Journal of Agricultural and Applied Economics, 33,2(August 2001):353–365

An Analysis of the Effects of Feed Ingredient Price Risk on the Selection of Minimum Cost Backgrounding Feed Rations

Brian K. Coffey

ABSTRACT

The traditional minimum cost feed ration linear programming model is expanded to permit risk management responses to price variability associated with feeding a particular ration across time. The cost minimizing objective function also considers feed costs in a mean-variance (E-V) framework. The model is specified using NRC nutrient requirements and an historic *Feedstuffs* price series. A decision-maker can choose his/her optimal ration by making tradeoffs between price risk and net income. The results should provide a basis for decision tools that allow livestock producers to manage the net income risk involved in the selection of a feed ration.

Since feed is a primary input for livestock producers, feed expenses greatly affect a producer's net income and, similarly, variation of feed expenses affects the producer's net income variability (i.e., net income risk). Kentucky Agricultural Statistics Service (KASS) data from 1998–99 give perspective to this effect. KASS reported that in the Appalachian Region of the United States (which includes KY, NC, TN, VA, WV), expenditures on feed comprised 23.6 percent of total farm expenditures, representing the single greatest farm expense. The importance of feed price variability is enhanced by the fact that the decision to feed a certain ration is often made in advance of the actual purchase and can influence production costs over an entire feeding program. This is because a producer will usually prefer to feed a consistent ration to a particular group of livestock for the entire time that they are on feed and, depending on the size of the operation, may make multiple purchases of feed ingredients during the feeding period. Therefore, one would expect variation of feed ingredient prices over the feeding period to be included in the rational decision-making process of choosing a feed ration. Generally, decision-making tools that are available to aid producers in performing this critical assembly of feed rations have chosen the optimal ration based solely on cost minimization.

Linear programming formulations that assume feed ingredient prices to be known with certainty have traditionally been used to identify minimum-cost feed rations. In general, these formulations minimized the cost of a ration subject to nutritional and volume require-

Brian Coffey is a graduate research assistant, Department of Agricultural Economics, University of Kentucky, Lexington, KY 40546–0276.

The author would like to thank Carl R. Dillon, John D. Anderson, and Eric S. Vanzant for assistance, support and helpful critical advice throughout the development of this paper.

This paper is a winner of the SAEA Graduate Student Paper Distinguished Professional Contribution Award.

ments. Assuming that a producer makes multiple purchases over the course of a feeding program, the aforementioned model merely minimizes the expected mean cost of the ration over the feeding period. It is reasonable that a producer would be willing to forego some net income (i.e., choose a feed ration with a higher mean cost) to reduce the variability of net income (i.e., variability of the cost of the feed ration). This is consistent with economic literature dating back to 1959 when Markowitz observed that while a linear programming model indicates that investors would always invest in funds with the highest expected returns, investors in the real world do not behave in this manner. He concluded that this is due to some aversion to the risk associated with the funds available and thus included this risk aversion in a model formulation. Freund made similar contributions. The result of their efforts is a technique that attempts to maximize profits subject to risk aversion. This technique is known as *expected value variance (E-V) analysis*. The logic associated with this technique along with the resulting model formulations have been widely applied to agricultural decision-making (Anderson, Dillon, and Hardaker; Boisvert and McCarl; and Hardaker, Huirne, and Anderson). A minimum-cost livestock feed ration model can be manipulated to contain such a formulation so that an optimal feed ration is based on variability and magnitude of feed ingredient prices, thus introducing a means of risk management into the process of selecting a feed ration.

The general objective of this study is to provide insight into how livestock producers can manage input price risk. Specifically, this study analyzes how the inclusion of feed ingredient price risk into the selection of an optimal feed ration for a backgrounding operation affects the composition of the ration. These effects can be quantified across different production goals and sizes of livestock. Average daily gain (ADG) and body weight (W) of animals will be varied. Information resulting from this study will serve two purposes. First, it will provide a basis in the agricultural economics literature for the consideration of feed ingredient price risk in the selection of an optimal feed ration. Second, these results of the experiments could serve as a starting point for more advanced decision-making tools for large-scale livestock producers such as feedlots and dairies. Specifically, tools can be designed to consider managing price risk of feed ingredients, as well as the level of these prices, when selecting a ration. This type of risk-management tool would likely combine the classic feed ration linear programming model with E-V analysis, as does the methodology of this study.

Literature associated with both minimum-cost feed rations and E-V analysis will be presented and discussed. Then the economic model that combines the two formulations is laid out and defined. Finally, the results of this model are presented for analysis and discussion with conclusions following this discussion.

Background

The background information presented at this point will illustrate the use of linear programming as a mechanism for identifying minimum-cost feed rations. Considerable attention will also be given to reviewing E-V analysis as a method of simulating decision-making in an uncertain environment, with an emphasis on how it has been applied to agriculture. The background information will also reaffirm the earlier discussion as to why the combination of the two widely published methodologies (minimum-cost ration balancing and E-V analysis) is appropriate.

