Impact of Changing Seasonal Rainfall Patterns on Rainy-Season Crop Production in the Guinea Savannah of West Africa

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Abstract

Rainy-season farming is a major source of income for the rural population in the Guinea Savannah zone of West Africa. Farming systems in the region are dominated by rain-fed production of cereals, but include also leguminous crops and oilseeds. A recent World Bank study has identified high potentials for competitive agricultural production and agriculture-led growth in the Guinea Savannah zones of Sub-Saharan Africa. This optimistic outlook is conditional on appropriate investment strategies, policy reforms, and institutional changes. Furthermore, the World Bank warns that global climate change could pose a potential constraint for agricultural growth due to likely reductions in rainfall levels and significant increases in rainfall variability. This could lead to serious dry spells and a drop of crop yields. The study regions are the département Atakora in Benin, the région Sud-Ouest in Burkina Faso, and the Upper East Region in Ghana. Climate projections and trend estimates for these regions show very heterogeneous results for level and variability of monthly rainfall patterns. Therefore, we want to investigate which potential future developments pose the greater threat for agricultural production in the study regions. We develop a set of regional agricultural supply models, each representing 10-12 cropping activities and roughly 150.000 ha of agricultural area. We distinguish two stages of crop production: The planting stage from April to June and the yield formation stage between June and November. Preliminary results suggest that drought events during the planting stage have a more severe impact on the output of individual crops than drought events during the second stage. In contrast, the impact on total farm revenues appears to be more prominent during the second stage, when farmers have a limited capability to adjust their production plan. A clear if not surprising result is the larger vulnerability of crops with growth cycles ranging from the very beginning to the very end of the rainy season. The observed diversity of cropping activities serves the purpose to reduce the vulnerability to adverse rainfall events within a certain range. However, some extreme events are associated with very poor harvests of specific cash crops, thus severely affecting the income of the farming sector. A comprehensive picture will be obtained once the climate change scenarios are completed and the model results are tested and validated for various settings.

Keywords: Climate change, West Africa, agricultural production, stochastic production frontier, highest posterior density estimation.

JEL classifications:

1 INTRODUCTION

Rainy-season farming is a major source of income for the rural population in the Guinea Savannah zone of West Africa. The farming systems in the region are dominated by rain-fed production of cereals ("cereal-root crop mixed farming systems" (Dixon et al. 2001)), but include also leguminous crops (cowpeas, Bambara beans) and oilseeds (groundnuts, cotton, soybeans). In a recent study, the World Bank (2009) has identified high potentials for competitive agricultural production and agriculture-led growth in the Guinea Savannah zones of Sub-Saharan Africa, possibly to an extent that the associated farming systems may turn into a "bread basket" for Africa as a whole (Dixon et al. 2001). This optimistic outlook, however, is conditional on appropriate investment strategies, policy reforms, and institutional changes (Worldbank 2009). Furthermore, global climate change "is likely to reduce the level of rainfall in Guinea Savannah zones in West Africa and significantly increase rainfall variability across the continent" (Worldbank 2009, p14) and therefore perceived as a potential constraint for agricultural growth. The main objective of this paper is to closer examine climate-change induced constraints for agricultural production in the Guinea Savannah zones of selected West African countries at sub-national level. Our study regions are the département Atakora in Benin, the région Sud-Ouest in Burkina Faso, and the Upper East Region in Ghana. For these regions, Hulme et al. (2001) present decreasing trends for mean rainfall levels for the period from June to August under different climate change scenarios. The results by Neumann et al. (2007) based on long time series from weather stations in the Volta Basin indicate more heterogeneous trends for level and variability of monthly rainfall. For instance trends for both, level and variance of April precipitation in Upper East Ghana are significantly declining, while trends for the same indicators in May are increasing. From these findings, we derive the following research questions that we address in this paper: Which potential future developments pose the greater threat for agricultural production in the study regions? Is the variability of the onset of the rainy season or are dry-spells during the rainy season more dangerous for farmers? Which crops are affected? How can/do farmers adapt?

To answer these questions, we started with developing a database on crop production, actual and attainable yields, area allocation, and composition of agricultural revenues for the study regions over the last 25 years and for 10-12 crops, depending on the region. This database is then used to estimate the structural parameters of a supply model for the regional agricultural sector, which distinguishes area allocation and yield formation periods as two stages of decision-making. The residuals of the deterministic part of the supply model estimation are then regressed against monthly rainfall indicators.

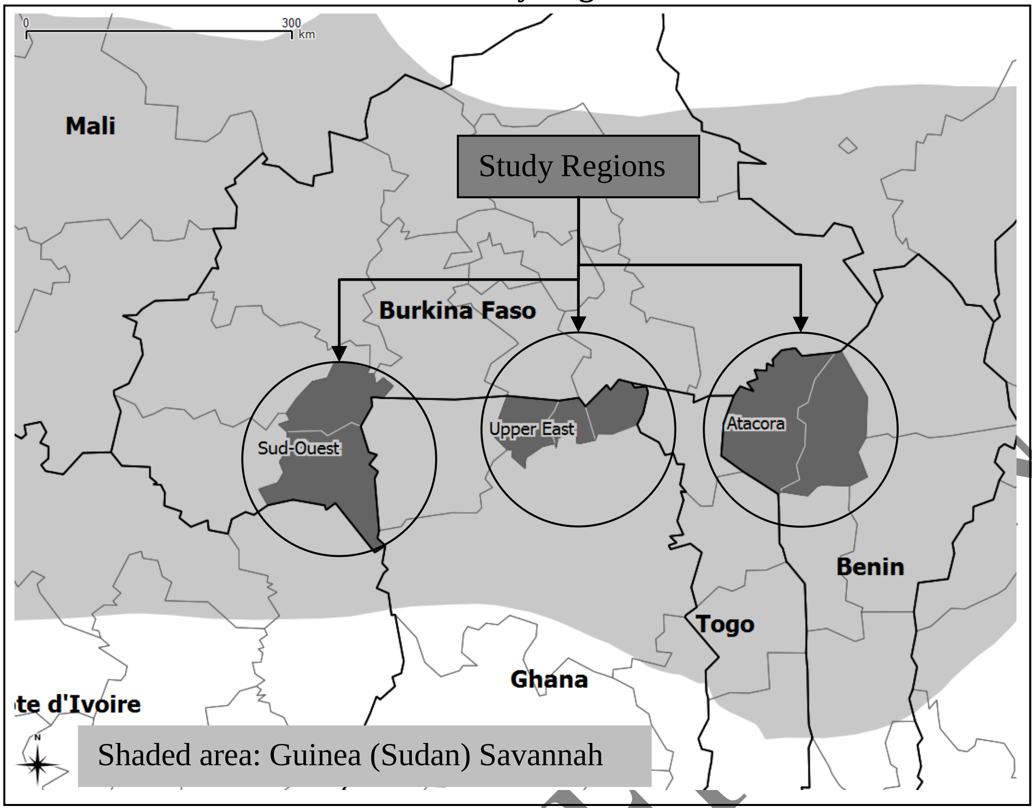
The remainder of this paper is organized in the following way: In the next section 2, we provide an overview on the agricultural sector in the case study regions. The database is described in section 3. The characteristics of the supply model and the estimation procedure are then outlined in section 4, major results shown in section 5. We conclude with a discussion in section 6.

