Farm Level Tradeoffs in the Regulation of Greenhouse Gas Emissions

Authors:
Amber Remble (Purdue University)
Wolfgang Britz (University of Bonn; Bonn, Germany)
Roman Keeney (Purdue University)

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The US budget situation has come to dominate legislative action in an unprecedented manner. Massive federal spending aimed at curbing the economic impacts of recession brought on by the financial crisis has left staggering debt and costs of debt service and entrenched the budget hawks within the legislature. This has been especially evident in the case of US farm policy, where drastic spending reform is being thrust upon a sector attaining spectacular economic performance.

With decreased spending in the farm sector and little political will to distribute money into a farm sector evidencing record income performance, the likelihood of meeting environmental objectives through direct regulation increases sharply. Thus, we face a situation where US farmers who have become accustomed to being paid for providing environmental service through land set asides, buffers, or carbon sequestration may be asked to bear the cost of externality outputs associated with their operations.

The European experience with supporting farm incomes is quite similar to that of the US, however in terms of environmental and greenhouse gas policy farm operations in the EU are subjected to something more akin to industrial pollution policy. With this alternative approach comes a host of secondary (to the operator lost income) impacts as consequence of the regulation policy. The objective of this paper is to examine the direct and indirect impacts of regulation at the farm level. We adapt a highly detailed single farm model calibrated to conditions in northwest Germany for examining the tradeoffs in increased regulation of GHG emissions stemming from the farm. The model contains a number of GHG indicators that are tied to farm level practices. The original model was developed for estimating GHG reduction potentials of German dairy farms and has been altered here to deal with arable crop farming. We focus specifically on the tradeoff between farm income and GHG mitigation as the primary tradeoff and then extend the analysis to look at various forms of nitrogen based pollution as well as foregone output (in terms of caloric content).

Subsequent sections describe the model structure and results. We first review studies of nitrogen response of arable crops (cereal and oilseed) for the region in an effort to support model results with best agronomic evidence. These are finally summarized into a nitrogen response function that underlies arable crop technology in the model. From there we move to the set of model equations necessary for understanding farm decision making and outcomes. We close the paper with a set of preliminary conclusions and thoughts about the prospects for farm level regulation aimed at reducing greenhouse gas emissions.

Nitrogen Response

Accounting for greenhouse gas emissions from arable crop farming requires detailed information on production technique. All information for costs of production and GHG emission are derived from KTBL (2010) and IPCC (2006) respectively. Table 1 lists the input costs (seed, fertilizer, herbicides, fungicides, insecticides, growth control, water, and hail insurance) associated with each cropping activity, tillage method, and intensity level. The “normal” intensity level is associated with the economically optimal level of fertilizer (calculated below) and 80% of this level. The “low” intensity level refers to 60% and 40% of the economically optimum fertilizer and “very low” intensity is associated with 20% of the economically optimal fertilizer level.

Critical to the analysis of emissions reduction and tradeoffs at the farm level is an underlying nitrogen response technology that represents the reduction in output (and thus revenue) that the farmer faces when lowering input to meet a regulatory target. In one sense, a regulation that causes a farmer to reduce nitrogen away from the optimal application level raises the effective price of nitrogen so that the value of marginal product will be equal to the marginal input cost of nitrogen fertilizer plus an implicit tax imposed per unit. The sum of these implicit taxes is the abatement cost of reduction. In practice, the farm operator will avail herself of a host of choice variables for achieving a given GHG reduction such that having best information on yield response to nitrogen is a necessary but insufficient condition for analysis of mitigation costs.

Several studies estimating cereal yield as a function of available nitrogen were consulted and are listed in table 2. The three agronomic articles that executed field studies estimated a quadratic functional form to their data while the simulated experiment conducted by Godard et. al. fitted an exponential function. The equation parameters of each study are listed in table 3. Gandorfer and Rajsic estimated two equations associated with the two locations of their field studies. Meyers-Andres et. al. completed 15 experiments of nitrogen fertilizer levels at two four different sites and with six different cultivars. Table 3 reports the estimated parameters of yield response of the six wheat cultivars to nitrogen fertilizer at 'Site A'.

Predicted wheat yields were calculated using the reported yield response equations to various nitrogen fertilizer inputs ranging from 0 to 360 kg and are illustrated in figure 1. The parameters of the two equations from the Gandorfer et. al. results were averaged to graph one response curve for that study, while the six equation parameters from the Meyer-Aurich et. al. were averaged as well. Subsequently, an average curve was calculated and graphed for all four studies, also demonstrated in figure 1.

Functional Form Considerations: Assumption of an intercept in nitrogen response

The predicted yields calculated from the range of nitrogen fertilizer input levels using the average parameters of the four studies were used to estimate a new quadratic yield response function. Because the yields were predicted from average response function, naturally the fitted equation has large R^2 and p-values.

Parameter	**Estimate**
Intercept	5.168173262
Nitrogen	0.029816778
Nitrogen Squared	-0.0000631

$$Yield = 5.16817 + 0.02981N - 0.00006N^2$$

To implement and exercise appropriate nitrogen fertilizer input levels into the mathematical programming cropping model, the economically optimal nitrogen input level was derived using the nitrogen to wheat (for feed) price ratio reported in Meyer-Aurich et. al (0.00766), which is similar to that price ratio used in Gandorfer and Rajsic (0.00771).

$$\frac{\partial Y}{\partial N} = 0.02981 - 0.00012N = 0.00766$$

$$\Rightarrow N^* = 180$$

At the optimal nitrogen level, winter wheat yield is predicted as 8.49 tons per hectare, as shown by the point of tangency between the average yield response curve and price ratio line illustrated in figure 2.Based on this analysis, we consider in our programming model a set of five production techniques beginning with an available nitrogen amount of 200 kg/ha and 40 kg/ha increments down to a level of 40 kg/ha. This allows us to trace out the true nonlinear response function using linear combinations of the five modeled techniques.

Table 4 shows the changes in wheat yield as we incrementally reduce the fertilizer rate by 10% from the economically optimum level of 200 kg/ha. Reducing the fertilizer rate by 10% has minimal downward impacts on the yield change (2-5%). The low productivity loss due to relatively large input reductions will most likely result in large changes in producer behavior when facing greenhouse gas abatement laws. The small changes in yield resulting from 10% decreases in fertilizer level are similar when a bound is placed on the minimum output using the intercept (last two columns of table 4). This proportional change in yield response is illustrated in figure 3. This graph allows us to see the proportionally small losses in terms of productivity for sizeable input reductions.

Estimation With No Intercept

To comply with linear programming spirit of essential inputs, the quadratic model was also estimated in such a way that forced the intercept to be equal to zero. Thus a zero nitrogen fertilizer input level results in zero wheat yield. The parameters of the estimated model changed slightly and are reported below. The new estimates also result in a new economically optimal nitrogen input level (200 kg per hectare), which is a slightly higher level than the alternative method (with intercept) and presents another option for the 'normal' input level.

