w_i(t)$, and the water table height in future periods, $h_i(\tau)$ ($\tau > t$). Rather, each user believes $h_i(t)$ to be exogenously determined at each t due to the pumping decisions of others. This behavioral assumption leads to the largest possible divergence between the competitive and optimal rents, so that the rent difference is an upper bound on the common-property welfare losses.

Under these assumptions, each user solves a sequence of time-independent optimization problems. At each t, user i solves $\max_{w_i} \pi_i(w_i, h_i)$, so that competitive water use satisfies

$$\frac{\partial \pi_i}{\partial w_i}(w_i, h_i) = 0 \qquad i = 1, \ldots, n. \tag{11}$$

While users do not anticipate changing water table heights in their decisions, the water use in all cells will nevertheless induce changes in h_i through time. A long-run competitive equilibrium can be understood as a steady state of the resource and water-use rates. By definition, a steady state will be achieved when water table heights remain constant in all cells, $\dot{h}_i = 0 \, \forall i$. By equation (3), pumping rates and water table heights rates must then satisfy

$$w_i = r + \sum_{j=1}^{n} \theta_{ji}(h_j - h_i) \qquad i = 1, \ldots, n. \tag{12}$$

We define a *steady-state competitive equilibrium* as a set of pumping rates, $\hat{\mathbf{w}} = (\hat{w}_1, \ldots \hat{w}_n)$, and water table heights, $\hat{\mathbf{h}} = (\hat{h}_1, \ldots, \hat{h}_n)$ that satisfy the $2n$ equations in (11) and (12). We note that the competitive steady state, as well as the optimal steady state we define below, are likely to involve spatially differentiated water table heights and pumping rates. These differences are sustained in what is known as a "flux equilibrium" because it involves continuos lateral resource flows (Sanchirico and Wilen, 2005).

Optimal pumping

Again following previous work (Koundouri, 2004), we now consider the optimal planning problem, in which a planner maximizes the discounted stream of aggregate rents. This problem takes the form

$$\max_{\{w_i,h_i\}} \quad \int_0^\infty \left[\sum_{i=1}^n \pi_i(w_i,h_i)\right] e^{-\delta t} dt$$

subject to (3), where $\delta > 0$ is the planner's discount rate. The current-value Hamiltonian for this problem is

$$\tilde{H} = \sum_{i=1}^n \pi_i(w_i,h_i) + \mu_i \left[r - w_i + \sum_{j=1}^n \theta_{ji}(h_j - h_i)\right],$$

where μ_i is the current-value costate variable pertaining to the water table height in cell i. The Maximum Principle conditions include

$$\frac{\partial \pi_i}{\partial w_i}(w_i,h_i) - \mu_i = 0 \qquad i = 1,\ldots,n \tag{13}$$

$$\dot{\mu}_i - \delta\mu_i = -\frac{\partial \pi_i}{\partial h_i}(w_i,h_i) - \sum_{j=1}^n \theta_{ji}(\mu_j - \mu_i) \qquad i = 1,\ldots,n \tag{14}$$

as well as equation (3).

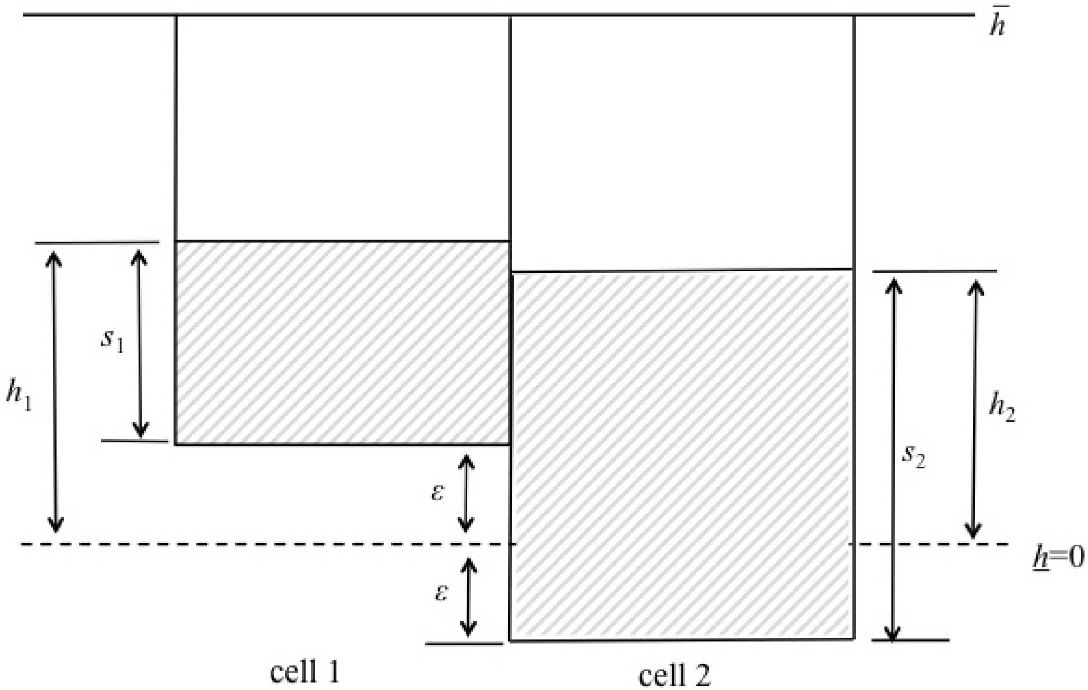
A comparison of (13) to (11) immediately reveals the nature of the common pool externality. The planner sets extraction at each period where marginal rents in cell i are equal the costate variable, μ_i, which is the marginal user cost of the resource or the the implicit value of a unit conserved in cell i for future periods. Under competitive pumping, however, the marginal user cost is ignored and pumping rates are set where marginal rents are equal to zero.