2 OVERVIEW ON THE CASE STUDY REGIONS

General overview

The three case study regions are administrative units at first sub-national level of their respective countries. They are located in the Savannah Belt of West Africa (see FIGURE 1), within a similar agro-ecological zone (AEZ) characterized by a growing period between 150 and 210 days (Fischer et al 2002). Sud-Ouest Burkina Faso and Atacora are substantially larger in terms of total area than Upper East Ghana (roughly 1600 and 2100 thousand hectares as opposed to some 880 thousand hectares). In contrast, the population density is much higher in Upper East Ghana, so that roughly 600 thousand persons live in each of the two francophone regions while more than 1 million persons were estimated as inhabitants of Upper East Ghana during 2009 (GLSS5). Agriculture is the main occupation, with an employment share between almost 90% in Sud-Ouest Burkina (EA07) and around 70% in Upper East Ghana (GLSS05). Smallholder farming with low levels of mechanisation is dominant in all three regions with average family sizes of 5 persons and 2 hectares of arable land. Irrigation plays only a minor role as e.g. only 2 thousand hectares of irrigated land are recorded for Upper East Ghana between 2003 and 2009 by the Ministry of Food and Agriculture of Ghana (MOFA), but in total around 350 thousand hectares were harvested during the dry season in the same period (See FIGURE 2).

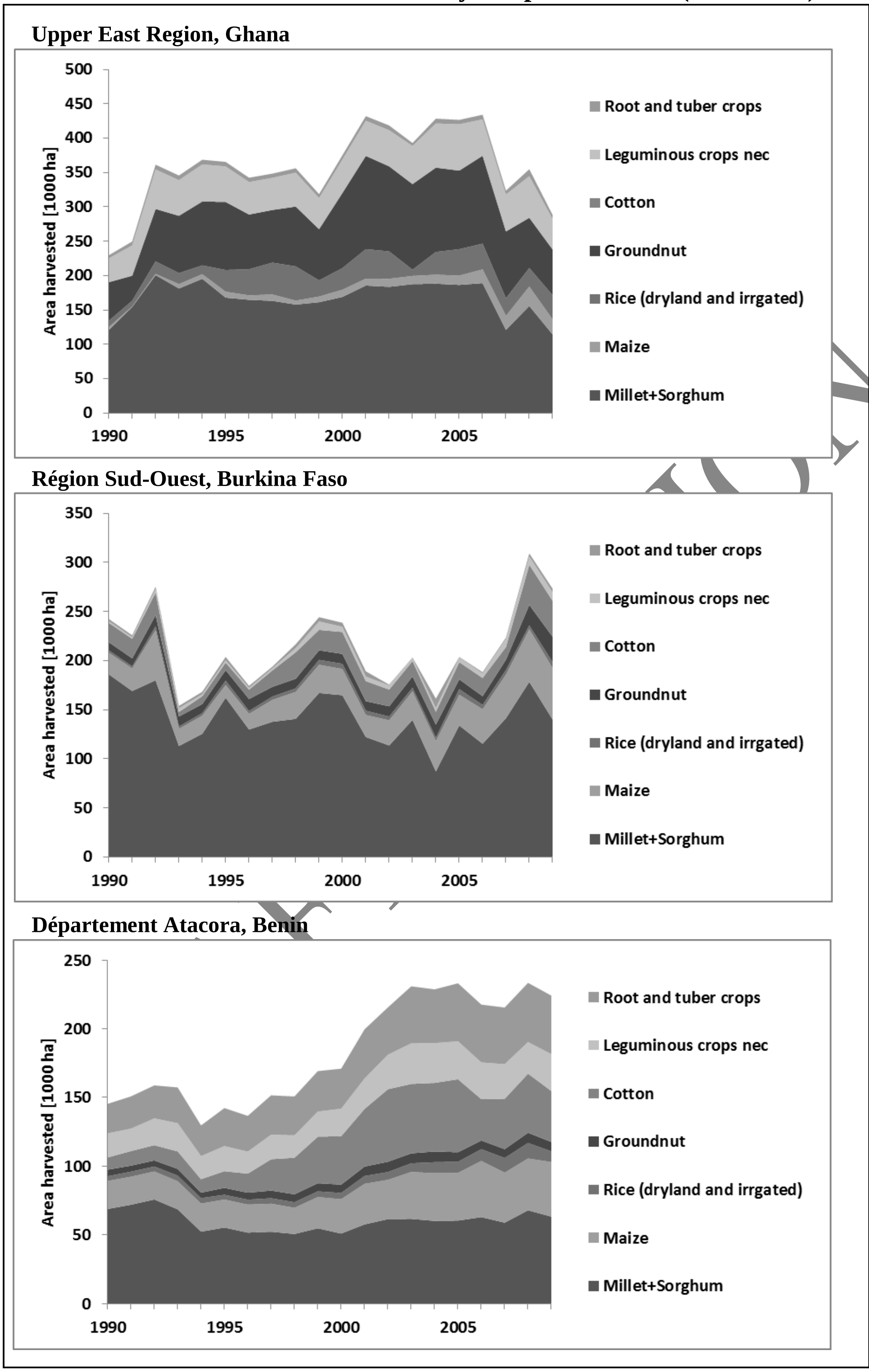
FIGURE 1. Location of Study Regions



Source: Own map, based on www.maplibrary.org

With regard to the allocation of areas across different cropping activities (FIGURE 2), one can observe that the agricultural sector in all three regions meets the characteristics of “cereal-root crop mixed farming systems” (Dixon et al. 2001): Cereals account for around 50% of the total harvested area between 1990 and 2009 in the three study regions, with millet and sorghum as major crop categories. However, the species and cultivars within these categories may vary largely between the regions (different shares of white and red sorghum, different types of millet). Some further differences in the composition of the overall cereal area are notable. Maize production in Upper East Ghana increased 10.fold between 1990 and 2010 from 4 thousand to almost 40 thousand hectares, but remained at an average share of 5% during the last decade. Maize is typically produced in the Northern and Brong Ahafo regions of Ghana, but farmers in Upper East started to expand maize areas due to its comparatively short growing cycle and the experienced shortening of the rainy season. Rice production in contrast is more common in Upper East Ghana with an average share of 8% in the last decade, during which the rice share remained at a 2% level in Sud-Ouest Burkina. The availability of irrigated areas (2 thousand hectares in Upper East) can only partly explain this. Groundnut production is again a dominant farming activity in Upper East Ghana and not substantial in the two francophone regions. Cotton on the other hand is very important as cash crop in Sud-Oest and Atacora (also due to the existence of marketing and processing organisations) and can hardly be found in Upper East, although the area was expanded since 2010.