The use of linear programming to select minimum-cost feed rations has a long and well-established history. One of the earliest examples is a study in which Stigler considered the minimum-cost diets that exactly meet the nutrient requirements for human subsistence. At the time of this study linear programming was far from being fully developed. However, the basic concept of satisfying a set of nutritional constraints while minimizing the cost of the diet is evident in Stigler's work. McCarl and Spreen write that traditional minimum-cost feed ration models are set up in this very way. That is, the cost of the total ration is the objective function and is minimized subject to nutritional constraints. The constraints are such that the nutritional contributions of each feed ingredient multiplied by the amount of that feed ingredient to be included in the ration must fall below certain upper-limit nutritional constraints and above certain lower-limit nutritional constraints. Waugh was among the first to actually apply this model to the formulation of minimum-cost livestock feed rations. Specifically, Waugh laid out a procedure using linear programming (a concept that was still somewhat new even at that time) to formulate minimum-cost dairy rations. Waugh writes that his rations may or may not be practical. He goes on to say, however, that if all prices and nutritional compositions of feeds are known and specified within the model, the resulting ration is indeed the absolute minimum cost ration that will fulfill dairy cattle requirements. McCarl and Spreen write that, after Waugh's efforts, the determination of minimum-cost feed rations for livestock has been one of the most common uses of linear programming. Thomas et al. offer a more recent example of a model that also examines minimum-cost dairy rations. In addition to a pronounced presence in academic literature of the basic minimum-cost feed ration methodology and the resulting applications to livestock production decisions, there is also plethora of software packages available that are designed for applied use by producers.

One example of incorporating minimum-cost feed rations into more broad beef production decisions is the analysis of finishing cattle in Florida by Prevatt et al. Prevatt et al. attempted to determine the feasibility of backgrounding and finishing cattle in Florida. Minimum-cost feed rations for backgrounding and finishing were determined based on available local and imported feeds. The study found that the variation of feed costs (due to either transportation cost of importing feed or scarcity of local feeds) over time drastically affected the variability of net returns to hypothetical backgrounding and finishing operations in Florida. Prevatt et al. hypothesized that acceptance of beef backgrounding and finishing operations would depend upon individual risk preferences. To illustrate this, several levels of required net returns to management along with required rates of return associated with the risk of the returns were investigated. Prevatt et al. concluded that acceptance of beef finishing operations in Florida would indeed vary across producers with different attitudes toward risk and that this risk was due, in no small part, to variation over time of feed ingredient prices.

E-V analysis is also very widely published in agricultural economic literature and deals with uncertainty of contributions to the objective function of a mathematical programming model, such as the prices of feed ingredients in a minimum-cost feed ration model. However, there has been considerable debate as to whether E-V analysis is a theoretically appropriate method to represent optimal decision making. It is generally agreed that expected utility theory (Von Neuman and Morgenstern) provides the theoretical base for risky choice. E-V analysis can be consistent with expected utility theory in three cases: (1) the underlying income distribution is normal (Freund), (2) the distributions of the decision variable differ only by location and scale (Meyer), and (3) the utility function is quadratic (Markowitz, Tobin). If any of these conditions are satisfied it is generally agreed upon that E-V analysis is indeed consistent with expected utility theory. There are additional empirical studies that strengthen this relationship by demonstrating the closeness of E-V analysis to the expected utility maximizing choices (Levy and Markowitz). Given this demonstrated consistency of E-V analysis with economic theory, it is an appropriate way to model an agricultural producer's response to uncertainty of input-output prices.

Many applications to agricultural decision-making have used the satisfaction of one or more of the aforementioned conditions to justify the use of E-V to model the decisions of producers when faced with net income risk. Dillon (1999) uses the technique to model a Kentucky producer's ability to manage risk associated with uncertainty of suitable field days and yields. In a separate study, Dillon (1992) models the adoption of wheat and soybean cultivars by Arkansas producers. In this study, some cultivars offer less yield variability at the expense of some decrease in expected yield and thus can be a risk-management tool for producers. Boisvert and McCarl show a variety of applications in *Agricultural Risk Modeling Using Mathematical Programming* and many other publications, some very recent, too numerous to mention here. The marked presence of E-V analysis in the agricultural risk-management literature is a strong indication of its appropriateness in dealing with uncertainty of returns and/or expenses.

The well-established history of the feed ration linear programming formulation along with the increasing acceptance of E-V analysis suggests that a mathematical programming formulation combining the two methodologies is a suitable means of addressing the uncertainty of feed ingredient prices. The only variable component of a producer's net returns under this formulation will be the prices of the feed ingredients. Thus by quantifying the risk associated with this component of expenses, risk of net returns is quantified. Such a model that analyzes the ability of producers facing variable feed ingredient prices to utilize the selection of a feed ration to manage the net income risk associated with their respective operations is outlined in the following section.