A fuller understanding of the externalities and their spatial structure can be seen in the optimal steady state. An optimal steady state is a a set of costate values, $\mu^* = (\mu_1^*,\ldots,\mu_n^*)$, a set of pumping rates, $\mathbf{w}^* = (w_1^*,\ldots,w_n^*)$ and a set of water table heights, $\mathbf{h}^* = (h_1^*,\ldots,h_n^*)$ satisfying equations (13), (14), and (3) when $\dot{h}_i = \dot{\mu}_i = 0$ for all $i = 1,\ldots,n$. Equation (13) implies that $\mu_i^* = \partial\pi_i/\partial w_i$. Substituting this relationship into (14), imposing $\dot{\mu}_i = 0$, and rearranging yields

$$\frac{\partial \pi_i}{\partial w_i} = \frac{1}{\delta}\left[\frac{\partial \pi_i}{\partial h_i} + \sum_{j=1}^n \theta_{ji}\left(\frac{\partial \pi_j}{\partial w_j} - \frac{\partial \pi_i}{\partial w_i}\right)\right] \qquad i = 1,\ldots,n. \tag{15}$$

Equation (15) is a version of the Fundamental Equation of Renewable Resources, which determines the optimal steady state as a balance between the marginal gains and losses of changing the resource stock in each cell. The left side of the equation represents the monetary asset value of a one-time extraction in cell i that would permanently lower its height by one unit. The right-hand side represents the value of the last unit of height kept in the aquifer. The terms inside brackets are a flow of extra earnings that would accrue each period in the future, so they are multiplied by the capitalization factor, $1/\delta$, to convert them to an asset value. $\partial\pi_i/\partial h_i$ represents the direct benefit of an increase in height in cell i, which comes from both revenue gains and cost savings as shown in equation (6). The sum inside the brackets arises from the spatial connections to other cells. An increased height in cell i changes the gradients toward other cells in a way that that increases net outflow from cell i: the outflow to surrounding cells at lower water table heights quickens while the

Figure 2: Physical characteristics in the 2-cell model.



inflow received from surrounding cells with higher heights slows. In a steady-state, each unit of flow into a cell is pumped. Each term in the sum then measures the social gain from the increased net flow from cell i to j, as θ_{ji} is the quantity transferred and the term in parentheses is the difference in marginal benefits of pumping between the two cells.

The expression on the right side of equation (15) is the optimal steady state value of the user cost , μ_i^*, which varies across cells. By (7) and (8), the shallowest cells in the aquifer will have the smallest values of $\partial\pi_i/\partial w_i$ and the largest values of $\partial\pi_i/\partial h_i$. These facts imply that the shallowest cells will be assigned the largest marginal user costs, μ_i^*, in the planner's solution, and that the gap between the competitive and planning solutions will be greatest in those cells. In this general multi-cell setting, further results about the effects of heterogeneity on the spatial externalities cannot be derived. Sharper results, however, can be obtained from a 2 cell model with particular functional forms, which is presented in the next section.