Parameter	**Estimate**
Intercept	0.000000
Nitrogen	0.086437
Nitrogen Squared	-0.000190

$$Yield = 0.086437N - 0.000190N^2$$

$$\frac{\partial Y}{\partial N} = 0.086437 - 0.00038N = 0.00766$$

$$\Rightarrow N^* = 204.7917 \approx 200$$

A 'low' nitrogen input level is calculated again as 40% of the economic optimum, or 80 kg per hectare, while a 'very low' input level at 20% of the economic optimum would be 40 kg per hectare. Table 5 illustrates the marginal returns from an incremental decrease from the optimal nitrogen level. The 'low' nitrogen level results in about 60% of the yield with 40% of the input. Additionally, the 'very low' nitrogen level results in 33% of the optimal yield from 80% decrease in the input.

While we have calculated our original results based on the model that has an intercept, we will conduct sensitivity using this approach to yields which has the more drastic yield penalties for nitrogen reductions.

Farm Optimization Model

The model used for the experiments on marginal abatement costs if a single-farm, dynamic, mixed integer programming model based on the dairy optimization model created by Lengers and Britz (2012). The current model is being modified to include detailed specifications for alternative cropping activities in Germany. The modified version currently excludes the original dairy activities and will be used to assess marginal abatement costs to German producers when facing greenhouse gas mitigation policy tools. The results of the preliminary trial of this experiment are yet to come and further development in model details and experiment specifications are expected to made. This paper solely outlines the important building blocks used to build a sufficient model to capture the characteristics and changes in producer behavior.

We assume that the objective of a risk-neutral, profit-maximizing farmer is to maximize the net present value of farm profits minus household expenditures. The firm is assumed to be liquidated at the end of the planning horizon, such that machinery and land are sold and credits are paid back. Any remaining equity is discounted to net present value using the assumed discount rate α:

$$A*[1-(\alpha/100)^{\sum y}]/\sum y$$

A special characteristic, due to the model construction and choice of fully dynamic mixed integer optimization, ist hat the decision maker is assumed to be fully informed of future states of nature. Hence, the model results always show the optimal farm plan over the chosen planning horizon with given assumed future states of nature (best-practice simulations).

Crop production

The cropping activities in the model include production options for winter cereals, summer cereals, and winter rapeseed. The farmer chooses how many hectares of each crop to produce at the beginning of the year. Output Q of each crop c in year y and state of nature s (discussed in a later section) is therefore determined by the number of hectares H planted and a yield parameter θ:

$$Q^{y,s} = \sum_{c,i,t} H_{i,t}^{c,y,s} * \theta_{i,t}^{c,y,s}$$

In addition to choosing the crop type and quantity to plant, the producer also has the option to grow each crop at various levels of intensity i (normal, low, and very low). The producer must also choose among pre-planting tilling options t for the soil. These tilling options include plough, minimum tillage, no tillage, and an ecological method, all of which will have impacts on the crop yield. The number of hectares planted for each crop is restricted by a maximum rotation constraint:

$$\sum_{c} H_{i,t}^{c,y,s} \leq L_{l}^{y,s} * \psi^{c}$$

Where L is the cropped land of land type l in each year y and state of nature s, and ψ is the maximum rotation share of each crop c. The variable costs of the cropping activities in the model are composed of costs for a) crop production z, b) synthetic fertilizer r and application f, c) diesel p_{diesel}, and d) machinery v. Fertilizer application is summed over synthetic fertilizer type j and months m in the year. The total cost of diesel is a product the tractor hours, the amount of diesel spent per tractor hour *m*, and the price of diesel. Income from annual single farm payments per hectare σ, which vary by land type *l*, are deducted from these variable costs.

$$\begin{aligned} VC^{y,s} &= \sum_{c,i,t} H_{i,t}^{c,y,s} * z_{i,t}^{c,y,s} \\ &+ \sum_{c,i,t,j,m} F_{i,t,j,m}^{c,y} * r_j^{y,s} + f_j \\ &+ M_{tractor}^{y,s} * m_{tractor,hour} * p_{diesel}^{y,s} \\ &+ \sum_{\mathrm{k}} M_{\mathrm{k}}^{y,s} * v_k \\ &- \sum_{\mathrm{l}} L_l^{y,s} * \sigma_l^y \end{aligned}$$

Nitrogen balance

Synthetic fertilizers are implemented in the model to supply the nitrogen demand of the various crops. There are two types of synthetic fertilizers available: AHL (ammonium urea solution) and ASS (ammonium sulfate fertilizer). Each synthetic fertilizer has a specific price per kg of nitrogen as well as application requirements of labor, tractors, and sprayers.

As an agronomic restriction, a nitrogen balance equation is imposed for each crop to prevent an excess of applied nitrogen per ha above the nitrogen demand needed for fructification, straw, and root growth. Furthermore, the nitrogen balance equations ensure that the nutrient demand is equivalent to the supply for a specific yield level of each crop. The nitrogen demand for each crop *c* in year *y* is represented on the left hand side of the equality of the following equation:

$$\eta_i^{c,y} H_i^{c,y} + N_i^{c,y} = \sum_{j,m} F_{j,m} * \left[1 - \gamma_j\right]$$

This per crop demand is determined by the number of hectares *H* planted and the per hectare nitrogen need for that crop η, both of which vary by intensity level *i* at which that crop is planted and grown. Additionally, a nitrogen surplus variable *N* is included to account for potential deviations in annual plant availability and soil mobilization that might alter the nitrogen demand from year to year. This surplus also varies by intensity level and is calculated as a predetermined percentage of the per crop nitrogen demand. The level of this surplus however is capped at a maximum level expressed in the following equation:

$$N_i^{c,y} \le H_i^{c,y} * \min\left[100, \eta_i^{c,y} * 0.2\right]$$

The supply of fertilizer is represented on the right hand side of the balance equation and is a function of the amount of fertilizer applied to the field *F* summed over the months *m* in each period and the fertilizer type *j*. An application loss rate of nitrogen γ for each synthetic fertilizer type is included in the supply equation to account for application incidence.

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Land allocation

The farmer is allowed to adjust his use of available land from year to year. There are two types of land in the model, arable and permanent grass land. Total land *TL* of type *l* consists of the initial land endowment *IL* plus land purchased *BL*:

$$TL_l^y = IL_l + \sum_y BL_l^y$$

Total land can either be used for crop production or can be rented out. However, permanent grass land cannot be used for cropping land:

$$L_{arable}^{y,s} = \sum_{c,i,t} H_{i,t}^{c,y,s}$$

Land used for cropping *L* and renting out *RL* must not exceed total land available in each year *y*:

$$L_l^{y,s} + RL_l^y \leq TL_l^y$$

As mentioned earlier, the amount of land used per crop is restricted by the cropping rotation share. More model specifications are located in Appendix I.