Data and Methods

The methodology of this study uses an E-V mathematical programming framework to replicate the selection of a feed ration by a beef backgrounder facing the uncertainty of feed ingredient prices as discussed earlier in the paper. In basic production theory, prior to development of any risk analysis framework, a producer would know with certainty the prices of all inputs. The relevant isoquants could be mapped out and the optimal combination of inputs would also be known with certainty. In the real world this is obviously not the case. Input prices are uncertain and this uncertainty will affect producers differently, depending on their attitude toward risk. There have been adjustments to neoclassical economic theory to reflect these responses to risk. The methodology in this particular study assumes that the producer will make choices that will minimize total feed costs subject to his or her aversion to feed ingredient price risk and that this is the equivalent of maximizing utility[1]. Under this methodology the producer's objective function consists of total feed cost plus a penalty reflecting aversion to the temporal variability of feed costs. This objective function is minimized to determine the optimal feed ration. The penalty used in the objective function is determined by the variability of feed costs and a risk-aversion parameter that represents an individual's attitude toward risk. This approach provides a framework with which to address the management of feed ingredient price risk by livestock producers.

Risk-Aversion Parameters

It is necessary to specify, numerically, the aforementioned risk parameters. Risk-aversion parameters will be estimated using the technique offered by McCarl and Bessler. The formula is as follows:

$$(1)\quad \Phi = \frac{2Z_\alpha}{S_y},$$

where Φ = risk-aversion parameter, Z_α = the standardized normal one-tailed Z value at a specified level of significance (α), and S_y is the relevant standard deviation in a risk-neutral scenario. In this study, S_y was calculated using 500-pound medium-frame steers being fed to achieve two pounds of average daily gain (ADG) by a producer with a risk-neutral attitude. This class of livestock was chosen since it is very common among Kentucky backgrounders and should, when coupled with sufficient alterations in the level of significance, adequately represent attitudes toward price variability across all sizes of livestock and all target average daily gains.

[1] Given the ability to substitute among feed ingredients, and the fact that feed costs are such a major portion of total expenses, minimizing feed costs is a powerful tool that a producer can use to help maximize net income.

The Economic Model

The E-V model is designed to choose optimal rations on a pounds-per-head-per-day basis. The mathematical specification of the model is as follows:

$$(2) \qquad \min \overline{FC} + \Phi \sum_t \left(\frac{1}{T-1}\right)(FC_t - \overline{FC})^2,$$

subject to:

$$(3) \qquad \sum_t \frac{1}{T} FC_t - \overline{FC} = 0,$$

$$(4) \qquad \sum_j p_{t,j} F_j - FC_t = 0 \qquad \forall\ t,$$

$$(5) \qquad \sum_j a_{i,j} F_j \geq LL_i, \qquad \forall\ i, \quad \text{and}$$

$$(6) \qquad F_j \geq 0 \qquad \forall\ j.$$

Indices include:

- t = time period (i.e., week);
- j = individual feed ingredients and may represent corn, soybean meal (44 percent crude protein), soybean meal (49 percent crude protein), corn gluten feed, distiller's dried grain, brewer's dried grain, dehydrated alfalfa, hominy, or wheat middlings; and
- i = individual nutrients and may represent net energy for maintenance (NEm), net energy for gain (NEg), protein, Calcium, or Phosphorous.

In this formulation, FC_t is the total feed ration cost in time period t and $\overline{FC}$ is mean total feed costs over T time periods. Time period t is in weeks with a total of 969 (T) weeks being considered. Φ is the risk-aversion parameter and is derived by the method presented earlier. Price of the j^{th} feed ingredient in time t is shown by $p_{t,j}$. F_j is a decision variable representing the amount of the j^{th} feed ingredient to be included in the ration and must be non-negative. The contribution of the i^{th} nutrient by the j^{th} feed ingredient to the ration is represented by $a_{i,j}$. LL_i represents the lower limit requirement for the i^{th} nutrient in the total feed ration.

This particular formulation minimizes $\overline{FC}$ subject to aversion to variability in FC_t. Inclusion of this risk aversion involves assessing a penalty to feed rations that are more variable in terms of FC_t. This penalty is the variance of FC_t times Φ. The quadratic variance term obviously introduces non-linearity into the objective function. The availability of non-linear programming (NLP) solvers makes it relatively easy to deal with this non-linearity. McCarl and Spreen suggest that in most cases it is no longer necessary to attempt to transform the objective function into a linear form and in fact it is often more efficient to allow the solver to deal with the non-linearity. Consequently, there is also non-linearity in the constraints of this model. Specifically, non-linearity is present in the specification of the protein requirement. This is a much more difficult problem to address. Until relatively recent years solvers would routinely "bog down" upon the introduction of such a constraint. A brief explanation should be given as to why the non-linearity is present and its importance to the model.