FIGURE 2. Harvested Areas by Crops 1990-2009 (in 1000 ha)



Sources: See table 1

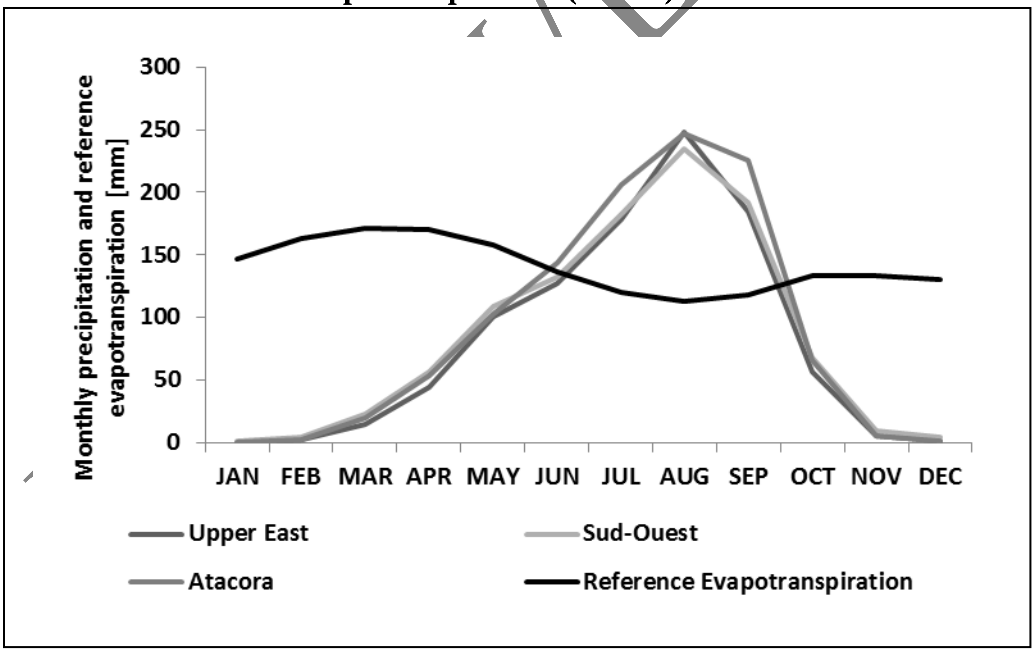
Leguminous crops (cowpea, Bambara beans and soybeans) appear to be rarely produced in Sud-Ouest Burkina, which seems to be implausible given the high shares in the other two regions, but Gleisberg-Gerber (2012) reports a similar finding from

farm-household surveys in 2006/2007. The area under soybeans is small in all three regions and is therefore not reported here as an individual category, but production has increased over the last decade and is therefore included in the database (see next section). The category “Root and tuber crops” in FIGURE 2 has the largest proportion in Atacora, where it refers to yams and cassava, while the small area in Upper East consists mainly of sweet potatoes. In general, the three regions exhibit a typical pattern of agricultural production for the Savannah regions of West Africa.
It should be noted here some of the larger inter-annual changes of harvested areas have to be attributed reforms of administrative boundaries since the 1980s and changing survey or reporting practices.

Cropping calendars and seasonal rainfall patterns

Planting activities in the present start typically in May, sometimes already in late April in the more southern parts of the three case study regions. An earlier onset of the growing season was more common in the past (Laux at al. 2008) and Neumann et al (2007) report significant declining trends for monthly precipitation in April since the 1960s. Thus, a shift of seasonal climatic patterns in the study regions is already visible and projected to continue (Ericksen et al. 2011). In general, monthly precipitation doubles between April and May (FIGURE 3) and continues to increase until its peak in July/August, falling then back to levels below 50 mm per month on October/November. This pattern is similar in all three case study regions (FIGURE 3).

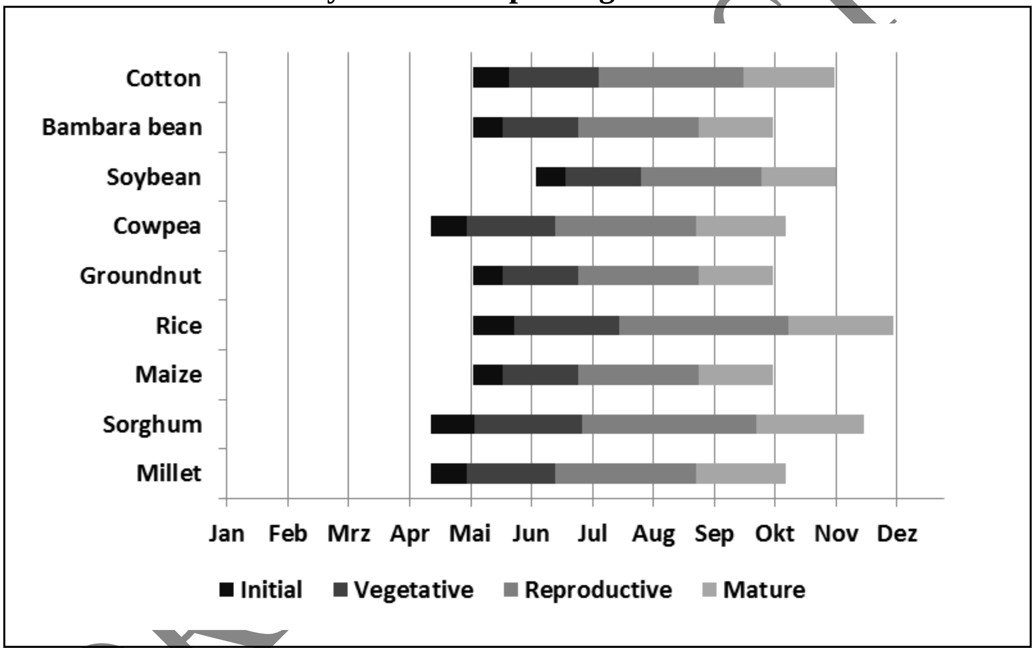
FIGURE 3. Average Monthly Precipitation and Reference Evapotranspiration (in mm)



Sources: FAO, WorldClim

Based on this inter-regional similarity, we compiled an average cropping calendar from various sources for the typical rainy-season crops as shown in FIGURE 4. We followed the CropWat terminology (Allen et al. 1998) for the growing stages (k1: initial stage, k2: vegetative or development stage, k3: reproductive or mid-season stage, and k4: maturation or late season stage) and use average coefficients for the length of these stages from Fischer et al (2002). The earliest planting date and the length of the total growing cycle were taken from the FAO Crop Calendar (http://www.fao.org/agriculture/seed/cropcalendar/welcome.do), various project reports, and from own field experiences. The general pattern is that millet and sorghum are planted first, as early as possible, albeit for two different reasons: Short-term varieties of millet can be harvested already in July when planted early and help to last through the lean months until the upcoming major harvest. Red sorghum (or Guinea corn) has a longer growing cycle than other crops, although short-term varieties were introduced in Ghana in recent years and taken up readily by farmers due to the shortening of the rainy season. Cowpea is often intercropped with cereals and sown at the same time, also as a provider of nitrogen. Most other crops are then sown or planted in May, so that they have reached the reproductive stage (k3) in July and August, when average monthly precipitation is highest and can meet the likewise higher evapotranspiration requirements of the crops (Allen at al 1998). Maturation stages fall in the periods of declining rainfall, and the harvest commences by end of September and continues in some cases until late November (apart from the mentioned short-term varieties of millet and also maize).