Initial Experiment

The initial experiment in its most basic form will test the changes in cropping activities, as well as intensities and tillage methods, synthetic nitrogen fertilizer application rates, and cash flows (profits) when the producer is forced to reduce greenhouse gas emissions by 30 percent incrementally over a 15 year time frame.

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Table 1. Model Input Costs for Cropping Activities

CROP	TILLAGE	INTENSITY	SEED	KAS	PK_18_10	LIME	HERB	INSECT	GROWTHCONTR	WATER	HAILINS
Winter Cereal	Plough	Normal	180	640	400	1	44	13	2	1.5	0.87
		Low	170	640	360	1	30	13	2	1.5	0.79
		Very Low	160	640	320	1	15	13	2	1.5	0.62
	Minimum Tillage	Normal	180	640	400	1	44	13	2	1.5	0.87
		Low	170	640	360	1	30	13	2	1.5	0.79
		Very Low	160	640	320	1	15	13	2	1.5	0.62
	No Till	Normal	180	640	400	1	55	13	2	1.5	0.87
		Low	170	640	360	1	37	13	2	1.5	0.79
		Very Low	160	640	320	1	18	13	2	1.5	0.62
Summer Cereal	Plough	Normal	140	310	320	1	39	*	*	0.6	0.61
		Low	130	310	290	1	26	*	*	0.6	0.55
		Very Low	120	310	250	1	13	*	*	0.6	0.49
	Minimum Tillage	Normal	140	310	320	1	39	*	*	0.6	0.61
		Low	130	310	290	1	26	*	*	0.6	0.55
		Very Low	120	310	250	1	13	*	*	0.6	0.49
	No Till	Normal	140	310	320	1	48	*	*	0.6	0.61
		Low	130	310	290	1	30	*	*	0.6	0.55
		Very Low	120	310	250	1	12	*	*	0.6	0.49
Winter Rapeseed	Plough	Normal	200	440	360	1	64	16	*	0.9	2.40
		Low	190	440	320	1	40	16	*	0.9	2.16
		Very Low	180	440	280	1	20	16	*	0.9	1.94
	Minimum Tillage	Normal	200	440	360	1	64	16	*	0.9	2.40
		Low	190	440	320	1	40	16	*	0.9	2.16
		Very Low	180	440	280	1	20	16	*	0.9	1.94
	No Till	Normal	200	440	360	1	77	16	*	1.2	2.40
		Low	190	440	320	1	40	16	*	1.2	2.16
		Very Low	180	440	280	1	20	16	*	1.2	1.94

Table 2. Nitrogen Response Curve Studies

Authors	Location	Experiment	Experiment Years	Functional Form
Gandorfer & Rajsic	South Germany (Betzendorf & Wolfsdorf)	Field Trials	2000-2002	Quadratic
Godard et. al.	France (Picardy & Midi-Pyrenees)	Simulation: STICS crop model and AROPAj supply model		Exponential
Meyer-Andres et. al.	Northern Germany	Field Trials; 5 cultivars	1999-2001	Quadratic
Sieling et. al.	Northwest Germany (Kiel)	Field Trials	1993-1999	Quadratic

Table 3. Estimated Parameters from Previous Studies (Y = a + b*N + c*N*N)

Authors	Equation	Parameters		
Quadratic Functions		**a**	**b**	**c**
Gandorfer & Rajsic	Betzendorf	4.547	0.02164	-0.000065
	Wolfsdorf	4.682	0.03674	-0.000089
Meyer-Andres et. al.	Contur B ('99)	3.330	0.0397	-0.00006
	Flair B	5.592	0.0400	-0.00007
	Vivant B	2.144	0.0265	-0.00005
	Contur B ('00)	4.376	0.0518	-0.00012
	Batis A	6.981	0.0289	-0.00010
	Drifter B	6.750	0.0332	-0.00006
Sieling et. al.		7.55	0.01981	-0.000047
Exponential Functions		Y max	Y min	T
Godard et. al.		10.186	3.319	-0.00655

Table 4. Diminishing Returns to Nitrogen

	Nitrogen (kg/ha)	Percent Change in N from Optimum	Yield (tons/ha)	Percentage Change in Yield from Optimum	Yield – Intercept (tons/ha)	Percentage Change in Yield-Intercept
Economic Optimum	**180**	**1.00**	**8.49**	**1.00**	**3.32**	**1**
	162	0.90	8.34	0.98	3.18	0.96
	144	**0.80**	**8.15**	**0.96**	**2.99**	**0.90**
	126	0.70	7.92	0.93	2.76	0.83
	108	**0.60**	**7.65**	**0.90**	**2.48**	**0.75**
	90	0.50	7.34	0.86	2.17	0.65
Low	**72**	**0.40**	**6.99**	**0.82**	**1.82**	**0.55**
	54	0.30	6.59	0.78	1.43	0.43
Very Low	**36**	**0.20**	**6.16**	**0.73**	**0.99**	**0.30**
	18	0.10	5.68	0.67	0.52	0.16

Table 5. Diminishing Returns to Nitrogen (no intercept) (change this description)

	Nitrogen (kg/ha)	Percent Change in N from Optimum	Yield (tons/ha)	Percentage Change in Yield from Optimum
Economic Optimum	**200**	**1.00**	**9.59**	**1**
	180	0.90	9.33	0.97
	160	0.80	8.91	0.93
	140	0.70	8.33	0.87
	120	0.60	7.60	0.79
	100	0.50	6.72	0.70
Low	**80**	**0.40**	**5.68**	**0.59**
	60	0.30	4.49	0.47
Very Low	**40**	**0.20**	**3.14**	**0.33**
	20	0.10	1.65	0.17