2-Cell Specification

We now consider a specification with $n = 2$ adjacent cells as depicted in Figure 2, where cell 1 is arbitrarily defined to be the shallower cell. To simplify notation, we define $\epsilon \equiv -\epsilon_1 = \epsilon_2$, so that the saturated thicknesses are $s_1 = h_1 - \epsilon$ and $s_2 = h_2 + \epsilon$ (equation (1)). Similarly, $\theta \equiv \theta_{12} = \theta_{21}$, so that the equations of motion, (3), become

$$\dot{h}_1 = r - w_1 + \theta(h_2 - h_1) \tag{16}$$
$$\dot{h}_2 = r - w_2 + \theta(h_1 - h_2) \tag{17}$$

The benefit function is specified as $F(w,s)=\bar{\pi}+\alpha w+\beta s-\frac{\gamma}{2}s^2$, where $\alpha>0$ is the constant marginal benefits of irrigation, and $\beta>0$, $\gamma\geq 0$ are parameters capturing the effects of saturated thickness. In what follows, γ, which indicates the curvature in benefits with respect to saturated thickness, will play an important role. To avoid unbounded solutions with constant marginal benefits, we impose the feasibility constraint that $w\in[0,\bar{w}]$ in both cells, where $\bar{w}>r$ can be interpreted either as the irrigation requirement for the crops grown or the maximum authorized use on water rights. The cost function is specified from the standard engineering equation $C(w,z)=\phi zw$, where $\phi>0$ is the energy cost of lifting one unit of water one unit of distance. The definitions above imply rent functions of

$$\pi_1(w_1,h_1) = \bar{\pi}+\alpha w_1+\beta(h_1-\epsilon)-\tfrac{\gamma}{2}(h_1-\epsilon)^2-\phi(\bar{h}-h_1)w_1 \tag{18}$$
$$\pi_2(w_2,h_2) = \bar{\pi}+\alpha w_2+\beta(h_2+\epsilon)-\tfrac{\gamma}{2}(h_2+\epsilon)^2-\phi(\bar{h}-h_2)w_2 \tag{19}$$

We first consider the competitive equilibrium. Each farmer solves $\max_{w_i}\{\pi_i(w_i,h_i):w_i\in[0,\bar{w}]\}$. At an interior solution, the optimal value of w_i satisfies the first order condition

$$\alpha-\phi(\bar{h}-h_i)=0, \quad i=1,2$$

which in turn implies that

$$h_1=h_2=\bar{h}-\frac{\alpha}{\phi}. \tag{20}$$

In the competitive equilibrium steady state, pumping rates in the two cells must be such that $\dot{h}_1=\dot{h}_2=0$. By substituting this condition along with equation (20) into (16) and (17), we obtain the result that steady-state competitive pumping rates are

$$w_1=w_2=r. \tag{21}$$

Thus, even though the two cells are asymmetric with respect to their depths, the competitive equilibrium steady state is symmetric with respect to both water table heights and pumping rates.

The planner's problem is

$$\max_{w_1,w_2 h_1,h_2} \quad \textstyle\int_0^\infty[\pi(w_1,h_1)+\pi_2(w_2,h_2)]e^{-\delta t}dt,$$

subject to (16), (17), and $w_i\in[0,\bar{w}]$ $(i=1,2)$. The Maximum Principle conditions with respect to w_i and h_i are a special case of those in equations (13) and (14), which lead to an optimal steady-state condition corresponding to equation (15):

$$\alpha-\phi z_i=\frac{1}{\delta}\left[\beta-\gamma s_i+\phi w_i-\theta(\phi z_j-\phi z_i)\right],\ i=1,2. \tag{22}$$

In steady state, $\dot{h}_1=\dot{h}_2=0$ and equations (16) - (17) imply that pumping rates must satisfy

$$w_i=r+\theta(h_j-h_i) \quad i=1,2. \tag{23}$$

Substituting (23) into (22) and simplifying yields an expression for the difference in the optimal steady-state cell heights:

$$h_1^*-h_2^*=\frac{2\gamma\epsilon}{\gamma+4\phi\theta+\phi\delta}\geq 0, \tag{24}$$

where the inequality follows from the assumptions that all parameters are all non-negative. Thus, in the optimal steady state, a difference in cell heights is maintained in a flux equilibrium. In the special case of $\epsilon = 0$ where the cells are symmetric, (24) implies that $h_1^* = h_2^*$. Equal cell heights also results from the case of $\gamma = 0$, where there is no curvature in benefits with respect to saturated thickness.

Numerical application

In this section we apply the 2-cell specification to numerically analyze the effects of differing cell depths in the northwest Kansas portion of the Ogallala aquifer. Our parameter values are broadly representative of the region but are chosen to most closely represent Sheridan County, which contains a relatively deep and productive patch of the aquifer with highly intensive groundwater use and rapid water level decline rates (Steward et al., 2009). We abstract from particular sites for our two cells, rather interpreting them as relatively shallow and deep parts of an intensively irrigated area in the county. We assume that water rights are fully appropriated in both cells, so that the full area overlying the two cells is irrigated. We also assume that there are a large number of users in each cell and that their competitive behavior is approximated by myopic pumping.