FIGURE 4. Average Planting Months and Growing Stages for Major Dry-Season Crop Categories



Sources: FAO, CropWat, own field experience

3 DATABASE

Sources

Farm-level survey data are usually collected for single years and may not report weather conditions. However a nearby weather station may be identified, but would then be used for a larger number of farmers and therefore effectively decrease the number of data points. For the purposes of this study, it appeared to be more appropriate to compile sufficiently long time series for agricultural production, prices, and precipitation with a sufficient amount of overlap. We screened the availability of sub-national datasets from various international providers (e.g. FAO CountryStat, AgroMaps) and from national statistical organisations (e.g. Ministry of Food and Agriculture, Ghana; AGRISTAT Burkina Faso; Ministère de l'Agriculture de l'Elevage et de la Pêche, Benin) and ensured consistency by comparing data for overlapping years. For weather data, we could obtain the data from the measurement stations also used by Neumann et al (2007) from the GLOWA-VOLTA project. In addition, weather data from national statistical organisations were obtained and compared to regional averages from WorldClim (http://www.worldclim.org/). The sources of the combined database are summarized in TABLE 1:

TABLE 1. Used Data Sources.

Country	Region	Indicator	Source
Benin	Atacora	Area harvested and production	MAEP (1993 – 2010)
			CountryStat (1988-1993)
		Precipitation	MAEP (1996-2009)
		Population	RGPH3
		Prices	GIEWS
Burkina Faso	Sud-Ouest	Area harvested and production	AGRISTAT (1984-2006), CountryStat (2000-2009)
		Precipitation	GLOWA-VOLTA Project (1961-2006, with gaps)
		Population	EA07
		Prices	SONAGESS, GIEWS, FAO
Ghana	Upper East	Area harvested and production	MOFA (1987-2010), AgroMaps (1997-2000)
		Precipitation	GLOWA-VOLTA Project (1961-2006, with gaps), MOFA (200-2010, Upper East)
		Population	GLSS05
		Prices	MOFA, FAO, GIEWS

District groupings

Because the targeted administrative units in the three countries differ substantially in agricultural area and population, we split them into comparable units with roughly 300000 inhabitants and 100000-150000 ha of agricultural area, based on sub-catchments of the Volta basin and existing administrative borders at lower hierarchical levels. In this, we follow roughly the concept of the European Commission's "Nomenclature of territorial units for statistics (NUTS)" (EUROSTAT 2011). Major criterion for the NUTS groupings is also population, e.g. between 150000 and 800000 inhabitants for a NUTS3 region. We use location within a Volta sub-catchment as additional criterion as this is invariant towards potential future administrative reforms.

TABLE 2. District Groupings

Level 0	Level 1	Level 2	Code	Commune/Province/District
Benin	Atakora	Atakora - West (Oti)	BJ.AK.OT	Boukoumbé
				Cobly
				Matéri
				Natitingou
				Tanguiéta
				Toffo
		Atakora - East (Mekrou)	BJ.AK.ME	Kérou
				Kouandé
				Péhunco
Burkina Faso	Sud-Ouest	Sud-Ouest -	BF.SO.BG	Bougouriba

		Bougouriba + Ioba		Ioba
		Sud-Ouest - Poni + Noumbiel	BF.SO.PN	Noumbiel
				Poni
Ghana	Upper East	Upper East Region - West (Tono)	GH.UE.TN	Builsa
				Kassena-Nankana
		Upper East Region - Center (Vea)	GH.UE.VA	Bongo
				Talensi-Nabdam
		Upper East Region - East (White Volta)	GH.UE.WV	Bawku Municipal
				Bawku West
				Garu-Tempane

Consolidation

To guarantee that certain accounting conditions for the district groupings are fulfilled, we adjust the raw data such that e.g. agricultural areas add up to higher hierarchical levels and that gross output data are products of yields and areas.

4 METHODS

Agricultural production as a two stage process

Based on examination of the literature on rainy-season crop production in West Africa and own field experience, we decided to model agricultural production as a two-stage process with output dynamics (Antle, 1983a). The first stage represents the planting season from April to June, during which farmers decide on the allocation of area and starts with initial maintenance activities (first weeding). Planted areas are then treated as an input for the second stage, which represents the output formation between June and November, depending on the crop cycles (see FIGURE 4). By distinguishing these two stages of decision making, we account for the fact that adverse weather events may affect agricultural production in two ways: In the area allocation stage, dry conditions can prevent a farmer from actually planting his fields and cause a total allocated area below the potential availability of land and labor. In turn, this means that more labor is available for the actually planted area in the second stage and thus creates higher yields due to increased labor intensity. Nevertheless, droughts during the yield formation phase can again compensate this effect adversely.

Choice of functional form

The straightforward approach would now be to estimate a production system as described above based on observations for outputs, allocated inputs, and weather indicators using established econometric techniques. The first obstacle for this is that intermediate inputs are not observed for the same years as output and weather. While survey data are available for distinct years in the study regions of Burkina Faso (Gleisberg-Gerber 2012) or Ghana (Yilma 2005), they do not permit to recover the inputs allocated in other years, especially when the time series go back to the 1980s. As our interest is in identifying the impact of historical monthly rainfall-events on allocated areas and crop yields, we have $m(c)+z(c)+1$ variables on the right-hand side (if a constant is included). The number of relevant months during the rainy seasons differs from crop to crop (see the crop calendar in FIGURE 4), but may range between 5 to 8 months (April to November) in the most extreme case. So, for a single compound input, we would need at least 5+1+1=7 parameters to be estimated for a e.g. a Cobb-Douglas specification of the yield functions. In case a fully flexible functional form should be desired, the estimation of ½(7+1)(7+2) = 36 (Chambers 1988) parameters is required. Some specifications of flexible functional forms (e.g. Generalized Leontieff, Diewert 1974) may require a lower number of parameters, but since our sample includes at best 26 observations (cereals in Burkina Faso) and 8 at worst (soybeans in Ghana), we are not in a position to robustly estimate a flexible functional form without any additional restrictions on the structure. An obvious source for the a-priori determination of the structure of yield functions is the literature on farm- or field-level yield functions (e.g. Frank et al, 1990). Here, functional forms like Mitscherlich-Baule (Frank et al 1990) or von-Liebig (Paris 1989) are found to be preferable over e.g. quadratic specifications as they permit the existence of yield plateaus, i.e. parts of the yield function where marginal productivity of inputs converges to zero but remains positive. Such plateaus refer to the potential or attainable yield a certain crop may obtain under "clearly specified agro-ecological, edaphic and management conditions." (Harmsen 2000). Comparisons of alternative functional forms (Paris 1989, Frank et al 1990) also showed that the substitutability between some inputs may be rather low or even zero as in the case of von-Liebig specifications. When moving from field- or farm-scale to regional sector scale, the substitutability of inputs may appear to be less restricted due to the heterogeneity of farms and management practices (and other conditions) within the regional agricultural sector. But the existence of yield plateaus on farm level should be reflected in a sector production function as the potential yield under the most advantageous conditions in the region.