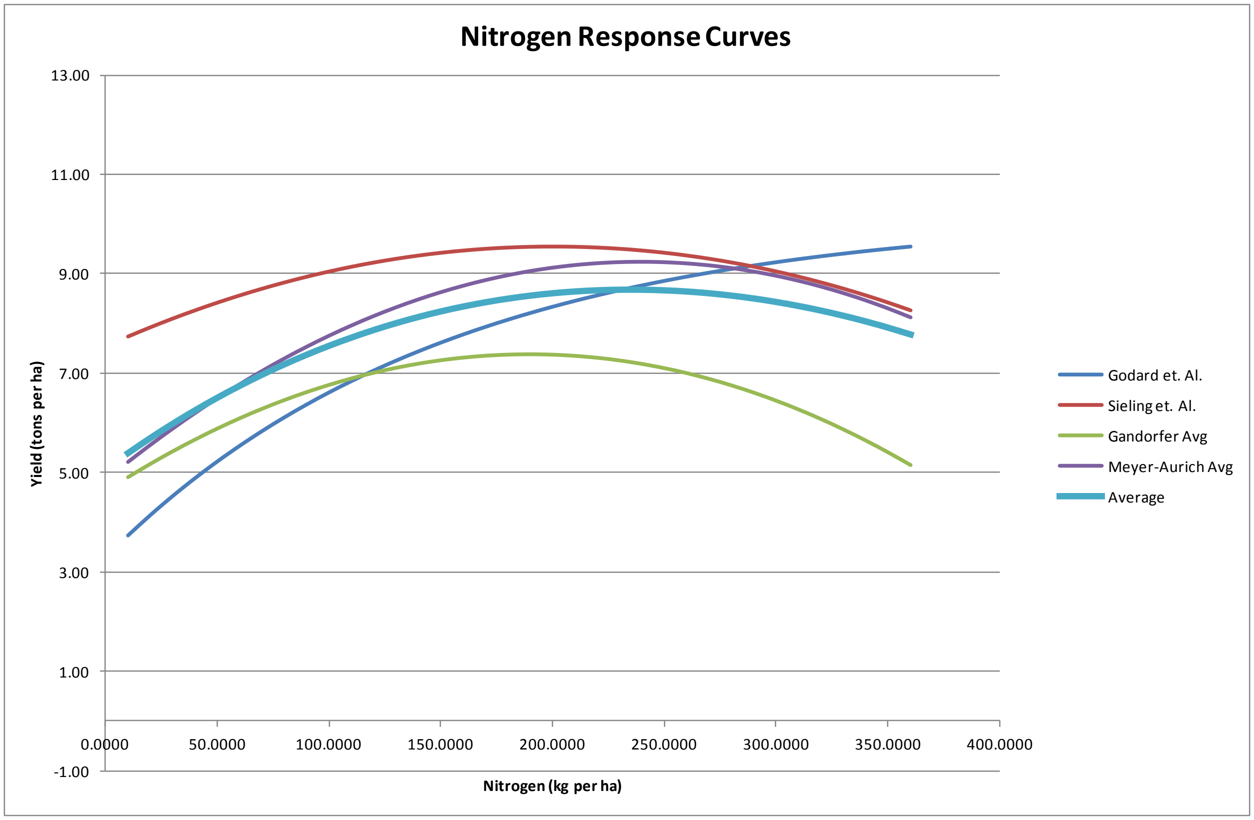


Figure 1. Yield Response Curves for Nitrogen Inputs from Past Studies

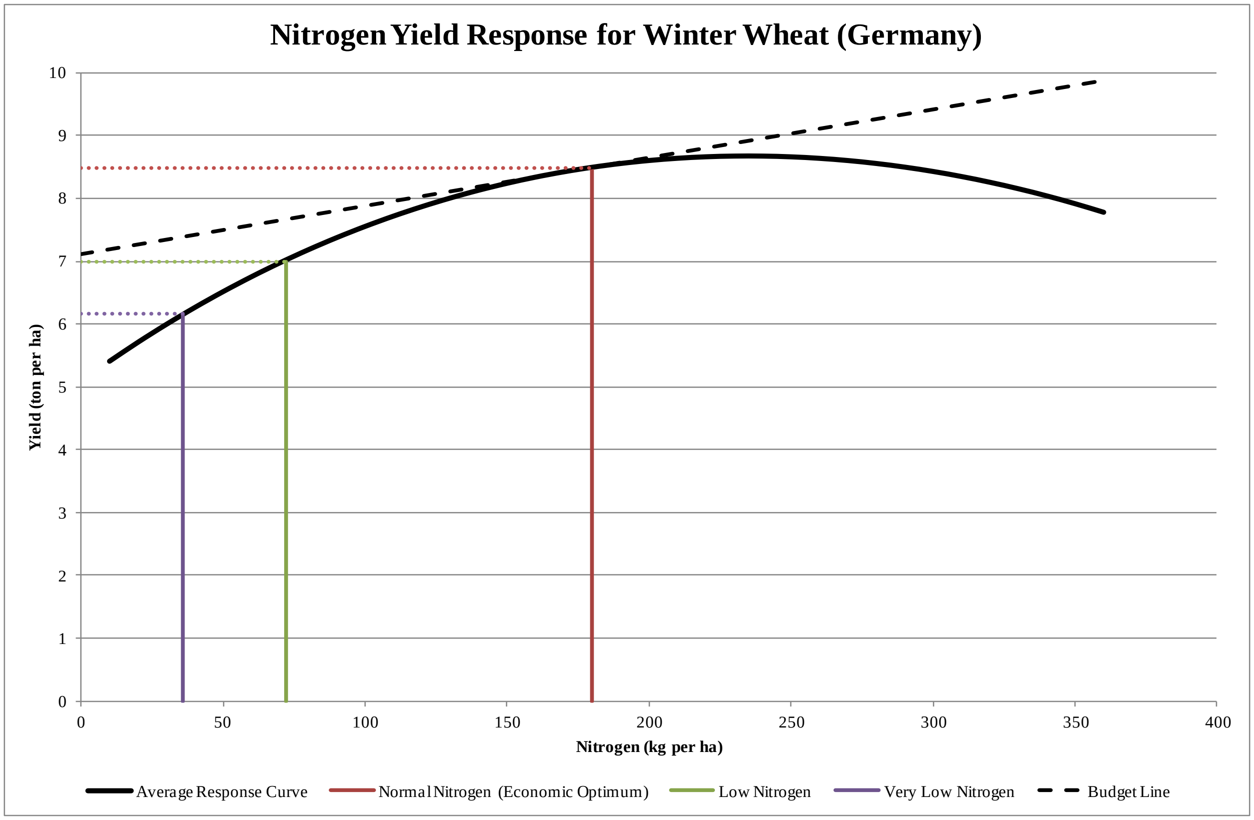


Figure 2. Average Yield Response with Three Levels of Nitrogen Input

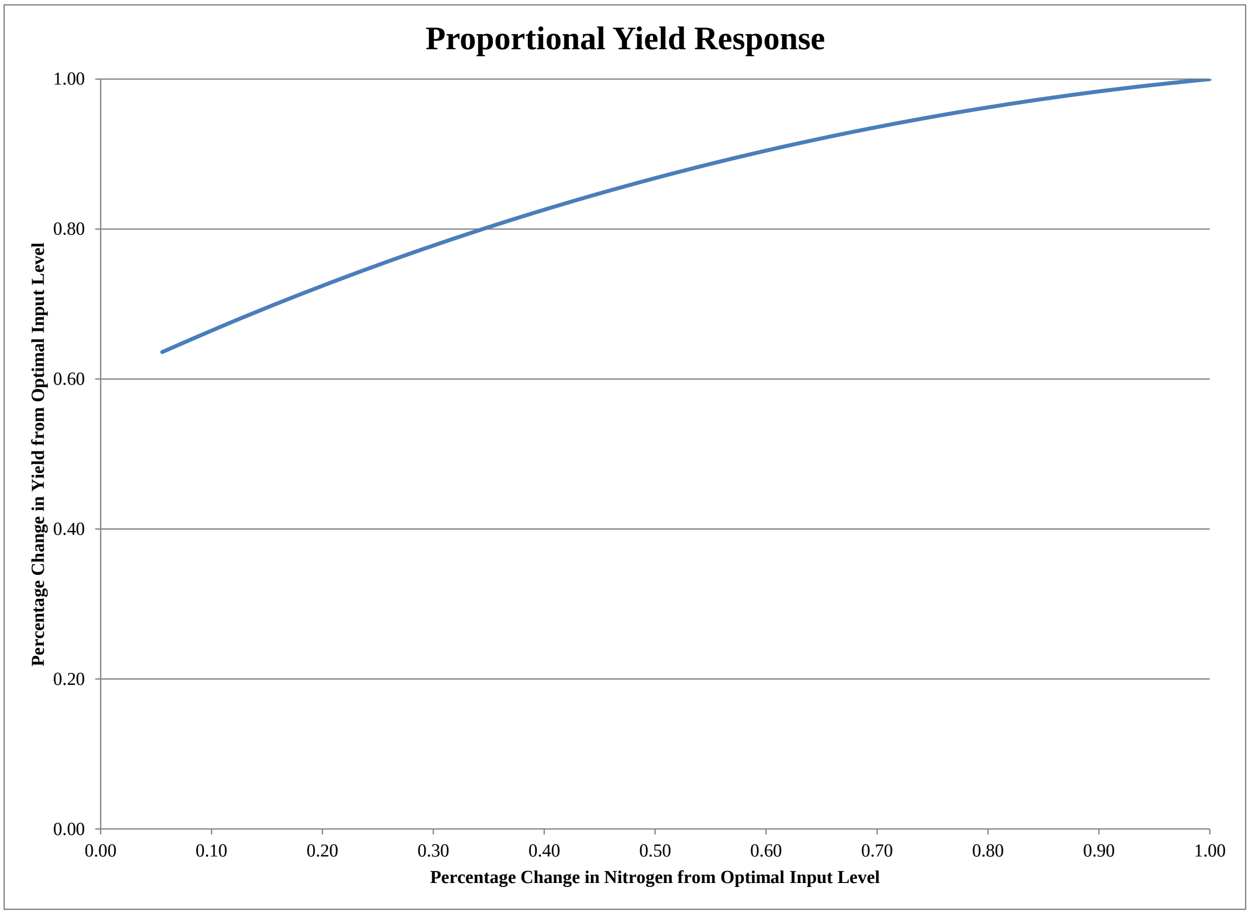


Figure 3. Proportional Yield Response to Various Nitrogen Input Levels

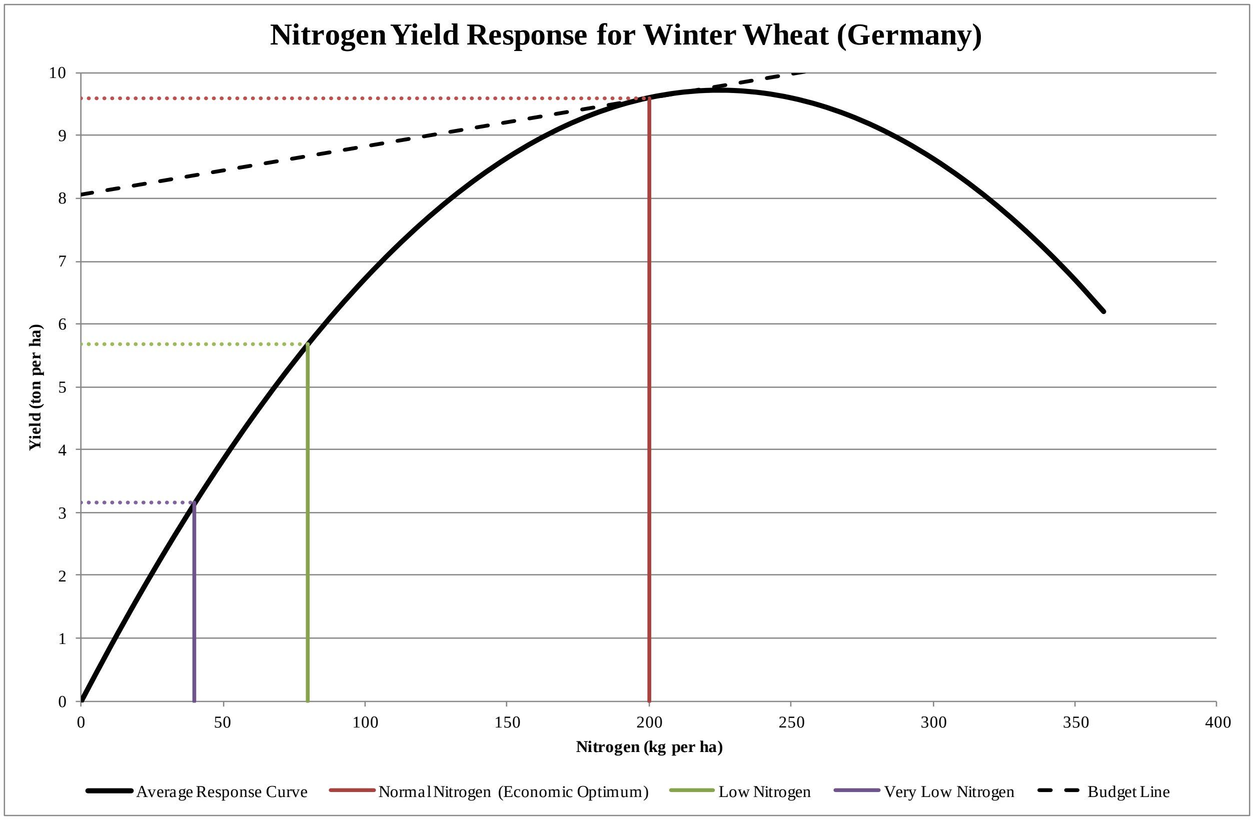


Figure 4. Yield Response with No Intercept

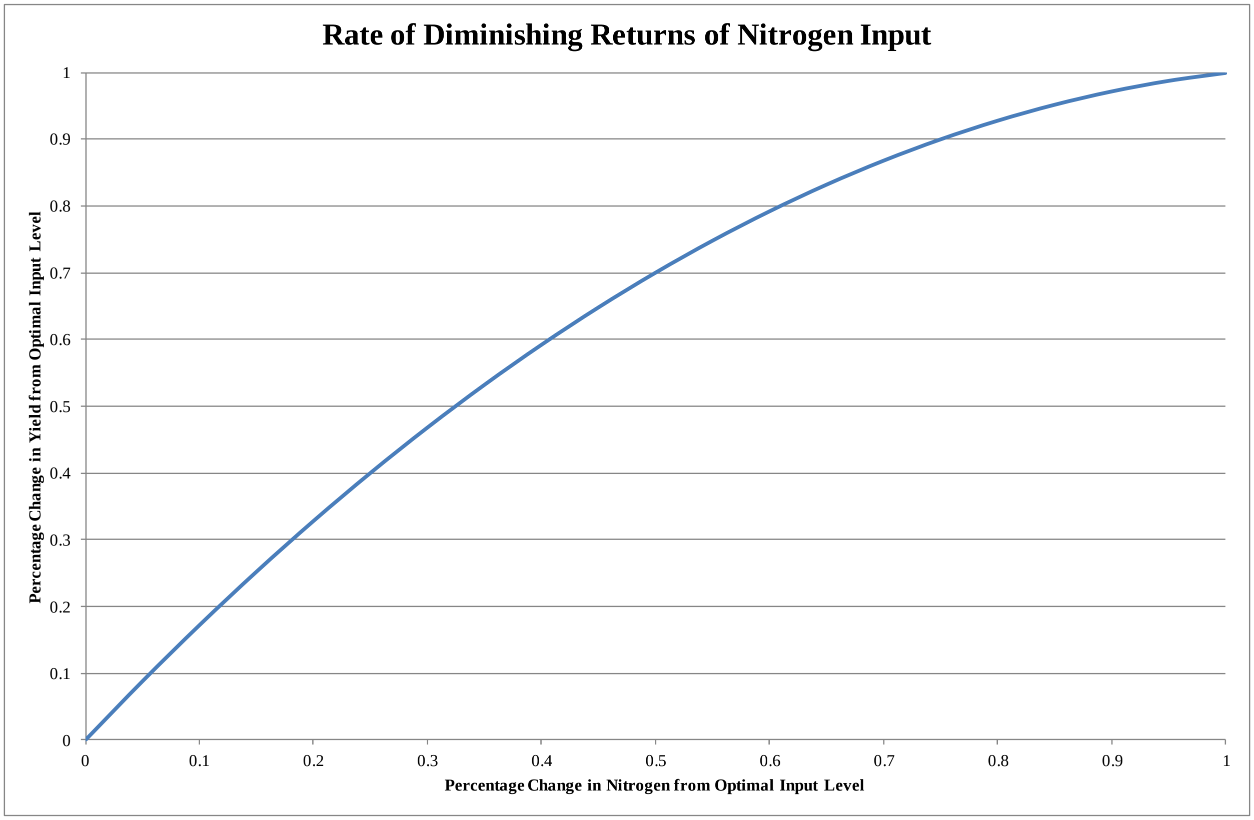


Figure 5. Proportional Yield Response to Various Nitrogen Input Levels (no intercept)

Appendix I

Other Model Specifications

I. *Objectives*

$$
\begin{aligned}
A^{y} = {} & A^{y-1} \\
& + \mathrm{NetCashFlow}^{y} \\
& + \sum_{b} \mathrm{B}_{b}^{y} \\
& - \mathrm{HHInc}^{y} \\
& - \mathrm{Investments}^{y}
\end{aligned}
$$

The gross margin for each state of nature is defined as revenue from sales, income from renting out land, and working off farm minus costs of buying intermediate inputs and other variable costs not explicitly covered. For off-farm work, and weekly work time in hours is given, it is assumed that 46 hours weeks are worked throughout the year, so that income is defined by multiplying the two terms with the hourly wage.

$$