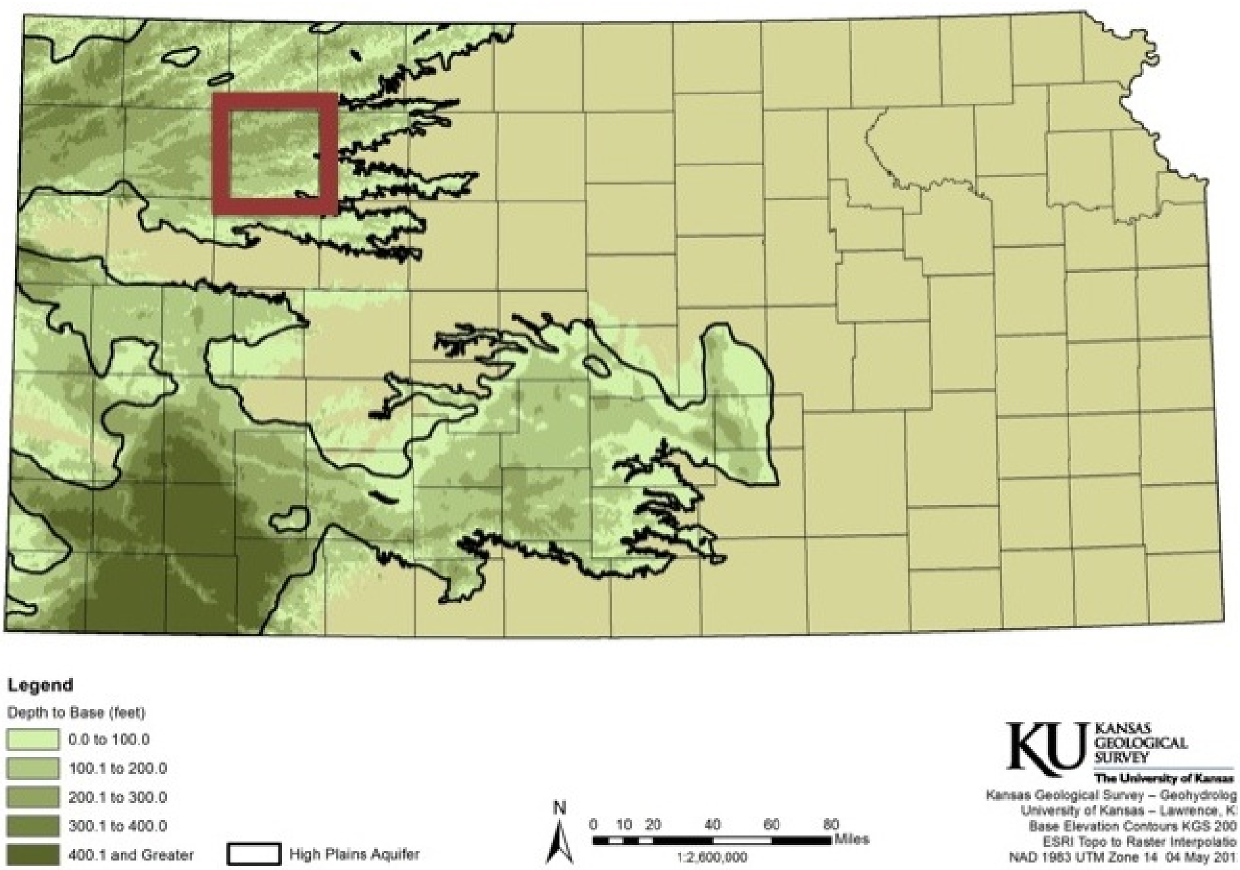
Table 1 presents the model parameter values. Specific yield varies spatially, but in northwest Kansas the most typical values are between 0.05 and 0.1 (Hecox et al., 2002). Here, we set specific yield near the middle of this range, $0.083 \approx \frac{1}{12}$, as a convenient choice to convert water stored in the aquifer to volumes of water pumped and applied to crops. Given the assumption that the cell surfaces are fully irrigated, the water stored in a 1-foot thick slice of the aquifer becomes one inch of irrigation water uniformly applied to the surface. In what follows, we make this transformation implicit by measuring irrigation in inches and water table heights in the aquifer in feet. We consider the two extreme cases for the speed of lateral groundwater flow, with $\theta = 0$ representing hydraulically independent cells and $\theta = 0.5$ representing the case where the height difference between cells is fully dissipated in one period given symmetric pumping rates (Saak and Peterson, 2007). The depth to the aquifer base varies across Sheridan County from under 100 to over 200 feet (Figure 3). As our cells are assumed to be situated in the deepest and most productive areas of the county, we set the mean depth to $\bar{h} = 220$ feet and allow each cell to vary from this value by increments ranging from $\epsilon = 0$ to $\epsilon = 20$. Recharge is set at 0.75 inches per year based on (Sophocleous and Schloss, 2000).