This model uses the 1984 National Research Council (NRC) nutritional requirement prediction equations to specify LL_i. Requirements for nutrients other than protein are scalars based either directly or indirectly on W and ADG. However, the specification of the protein requirement is much more complex. Protein requirement is dependent upon, among other things, the amount of metabolic fecal protein loss. This fecal loss of protein is a function of the estimated dry matter intake (DMI) of the animal. The method of estimating DMI, as recommended by the NRC, requires that net energy for maintenance of the actual diet (NEm_d) that will fed be calculated and converted to Megacalories per kilogram (Mcal/kg). This somewhat circular procedure for specifying the protein requirement involves introducing DMI and NEm_d as decision variables. There are interrelationships between these and the decision variable F_j that introduce non-linearity into the constraints. Although the difficulties discussed earlier make this approach somewhat intimidating, it is a very robust approach and thus was pursued. The robustness comes from the fact that the specifications of the requirements are entirely endogenous to the model. This means that given only W and the target ADG, the model can calculate a balanced feed ration. The constraints, treated as components of constraint 4, necessary to specify the protein requirement are outlined below:

$$\text{(4a)} \quad \frac{\sum_j a_{NEm,j} F_j}{\sum_j F_j} - NEm_d = 0$$

$$\text{(4b)} \quad [W^{.75}(.1493NEm_d - 0.460NEm_d^2 - 0.196)] - DMI = 0$$

$$\text{(4c)} \quad \frac{(3.34DMI + 2.75W^{.5} + 2W^{.6} + GP)}{.594} - LL_p \leq 0$$

GP is grams of protein deposited into the muscle and is a scalar based on W and ADG, such that $GP = (268 - 29.4(.0557(WEIGHT^{.75})(ADG^{1.097})/ADG))ADG$. All other symbols maintain their previous definitions. Given the power of recent solvers available for use with General Algebraic Modeling Systems (GAMS), this non-linearity was determined to pose no serious limitations upon the model. Allowing the requirements to be endogenized also makes the model somewhat unique. Due to the limitations of previous solvers, practically all minimum-cost feed ration models and ration balancing models using the NRC approach estimate either DMI, NEm_d, or both, exogenously. The approach presented here will be more exact and closer to a true optimization. This selection of an optimal feed ration will be carried out for various scenarios intended to represent different production goals (i.e., different target ADG's), different sizes of cattle, and different attitudes toward risk.

Analytical Procedure

To account for different levels of risk aversion, Z_α is varied in the formula for the calculation of risk-aversion parameters presented earlier. Significance levels of 0.50, 0.75, 0.80, and 0.85 were used to represent 50-percent, 75-percent, 80-percent, and 85-percent levels of risk aversion, respectively. This represents an individual's preference to realize the same or lower feed costs 50 percent, 75 percent, 80 percent, or 85 percent of the time. These risk-aversion parameters are shown in Table 1. The calculation of FC is based on weekly prices of individual feed ingredients taken from an historic price series collected from *Feedstuffs*, for the Chicago market, between 1981 and 1999. All prices were left in nominal terms in the interest of simulating real-world conditions in which producers face the risks associated with nominal prices of inputs. Descriptive statistics for the price series of all feed ingredients being considered are presented in Table 2 and nutritional compositions of these ingredients are shown in Table 3. These nutritional values are taken from Preston's "Feed Composition Guide" in *BEEF*. The ingredients with more price variability will be less attractive as components of the optimal balanced feed ration at higher levels of risk aversion.

Table 1. Risk-Aversion Parameters

α	Z_α	Parameter Value
0.50	0.000	0.000
0.75	0.675	24.780
0.80	0.842	30.923
0.85	1.037	38.085

The rations are balanced for different production goals, sizes of livestock, and previously listed levels of risk aversion using LL_i constraints for protein, calcium, phosphorous, net energy for maintenance (NEm), and net energy for gain (NEg). As mentioned, all nutritional requirements were obtained using the approach outlined in the 1984 NRC prediction equations for the nutritional requirements. The 1984 version was chosen over the more recent editions, in part due to the use of crude protein (as opposed to metabolic protein) in specifying the protein requirements. This avoids certain technical complexities. These complexities warrant consideration in practical ration

Table 2. Descriptive Statistics of Feed Ingredient Price Series Available to the Model

	Mean ($/ton)	Standard Deviation ($/ton)	C.V.[1] (%)	Max ($/ton)	Min ($/ton)
Brewer's Dried Grain	97.43	25.49	26.16	170.00	46.00
Corn Gluten Feed	99.02	19.13	19.32	145.00	50.00
Corn	92.88	21.09	22.70	187.50	45.00
Dehydrated Alfalfa	124.59	13.45	10.80	159.00	96.00
Distiller's Dried Grain	125.54	23.07	18.38	185.00	70.00
Hominy	87.22	17.82	20.43	160.00	48.00
Soybean Meal (44%)[2]	183.06	38.43	20.99	318.00	107.00
Soybean Meal (49%)[2]	196.45	39.33	20.02	331.00	115.00
Wheat Middlings	75.93	21.12	27.81	150.00	35.00

Source: Ingredient Market Report. *Feedstuffs*. Various issues 1981 to 1999.
1. C.V. = coefficient of variation and is the standard deviation expressed as a percentage of the mean.
2. 44% and 49% represent the estimated crude protein available in each type of soybean meal.

balancing applications and nutritional research but would add very little to this specific discussion. Other desirable traits of a model specified with 1984 guidelines, such as robustness, were previously addressed. The livestock classification of medium frame steers was used in all cases. W was varied from 400 to 800 pounds in 100-pound increments to accounts for the growth of animals in a typical backgrounding program. ADG was varied across 1.0, 2.0, and 3.0 pounds per day. Nutritional requirements for all sizes of livestock considered under each target ADG are shown in Table 4. It is important to note that these requirements are reported as calculated by the model. The results of this approach are discussed in terms of general qualitative trends in the composition of the feed ration across W, ADG, and Z_α, as well as specific quantitative examples, in the following section.