A similar problem occurs for the area allocation functions. A family of functional forms that fulfils these properties are "Constant Elasticity of Substitution" (CES) functions as introduced by Arrow et al (1963) and "Constant Elasticity of Transformation" (CET) functions (Powell and Gruen 1968, van der Mensbrugghe 2005). CES and CET functions are frequently used in computable general equilibrium models (CGE, see e.g. Lofgren et al 2000, van der Mensbrugghe 2005) and are also discussed in the context of agricultural sector models (Howitt 1995, Heckelei and Wolff 2003). Perroni and Rutherford (1995) show that nests of CES functions are, while not being fully second-order flexible, fulfil the requirement of regular flexibility, meaning that they are sufficiently versatile to mimic a wide variety of technologies but are globally concave and monotonous. The advantage of the comparable low number of parameters to be estimated is partly offset by the fact that CES functions are non-linear in parameters, thus requiring non-linear estimation techniques. Heckelei and Wolff (2003) propose a method to estimate parameters of CES functions based on generalized maximum entropy approaches (GME, Golan et al 1997). Such approaches permit the inclusion of behavioural and other constraints (e.g. first-order conditions for profit maximization) and are applicable also to estimation problems with very low or even negative degrees of freedom. To summarize the consideration above, we decided to use a combination of CET and CES functions for the representation of the rain-dependent yield functions because they:

- are globally well behaved if appropriately parameterized
- permit low elasticities of substitution
- require a minimum amount of parameters to be estimated
- permit the existence of almost-zero marginal productivity (yield plateau)

Furthermore, we can build on established estimation methods even in case of ill-posed problems.

Formal representation of area allocation

We suppose that the representative farmer's decision problem at time t in district d during the allocation stage is to maximize total expected revenues R^e by allocation areas A across c cropping activities, each associated with a crop-specific expected revenue r^e per hectare. We assume here that the farmer does not know in stage 1 if he will really harvest the allocated area and add an expectation index to area:

$$\max_{A} R^e = \sum_{c} r^e_{d,c,t} A^e_{d,c,t} \tag{1}$$

The farmer is endowed with a certain amount of a compound fixed but allocable input (labour and capital, but dominantly labour in the study regions) Z, which is used for planting activities based on a CET technology, where $\tau > 0$ denotes the time-invariant elasticity of transformation between al crops and θ the time-dependent share parameters of the c-th crop:

$$\bar{Z}_{d,t} = \left(\sum_{c} \theta_{d,c,t} {A^e_{d,c,t}}^{\frac{\tau_d + 1}{\tau_d}} \right)^{\frac{\tau_r}{\tau_d + 1}} \tag{2}$$

The first-order conditions for a maximum of (1) subject to (2) is:

$$r^e_{d,c,t} = w^1_{d,t} {\bar{Z}_{d,t}}^{-\frac{1}{\tau_d}} \theta_{d,c,t} {A^e_{d,c,t}}^{\frac{1}{\tau_d}} \tag{3}$$

where w^1 denotes the wages payable for the usage of Z during the first stage, which we assume to follow the general dynamics of the average producer price index in a country.

Expectations for per-hectare revenues

While available agricultural labour and first-stage wages can be derived from the database, and harvested areas are directly recorded, it is not possible to directly observe expected revenues and originally planted areas. A model for the formation of expectations frequently found in the literature is the adaptive expectations model by Nerlove (1958), in which expectations are based on past observations, adjusted by the last expectation error. We follow the application of this model by Leaver (2004), who uses natural logarithms of expected prices instead of their absolute values:

$$\begin{aligned} \ln\left(r^e_{d,c,t}\right) &= \ln\left(r^e_{d,c,t-1}\right) + \beta_d \left(\ln\left(r^o_{d,c,t-1}\right) - \ln\left(r^e_{d,c,t-1}\right)\right) \\ &= \beta_d \ln\left(r^o_{d,c,t-1}\right) + \left(1 - \beta_d\right) \ln\left(r^e_{d,c,t-1}\right) \end{aligned} \tag{4}$$

where β denotes the expectation coefficient ($0 \le \beta \le 1$) for the representative farmer in district d and r^o observed revenues. We assume that this expectation parameter is not specific for any particular crop. As in the case of expected revenues is it usually not possible to observe planned or desired areas but only harvested, which may deviate from original production plans. In the adaptive expectations model as used here, the relation between desired areas and past realized areas is (Leaver 2004):

$$\begin{aligned} \ln\left(A^o_{d,c,t}\right) &= \ln\left(A^o_{d,c,t-1}\right) + \lambda_d \left(\ln\left(A^e_{d,c,t}\right) - \ln\left(A^o_{d,c,t-1}\right)\right) \\ &= \lambda_d \ln\left(A^e_{d,c,t}\right) + \left(1 - \lambda_d\right) \ln\left(A^o_{d,c,t-1}\right) \end{aligned} \tag{5}$$

where λ denotes the adjustment coefficient ($0 \le \lambda \le 1$). We take now the logarithms of both sides of equation (3), solve for expected areas, and substitute expected areas in equation (5) by the resulting expression. Following the further steps indicated by Leaver (2004) (see also Annex 1) and collecting terms, we end up with:

$$\begin{aligned}\ln\left(A^{o}_{d,c,t}\right) &= \lambda_d \beta_d \tau_d \ln\left(r^{o}_{d,c,t-1}\right) - \lambda_d \beta_d \tau_r \ln\left(w^{1}_{d,t}\right) - \lambda_d \beta_d \tau_d \ln\left(\theta_{d,c}\right) \\ &+ \lambda_d \ln\left(\bar{Z}_{d,t}\right) - \lambda_d\left(1-\beta_d\right)\ln\left(\bar{Z}_{d,t-1}\right) \\ &- \left(1-\lambda_d\right)\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-2}\right) + \left(\left(1-\beta_d\right)+\left(1-\lambda_d\right)\right)\ln\left(A^{o}_{d,c,t-1}\right)\end{aligned} \tag{6}$$

In equation (6), all variables are in principle observable, with the exception of observed revenues. This problem is addressed in the following step.

Production and yield functions

As discussed above, we opt to represent actual production Q at time of harvest as a CES function of area, total intermediate inputs V per crop and total fixed but allocable inputs per crop Z:

$$Q_{d,c,t} = \gamma_{d,c,t}\left(\delta^{A}_{d,c} A_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}} + \delta^{V}_{d,c} V_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}} + \delta^{Z}_{d,c} Z_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}}\right)^{\frac{\sigma_{d,c}}{\sigma_{d,c}-1}} \tag{7}$$

The δ's are share parameters of the respective inputs, ranging between 0 and 1, σ denotes the elasticity of substitutions (0< σ <1, and γ is a shift parameter that represents Hicks-neutral technological change. The area-intensive form of (7) or yield function is then:

$$y_{d,c,t} = \gamma_{d,c,t}\left(\delta^{A}_{d,c} + \delta^{V}_{d,c} v_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}} + \delta^{Z}_{d,c} z_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}}\right)^{\frac{\sigma_{d,c}}{\sigma_{d,c}-1}} \tag{8}$$

Here, y denotes yield in quantities per hectare, v and z intermediate and fixed but allocable input quantities per hectare. If area is valued at its marginal productivity and based on output prices p, the second-stage willingness to pay for harvestable (i.e. planted) area is:

$$r_{d,c,t} = p_{d,c,t}\gamma_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}}\delta^{A}_{d,c} y_{d,c,t}^{\frac{1}{\sigma_{r,c}}} \tag{9}$$