The economic parameters include the marginal cost of pumping per foot of lift, ϕ, and the coefficients of the benefits function, $(\bar{\pi}, \alpha, \beta, \gamma)$. The pumping cost parameter was computed from the tables and formulas in Martin et al. (2011), assuming a system operating pressure of 20 psi and a natural gas price of \$15/mcf. In this specification, α becomes a somewhat arbitrary parameter because, with known values of ϕ and $\bar{h}$, it is linearly related to the steady state competitive cell height, $\hat{h}_1 = \hat{h}_2 = \bar{h} - \alpha/\phi$ (equation (20)). The role it plays in the model is one of inducing an equilibrium pumping cost. For water table heights such that marginal pumping cost, $\phi(\bar{h} - h_i)$, is less than α, farmers will optimally set pumping at the maximum rate, $w_i = \bar{w}$; if marginal pumping costs are above α, pumping would be set to zero; and only when marginal pumping cost equals α can we obtain an interior equilibrium or the "singular value" determining the steady state. Here, we set $\alpha = 6.8$, which implies a competitive steady state cell height of $\hat{h}_1 = \hat{h}_2 = 50$ feet. For the largest degree of asymmetry ($\epsilon = 20$) this implies a steady state saturated thickness of 30 feet in the shallow cell, a value that farmers in the region commonly assume to be a minimum viable threshold to support irrigation.

Table 1: Parameter values

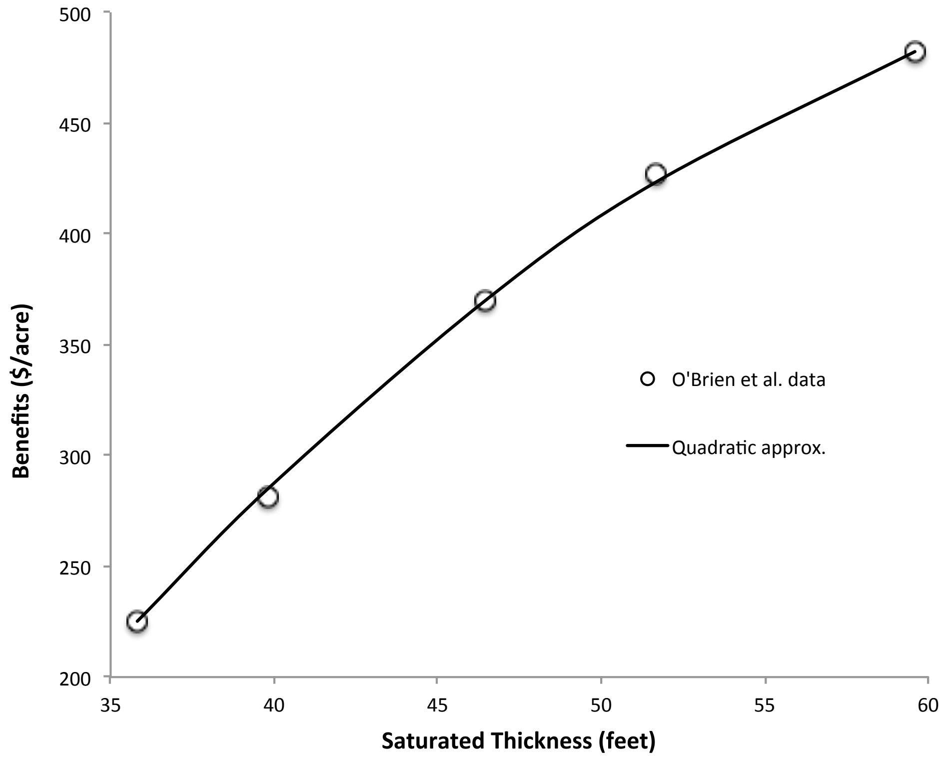
Parameter	Units	Value
Aquifer specific yield		0.083
θ		0,0.5
$\bar{h}$	feet	220
ϵ	feet	$0, 1, \ldots, 20$
r	inches	0.75
ϕ	\$/acre-inch/foot	0.04
α	\$/acre-inch	6.80
β	\$/foot	31.213
γ	\$/foot^2	0.4274
$\bar{\pi}$	\$/acre	-618.94
δ		0.04

Figure 3: Variation in depth to aquifer base



http://www.kgs.ku.edu/HighPlains/HPA_Atlas/Aquifer%20Basics/index.html#Depth%2520to%2520Base%2520of%2520HPA.jpg

