Unbiased and Consistent Estimation of Risk Preferences: A Monte Carlo Simulation

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Introduction

Agents' risk attitudes directly impact their decision making. A significant amount of effort in the literature has been devoted to estimating risk preferences from agents' production decisions. However, whether risk preferences can be indeed recovered is being debated in the literature. We conduct a Monte Carlo experiment to investigate this issue and discuss potential factors that might affect estimation performance.

The Experiment Design

The experiment design in this study largely follows Lence's (2009) setup with some modifications. Producers are assumed to maximize their expected utility (EU) conditional on random, end-of-period wealth:

(1) $\widetilde{W}(x) \equiv \tilde{p}\tilde{y} - r'x + W_0$,

where $\tilde{p}$ denotes the end-of-period output price and $\tilde{y}$ output, both of which are stochastic; r is input price vector; W_0, the initial wealth, is generated from $W_0 = 18.9 + 69.2z$, where the random variable z falls in the interval [0, 1] and follows the standard $Beta$ (0.87, 1.27) distribution. The production function is:

(2) $\tilde{y} = \alpha_0 x_A^{\alpha_A} x_B^{\alpha_B} \tilde{e}_y$,

Parameters α_0, α_A and α_B are set as 3, 0.2, and 0.6, respectively; $\tilde{e}_y$ follows a log-normal distribution with mean one and variance of 0.0961.

The price is generated from the following process:

(3) $\ln(\tilde{p}) = -0.0659 + 0.5\ln(p_0) - 0.3\ln(\tilde{e}_y) + \tilde{e}_p$,

where p_0 is the initial price; $\tilde{e}_p$ follows a zero-mean normal distribution with a standard deviation of 0.3. The prices p_0, r_A, r_B are drawn from a log-normal distribution with unconditional mean of one (logarithms of prices have mean -0.03125, variance 0.0625).

The utility function $U(\cdot)$ takes the hyperbolic absolute risk aversion (HARA) form:

(4) $U(W) = (1-\gamma_1)^{-1}(\gamma_0 + W)^{1-\gamma_1}$,

where γ_0 and γ_1 are parameters to be recovered; and $\gamma_0 + W > 0$. $[\gamma_0, \gamma_1]$ take values of [-5, 2], [0, 3], and [43, 6] under assumptions of decreasing relative risk aversion (DRRA), constant relative risk aversion (CRRA), and increasing relative risk aversion (IRRA), respectively. Producers maximize EU by choosing the optimal amounts of inputs, $\boldsymbol{x}^*$:

(5) $\max_x E\{U[\widetilde{W}(x)]\}$.

$\boldsymbol{x}^*$ are solved using the numerical quadrature method, which, together with the output and price information, form a typical set of production data for risk preference estimation. Notice that the DRRA case may result in considerable amount of corner (non-optimal) solutions and therefore data contamination. [-5, 2] was set to alleviate such contamination.