Estimation of CET/CES elasticities and expectation parameters

Based on this, we replace r^{o} with Equation (9) and collect terms to obtain Equation (10), in which observed area depends on lagged prices, yields, harvested areas, crop-specific constants and trends, Z, and first-stage wages:

$$\begin{aligned}\ln\left(A^{o}_{d,c,t}\right) &= \lambda_d \beta_d \tau_d \ln\left(p^{o}_{d,c,t-1}\right) + \lambda_d \beta_d \frac{\tau_d}{\sigma_{r,c}} \ln\left(y^{o}_{d,c,t-1}\right) \\ &+ \left(\left(1-\beta_d\right)+\left(1-\lambda_d\right)\right)\ln\left(A^{o}_{d,c,t-1}\right) - \left(1-\lambda_d\right)\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-2}\right) \\ &+ \lambda_d \beta_d \tau_d \frac{\sigma_{d,c}-1}{\sigma_{d,c}}\left(\gamma^{1}_{d,c} + \gamma^{2}_{d,c} t\right) + \lambda_d \beta_d \tau_d \ln\left(\frac{\delta^{A}_{d,c}}{\theta_{d,c}}\right) \\ &- \lambda_d \beta_d \tau_r \ln\left(w^{1o}_{d,t-1}\right) + \lambda_d \ln\left(\bar{Z}_{d,t}\right) - \lambda_d\left(1-\beta_d\right)\ln\left(\bar{Z}_{d,t-1}\right)\end{aligned} \tag{10}$$

Equation (10) can be re-written as:

$$\begin{aligned}\ln\left(A^{o}_{d,c,t}\right) \quad &= \alpha^{p}_{d}\ln\left(p^{o}_{d,c,t-1}\right)+\alpha^{y}_{d,c}\ln\left(y^{o}_{d,c,t-1}\right)\\ &+\alpha^{A-1}_{d}\ln\left(A^{o}_{d,c,t-1}\right)-\alpha^{A-2}_{d}\ln\left(A^{o}_{d,c,t-2}\right)\\ &+\alpha^{C}_{d,c}+\alpha^{T}_{d,c}t-\alpha^{w}_{d,c}\ln\left(w^{1}_{d,t-1}\right)\\ &+\alpha^{Z}_{d}\ln\left(\bar{Z}_{d,t}\right)-\alpha^{Z-1}_{d}\ln\left(\bar{Z}_{d,t-1}\right)\end{aligned} \tag{11}$$

The linear parameters α^0 are the appropriate combinations of parameters in Equation (10). Equation (11) can be estimated by adding an error term u^{OLS} and using ordinary least squares (OLS). However, there is no guarantee that the estimated parameter values comply with the theoretical requirements implied by Equation (10). We therefore add a second estimation step based on a Generalized Cross-Entropy procedure (GCE, Golan et al 1996), in which we impose theoretical constraints on parameter values. The required a-priori probabilities for parameters α^0 and error term u^{GCE} are derived from the OLS regression. Details for the procedure are given in Annex 2, the results are shown in the next section, TABLE 3. The estimation procedure was implemented as a sequence of non-linear optimization problems in the General Algebraic Modelling System (GAMS) and solved with the numerical algorithm CONOPT. Based on the estimated values for the elasticities of transformation and substitution (τ and σ) and the expectation and adjustment coefficients (β and λ), it is now possible to parameterize the yield functions (8) in the next step.

Calibration of yield function parameters

While it was possible to use available information for yields, output prices, and harvested areas to estimate some of the needed structural parameters, we are not in a position to estimate either production or cost functions directly as we do not have comparable time series for observations on used inputs (apart from area), and neither an equivalent set of input prices, namely the marginal values for land. In the framework of Computable General Equilibrium modelling (CGE), it is common practice to calibrate the share parameters (δ, θ) of production or transformation functions to a benchmark that consists of prices and quantities for a single year or a representative time period (Rutherford 2002, van der Mensbrugghe 2005). For the calibration of the production function (7) – or the yield function (8), respectively – we compiled agronomic information on variable input quantities (fertilizer) per hectare, average fertilizer prices, cost shares of variable inputs in crop budgets, and the share of labour requirements during the second half of the rainy season for crop maintenance and harvesting. Major sources for these data were surveys from Gleisberg-Gerber (2012) in Sud-Ouest Burkina, Yilma (2005) in Upper East Ghana, and the findings by Kuhn at al. (2011) for Benin. For Ghana, we used also a Social Accounting Matrix with a detailed agricultural sector from 2005 (Breisinger et al 2007) to supplement the survey data from Yilma (2005). Additional information on seasonal labour requirements by cropping activities was obtained from Ngeleza at al. (2011). We used the period between 2002 and 2006 as benchmark because the available information referred to this time period and because it is just before the price surges in 2007 and the flood in Ghana. The benchmark values for used inputs are expressed as cost shares ρ:

$$\bar{\rho}^{A}_{d,c}=\frac{\bar{r}_{d,c}\bar{A}_{d,c}}{\bar{p}_{d,c}\bar{Q}_{d,c}};\bar{\rho}^{V}_{d,c}=\frac{\bar{q}_{d,c}\bar{V}_{d,c}}{\bar{p}_{d,c}\bar{Q}_{d,c}};\ \bar{\rho}^{Z}_{d,c}=\frac{\bar{w}^{2}_{d,c}\bar{Z}_{d,c}}{\bar{p}_{d,c}\bar{Q}_{d,c}}; \tag{12}$$

With known elasticities of substitution from the previous step, first-order conditions for inputs can be solved for the unknown share and shift parameters. Because share parameters δ have to add up to unity, this gives in our case four equations and four unknowns. While calibrating the production function parameters in the described manner (Rutherford 2002, van der Mensbrugghe 2005), we noted that the resulting parameters were not consistent with information on maximum attainable yields (y^{max}) from other sources, e.g from Nin-Pratt et al (2010): When solving Equation (9) for the shift- and share parameters and include area cost shares, we obtain Equation (12):

$$\bar{\rho}^A_{d,c}\,\bar{y}_{d,c}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}} = \bar{\gamma}_{d,c}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}}\,\delta^A_{d,c} \tag{12}$$

The limit of the yield function Equation (8) in the benchmark period is:

$$\lim_{v_{d,c}\to\infty, z_{d,c}\to\infty} \bar{y}_{d,c,t} = \bar{\gamma}_{d,c}\,\delta^{A\;\frac{\sigma_{d,c}}{\sigma_{d,c}-1}}_{d,c} \equiv \bar{y}^{\max}_{d,c} \tag{13}$$

Combining Equations (12) and (13) gives a relationship between benchmark cost shares and attainable yields:

$$\bar{\rho}^{A\;\frac{\sigma_{d,c}}{\sigma_{d,c}-1}}_{d,c}\,\bar{y}_{d,c} = \bar{y}^{\max}_{d,c} \tag{14}$$

The left side of Equation (14) was in some cases absurdly larger than plausible levels for attainable or even potential yields in the study regions. However, adjusting the benchmark cost shares for area to generate plausible, albeit high levels for attainable yields was not satisfying and caused problems when adjusting the cost shares of the other inputs. We resolved this by using average yield gaps and their standard deviations from Nin-Pratt et al. 2010 to derive plausible ranges for maximum attainable yields. We assumed that outcomes for y^{max} within this range are equally likely with a probability π of 1/3 for lowest, centred, and highest support point $\hat{y}_s$. Together with benchmark cost shares, we set up a Cross-Entropy procedure of the following type:

$$\min_{\pi,\rho} CE = \sum_{d,c}\left[\sum_i \tilde{\rho}^i_{d,c}\ln\left(\frac{\tilde{\rho}^i_{d,c}}{\bar{\rho}^i_{d,c}}\right) + \sum_s \tilde{\pi}_{s,d,c}\ln\left(\frac{\tilde{\pi}_{s,d,c}}{\bar{\pi}_{s,d,c}}\right)\right] \tag{15}$$

$$\textit{subject to:}$$

$$\sum_i \tilde{\rho}^i_{d,c} = 1; 0 \le \tilde{\rho}^i_{d,c} \le 1;$$

$$\sum_s \tilde{\pi}_{s,d,c} = 1; 0 \le \tilde{\pi}_{s,d,c} \le 1;$$

$$\tilde{\rho}^{A\;\frac{\sigma_{d,c}}{\sigma_{d,c}-1}}_{d,c}\,\bar{y}_{d,c} = \sum_s \tilde{\pi}_{s,d,c}\hat{y}_{s,d,c}$$