Figure 4: Calibrated benefits versus saturated thickness



The remaining parameters require knowledge of the relationship between aquifer saturated thickness and irrigation returns. O'Brien et al. (2001) is among the few studies that have quantified this relationship. Using weather data from Sheridan County, a crop model, and a linked irrigation scheduling model, irrigated corn yields were simulated under five different well pumping capacities ranging from 15 L/s (237 gal/min) to 37 L/s (586 gal/min). The results revealed an increasing, concave relationship between yield and well capacity. For our purposes, the well capacities must be translated back to the saturated thickness of the aquifer, which was accomplished by interpolating from the results in Hecox et al. (2002).[1] Figure 4 displays the resulting data points plotted as irrigation benefits versus saturated thickness, where benefits are calculated as O'Brien et al.'s predicted yields multiplied by an assumed corn price of \$6/bu less non-water variable production costs of \$576/acre (Dumler et al., 2010). We then calibrated the parameters β and γ to approximate the shape of the data, using the conditions that the fitted line pass through the endpoints of the data and with enough curvature to pass through the middle observation. The reported value of $\bar{\pi}$ is the intercept of the benefits function implied by the shape parameters, which plays only a passive role in the analysis. Finally, we set the discount rate to $\delta = 0.04$.

Competitive and optimal steady states were simulated for each combination of ϵ and θ in Table 1. The competitive solution was calculated directly from equations (20) and (21), while the opti-

[1]Hecox et al. calculated and reported the minimum saturated thickness required to support various pumping capacities, under different assumptions about aquifer properties, pumping duration, and well spacing. We interpolated from their results based on a hydraulic conductivity of 100 feet per day, 90 days of pumping, and wells spaced on quarter-section centers. These conditions are representative of groundwater use in Sheridan County.

Table 2: Simulated water table height and saturated thickness, competitive and optimal solutions

	Competitive				Optimal ($\theta = 0$)				Optimal ($\theta = 0.5$)			
	Height (ft.)		Sat. Thickness (ft.)		Height (ft.)		Sat. Thickness (ft.)		Height (ft.)		Sat. Thickness (ft.)	
ε	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2
0	50	50	50	50	73.0	73.0	73.0	73.0	73.0	73.0	73.0	73.0
1	50	50	49	51	74.0	72.0	73.0	73.0	73.9	72.2	72.9	73.2
2	50	50	48	52	75.0	71.0	73.0	73.0	74.7	71.3	72.7	73.3
3	50	50	47	53	76.0	70.0	73.0	73.0	75.5	70.5	72.5	73.5
4	50	50	46	54	77.0	69.0	73.0	73.0	76.4	69.7	72.4	73.7
5	50	50	45	55	78.0	68.0	73.0	73.0	77.2	68.8	72.2	73.8
6	50	50	44	56	79.0	67.0	73.0	73.0	78.1	68.0	72.1	74.0
7	50	50	43	57	80.0	66.0	73.0	73.0	78.9	67.1	71.9	74.1
8	50	50	42	58	81.0	65.0	73.0	73.0	79.7	66.3	71.7	74.3
9	50	50	41	59	82.0	64.0	73.0	73.0	80.6	65.5	71.6	74.5
10	50	50	40	60	83.0	63.1	73.0	73.1	81.4	64.6	71.4	74.6
11	50	50	39	61	84.0	62.1	73.0	73.1	82.3	63.8	71.3	74.8
12	50	50	38	62	85.0	61.1	73.0	73.1	83.1	62.9	71.1	74.9
13	50	50	37	63	86.0	60.1	73.0	73.1	83.9	62.1	70.9	75.1
14	50	50	36	64	87.0	59.1	73.0	73.1	84.8	61.3	70.8	75.3
15	50	50	35	65	88.0	58.1	73.0	73.1	85.6	60.4	70.6	75.4
16	50	50	34	66	89.0	57.1	73.0	73.1	86.4	59.6	70.4	75.6
17	50	50	33	67	90.0	56.1	73.0	73.1	87.3	58.7	70.3	75.7
18	50	50	32	68	90.9	55.1	72.9	73.1	88.1	57.9	70.1	75.9
19	50	50	31	69	91.9	54.1	72.9	73.1	89.0	57.1	70.0	76.1
20	50	50	30	70	92.9	53.1	72.9	73.1	89.8	56.2	69.8	76.2