\begin{aligned}
\pi^{y,s} = {} & \mathrm{SaleRev}^{y,s} \\
& + \sum_{l} \mathrm{RentH}_{l}^{y} * \mathrm{RentP}_{l}^{y} \\
& + \mathrm{Hourly}^{y,s} * u^{y,s} * m \\
& + \mathrm{OffWork}_{\mathrm{type}}^{y} * WorkTime_{type} * 46 * u_{type}^{y,s} \\
& - \mathrm{BInputs}^{y,s} \\
& - \mathrm{VC}^{y,s}
\end{aligned}
$$

The net cash flow is defined as the sum of the gross margin in each state of nature, interest gained on cash, interest paid on outstanding credits, and paying back credits. For the last year, where it is assumed that the firm is liquidated, the following terms are added. It is assumed that all physical structures are removed and sold.

$$
\begin{aligned}
CashFlow^{y} = & \sum_{s}(CashBalance * \phi^{s}) \\
& + (A^{y-1} * (\text{interest gained} / 100)) \\
& - \sum_{\mathrm{b}}(\mathrm{B}_{\mathrm{b}}^{\mathrm{y}} * \beta_{b} / 100) \\
& - \text{Repayments on past credits} \\
& - \sum_{\mathrm{b}} B_{b}^{y} \\
& + \sum_{\mathrm{k}} \mathrm{MachineInvestment}_{\mathrm{k}}^{\text{last year}} / LifetimeMachine_{k} * P_{k}^{y} * (2/3) \\
& + \sum_{\mathrm{l}} TL_{l} * PLand_{l}^{y} - 4 * \mathrm{PRent}_{\mathrm{l}}^{\mathrm{y}}
\end{aligned}
$$

II. Labor use

The model considers labor needs for each month *m* and state of nature *s*. Labor needs for the farm operation *G* are related to certain farm activities on the field. The labor needs are determined by the number of hectares per crop *H* and the required labor hours per hectare and per month for that crop ρ in addition to the labor hours needed for fertilizer application μ:

$$
\begin{aligned}
G^{y,s,m} = & \sum_{c,i,t} H_{i,t}^{c,y,s} * \rho_{i,t}^{c,m} \\
& + \sum_{c,i,t} F_{i,t,j}^{c,y,s,m} * \mu_{j}
\end{aligned}
$$

Farm family members can allocate their labor time off the farm on a full-time, half-time, or hourly basis. Full- and half-time off-farm employment is represented as integers. Commuting time is accounted for in the model for full- and half-time work. It is assumed that wages for full-time positions exceed wages of half-time positions which exceed wages from menial work completed on an hourly basis. Thus total labor demand is represented in the model as labor requirements for cropping *G,* full- and half-time labor off the farm, and hourly work off the farm:

$$
G^{y,s,m} + full^{y,m} + half^{y,m} + hourly^{y,s} \leq TG^{y,m}
$$

It is required that the labor demand does not exceed the total labor available TG in each month *m* of that year *y*.