Results and Discussion

The compositions of all optimal rations calculated for each combination of W, ADG, and Z_α are shown in Table 5. The corresponding mean costs and standard deviations of cost are presented in Table 6. From the nine available ingredients the model chose only five to satisfy the requirements for all W, ADG and Z_α.

Table 3. Dry Matter Basis Nutritional Composition of Feed Ingredients

	Dry Matter (%)	NEg[1] (Mcal/cwt)	NEm[1] (Mcal/cwt)	Crude Protein (%)	Calcium (%)	Phosphorous (%)
Brewer's Dried Grain	92	61	92	29	0.30	0.62
Corn Gluten Feed	90	58	88	23	0.12	0.88
Corn	88	64	96	9	0.02	0.3
Dehydrated Alfalfa	92	31	62	19	1.42	0.25
Distiller's Dried Grain	90	68	100	28	0.25	0.75
Hominy	90	67	99	11	0.04	0.75
Soybean Meal (44%)[2]	91	61	92	51	0.4	0.72
Soybean Meal (49%)[2]	92	64	96	55	0.28	0.7
Wheat Middlings	89	59	89	19	0.15	1.02

Source: Preston, R.L. "Feed Composition Guide." *BEEF* Vol. 33, No. 8, January 1997.
1. NEg = Net Energy Required for Gain, NEm = Net Energy Required for Maintenance.
2. 44% and 49% represent the estimated crude protein available in each type of soybean meal.

Table 4. Nutrient Requirements for Medium-Frame Steers

Body Weight (lbs)	ADG[1] (lbs/day)	Crude Protein[2] (g/day)	Calcium (g/day)	Phosphorous (g/day)	NEg[3] (Mcal/day)	NEm[3] (Mcal/day)
400	1.0	497.52	18.03	10.00	1.16	3.81
	2.0	638.45	29.80	13.80	2.45	3.81
	3.0	775.01	41.28	17.51	3.87	3.81
500	1.0	545.40	18.55	11.22	1.37	4.51
	2.0	674.47	29.31	14.70	2.93	4.51
	3.0	798.54	39.73	18.06	4.58	4.51
600	1.0	589.82	19.11	12.43	1.57	5.16
	2.0	707.45	28.92	15.61	3.36	5.16
	3.0	819.59	38.34	18.65	5.24	5.16
700	1.0	632.29	19.71	13.67	1.76	5.80
	2.0	738.68	28.60	16.55	3.77	5.80
	3.0	839.15	37.06	19.28	5.88	5.80

1. ADG = average daily gain.
2. Since crude protein requirements are based on the actual diet chosen by the model, they will vary slightly across risk aversion levels. However, this variation is small enough to ignore in all cases. Only the crude protein requirements calculated by the risk neutral scenario are reported here.
3. NEg = Net Energy Required for Gain, NEm = Net Energy Required for Maintenance.

These are dehydrated alfalfa, wheat middlings, brewer's dried grain, hominy, and corn gluten feed. As few as two of the ingredients were sufficient in some cases, while some rations contained all five.[2] These basic trends in the composition of the feed rations provide for interesting comparison of the available feed ingredients.

Of the feeds available it seems that some are appropriate only under certain scenarios and some feed ingredients actually offer risk-management opportunities. Dehydrated alfalfa is the only ingredient present in all rations with wheat middlings and hominy being the next most common ingredients. For every risk-neutral scenario concerning any W and ADG the rations were composed of dehydrated alfalfa and wheat middlings. As the model was solved across risk-aversion levels, corn gluten feed (CGF) and/or hominy entered the rations. Brewer's dried grain also entered some rations in small quantities. In the rations containing corn gluten feed, the amount of corn gluten feed in the ration invariably increases as the aversion to risk increases. As ADG is increased holding W and Z_α constant, the amount of CGF in the diet also increases, with only a few exceptions. The same is true for increasing W, *ceteris paribus*. Conversely, for rations containing wheat middlings, the amount of wheat middlings in the ration decreases as aversion to risk increases. The results of changes in amount of wheat middlings are mixed when ADG and W are individually varied. This extremely contrasting behavior of CGF and wheat middlings is understandable upon closer inspection of the two feeds. The nutrient compositions of the two feeds are very similar but the price series have noticeably different characteristics (see Tables 2 and 3). The mean price of wheat middlings is nearly 25 percent lower than that of CGF. Since the nutrient compositions are so similar this means that nutrients contained within wheat middlings are a better buy when only mean

[2] It should be noted that while they are technically correct and meet basic nutritional needs these rations may or may not be practical. For instance, the amount of roughage in the diet is not explicitly addressed. (However, the presence dehydrated alfalfa may very well supply sufficient roughage.) Since the focus of this discussion is on the price risk associated with the rations, these possible impracticalities were ignored.