The constrained minimization problem (15) was again implemented in GAMS as non-linear problem and solved with CONOPT. The results are shown in TABLE … in the results section. Based on the estimated cost shares, we calibrate shift and share parameters to the benchmark and estimate exponential yield trends for the full time series (the growth rate of the shift parameter). After this final step, the yield functions are fully parameterized.

Expected revenues and input allocation

Time series for actual revenues per hectare (r^o) can now be derived from Equation (9). Values for expected revenues (r^e) are generated by using the expectations coefficient β from Equation (4):

$$\ln\left(r^e_{d,c,t}\right) = \sum_{i=1}^{n} \beta_d \left(1-\beta_d\right)^{i-1} \ln\left(r^o_{d,c,t-i}\right) \tag{16}$$

In general, it was possible to restrict the lagged periods to 3 or 4 as the estimated values for β were above 0.8 (see TABLE…). We assume that variable input prices (q) are known at the beginning of the growing period. Under assumption of cost minimization, we can then derive allocated variable inputs per hectare based on:

$$v_{d,c,t} = \left(\frac{r^e_{d,c,t}}{q_{d,t}} \frac{\delta^V_{d,c}}{\delta^A_{d,c}}\right)^{-\sigma_{r,c}} \tag{17}$$

Finally, we suppose that expected allocation of fixed but allocable inputs equals the benchmark value because farmers will expect an average amount of workload for maintenance and harvest.

$$z^e_{d,c,t} = \bar{z}_{d,c} \tag{18}$$

Expected yields are then:

$$y^e_{d,c,t} = \gamma_{d,c,t}\left(\delta^A_{d,c} + \delta^V_{d,c} v_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}} + \delta^Z_{d,c} \bar{z}_{d,c,t}^{\frac{\sigma_{d,c}-1}{\sigma_{d,c}}}\right)^{\frac{\sigma_{d,c}}{\sigma_{d,c}-1}} \tag{19}$$

Impact of seasonal rainfall

The purpose of the previous steps was to provide the ground for estimations of seasonal rainfall impacts. In the case of allocated area, we use the error term of the consolidated allocation function (10) u^{GCE} as dependent variable, while in the case of yields, we use the ratio of y^e and y^o as dependent variable. In both cases, we use sums of monthly rainfall (N) during the growing periods as independent variables. We assume that the crop-specific effects are uniform across the regions. The estimation equations are specified as:

$$u^{GCE}_{d,c,t} = g\left(\ln\left(N_{month\in CropCal}\right); \alpha^A_{c,month\in CropCal}\right) \tag{20}$$

$$\ln\left(\frac{y^o_{d,c,t}}{y^e_{d,c,t}}\right) = f\left(\ln\left(N_{month\in CropCal}\right); \alpha^y_{c,month\in CropCal}\right)$$

RESULTS ARE NOT YET SHOWN BECAUSE NEW MEASUREMENT STATIONS WERE RECENTLY INCLUDED INTO THE SAMPLE

5 RESULTS

Area allocation function estimates

The estimation of the area allocation functions with OLS has a prominent role as the theoretical model is regressed on the sample data without any restriction on parameters. The results from the OLS regression are shown in TABLE 3. Lagged price enters with a positive sign as dictated by theory and is significant at a 99% confidence level. This is a very crucial result as it indicates that farmers in the case study region do react to price signals. It should be mentioned here that the price and production data were collected from different sources and there is no evidence that the correlation could have an artificial origin. Lagged yields are heterogeneous with regard to signs and significance levels, but only cowpea and cotton show significance with signs violating the underlying theoretical model. In contrast, lagged areas are significant and have positive sign as could be expected from theory. Areas lagged twice are not significant but show a correct sign. In general, we find that the parameter test statistics give an indication for the empirical validity of the theoretical model and we conclude that a restricted estimation of the parameters with Cross Entropy methods would not violate the information contained in the sample.

TABLE 3. OLS Parameter Results of Area Allocation Function

Variable	Crop	coeff.	s.e.	t-value	Pr
Price[t-1]		**0.28**	**0.10**	**2.77**	**0.01**
Yield[t-1]	Rice	**0.52**	**0.16**	**3.30**	**0.00**
	Maize	-0.06	0.13	-0.49	0.35
	Millet	0.12	0.18	0.69	0.31
	Sorghum	-0.03	0.19	-0.18	0.39
	Sweet potato	**0.20**	**0.09**	**2.34**	**0.03**
	Yams	**0.38**	**0.17**	**2.19**	**0.04**
	Cassava	0.59	0.47	1.27	0.18
	Cowpea	***-0.30***	***0.12***	***-2.54***	***0.02***
	Bambara bean	0.10	0.15	0.64	0.32
	Soybean	0.03	0.16	0.20	0.39
	Groundnut	0.06	0.17	0.39	0.37
	Cotton	***-0.94***	***0.23***	***-4.08***	***0.00***
Area[t-1]		**0.21**	**0.03**	**7.81**	**0.00**
Area[t-2]		-0.02	0.03	-0.67	0.32
Z[t]		0.10	0.16	0.61	0.33
Z[t-1]		0.28	0.17	1.65	0.10
w1[t]		**-0.20**	**0.09**	**-2.35**	**0.03**
w1[t-1]		-0.10	0.09	-1.16	0.20

Source: Own results

To evaluate the overall performance of OLS and GCE estimation, we calculated the F-statistics for both estimation models TABLE 4. While the F-Statistic decreases remarkably after imposing theoretically motivated constraints on the parameters, the level of significance remains high enough to reject the null-hypothesis.