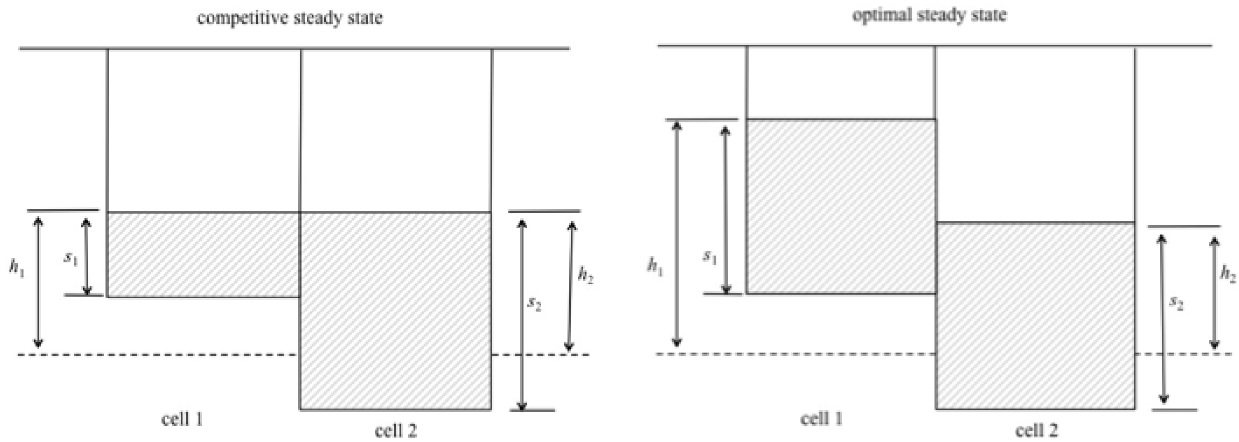
mal steady state was obtained from numerical solution of the system of equations (22) and (23). Each solution consists of pumping rates and water table heights in each cell, from which saturated thickness levels and economic rents were also computed.

Table 2 reports the simulated water table height and saturated thickness for the full set of combinations of θ and ϵ. The competitive solution is independent of θ, and always yields a steady-state water table height of 50 feet in both cells. Identical heights, however, implies different saturated thickness levels, with the gap depending on the assumed level of ϵ. At the maximum asymmetry of $\epsilon = 20$, the gap in saturated thickness, by definition, is 40 feet.

While the competitive solution has equal water table heights with unequal thickness, the optimal solution has the opposite pattern. As illustrated in Figure 5, the optimal planning solution is to maintain similar saturated thicknesses in the two cells, which implies an optimal gap in water table heights. In the case of $\theta = 0$, this gap is maintained simply because there is assumed to be no hydraulic connection between the cells. At each level of ϵ, the optimal saturated thickness in the two cells are virtually identical to each other (Table 2); a slight difference between cells emerges with extreme asymmetry, which reflects the large difference in pumping lifts affecting pumping rates. In the case of $\theta = 0.5$, the optimal difference in cell heights must be maintained in a flux equilibrium, and the difference in heights is not as large.

The economic rents and implied welfare losses from common pool externalities are in Table 3. As expected, rents in each cell are larger under optimal management than under competitive pumping, with the difference reflecting the common-pool welfare losses. As also expected, rents are always higher in cell 2 than in cell 1 because of the resource endowment advantage. Rents in the two cells are very similar, however, in the case of optimal management when $\theta = 0$. In this case the profits

Figure 5: Competitive versus optimal solutions



in the two cells can be managed independently, yielding similar levels of saturated thickness (Table 2) and hence similar rents. The inequality in rents is largest under competitive pumping, implying that optimal management will not only improve total gains but would reduce inequality as well.

Welfare losses are computed as the difference in rents between the optimal solutions and can be equivalently termed the gains from optimal management. Combined welfare losses are larger for the case of hydraulically connected cells ($\theta = 0.5$) than for independent cells ($\theta = 0$). This difference reflects the spatial portion of the externality due to the connection between cells. In the case of $\theta = 0$ there are still common pool externalities within each cell, with associated (combined) welfare losses ranging from \$215/acre to \$386/acre. When cells are hydraulically connected, the common pool externalities are exacerbated, resulting in joint welfare losses from \$215/acre to \$404/acre. In essentially all cases, farmers in both cells would gain from optimal management policies, although the larger gain would be felt by those in the shallower cell.

Conclusions

This paper has studied the divergence in the planning and equilibrium solutions for a multicell aquifer with heterogeneity in cell depths. Using this framework we characterized the effects of the standard common pool resource externalities and the separate effects brought about by resource asymmetry. In particular, we determined the degree of over-pumping (or under-pumping) in each cell as a function of the asymmetry, as well as the effects of asymmetry on joint welfare losses and the distribution of these losses across the two users.

For policy purposes, this framework provides insight on how policies to control groundwater use should be targeted. Our numerical application to northwest Kansas reveals that even for modest levels of asymmetry across locations, policies will have different welfare impacts on users. Further, in this case, efficiency-driven policies to improve total welfare will also reduce the gap in earnings between users with different resource endowments.

Table 3: Economic rent and welfare losses.