Table 5. Feed Rations Across Body Weight, ADG, and Risk-Aversion Levels

Body Weight (lbs)	ADG[1] (lbs/day)	Risk Aversion (%)	(Pounds/head/day on an As Fed Basis)				
			DehyAlf[1]	WhMids[1]	BDG[1]	HOM[1]	CGF[1]
	1.0	50	2.50	5.21			
		75	2.75	2.01	0.04	2.61	
		80	2.76	1.44	0.15	2.81	0.22
400		85	2.80	0.82	0.05	2.96	0.75
	2.0	50	4.37	6.39			
		75	4.83	0.60		4.76	
		80	4.85	0.11		4.94	0.27
		85	4.83			4.65	0.72
	3.0	50	6.16	7.74			
		75	6.76	0.07		6.19	0.15
		80	6.73			5.74	0.75
		85	6.71			5.36	1.20
	1.0	50	2.46	6.53			
		75	2.83	1.62	0.10	3.98	
		80	2.86	0.90	0.06	4.18	0.53
500		85	2.89	0.25		4.35	1.03
	2.0	50	4.08	8.33			
		75	4.72	0.26		6.58	0.10
		80	4.70			6.26	0.73
		85	4.68			5.91	1.15
	3.0	50	5.63	10.35			
		75	6.40			7.66	1.01
		80	6.39			7.59	1.09
		85	6.37			7.24	1.51
	1.0	50	2.42	7.76			
		75	2.92	1.20	0.09	5.28	0.11
		80	2.97	0.35		5.50	0.78
600		85	2.98			5.60	1.00
	2.0	50	3.83	10.16			
		75	4.60			7.90	0.59
		80	4.57			7.43	1.13
		85	4.55			7.08	1.54
	3.0	50	5.15	12.79			
		75	6.08			9.25	1.51
		80	6.04			8.79	2.05
		85	6.04			8.69	2.17
	1.0	50	2.40	8.94			
		75	3.04	0.69		6.54	0.37
		80	3.07			6.75	0.81
700		85	3.06			6.63	0.95
	2.0	50	3.60	11.91			
		75	4.49			9.02	0.97
		80	4.45			8.56	15.2
		85	4.43			8.21	1.93
	3.0	50	4.69	15.14			
		75	5.78			10.79	2.00
		80	5.75			10.33	2.54
		85	5.72			9.98	2.95

[1] ADG = Average Daily Gain, DehyAlf = dehydrated alfalfa, WhMids = Wheat Middlings, BDG = brewer's dried grain, HOM = hominy, CGF = corn gluten feed.

Table 6. Means and Standard Deviations of Ration Cost Across Livestock Classes, ADG, and Risk-Aversion Parameters

Body Weight (lbs)	ADG[1] (lbs/day)	Risk Aversion (%)	(Dollars/head/day) Mean	Standard Deviation
400	1.0	0	0.354	0.062
		75	0.363	0.051
		80	0.367	0.050
		85	0.374	0.048
	2.0	0	0.514	0.081
		75	0.531	0.065
		80	0.535	0.064
		85	0.539	0.063
	3.0	0	0.678	0.101
		75	0.701	0.082
		80	0.707	0.081
		85	0.711	0.080
500	1.0	0	0.401	0.075
		75	0.416	0.060
		80	0.423	0.058
		85	0.430	0.056
	2.0	0	0.571	0.099
		75	0.596	0.077
		80	0.602	0.075
		85	0.606	0.075
	3.0	0	0.744	0.125
		75	0.782	0.096
		80	0.783	0.095
		85	0.787	0.095
600	1.0	0	0.446	0.088
		75	0.468	0.068
		80	0.476	0.065
		85	0.480	0.064
	2.0	0	0.624	0.117
		75	0.660	0.088
		80	0.665	0.087
		85	0.669	0.086
	3.0	0	0.806	0.148
		75	0.857	0.110
		80	0.862	0.109
		85	0.863	0.109
700	1.0	0	0.489	0.100
		75	0.519	0.075
		80	0.525	0.073
		85	0.526	0.073
	2.0	0	0.676	0.134
		75	0.721	0.099
		80	0.726	0.098
		85	0.729	0.098
	3.0	0	0.867	0.171
		75	0.929	0.125
		80	0.934	0.124
		85	0.938	0.124

[1] ADG = Average Daily Gain.

cost of the ration over time is considered. However, CGF has a coefficient of variation of 19.13 percent compared to 27.81 percent for wheat middlings. Since this measure takes into account mean and standard deviation, it reveals that this low mean cost comes at the expense of enduring more variability in the price paid for the feed ration. Thus introducing CGF in place of wheat middlings is a means of managing price risk associated with the feed ration across levels of risk aversion.