Risk Preferences	Sample Size	HARA: Utility $\hat{\gamma}_0$	HARA: Utility $\hat{\gamma}_1$	HARA: Technology $\hat{\alpha}_0$	HARA: Technology $\hat{\alpha}_A$	HARA: Technology $\hat{\alpha}_B$	Restricted HARA: with known γ_1, $\hat{\gamma}_0$	Restricted HARA: with known γ_0, $\hat{\gamma}_1$
DRRA	100	3.292 (-18.06,309.70)	5.236 (0.51,48.54)	2.833 (2.55,3.13)	0.205 (0.19,0.22)	0.616 (0.57,0.67)	-10.521 (-20.60,70.05)	3.205 (-0.21,8.67)
	500	-0.011 (-16.30,137.50)	2.805 (0.84,12.86)	2.855 (2.73,2.98)	0.201 (0.19,0.21)	0.604 (0.58,0.63)	-6.809 (-14.46,18.32)	2.268 (1.14,3.65)
	1,000	-2.002 (-15.74,72.56)	2.419 (1.00,7.29)	2.861 (2.78,2.95)	0.201 (0.20,0.21)	0.601 (0.59,0.62)	-6.021 (-12.61,10.31)	2.147 (1.23,3.21)
	10,000	-5.015 (-10.66,4.44)	2.036 (1.54,2.67)	2.866 (2.84,2.89)	0.200 (0.20,0.20)	0.600 (0.60,0.61)	-5.089 (-8.23,-0.95)	2.006 (1.76,2.30)
CRRA	100	6.114 (-17.86,375.71)	6.836 (0.79,67.67)	2.835 (2.57,3.13)	0.205 (0.19,0.22)	0.616 (0.57,0.67)	-7.470 (-19.67,65.94)	4.738 (0.35,11.96)
	500	4.267 (-15.44,168.30)	4.001 (1.28,19.24)	2.855 (2.74,2.98)	0.201 (0.19,0.21)	0.604 (0.58,0.63)	-2.618 (-11.73,23.14)	3.414 (1.70,5.59)
	1,000	2.901 (-13.70,86.69)	3.510 (1.55,10.57)	2.861 (2.78,2.94)	0.201 (0.20,0.21)	0.602 (0.59,0.62)	-1.447 (-9.07,16.29)	3.323 (1.79,5.34)
	10,000	0.026 (-6.65,11.34)	3.018 (2.37,4.19)	2.867 (2.84,2.89)	0.200 (0.20,0.20)	0.600 (0.60,0.60)	-0.003 (-3.80,4.65)	2.991 (2.63,3.40)
IRRA	100	17.670 (-17.25,437.12)	8.556 (0.77,74.59)	2.839 (2.56,3.15)	0.205 (0.19,0.22)	0.616 (0.57,0.67)	17.843 (-9.42,152.18)	9.195 (1.90,22.66)
	500	27.959 (-12.21,276.72)	5.917 (1.57,28.18)	2.859 (2.74,2.97)	0.201 (0.19,0.21)	0.604 (0.58,0.63)	33.742 (6.78,116.27)	6.869 (3.29,11.42)
	1,000	35.132 (-6.86,237.67)	5.897 (2.12,21.22)	2.863 (2.78,2.95)	0.201 (0.20,0.21)	0.601 (0.59,0.62)	37.974 (15.72,84.53)	6.429 (3.99,9.31)
	10,000	42.870 (15.69,99.00)	5.984 (3.90,9.75)	2.866 (2.84,2.89)	0.200 (0.20,0.20)	0.600 (0.60,0.61)	42.975 (33.84,55.05)	6.021 (5.31,6.84)

Estimation

Recovery of the utility function parameters is based on the following first order conditions (FOC) of the EU maximization problem:

(6) $\varepsilon_{y,n} = [\log(y_n) - \log(\alpha_0) - \alpha_A \log(x_{A,n}^*) - \alpha_B \log(x_{B,n}^*)]$,

(7) $\varepsilon_{j,n} = (\gamma_0 + W_{1,n})^{-\gamma_1}(p_n \alpha_j x_{j,n}^{*-1} y_n - r_{j,n})(\gamma_0 + W_{0,n})^{\gamma_1}$, j=A,B,

where $W_{1,n} = W_{0,n} + p_n y_n - r_A x_{A,n}^* - r_B x_{B,n}^*$; the multiplicative term $(\gamma_0 + W_{0,n})^{\gamma_1}$ in (7) is a scaling factor used to avoid the solution of $+\infty$ for γ_1 of the original FOCs.

The GMM is used to estimate parameters $[\alpha_0, \alpha_A, \alpha_B, \gamma_0, \gamma_1]'$. Instruments used are $[1, W_{0,n}, p_{0,n}, r_{A,n}, r_{B,n}, x_{A,n}^*, x_{B,n}^*]'$.

Results and Conclusions

Two million observations were generated in each scenario (DRRA, CRRA, and IRRA) and used in the estimation at different sample sizes.

The table on the left reports the median and the 2.5% and 97.5% quantiles (in parentheses) of the results obtained from valid estimations (without singularity issues).

- All risk preference parameters in the flexible HARA utility function can be consistently estimated, though at a slower convergence rate for γ_0.
- Technology parameters can be estimated with high precision across all sample sizes (bias in α_0 is due to log transformation of distribution and can be corrected accordingly).
- The right panel of the figure below shows that the GMM objective function for γ_1 (for a sample of 1,000 obs.) has a steep curve, which means the algorithm will easily converge and produce estimates in a relatively small range. However, the left panel gives a fairly flat surface for a large set of γ_0, suggesting relatively large shifts in solutions may be produced (see wider ranges in the table). But the curve becomes much steeper at the sample size of 10,000 (not shown).
- Estimates converge faster if one parameter is set at true value (the last two columns). Parameters of the widely used power utility function (i.e., when γ_0 is known) can be estimated with good precision.