III. Machinery

Cropping production requires certain machinery hours to execute tilling, planting, spraying, and harvesting activities. The model makes use of eight machinery types *k* to complete these requirements: tractor, plough, chisel plough, sow machine, seedbed combination, circular harrow sow, weeder, and spring tine harrow, which become and inoperative and require replacement when their maximum operation hours are reached.

The available inventory for machinery $InvM$ in the current period y for each machinery type k is determined by the inventory of machinery at the end of the last period plus new machinery purchased in the current period BM. Machine inventory per period is represented in operation hours. To convert machinery purchased, a binary variable, into operation hours we scale this variable by δ which represents the physical lifetime in hours of each machinery type. The operating hours M per state of nature absorbed in the current period scaled by the probability of each state of nature φ are then subtracted to obtain an end of the period inventory of machinery operating hours by machine type.

$$\begin{aligned} InvM_k^y &= InvM_k^{y-1} \\ &\quad + BM_k^y * \delta_k \\ &\quad - \sum_s M_k^{y,s} * \phi^s \end{aligned}$$

The machinery hours demanded in each year is determined by the hours of specific machine type M required for the cropping activity H chosen in that period and the fertilizer application F needed for the crops. This machinery demand must not exceed the machinery operating hours in the current period.

$$\begin{aligned} &\sum_{c,i,t} H_{i,t}^{c,y,s} * M_{i,t,k}^{c} \\ + &\sum_{c,i,t,j,k} F_{i,t,j,k}^{c,y,s} * M_{i,t,j,k} \\ \leq &\ M_k^{y,s} \end{aligned}$$

The state of nature with the highest need in that year defines that period's machinery need.

IV. Farm Finance

The farmer can make investments by purchasing both land and machinery. Investments are implemented in the model as binary variables. The sum of investments in a given period y is thus defined as:

$$\begin{aligned} Investments^y &= \sum_l BL_l^y * P_l^y \\ &\quad + \sum_k BM_k^y * P_k^y \end{aligned}$$

Where BL and BM are purchased land and machinery respectively and P_l and P_k are the respective prices of each investment. These investments can be financed either from accumulated cash (equity) or credit. The model differentiates credits by repayment periods and interest rates. Credits are repaid in equal installments over the repayment period, so that the annuity drops from year to year. Accumulated cash draws interest. In order to keep the possible branching trees at an acceptable size, the re-investment points can be restricted to specific years.

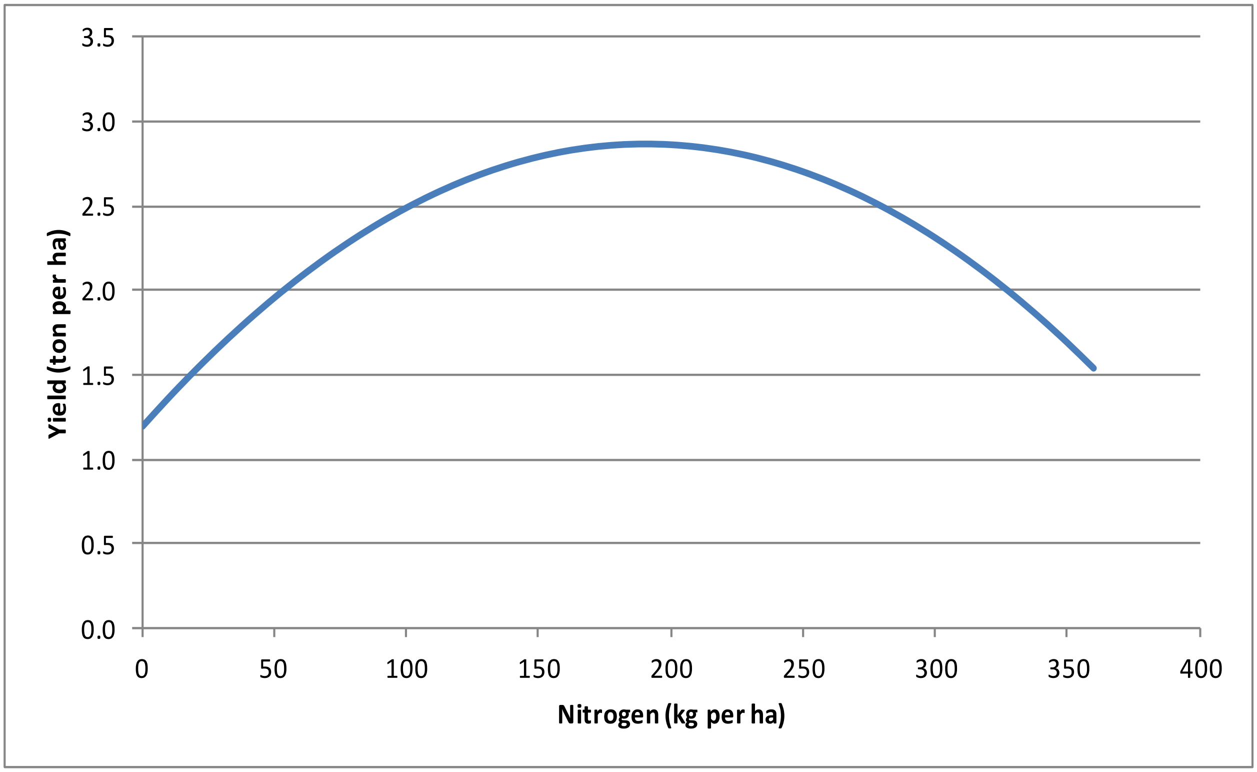
Appendix II

Rapeseed Nitrogen Response

Siadat et. al. (Iran)