In terms of the standard deviations of the ration costs, as W or ADG increases the variability of the optimal ration also increases. The effect from increasing ADG is usually more pronounced. For example, in the risk-neutral case of a 400-pound steer being fed for two pounds ADG, the standard deviation of the ration cost is $0.081 per head per day. Feeding a 500-pound animal for the same gain at the same level of risk aversion increases that standard deviation by $0.018 per-head per-day, while feeding the 400-pound steer for three pounds ADG results in a standard deviation that is $0.020 per-head per-day higher. The increases of CGF behave similarly. That is, increasing ADG results in a greater increase in the use of CGF in the ration than does an increase in W. These results indicate that the inclusion of CGF in the ration can also serve to manage risk associated with different production goals and different sizes of livestock as well as to accommodate different attitudes toward risk.

A representative case of utilizing the selection of a feed ration as a risk-management tool is a 600–pound steer being fed for two pounds ADG. This weight represents the midpoint of a backgrounding program that purchases steers at 500 pounds and feeds them for 100 days to

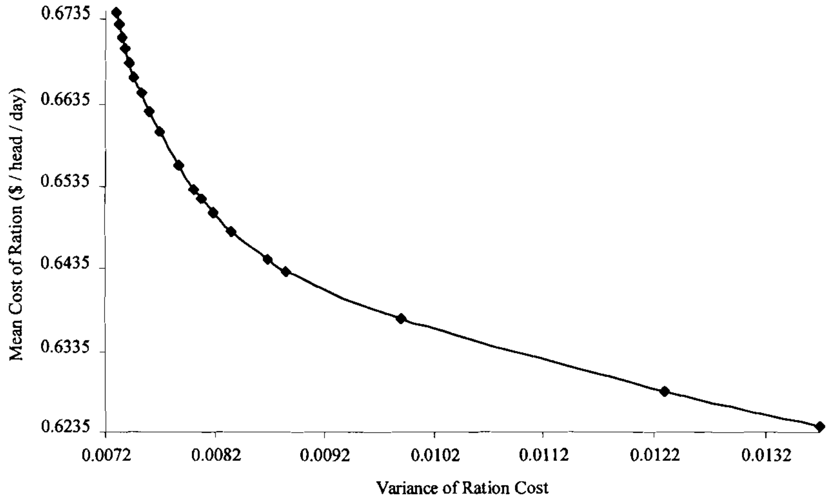

Figure 1. E-V frontier for a 600-pound, medium-frame steer being fed for 2 pounds average daily gain

be sold as 700-pound steers and thus can be used to approximate average feed costs for the entire feeding period. In the risk-neutral scenario, only dehydrated alfalfa and wheat middlings were fed. Wheat middlings comprised about 72 percent of the ration on an as-fed basis. The mean cost of this risk-neutral ration was $0.405 per-head per-day with the standard deviation being $0.051. Corn gluten feed entered the ration at the first reported level of risk aversion (75 percent) and at the highest level (85 percent) accounted for more than 11 percent of the ration with wheat middlings being omitted entirely. At this highest level of risk aversion the mean cost of the ration was $0.624 and the standard deviation $0.117. Much as expected, a producer feeding a 600-pound steer for two pounds ADG can choose different feed ingredients such that a lower variance of feed expenses is achieved at the expense of a higher mean ration cost for a specific situation. Admittedly, the reduction of variance in expenses of about $0.03 per-head per-day shown in this example seems relatively small. However, depending upon the scale of production this reduction can be quite noticeable. For example, assume a producer is backgrounding 100 steers over a 100-day feeding program. This producer would most likely make multiple feed purchases over the feeding program. Letting the feed cost of the 600-pound steer represent costs over the entire 100 days and making the assumptions that all 100 steers perform identically can give some perspective to the decrease in variance. In the risk-neutral case the producer would expect total feed costs to be $6240.00 and to fall between $5070.00 and $7410.00 about two-thirds of the time. If a producer chose the most risk-averse ration, he or she would expect feed costs to be $6690.00, but to fall between the more narrow range of $5830.00 to $7550.00 about two-thirds of the time. This case illustrates how producers with different attitudes toward risk would opt for different feed rations to include in identical feeding programs and can be extended to present the set of risk-efficient choices available.

Rations for all levels of risk aversion are nutritionally balanced and represent a risk-efficient choice given a producer's individual risk preferences. Traditionally, this set of available choices has been presented in a mean-variance framework as an E-V frontier. Presenting such a frontier to a producer can allow a risk-averse producer to see exactly what increases in mean costs are necessary to achieve a given variance of feed expense. Similarly, producers with attitudes near risk neutrality can realize what level of expense variability will be present at the lowest possible mean cost. It is a practical and fairly common approach to present such a frontier to a decision-maker and allow him or her to choose a point that best reflects his or her individual aversion to risk (McCarl and Spreen). The E-V frontier for this scenario is presented in Figure 1. For the sake of a smoother graph, the figure contains several levels of risk aversion in addition to the reported levels. This E-V frontier is presented as a set of risk-efficient expenses and thus appears as the mirror image of the more common presentation of a set of returns. In this presentation it is true that if point A lies to the southwest of point B, point A is risk dominant and point B will not appear on the frontier. That is, a point is not on the frontier if another point has either a lower mean cost or a lower variance of cost. Basically, the feed ration E-V frontier behaves much as expected. In this scenario the possibility of accepting higher expenses for the sake of less variable feed expenses definitely exists.