TABLE 4. F-Statistics of Area Allocation Function

	F-stat	Pr
OLS	6.19	0.00
GCE	2.61	0.00

FURTHER RESULTS ARE NOT YET SHOWN BECAUSE NEW MEASUREMENT STATIONS WERE RECENTLY INCLUDED INTO THE SAMPLE

6 CONCLUDING REMARKS

To our knowledge, the presented model is the first computational tool that permits impact assessment of climate change scenarios for agricultural production in the Guinea-Savannah of West Africa at sub-national level and for a large variety of crops. The use of agronomic information in the form of yield plateaus and crop calendars as well as the fact that structural parameters are estimated based on time series for the region improve the plausibility of simulation results. Well behaved functional forms ensure consistency of out-of-sample simulation results with economic theory. Future developments of the model will include the incorporation of weather events during the planting period on expected gross margins (Roberts and Schlenker 2009) and alternative assumptions of farmer's behavior, e.g. by including downside risk aversion (Antle 2010)

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Appendix 1. Derivation of Area Allocation Function

$$1)\ln\left(r^{e}_{d,c,t}\right)=\ln\left(r^{e}_{d,c,t-1}\right)+\beta_d\left(\ln\left(r^{o}_{d,c,t-1}\right)-\ln\left(r^{e}_{d,c,t-1}\right)\right)=\beta_d\ln\left(r^{o}_{d,c,t-1}\right)+\left(1-\beta_d\right)\ln\left(r^{e}_{d,c,t-1}\right)+\ln\left(N^{r}_{d,c,t}\right)$$

$$2)\ln\left(A^{o}_{d,c,t}\right)=\ln\left(A^{o}_{d,c,t-1}\right)+\lambda_d\left(\ln\left(A^{e}_{d,c,t}\right)-\ln\left(A^{o}_{d,c,t-1}\right)\right)=\lambda_d\ln\left(A^{e}_{d,c,t}\right)+\left(1-\lambda_d\right)\ln\left(A^{o}_{d,c,t-1}\right)$$

$$3)r^{e}_{d,c,t}=w^{1e}_{d,t}\bar{Z}_{d,t}^{-\frac{1}{\tau_d}}\theta_{d,c}N^{\theta}_{d,c,t}A_{d,c,t}^{\frac{1}{\tau_d}}\Leftrightarrow A_{d,c,t}^{\frac{1}{\tau_d}}=r^{e}_{d,c,t}w^{1e\ -1}_{d,t}\bar{Z}_{d,t}^{\frac{1}{\tau_d}}\theta^{-1}_{d,c}N^{\theta\ -1}_{d,c,t}$$

$\ln\left(r^{e}_{d,c,t}\right)$

$$4)\ln\left(A^{e}_{d,c,t}\right)=\tau_d\ln\left(r^{e}_{d,c,t}\right)-\tau_d\ln\left(\theta_{d,c}\right)-\tau_r\ln\left(N^{\theta}_{d,c,t}\right)-\tau_r\ln\left(w^{1e}_{d,t}\right)+\ln\left(\bar{Z}_{d,t}\right)$$

$4)\,in\,2)$:

$$5)\ln\left(A^{o}_{d,c,t}\right)=\lambda_d\tau_d\ln\left(r^{e}_{d,c,t}\right)-\lambda_d\tau_d\ln\left(\theta_{d,c}\right)-\lambda_d\tau_r\ln\left(N^{\theta}_{d,c,t}\right)-\lambda_d\tau_r\ln\left(w^{1e}_{d,t}\right)+\lambda_d\ln\left(\bar{Z}_{d,t}\right)+\left(1-\lambda_d\right)\ln\left(A^{o}_{d,c,t-1}\right)$$

lag 5) *one period and multiply by* $(1-b)$:

$$6)0=\lambda_d\left(1-\beta_d\right)\tau_d\ln\left(r^{e}_{d,c,t-1}\right)-\lambda_d\left(1-\beta_d\right)\tau_d\ln\left(\theta_{d,c}\right)-\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(N^{\theta}_{d,c,t-1}\right)$$
$$-\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(w^{1e}_{d,t-1}\right)+\lambda_d\left(1-\beta_d\right)\ln\left(\bar{Z}_{d,t-1}\right)+\left(1-\lambda_d\right)\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-2}\right)-\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-1}\right)$$

$3)\,in\,5)$:

$$7)\ln\left(A^{o}_{d,c,t}\right)=\lambda_d\beta_d\tau_d\ln\left(r^{o}_{d,c,t-1}\right)+\lambda_d\left(1-\beta_d\right)\tau_d\ln\left(r^{e}_{d,c,t-1}\right)+\lambda_d\tau_d\ln\left(N^{r}_{d,c,t}\right)-\lambda_d\tau_d\ln\left(\theta_{d,c}\right)-\lambda_d\tau_r\ln\left(N^{\theta}_{d,c,t}\right)$$
$$-\lambda_d\beta_d\tau_r\ln\left(w^{1o}_{d,t-1}\right)-\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(w^{1e}_{d,t-1}\right)-\lambda_d\tau_d\ln\left(N^{w}_{d,c,t}\right)+\lambda_d\ln\left(\bar{Z}_{d,t}\right)+\left(1-\lambda_d\right)\ln\left(A^{o}_{d,c,t-1}\right)$$

$7)\,less\,6)$:

$$\begin{aligned}8)\ln\left(A^{o}_{d,c,t}\right)\quad&=\lambda_d\beta_d\tau_d\ln\left(r^{o}_{d,c,t-1}\right)+\lambda_d\left(1-\beta_d\right)\tau_d\ln\left(r^{e}_{d,c,t-1}\right)+\lambda_d\tau_d\ln\left(N^{r}_{d,c,t}\right)-\lambda_d\tau_d\ln\left(\theta_{d,c}\right)-\lambda_d\tau_r\ln\left(N^{\theta}_{d,c,t}\right)\\&-\lambda_d\beta_d\tau_r\ln\left(w^{1o}_{d,t-1}\right)-\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(w^{1e}_{d,t-1}\right)-\lambda_d\tau_d\ln\left(N^{w}_{d,c,t}\right)+\lambda_d\ln\left(\bar{Z}_{d,t}\right)+\left(1-\lambda_d\right)\ln\left(A^{o}_{d,c,t-1}\right)\\&-\lambda_d\left(1-\beta_d\right)\ln\left(r^{e}_{d,c,t-1}\right)+\lambda_d\left(1-\beta_d\right)\tau_d\ln\left(\theta_{d,c}\right)+\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(N^{\theta}_{d,c,t-1}\right)+\lambda_d\left(1-\beta_d\right)\tau_r\ln\left(w^{1e}_{d,t-1}\right)-\lambda_d\left(1-\beta_d\right)\ln\left(\bar{Z}_{d,t-1}\right)\\&-\left(1-\lambda_d\right)\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-2}\right)+\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-1}\right)\end{aligned}$$

collecting terms

$$\begin{aligned}8)\ln\left(A^{o}_{d,c,t}\right)\quad&=\lambda_d\beta_d\tau_d\ln\left(r^{o}_{d,c,t-1}\right)-\lambda_d\beta_d\tau_r\ln\left(w^{1o}_{d,t-1}\right)-\lambda_d\beta_d\tau_d\ln\left(\theta_{d,c}\right)\\&+\lambda_d\ln\left(\bar{Z}_{d,t}\right)-\lambda_d\left(1-\beta_d\right)\ln\left(\bar{Z}_{d,t-1}\right)\\&+\lambda_d\tau_d\ln\left(N^{r}_{d,c,t}\right)-\lambda_d\tau_d\ln\left(N^{w}_{d,c,t}\right)-\lambda_d\tau_r\ln\left(N^{\theta}_{d,c,t}\right)+\lambda_d\tau_r\ln\left(N^{\theta}_{d,c,t-1}\right)-\lambda_d\beta_d\tau_r\ln\left(N^{\theta}_{d,c,t-1}\right)\\&-\left(1-\lambda_d\right)\left(1-\beta_d\right)\ln\left(A^{o}_{d,c,t-2}\right)+\left(\left(1-\beta_d\right)+\left(1-\lambda_d\right)\right)\ln\left(A^{o}_{d,c,t-1}\right)\end{aligned}$$