	Economic Rent ($/a)						Welfare Losses ($/a)					
	Competitive		Optimal ($\theta = 0$)		Optimal ($\theta = 0.5$)		$\theta = 0$			$\theta = 0.5$		
ε	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2	Cell 1	Cell 2	Combined	Cell 1	Cell 2	Combined
0	996	996	1103	1103	1103	1103	107	107	215	107	107	215
1	986	1005	1103	1103	1100	1106	117	98	215	114	101	215
2	975	1014	1103	1103	1097	1110	128	89	217	122	95	217
3	964	1023	1103	1103	1093	1113	139	80	219	129	90	219
4	953	1032	1103	1103	1090	1117	150	72	222	137	85	222
5	941	1039	1103	1103	1087	1120	162	64	226	146	81	227
6	929	1047	1103	1103	1084	1124	174	56	230	155	77	232
7	916	1054	1103	1103	1081	1127	187	49	236	165	73	238
8	903	1061	1103	1103	1078	1131	200	43	242	175	70	245
9	890	1067	1103	1103	1075	1134	213	36	250	186	68	253
10	876	1073	1103	1103	1072	1138	227	31	258	197	66	262
11	861	1078	1103	1103	1070	1142	241	25	267	208	64	272
12	847	1083	1103	1103	1067	1146	256	20	276	220	63	283
13	832	1087	1103	1103	1064	1150	271	16	287	233	62	295
14	816	1092	1103	1103	1062	1153	287	12	299	246	62	308
15	800	1095	1103	1103	1059	1157	303	8	311	259	62	321
16	783	1098	1103	1104	1056	1161	319	5	324	273	63	336
17	766	1101	1103	1104	1054	1165	336	2	338	287	64	352
18	749	1104	1103	1104	1051	1170	353	0	353	302	66	368
19	731	1105	1102	1104	1049	1174	371	-2	369	317	68	386
20	713	1107	1102	1104	1046	1178	389	-3	386	333	71	404

References

Athanassoglou, S., G. Sheriff, T. Siegfried, and W. T. Huh (2012). Optimal mechanisms for heterogeneous multi-cell aquifers. *Environmental and Resource Economics 52*(2), 265–291.

Brozović, N., D. L. Sunding, and D. Zilberman (2010). On the spatial nature of the groundwater pumping externality. *Resource and Energy Economics 32*(2), 154–164.

Dumler, T., D. O'Brien, B. Olson, and K. Martin (2010). Center-pivot-irrigated corn cost–return budget in western kansas. kansas state university agricultural experiment station and cooperative extension service. *Farm Management Guide, MF-585*.

Gisser, M. (1983). Groundwater: Focusing on the real issue. *The Journal of Political Economy*, 1001–1027.

Gisser, M. and D. A. Sanchez (1980). Competition versus optimal control in groundwater pumping. *Water Resources Research 16*(4), 638–642.

Hecox, G., P. Macfarlane, B. Wilson, and O. A. Assessment (2002). Calculation of yield for high plains wells relationship between saturated thickness and well yield. *Kansas Geological Survey Open File Report*, 24.

Koundouri, P. (2004). Potential for groundwater management: Gisser-sanchez effect reconsidered. *Water Resources Research 40*(6).

Martin, D. L., T. W. Dorn, S. R. Melvin, A. J. Corr, and W. L. Kranz (2011, February). Evaluating energy use for pumping irrigation water. *Proceedings of the 23rd Annual Central Great Plains Irrigation Conference*.

O'Brien, D., F. Lamm, L. Stone, and D. Rogers (2001). Corn yields and profitability for low-capacity irrigation systems. *Applied Engineering in Agriculture 17*(3), 315–321.

Peterson, J. M. and Y. Ding (2005). Economic adjustments to groundwater depletion in the high plains: Do water-saving irrigation systems save water? *American Journal of Agricultural Economics 87*(1), 147–159.

Pfeiffer, L. and C.-Y. C. Lin (2012). Groundwater pumping and spatial externalities in agriculture. *Journal of Environmental Economics and Management*.

Saak, A. E. and J. M. Peterson (2007). Groundwater use under incomplete information. *Journal of Environmental Economics and Management 54*(2), 214–228.

Saak, A. E. and J. M. Peterson (2012). Groundwater pumping by heterogeneous users. *Hydrogeology Journal 20*(5), 835–849.

Sanchirico, J. N. and J. E. Wilen (2005). Optimal spatial management of renewable resources: matching policy scope to ecosystem scale. *Journal of Environmental Economics and Management 50*(1), 23–46.

Sophocleous, M. and J. Schloss (2000). *An Atlas of the Kansas High Plains Aquifer*, Chapter Estimated annual groundwater recharge. Kansas Geological Survey.

Steward, D. R., J. M. Peterson, X. Yang, T. Bulatewicz, M. Herrera-Rodriguez, D. Mao, and N. Hendricks (2009). Groundwater economics: An object-oriented foundation for integrated studies of irrigated agricultural systems. *Water Resources Research 45*(5).

Wang, C. and E. Segarra (2011). The economics of commonly owned groundwater when user demand is perfectly inelastic. *Journal of Agricultural and Resource Economics 36*(1), 95.