Summary and Conclusions

The importance of feed expense, in terms of its effect on net income risk, to a livestock operation has been established. In the past there have been very few decision aids that give livestock producers the option of managing net income risk by choosing optimal feed rations that account for the price risk of the feed ingredients in the ration. The economic literature on this simultaneous consideration of feed cost minimization and risk management has also been quite sparse. The methodology of this study combines the classic minimum cost feed ration linear programming model with E-V analysis. The result of this combination is a model that should result in optimal feed rations. That is, minimum cost feed rations that are subject to an individual's risk aversion and thus represent utility maximization. This method of feed ration selection is also an option for livestock producers wishing to manage input price risk and thus manage, at least in part, net income risk.

The results show that livestock producers can manage input price risk by selecting combinations of feed ingredients that are less variable than their technical substitutes. Selecting these less variable rations will come at the cost of increasing the expected mean price of the ration and thus reducing net income. The amount of net income a producer is willing to forgo to realize a given level of input price stability is dependent upon that individual's attitude toward risk. To account for this, several different levels of risk aversion can be modeled, as was done in this study, and the resulting E-V frontier presented to a decision-maker for selection of the production decision that best suits his or her attitude toward risk. By doing this a livestock producer should be able to choose feed ingredients that simultaneously fulfill nutritional requirements of livestock and manage the net income risk of their respective operation.

References

Anderson, J.R., J.L. Dillon, and J.B. Hardaker. *Agricultural Decision Analysis.* Ames, Iowa: Iowa State University Press, 1977.

Boisvert, R.N., and B.A. McCarl. *Agricultural Risk Modeling Using Mathematical Programming.* Southern Cooperative Series Bulletin 356. Cornell University Agricultural Experiment Station, July 1990.

Dillon, C.R. "Microeconomic Effects of Reduced Yield Variability Cultivars of Soybeans and Wheat." *Southern Journal of Agricultural Economics.* 24,1(1992): 121–133.

Dillon, C.R. "Production Practice Alternatives for Income and Suitable Field Day Risk Management." *Journal of Agricultural and Applied Economics.* 31,2(1999): 247–261.

Freund, R. "The Introduction of Risk into a Programming Model." *Econometrica.* 21(1956): 253–263.

Hardaker, J.B., R.B.M. Huirne, and J.R. Anderson. *Coping With Risk in Agriculture.* New York, NY: CAB International, 1997.

Ingredient Market Report. *Feedstuffs.* Various issues 1981 to 1999.

Kentucky Agricultural Statistics Service. *Kentucky Agricultural Statistics 1998–99.* National Agricultural Statistics Service of USDA and Kentucky Department of Agriculture.

Levy, H. and H. Markowitz. "Approximating Expected Utility by a Function of Mean and Variance." *American Economic Review.* 69(1979): 308–317.

Markowitz, H.M. *Portfolio Selection: Efficient Diversification of Investments.* New York: John Wiley and Sons, Inc., 1959.

McCarl, B.A. and T.H. Spreen. *Applied Mathematical Programming Using Algebraic Systems.* 1997. Unpublished, available at the URL: http://agrinet.tamu.edu/mccarl/regbook.htm.

Meyer, J. "Two-Moment Decision Models and Expected Utility Maximization." *American Economic Review.* 77(1987): 421–430.

Preston, R.L. "Feed Composition Guide." *BEEF* Vol. 33, No. 8, January 1997.

Prevatt, J.W., B.E. Melton, T.H. Spreen, W.K. Mathis. "The Economics or Carcass Beef Production: An Appraisal of Florida's Feedlot Potential." *Southern Journal of Agricultural Economics.* 10,2(1978): 49–55.

Stigler, G.J.. "The Cost of Subsistence." *Journal of Farm Economics.* 27(1945): 303–314.

Thomas, C., M.A. DeLorenzo, D.K. Beede, and T.H. Spreen. "Predicting Daily Feed Costs for Dairy Management Models." *Journal of Dairy Science.* 27(1992): 3109–3121.

Tobin,J. "Liquidity Preference as Behavior Toward Risk." *Review of Economic Studies.* 25(1958): 65–86.

Von Neuman, J. and O. Morganstern. *Theory of Games and Economic Behavior.* Princeton, N.J.: Princeton University Press, 1947.

Waugh, F.V. "The Minimum-Cost Dairy Feed." *Journal of Farm Economics.* 33(1951): 299–310.