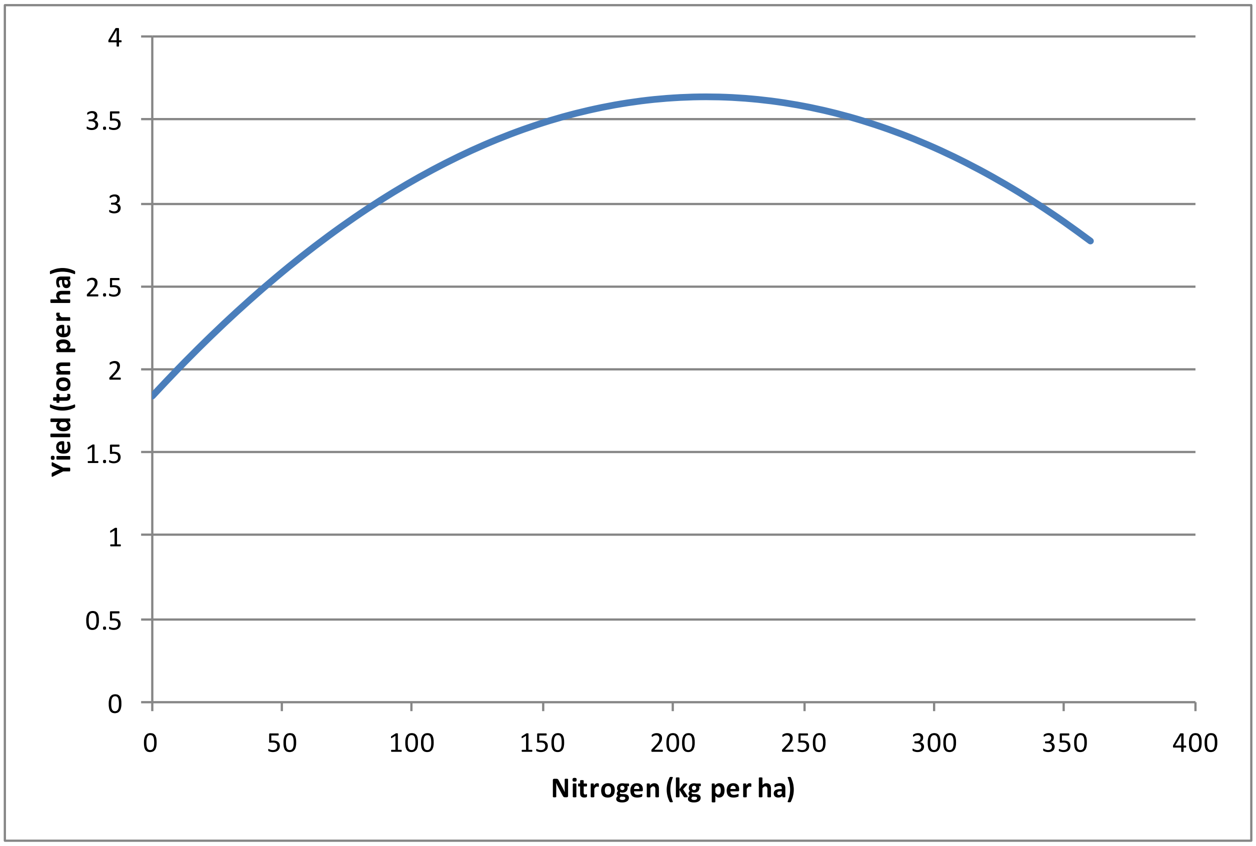
N	N sq	kg per ha	ton per ha
0	0	1235	1.235
100	10000	2282	2.282
160	25600	3078	3.078
220	48400	2730	2.73

$$Y = 1.1930 + 0.0176N - 0.00005N^2$$



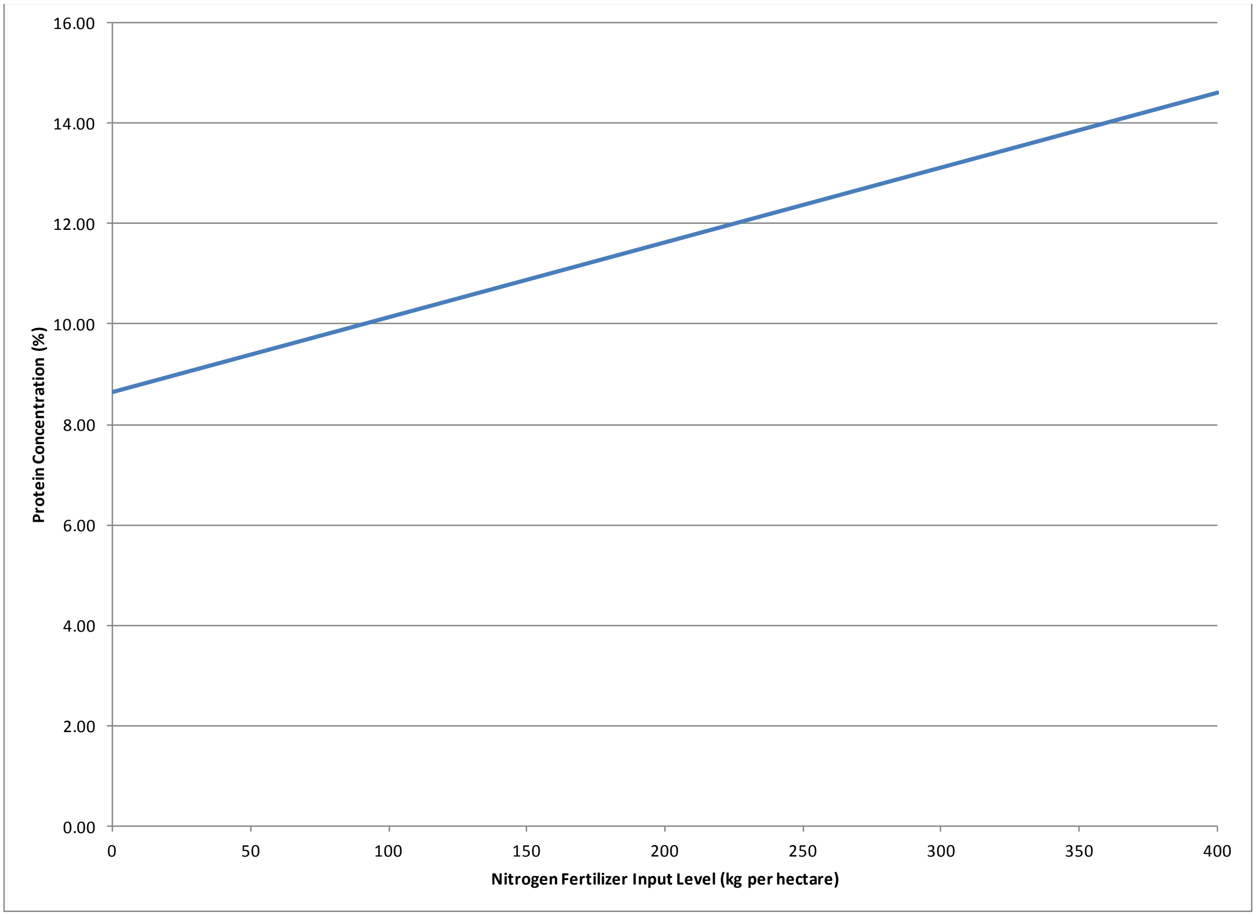
Sieling (NW Germany)

$$Y = 1.84 + 0.017N - 0.00004N^2$$



Protein Response

Meyers-Aurich (2010)
Average $Q = 8.643 + 0.015N$



Reiger: Grain Protein: 14.5%